Dynamic Network Model

Timothy Sudijono

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1 Sensor Model

We want to describe a model for collections of sensors that appear and disappear at random through a space. Our model accommodates for the following specifications:

- The sensors live on a manifold $\mathcal{M}$, which in this case we take to be $\mathcal{M} = \mathbb{S}^2$.
- The sensors have a random lifetime S distributed according to some measure τ on $\mathbb{R}_+$. For example, the probability the node dies before a given time $\mathbb{P}(S \leq t)$ is just $\tau([0, t])$.
- The sensors appear uniformly at random on the sphere, and suppose their interarrival times are exponentially distributed with parameter $1/\lambda$ (equivalently the number of sensors that have appeared in a given unit time interval is Poisson λ).
- Every sensor is independent, and for each sensor, its lifetime, location of appearance, and time of appearance are independent.

This can be viewed as a dynamic analog of MANETs and other wireless communication networks.

To define this sensor model, we adapt the definition of a *nonhomogeneous Poisson point process.*

Definition 1.1 (Nonhomogeneous Poisson Point Process)**.** *A nonhomogeneous Poisson process on some measurable space $(S, \mathcal{S})$ with intensity Λ is a point process N satisfying*

- *The number of points in a bounded Borel set $B \in \mathcal{S}$ is a Poisson random variable with mean $\Lambda(B)$.*
- *The number of points in n disjoint Borel sets forms n independent random variables.*

1.1 Modeling without sensor deaths

If we don't have to model sensor deaths we can take the measurable space to be

$$\left(\mathbb{R}_+ \times \mathbb{S}^2,\ \mathcal{B}(\mathbb{R}_+) \times \mathcal{B}(\mathbb{S}^2)\right).$$

The first coordinate models time, and the second models the location of the sensor. In this case, consider some Borel set, for simplicity of the form $I \times A$ (Note $I \subset \mathbb{R}, A \subset \mathbb{S}^2$). Then the number of sensors in this set, $N(I \times A)$ is a random variable, a Poisson random variable with mean

$$\lambda \operatorname{Leb}(I) \operatorname{Leb}(A).$$

1.2 Modeling sensor deaths

Modeling sensor deaths can be done by taking the measurable space in Definition 1.1 to be

$$\left(\mathbb{R}_+ \times (\mathbb{S}^2 \times \mathbb{R}_+),\ \mathcal{B}(\mathbb{R}_+) \times (\mathcal{B}(\mathbb{S}^2) \times \mathcal{B}(\mathbb{R}_+))\right).$$

In this case the last coordinate models the lifetime of the sensor. Then we can take the intensity measure defined in Definition 1.1 as

$$\Lambda(I \times (A \times J)) = \lambda \operatorname{Leb}(I) \operatorname{Leb}(A) \tau(J)$$

where $I \subset \mathbb{R}_+$ is the time interval in consideration, $A \subset \mathbb{S}^2$ is the patch on the sphere we look at, and J is the interval of lifetimes we consider. The number of sensors born at a time in I, at a location in A, and has lifetime J is then distribution according to a Poisson random variable with mean

$$\lambda \operatorname{Leb}(I) \operatorname{Leb}(A) \tau(J)$$

1.3 Misc.

The definition of a general point process is given below:

Definition 1.2 (Point Process)**.** *Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space, and $(\mathfrak{N}, \mathcal{N})$ the space of counting measures on a space $(S, \mathcal{B}(S))$. A point process ξ is a measurable map*

$$\xi : (\Omega, \mathcal{F}, \mathbb{P}) \to (\mathfrak{N}, \mathcal{N}),$$

The above definition essentially states that a point process is a *random measure* (a measure-valued random variable).

Another construction called *Marked Point Processes* might also be helpful: they are point processes but tag each of the points with a 'mark.'