High school Lecture 2

Qubit – and what can be done with it?

Classic computation => bit : 0 or 1

Quantum computation => qbit : $|0\rangle$ or $|1\rangle$ or $\alpha|0\rangle + \beta|1\rangle$

$$\underbrace{|\psi\rangle}_{\text{qubit state}} = \alpha|0\rangle + \beta|1\rangle$$

„qubit ~~is~~ in state ψ"

$\begin{cases} \alpha \neq 0 \\ \beta \neq 0 \end{cases}$ => superposition

~~1) $|\alpha|^2 + |\beta|^2 = 1$~~

~~1) $|\alpha|^2$ = pro~~

1) Qubit state is not affected until we measure it. ~~Before we mea~~

2) When we measure, we ~~k~~ get state either $|0\rangle$ or $|1\rangle$. Always. No more superposition!

3) After the measurement the previous state of the qubit is destroyed forever. There is no possibility to deduce it (one can try to recreate it from scratch, but there ~~can~~ may be no guarantee it will be the same)

Probability of measuring $|0\rangle$ is

$$P_{|0\rangle} = |\alpha|^2$$

Analogically:

$$P_{|1\rangle} = |\beta|^2$$

But we will for sure get either $|0\rangle$ or $|1\rangle$, so:

$$P_{|0\rangle} + P_{|1\rangle} = 1$$

$$\Downarrow$$

$$\boxed{|\alpha|^2 + |\beta|^2 = 1}$$ normalization

Questions:

1) Are the following proper Qubit states?

a) $|\psi\rangle = |0\rangle$

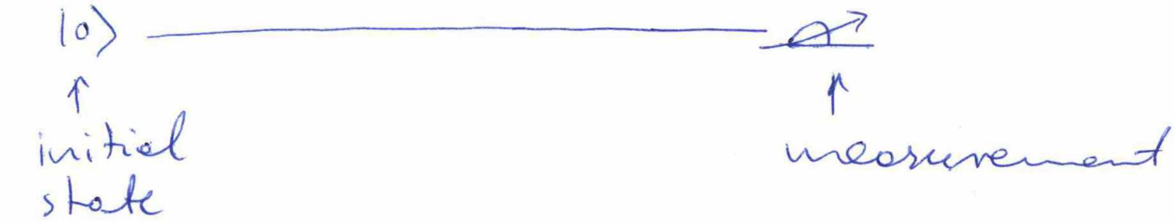
b) $|\psi\rangle = 2|0\rangle$

c) $|\psi\rangle = |0\rangle + |1\rangle$

d) $|\psi\rangle = \frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle$

2) ~~What can be~~ what state of qubit results in equal probability of measuring $|0\rangle$ and $|1\rangle$?

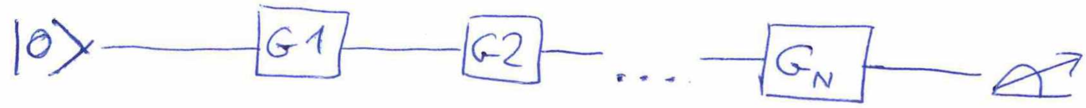
How we visualize qubits?



(always $|0\rangle$ by convention)

$\boxed{Q:}$ What will the measurement give us?

We can apply one of more "gates" to the qubit before measuring it:



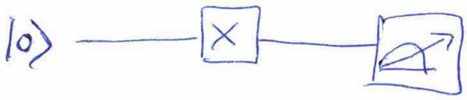
$\boxed{Q:}$ What will measurement be?

- The gates change state of a qubit to another state.
- There are also gates, which act on more than 1 qubit → but we'll not learn about them today.

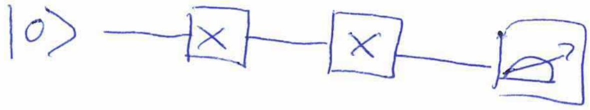
We'll learn about two specific, very important gates ~~and~~ now: $\boxed{H}$ and $\boxed{X}$

X - bit flip gate or negation gate

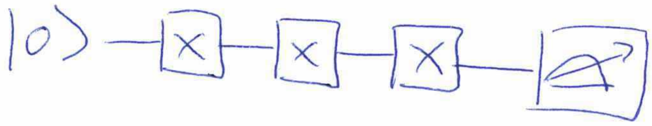
X - changes the state of qubit to opposite state



~~R~~ Result is 1.



Result is ?



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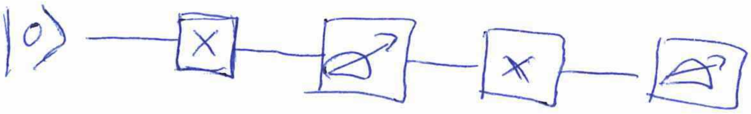

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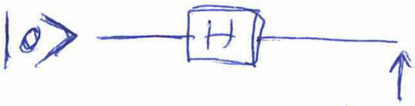
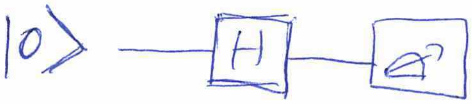
H - Hadamard gate

↳ Hadamard gate introduces superposition



$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$



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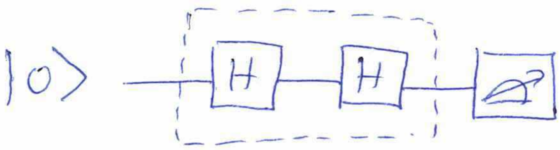
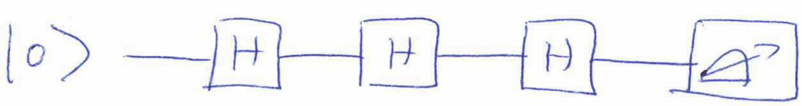


$\boxed{I} = \boxed{1}$



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Now, we learned how to interpret gates, but sometimes interpretation is difficult and we should rather „calculate".

$$\Downarrow$$

<u>Heisenberg formalism</u> - can be used for any states (not only single qubits)!

We ~~interpre~~ represent states as vectors:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \longrightarrow |\psi\rangle = \begin{bmatrix}\alpha \\ \beta\end{bmatrix} \Rightarrow \quad |0\rangle = \begin{bmatrix}1 \\ 0\end{bmatrix} \quad |1\rangle = \begin{bmatrix}0 \\ 1\end{bmatrix}$$

Now, we said that gates change state of qubit to some other state:

$$\begin{bmatrix}\alpha \\ \beta\end{bmatrix} \longrightarrow \begin{bmatrix}\alpha' \\ \beta'\end{bmatrix}$$

$$\Downarrow$$

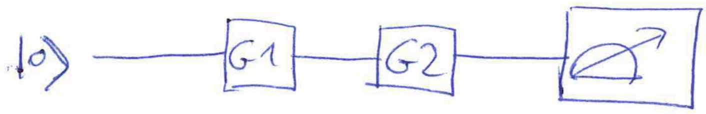
gates are represented by 2×2 matrices as only vectors multiplied by such matrices will give us also 2×1 vectors

$$\begin{bmatrix}\alpha' \\ \beta'\end{bmatrix} = \begin{bmatrix} & G & \end{bmatrix}\begin{bmatrix}\alpha \\ \beta\end{bmatrix}$$

$\uparrow$ we multiply vector by the gate matrix from the left!

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \cancel{H =} \quad H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\mathbb{1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} G2 \end{bmatrix} \begin{bmatrix} G1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

So, let's ~~proof~~ prove that $HH = \mathbb{1}$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{2} \cdot \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{matrix} 2 & 0 \\ 0 & 2 \end{matrix} = \frac{1}{2} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

What will be the state of

$|0\rangle$ — $\boxed{H}$ — ?

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\Updownarrow$$

$$\boxed{\frac{1}{\sqrt{2}} \left(|0\rangle + |1\rangle \right) \equiv |+\rangle}$$

$\boxed{Q:}$ $H|1\rangle = ?$

...

$$\boxed{\frac{1}{\sqrt{2}} \left(|0\rangle - |1\rangle \right) \equiv |-\rangle}$$

We have many other qubit gates, we'll learn about them later on.

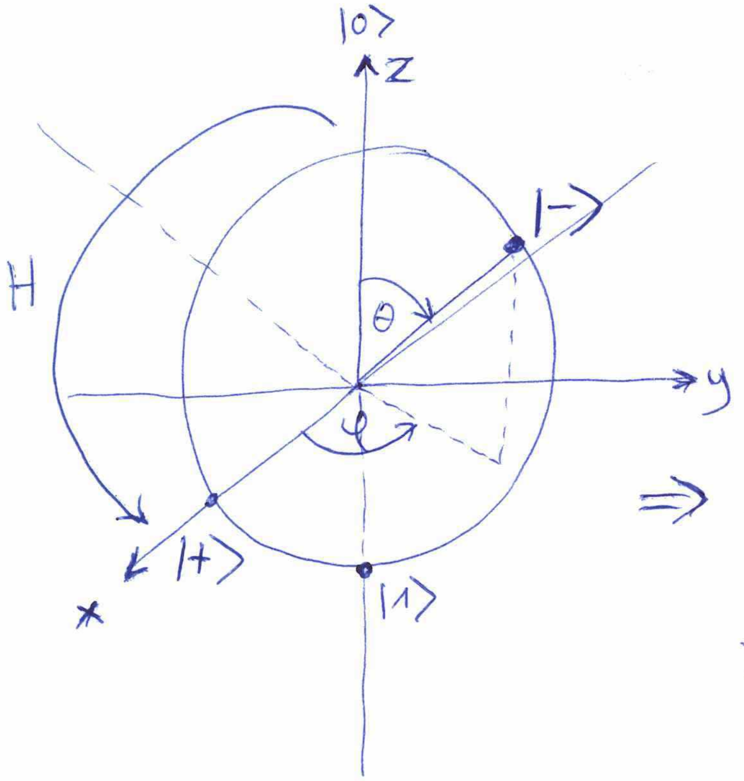
Homework:

1) Learn about Z gate

2) Prove that $X = HZH$

Bloch sphere

we did <u>NOT</u> manage to cover Bloch sphere



$\Rightarrow$ $H = 180°$ rotation around $x+z$ axis

$X = 180°$ rotation around x axis

$\downarrow$

by analogy

Y – $180°$ rotation around y axis

Z – $180°$ rotation around z axis

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$P(\varphi) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{bmatrix}$$

$$HZH = X$$

$$XX = \mathbb{1}$$

<u>Generalization to multiple qubits:</u>

2 qubits $\Rightarrow |\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$

$$|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{bmatrix}$$

General operations are now 4×4 matrices

n qubits $\Rightarrow \underbrace{|000\ldots0\rangle \ \ldots \ |111\ldots1\rangle}_{2^n \text{ states}}$

$\Downarrow$

2^n matrices

$\Downarrow$

we no longer can simulate and that's why we need quantum computers

Summary

Homework:

1) Create account on quantum experience
2) Run your first circuit on quantum simulator and real device
3) Play / experiment with gates

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4) Learn about Z gate.
5) Learn by heart matrix forms of $X, H, Z, \mathbb{1}$
6) Prove that $X = HZH$

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7) Complete the SkillsBuild exercises and other