Deutsch-Jozsa algorithm (1992r)

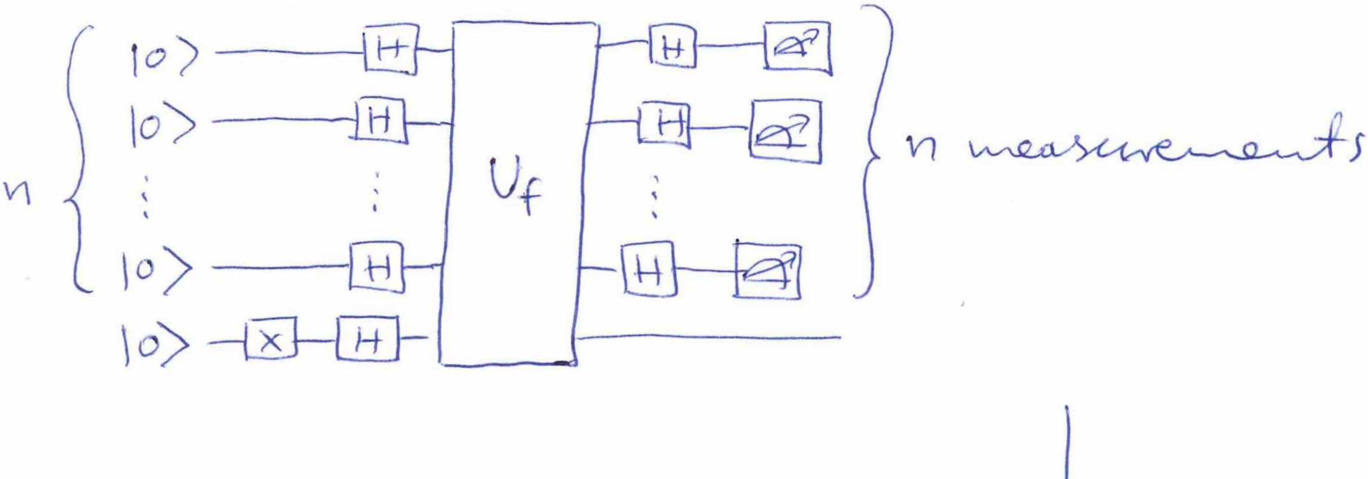
- generalization of Deutsch algorithm to n qubits (function has 2^n possible inputs, but is still binary in it's values)

$$f: \{0, 1, \dots, 2^{n-1}\} \longrightarrow \{0, 1\}$$

$$\Updownarrow$$

$$f: \{0, 1\}^{\otimes n} \longrightarrow \{0, 1\}$$

- in this case functions may be balanced, constant or neither balanced nor constant (balanced function is such, that has equal number of values 0 as 1)
- we will assume, that oracles <u>always</u> represent function either balanced or constant (not other)



↓

- if measurement gives $0^{\otimes n}$ (n zeroes), then function is constant
- any other measurement $\Rightarrow$ balanced (again assuming, that oracle always implements function either constant or balanced)

$\Downarrow$

- again we do <u>only 1 measurement</u>

but classically we would need on average $\frac{2^n}{2} + 1 = 2^{n-1} + 1$ measurement !!!

$\Downarrow$

Deutsch	1	vs	2 classically
D-J	1	vs	$2^{n-1}+1$ classically

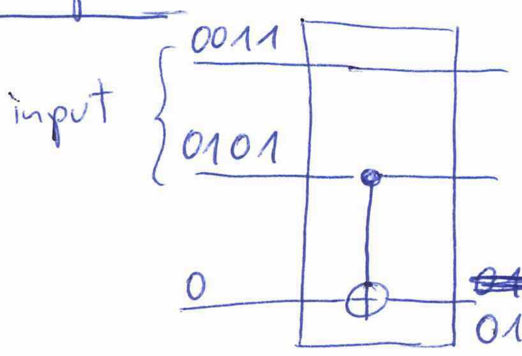
D-J example on 2 qubits

We can have 16 possible functions:

input	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0 0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0 1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1 0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1 1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
	C					A	B									D

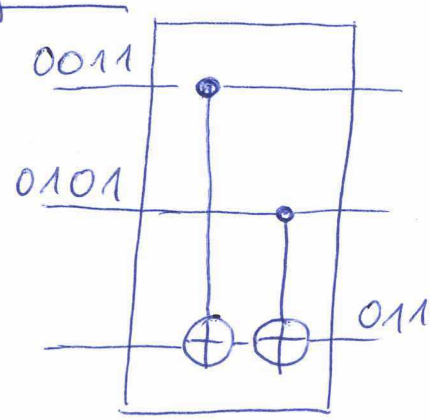
only these 8 are either constant or balanced

Example A



~~0110~~ 0101 $\Rightarrow f_6$ - balanced

Example B



→ this is addition modulo 2

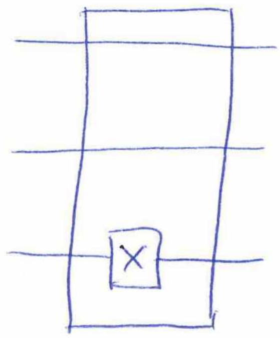
$$f(x_0, x_1) = x_0 \oplus x_1$$

0110 $\Rightarrow f_7$ - balanced

Example C

Empty oracle $\Rightarrow$ constant function f_1

Example D



$\Rightarrow$ constant function f_{16}