Outline of Results

Also includes questions and conjectures, and further desired results.

Frozen Measure

Results for contact process on Z. We apply the Harris paper and the literature on the K-Markov Approximation.

- **When the frozen measure has good continuity rates, the frozen local approximation also has good continuity rates.**
- **As a result, we immediately have existence of a stationary distribution for this process via the Harris paper, along with ergodicity.**
- **The K-Approximation converges to the full dynamics in the Ornstein distance** (It's a good approximation).

We can possibly generalize to have more applicable results.

While this investigation is not exactly applicable to answering the big question, the following question can aid in it: *Does perturbing the frozen measure change the stationary distribution slightly?* Further investigations need to be done for continuity rates being preserved for these processes.

Try working with the Ising model?

Nonlinear Markov Approximation

When considering the full local approximation without fixing the the external measure, we can reinterpret this process as a nonlinear Markov chain, motivated by the Harris paper from the previous section.

The question now motivating this study is the ergodicity of the Nonlinear Markov Chain, i.e. *under what conditions on the dynamics does the nonlinear Markov chain converge to a unique stationary distribution?*

- **We establish the continuity of this process in the total variation distance.**
- We also investigated this question in the weak topology, and it boiled down to showing weak convergence of a conditional expectation. There is some literature on this problem, but perhaps not directly applicable.

Numerically, it's difficult to simulate this so we consider the K-Approximation, similar to the K-Markov approximation in the previous section.

K-Approximation

Aside from numerical tractability, the K-Approximation works on the space of finite probability measures, and hence we have compactness in the total variation distance.

We have partial result on the ergodicity of the K-Approximation:

Under the condition that the conditional expectations in the Local Approximation are all well defined, there exists a stationary distribution of the K-Approximation, for every k.

The question then is *Do we have uniqueness of these stationary distributions*? Numerically the answer is no; the ergodic properties of the full dynamics in some processes are reflected in the K-Approximation.

Another interesting property to verify is the Doob property, or at least approximately. Numerically we have some evidence of this fact, and when the initial measure is symmetric then this property holds.

In the case we have unique ergodicity for each k, we have a sequence of stationary measures μ_k. Can we say these converge to any measure μ, and furthermore does μ a stationary distribution of the Nonlinear Markov Chain? We have some numerical evidence that this is the case. Theoretically, this seems to be a perturbation analysis question: how much does the stationary distribution change upon changing the dynamics slightly?

Numerical Results/ Investigations

Comparison Simulation

In what circumstances is the mean field approximation more effective than the Local Approximation/ K Approximation?

- Results comparing Mean field approximation and K Approximation for large T, different values of kappa and depth
- Results comparing mean field, k approx, dynamics, local approx for small T, on different graphs
- computed this for the following functionals: expected time spent at a state, probability of being at a state, for a single node and a node + neighbor.

Phase Transition Investigation

Idea is to see if ergodic properties match up between the full dynamics on the infinite tree (theoretically what's known about these processes) and the k-approximation. Is there uniqueness? Does the behavior change at known phase transitions?

Preliminary results (from last semester):

- For the contact process, seem to have unique ergodicity for small values of infection rate (other than the trivial one)
- For large values of the infection rate, we have multiple stationary distributions.
- For the Ising Model we seem to have unique ergodicity? Check this again.

We plan to do these investigations for the:

- Ising Model, dependent on the beta parameter. We can explicitly calculate the threshold for having a unique stationary distribution.
- Contact Process. Refer to the Pemantle paper. High infection rates have multiple stationary distributions.
- Majority dynamics / Voter Process. Depends on the initial bias parameter; we want to check if we see similar results to the Kanoria - Montanari paper.
- (If time) Potts, as an example of a process with more than two values for the state space.

Verification of some conjectures

- Approximate Doob conjecture/ convergence of the stationary measures of the k-approximation. Have some evidence for this already.
- Doob property for small T.