Deciphering Market Dynamics: ARMA-GARCH Modeling for Financial Forecasting

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AGENDA

1. Problem & Motivation
2. Data Collection
3. Methodology
4. Key Findings
5. Discussion

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Problem & Motivation

- **Problem:** Closing prices of financial securities are uniquely difficult to model
- **Goal:** Forecast closing prices of financial instruments up to 14 days in advance
- **Motivation:** Improve investment strategies; explore time series methods

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Data Collection

- Daily closing prices of PIMCO Active Bond Exchange-Traded Fund (BOND)
  - ETF = portfolio of diverse bonds → trends indicate broader market dynamics
- 5 year time period (Nov 2018- Nov 2023)

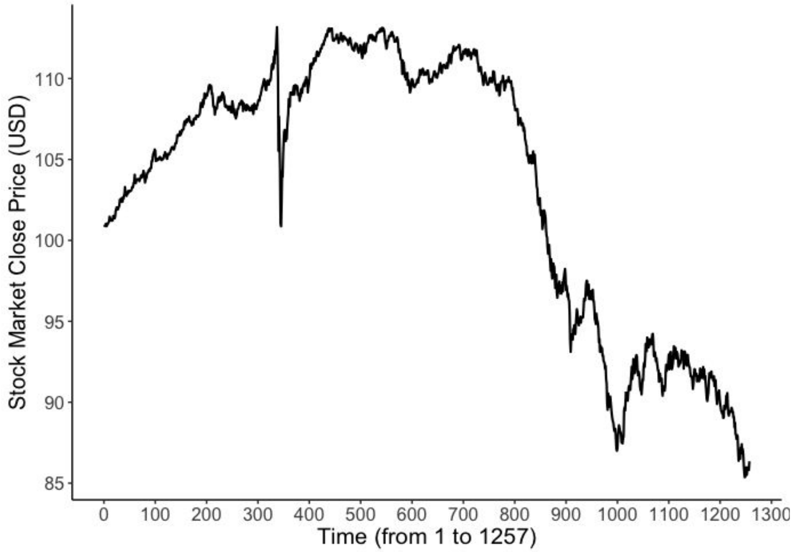

Figure 1: *Valuation of BOND over 5 years*

Methodology

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Polynomial Regression

- Predicts closing price from time and seasonal dummy variables, with interactions
- Log transformed response for stabilization of variance
- Limitation: Does not account for autocorrelation

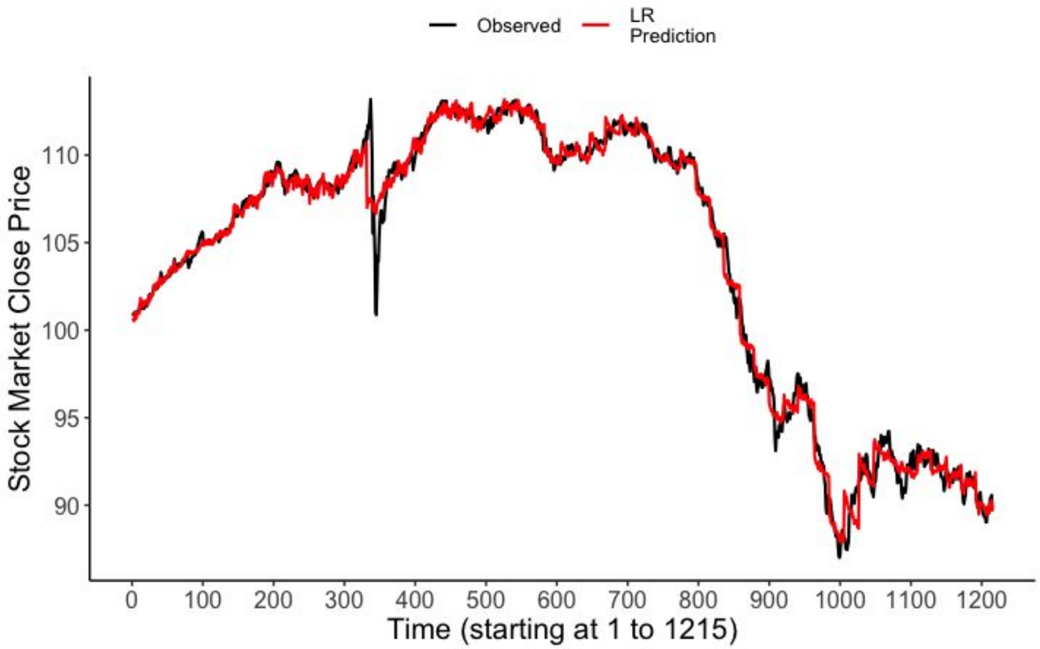


Figure 2: *Linear Regression Fit to Training Data; 4th Degree w.r.t. Time*

ARMA:
Autoregressive Moving Average

- Captures autocorrelation
- Predicts regression residuals based on past observations (autoregressive) and errors (moving average)
- Used with regression for point predictions
- Limitation: Performs best with constant variance time-series

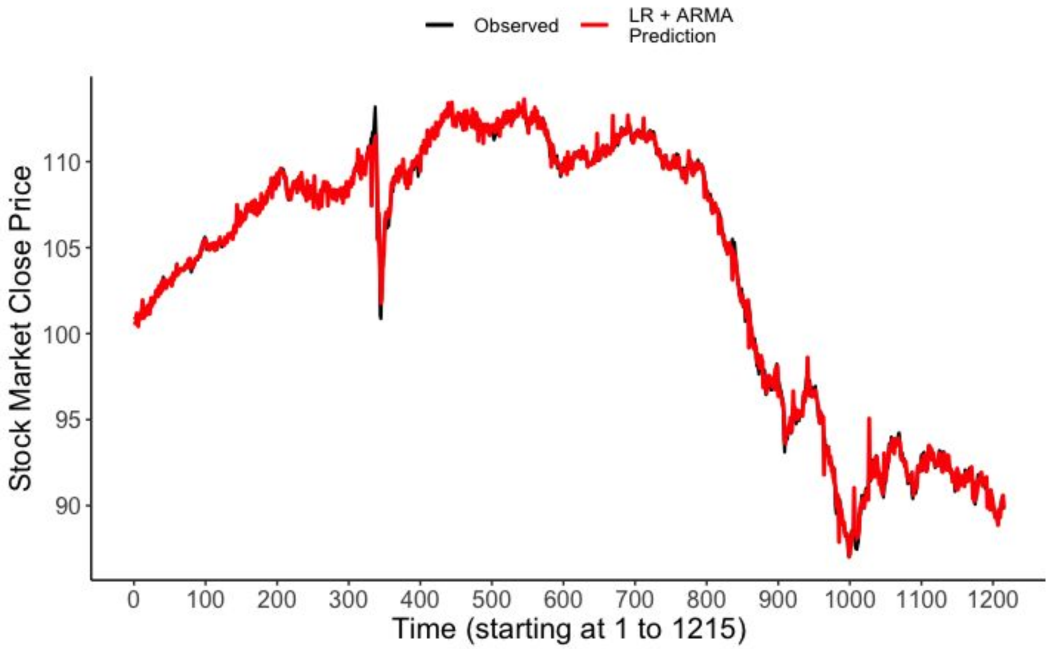
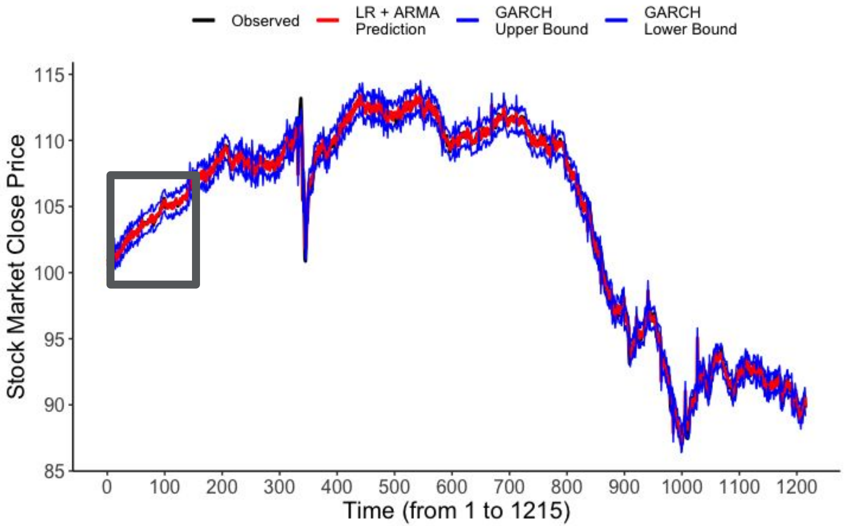


Figure 3: *Linear Regression Fit to Training Data; 4th Degree w.r.t Time, ARMA(2,1) Corrections*

GARCH:
Generalized Autoregressive Conditional Heteroskedasticity

- Captures non-constant variance (heteroskedasticity)
- Predicts variance based on past observations and squared residuals
- Used for prediction intervals



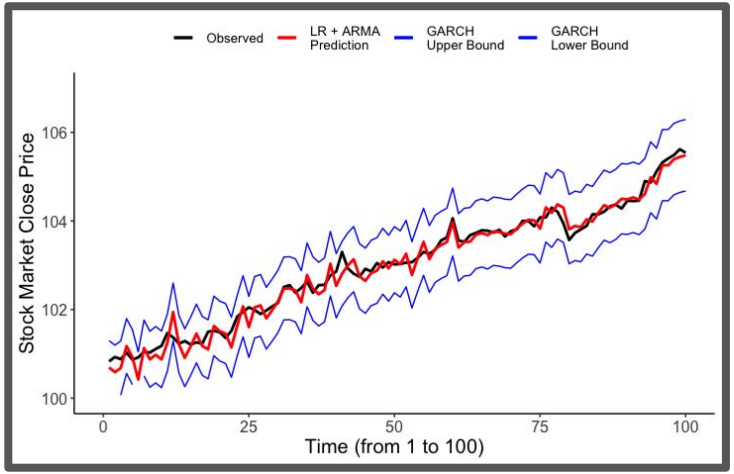


Figure 4: *Linear Regression Fit to Training Data with ARMA(2,1) Corrections and GARCH(1,1) bounds*

LSTM: Long Short-Term Memory Neural Network

- Retains and drops important information from past data (20 days)
- 451,501 parameters

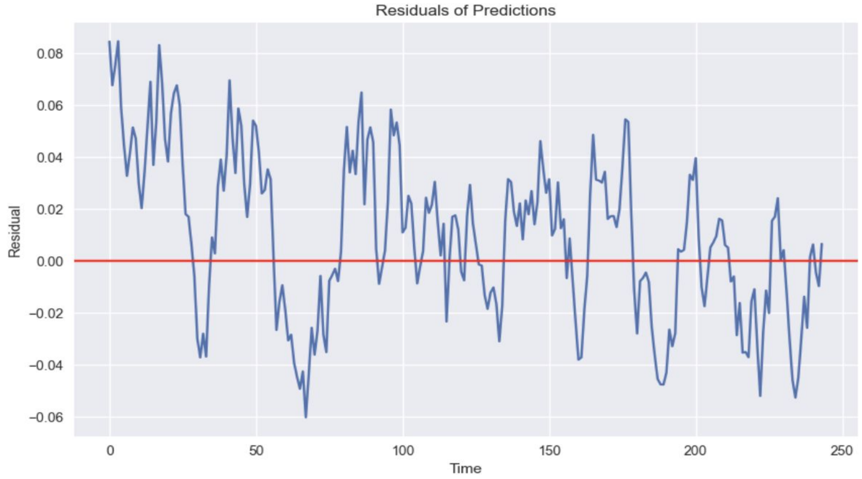


Figure 5: *LSTM Predictions on Test Set; Residuals of Predictions*

Evaluation Metrics

Hypothesis Testing

Ljung-Box Test:
$p < 0.05$ indicates autocorrelation

White Test:
$p < 0.05$ indicates non-constant variance

Error (MAE and RMSE)

MAE = linear score

RMSE = quadratic score

Low error = optimal

Mean Directional Accuracy (MDA)

Probability that model predicts direction of change accurately

Ideal value = 100%

Coverage Probability (CP)

Proportion of actual values contained in prediction interval

Ideal value = 95%

Key Findings

Strong Overall Model

- 4th degree polynomial, ARMA(2,1), GARCH(1,1)
- Low MAE and RMSE (0.004 and 0.005)
- High MDA (69%)
- Good CP (79%)

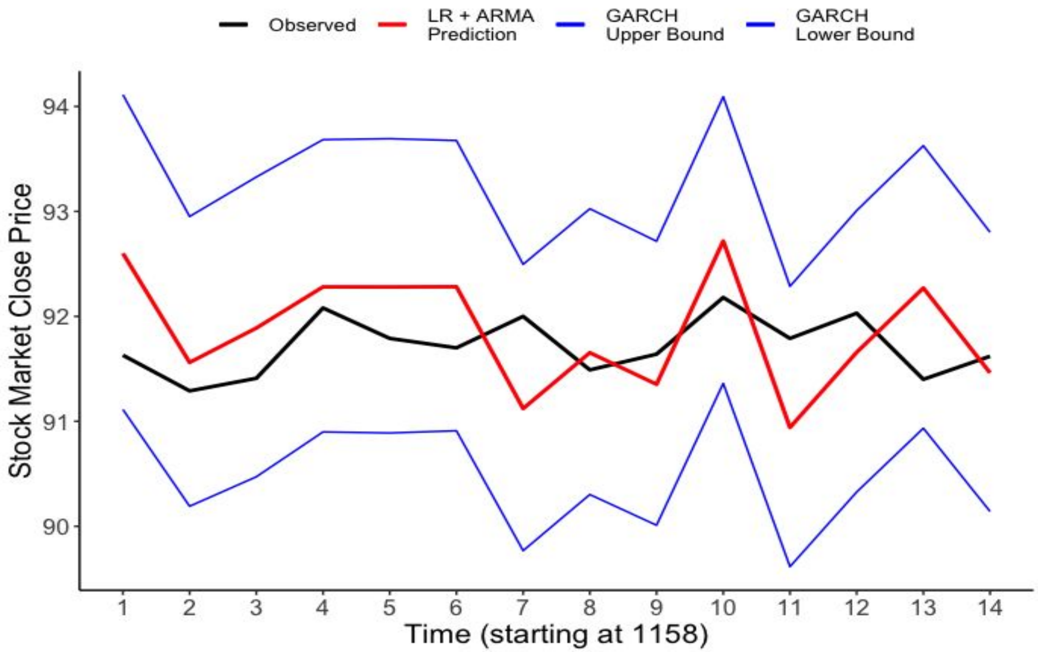


Figure 6: *Model that performs strongly across all metrics; 4th degree polynomial, ARMA(2,1), GARCH(1,1)*

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High Directional Accuracy

- 2nd degree polynomial, ARMA(3,2), GARCH(2,2)
- High RMSE & MAE (0.017 and 0.018)
- High MDA (77%)
- Good CP (86%)

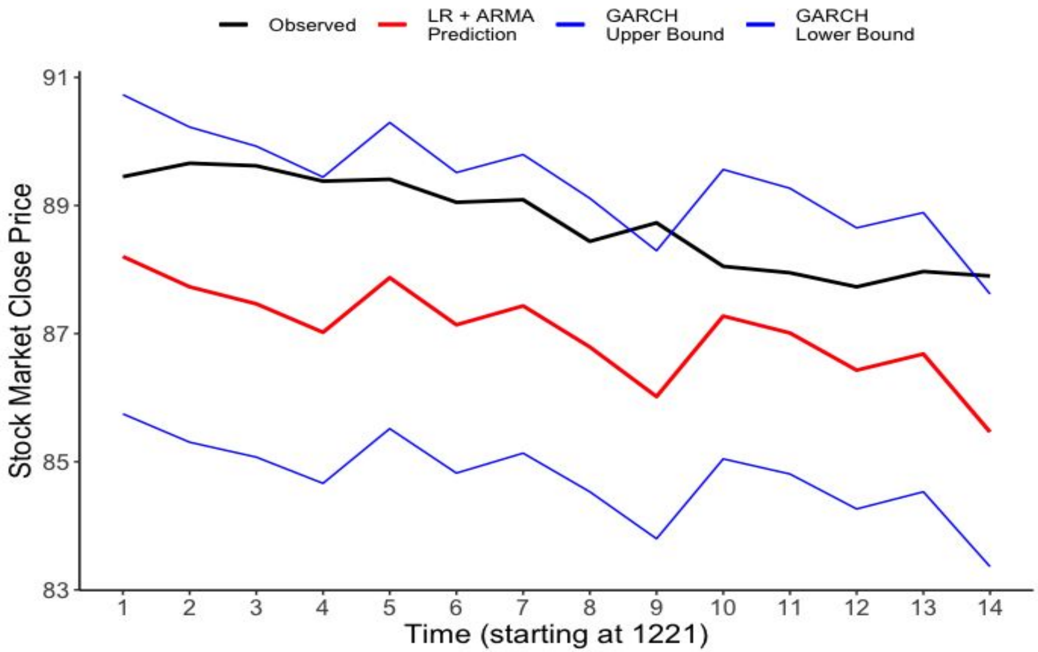

Figure 7: *Model with high directional accuracy but low accuracy; 2nd degree polynomial, ARMA(3,2), GARCH(2,2)*

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High Accuracy Model

- 3rd degree polynomial, ARMA(2,1), GARCH(2,2)
- Low RMSE & MAE (0.005 and 0.006)
- Low MDA (38%)
- Great CP (93%)

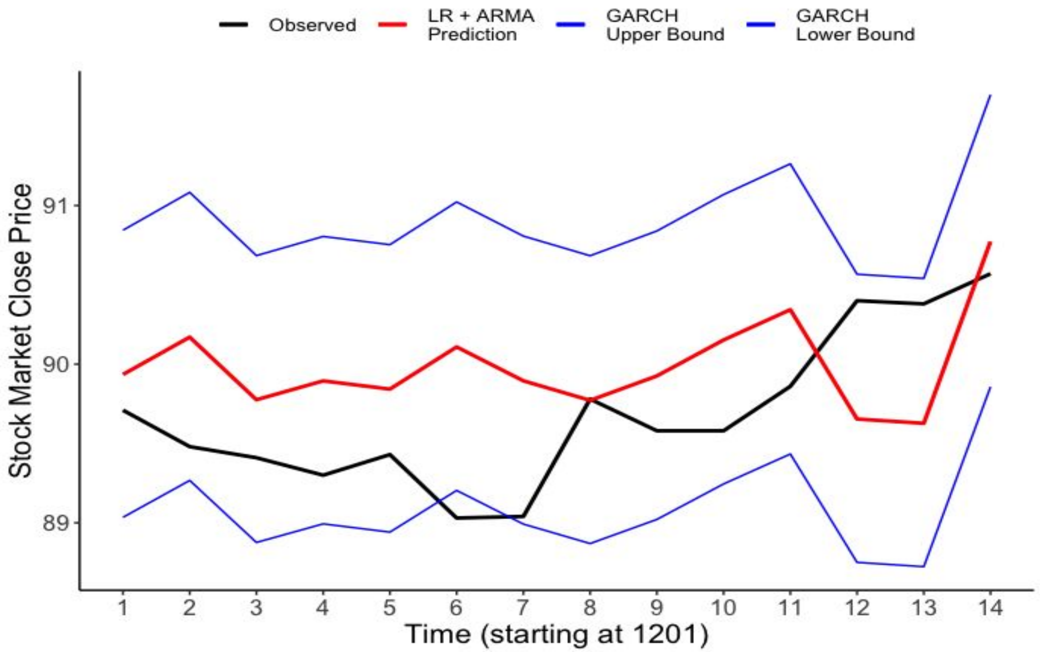

Figure 8: *Model with high accuracy but low mean directional accuracy; 3rd degree polynomial, ARMA(2,1), GARCH(2,2)*

Discussion

- The ‘**best**’ model depends on the objective
  - **Low Error (MAE & RMSE)** → precise daily predictions
  - **High Accuracy (MDA)** → understand overall market trends
  - **High coverage probability (CP)** → optimize risk management

- No singular model (combination of ARMA and GARCH parameters, polynomial degrees) optimizes all of these metrics

THANK YOU
Questions?

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Appendix

Table 1: *Examples of model evaluations; ARMA P refers to autoregressive parameter, ARMA Q refers to moving average parameter, GARCH 1 refers to autoregressive parameter, and GARCH 2 refers to generalized parameter; IS refers to in-sample and OS refers to out-of-sample.*

Model Number	Degree Polynomial	ARMA P	ARMA Q	GARCH 1	GARCH 2	Box P-Value	White P-Value	MAE (IS)	RMSE (IS)	MDA (IS)	MAE (OS)	RMSE (OS)	MDA (OS)	Coverage Prob. (OS)
2408	4	2	1	1	1	0.97	1.00	0.0029	0.0044	0.52	0.004	0.005	0.69	0.79
1195	2	3	2	2	2	1.00	1.00	0.0040	0.0061	0.53	0.017	0.018	0.77	0.86
2132	3	2	1	3	1	1.00	1.00	0.0032	0.0050	0.52	0.005	0.006	0.38	0.93
88	1	1	1	1	2	0.81	1.00	0.0052	0.0076	0.51	0.005	0.006	0.54	1.00
1805	3	2	1	1	3	1.00	1.00	0.0034	0.0054	0.50	0.032	0.094	0.77	0.93
1845	3	4	2	2	1	0.88	1.00	0.0034	0.0051	0.53	0.028	0.029	0.31	0.14
2319	3	2	1	3	3	1.00	1.00	0.0032	0.0050	0.52	0.308	0.320	0.62	0.07
1097	2	2	3	2	1	0.98	1.00	0.0034	0.0053	0.53	0.115	0.138	0.62	0.21
1117	2	3	2	2	1	1.00	1.00	0.0039	0.0060	0.54	0.123	0.151	0.69	0.36
635	1	0	2	3	2	0.00	0.99	0.0190	0.0226	0.50	0.046	0.051	0.54	0.43
3042	4	2	1	3	2	0.97	1.00	0.0029	0.0044	0.53	0.008	0.010	0.54	0.57
914	2	2	3	1	2	0.98	1.00	0.0035	0.0053	0.55	0.007	0.010	0.54	0.86
788	2	2	3	1	1	0.99	1.00	0.0036	0.0055	0.55	0.041	0.060	0.62	0.29
2511	4	2	1	1	2	0.97	1.00	0.0029	0.0044	0.52	0.156	0.174	0.85	0.00
2236	3	2	1	3	2	1.00	1.00	0.0035	0.0056	0.52	0.026	0.026	0.62	0.14