Exam 2 for CS 333201

10:10 - 12:00 a.m., May 3, 2017

1. (15%) Suppose that the distribution function of X is given by

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{4}, & 0 \le x < 1 \\ \frac{x+1}{4}, & 1 \le x < 2 \\ \frac{11}{12}, & 2 \le x < 3 \\ 1, & 3 \le x \end{cases}$$

   (a) (5%) Determine the probability mass function of X.

   (b) (10%) Calculate $P(1 < X \le 3)$, $P(1 \le X < 3)$, and $P\left(\frac{1}{2} < X < \frac{3}{2}\right)$.

2. (10%) Two coins are to be flipped. Assume that the flips are independent. The first coin will land on heads with probability 0.6, and the second with probability 0.75. Let X be the total number of heads that result.
   (a) (5%) Calculate $P(X = 1)$.
   (b) (5%) Find $E(X)$.

3. (15%) Consider a binary communication channel with error probability of 0.2. Assume the errors occur independently in each bit. Suppose that we want to transmit an important message consisting of one bit. To reduce the chance of error, we transmit 00000 instead of 0 and 11111 instead of 1.
   (a) (5%) What is the probability that the errors occur in at least one of the five bits?
   (b) (5%) If the receiver uses "majority voting" to decode this message, what is the probability that the message will be wrong when decoded?
   (c) (5%) Instead of using 5-bit words, if we transmit 000 for 0 and 111 for 1, what is the probability that the message will be wrong when decoded by "majority voting"?

4. (10%) Consider a binary communication channel with error probability of 0.2. Assume the errors occur independently in each bit.
   (a) (5%) What is the probability that the first error occurs in the 6^{th} bit?
   (b) (5%) What is the probability that the third error occurs in the 13^{th} bit?

5. (15%) If passengers arrive at the airport obey the Poisson model with the rate $\lambda = 9$ per hour.
   (a) (5%) What is the probability of no passenger in one hour?
   (b) (5%) What is the probability that there are exactly three passengers in an eight-hour interval?
   (c) (5%) What is the probability that in exactly three of the next eight hours no passengers will arrive?

6. (20%) Let X be a continuous random variable with probability distribution function

$$F(x) = \begin{cases} 0, & -\infty < x \le -1 \\ \frac{(x+1)^2}{4}, & -1 < x < 1. \\ 1, & 1 \le x < \infty \end{cases}$$

   (a) (10%) Calculate $E(X)$ and $Var(X)$.
   (b) (5%) Find the probability density of X^3.
   (c) (5%) Calculate $E(2X^2 + 3)$.

7. (15%) If X has the probability density function $f(x) = 3(1-x)^2,\ 0 < x < 1$.
   (a) (5%) Find the probability density function of $Y = (1-X)^3$;
   (b) (10%) Find the density function of $Z = e^Y$ and $E(Z)$.