Inequality progress

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1 Review of Tripods paper

The paper "A Distance Between Filtered Space" by Facundo deals with the stability of filtrations on finite spaces, not necessarily defined on the same set.

We define a filtration on a finite set X as the pair (X, F_X) where $F_X : 2^X \to \mathbb{R}^+$ such that $F_X(\sigma) \leq F_Y(\tau)$ whenever $\sigma \subseteq \tau$.

Let $\mathbb{X}, \mathbb{Y}$ be filtered spaces. The following proposition gives a stability result between filtrations on different spaces:

Prop. 1.1 (Thm 4.2)**.**

$$d_D(D_k(X), D_k(Y)) \leq d_{\mathcal{F}}(\mathbb{X}, \mathbb{Y})$$

where d_D is the bottleneck distance, and $d_{\mathcal{F}}$ is the tripods distance, defined as follows.

The idea of the tripods distance is to compare the filtrations on the same space. If Z is another filtered space, let $\phi_X : Z \to X$ be a *parametrization* of the space X, which is just a surjective map from Z to X. We defined

$$d_{\mathcal{F}}(\mathbb{X}, \mathbb{Y}) := \inf \{\max_{\tau \in 2^Z} |\phi_X^* F_X(\tau) - \phi_Y^* F_Y(\tau)|; \phi_X : Z \to X, \phi_Y : Z \to Y \text{ parametrizations}\}.$$

We define the pullback filtration $\phi_X^* F_X$ as the function on Z taking

$$\tau \mapsto F_X(\phi_X(\tau)) \forall \tau \in \text{pow}(Z)$$

2 Progress

Recall that we consider filtered spaces defined as (X, F_X) for X a weighted network and $F_X : 2^{V(X)} \to \mathbb{R}$ defined by

$$F_X(\sigma) = \max_{x, x' \in \sigma} (w_v(x), w_e(x, x'))$$

The idea is to find parametrizations such that

$$\max_{\tau \in 2^Z} |\phi_X^* F_X(\tau) - \phi_Y^* F_Y(\tau)|$$

is bounded by the TL^1 distance.

The idea is to use Z as the bipartite graph between X, Y, and incorporate the matching between the two graphs.

One candidate for ϕ_X, ϕ_Y is to send all $X \subseteq Z \to X$ identically and all $Y \subset Z \to X$ via the largest weight vertex in the bipartite matching.

This induces a filtration on Z via $\tau \in 2^Z$ born at $F_X(\phi_X(\tau))$. The issue is that any simplices without an edge between some of their vertices will be born at infinite time. That is, for some τ, we might have that $\phi_X(\tau)$ might be born at finite time and $\phi_Y(\tau)$ born at infinite time since $\phi_Y(\tau)$ might be a subset of vertices that isn't fully connected. We can easily come up with an example such that this is the case.

What are other candidates for the parametrization?