Exam 1 for CS 333201

10:10 - 12:00 a.m., March 22, 2017

1. (15%) Eight people are seated in a row. Find the probability if
   (a) (5%) there are 4 men and 4 women and no 2 men or 2 women can sit next to each other?
   (b) (5%) there are 5 men and they must sit next to each other?
   (c) (5%) there are 4 married couples and each couple must sit together?

2. (15%) If a person bought 20 Christmas cards and 10 envelopes (labeled 1,2,...,10). In how many ways can the person put the 20 cards into the envelopes if
   (1) (5%) the cards are distinct?
   (2) (5%) the cards are identical?
   (3) (5%) the cards are identical and no envelope can be left empty?

3. (20%) A person has 7 keys numbered 1 through 7 and each key can be used to open only one corresponding office (also numbered 1 through 7). The keys are selected one at a time, without replacement. Let the events E_i denote that the person can open the door of the ith office, $i = 1, \ldots, 7$.
   (a) (5%) Find $P(E_i)$ for $i = 1, \ldots, 7$.
   (b) (5%) Find $P(E_iE_j), i \neq j$.
   (c) (10%) Find the probability that the person can open at least one door.

4. (15%) There are three machines A, B, and C in a semiconductor manufacturing facility that make chips. They manufacture 20%, 35%, and 45%, respectively, of the total semiconductor chips, and of their outputs, 6%, 4%, and 2% of the chips are defective.
   (a) (10%) What is the probability that a random drawn chip from the combined output is defective?
   (b) (5%) If a chip is drawn randomly and is found defective, what is the probability that this defective chip was manufactured by machine B?

5. (15%) A pair of fair dice is rolled until a sum of either 5 or 7 appears.
   (a) (5%) What is the probability that a sum of 5 occurs in the first roll? What is the probability that a sum of 7 occurs in the first roll?
   (b) (10%) Find the probability that a sum of 5 occurs before a sum of 7.

6. (15%) $\{E_1, E_2, \dots, E_n\}$ is a set of events.
   (a) (8%) Prove De Morgan's first law, $(\bigcup_{i=1}^{n} E_i)^c = \bigcap_{i=1}^{n} E_i^c$, by elementwise proof.
   (b) (7%) Show that, if the set $\{E_1, E_2, \dots, E_n\}$ is independent, then $P(E_1 \cup E_2 \cup \dots \cup E_n) = 1 - \prod_{i=1}^{n}(1 - P(E_i))$

7. (5%) The figure blow shows a switch network in a digital communication link, where each of the switches α_i, $i = 1, \dots, 6$ is independently closed or open with probabilities p and $1 - p$, respectively. What is the probability that there exists at least one closed path from 1 to 2?