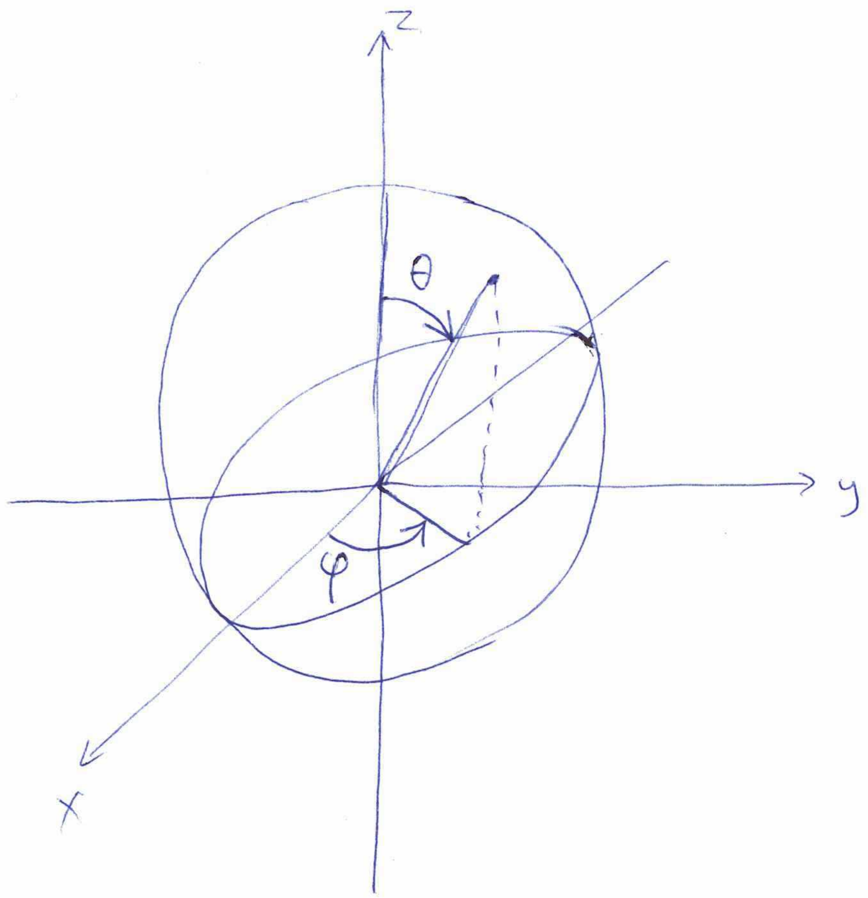
High School　　Lecture 3　　①

Bloch sphere, two qubits and Deutsch algorithm

Bloch sphere

- state of single qubit can be graphically represented on a so called Bloch sphere
- Bloch sphere is different that qsphere in ~~Qiskit~~ Quantum Composer



- two angles, θ, φ fully parametrize the state of qubit
- $\theta \in [0, \pi]$, $\varphi \in [0, 2\pi]$
- Positions of $|0\rangle, |1\rangle, |+\rangle, |-\rangle$
- How gates work : X, Z, Y, H, $R_z(\varphi) \to Z, S, T$

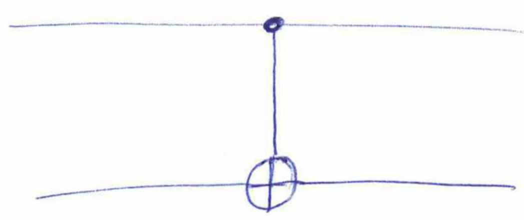
We can quickly check, that:

$$XX = \mathbb{1}$$
$$HH = \mathbb{1}$$

$$\boxed{|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle}$$

Two qubits

- Two qubits can give 4 possible outputs after measurements: 00, 01, 10, 11
- Basis: $|00\rangle, |01\rangle, |10\rangle,$
- We represent state of such system as 4-element vectors
- the only really gate acting on 2 qubits which we will use is CNOT



$0,0 \to 0,0$
$0,1 \to 0,1$
$1,0 \to 1,1 \Rightarrow |x, y\rangle \longrightarrow |x, x \oplus y\rangle$
$1,1 \to 1,0$
$\uparrow \; \uparrow \;\; \uparrow \;\; \uparrow$
$x \; y \;\; x \;\; x\oplus y$

- Każda operacja na n qubitach może być zapisana jako ~~zbio~~ sekwencja operacji 1-qubitowych i bramek CNOT

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$|a\rangle \otimes |b\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 b_1 \\ a_1 b_2 \\ a_2 b_1 \\ a_2 b_2 \end{pmatrix}$$

i.e.:

$$|1\rangle \otimes |0\rangle \equiv |10\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Deutsch algorithm

Let's consider binary function:

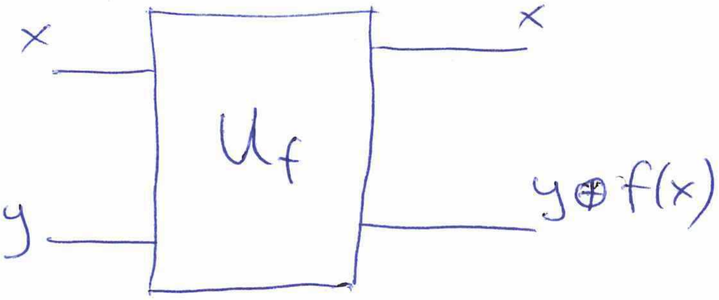
$$f: \{0,1\} \to \{0,1\}$$

	f_1	f_2	f_3	f_4
0	0	0	1	1
1	0	1	0	1

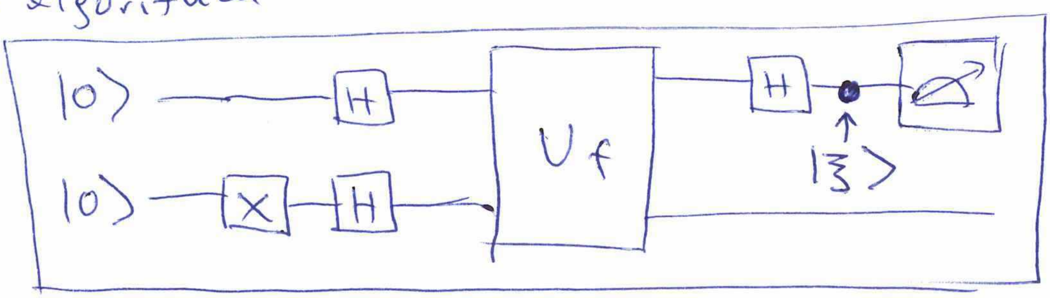
$\Rightarrow$ 4 such functions

- 2 are constant (same values for all inputs $\Rightarrow f_1$ and f_4)
- 2 are balanced (same number of 0s and 1s in outputs) $\Rightarrow f_2, f_3$

- we have block box implementing one of these functions, now how to determine which of them it is?
- We have to call the box tice, for inputs 0 and 1
- Deutsch algorithm requires quantum computer, but it can do this in just <u>one</u> call to the box implementing the function
- the box:



- the algorithm



$$|\xi\rangle = \text{const.} \left[\left((-1)^{f(0)} + (-1)^{f(1)} \right) |0\rangle + \left((-1)^{f(0)} - (-1)^{f(1)} \right) |1\rangle \right]$$

- if we measure first qubit in state $|0\rangle$ then it means that $f(0)=f(1) \Rightarrow f$ is constant
- if we measure first qubit in state $|1\rangle$ then it means, that $f(0)=-f(1)$

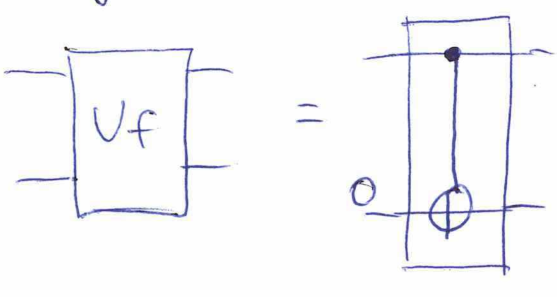
$\Downarrow$

f is balanced

$\Downarrow$

Classical complexity	quantum complexity
2	1

Examples



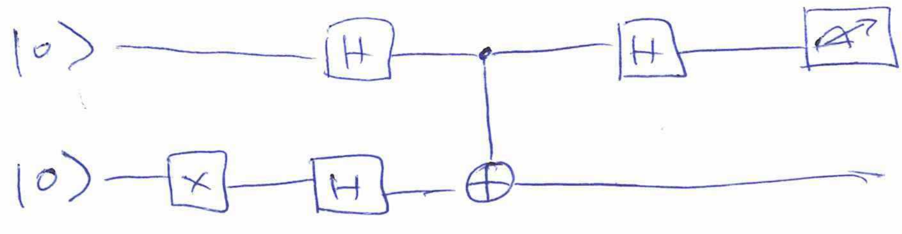
$\Rightarrow$

$\underline{0}0 \to 0\underline{0}$
$\underline{1}0 \to 1\underline{1}$

$\Downarrow$

$f: \begin{array}{l} 0 \to 0 \\ 1 \to 1 \end{array}$ (f_2)

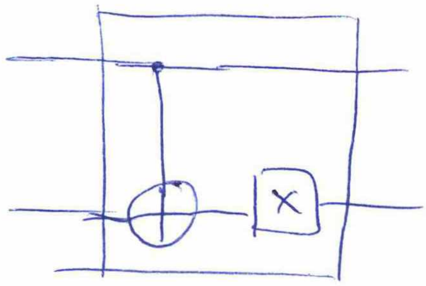
balanced



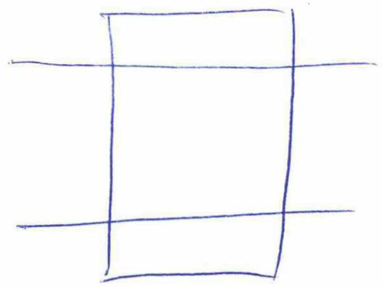
do this in quantum composer

$\downarrow$

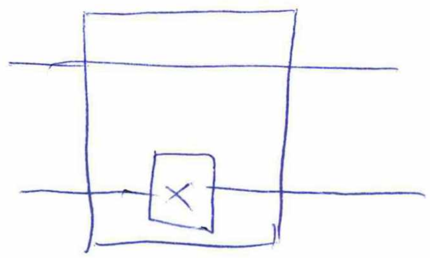
Other functions



$0 \to 1$
$1 \to 0$ $\Rightarrow f_3$, balanced



$0 \to 0$
$1 \to 0$ $\Rightarrow f_1$, constant



$0 \to 1$
$1 \to 1$ $\Rightarrow f_4$, constant

Generalization of the Deutsch algorithm is Deutsch-Jozsa algorithm, which is still not very practical, but we can have ~~a~~ (theoretically) huge gain in complexity reduction

↓

Homework for volunteers to explain this