Intervention Analysis of UKDriverDeaths Dataset

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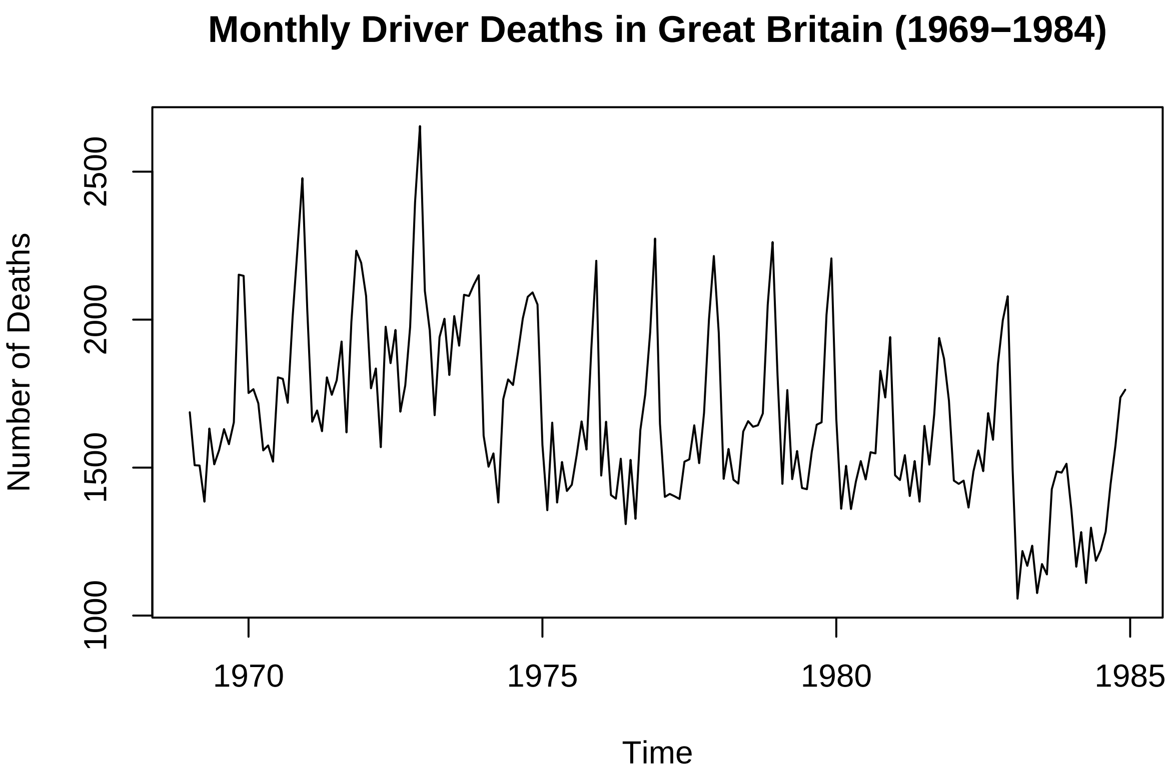
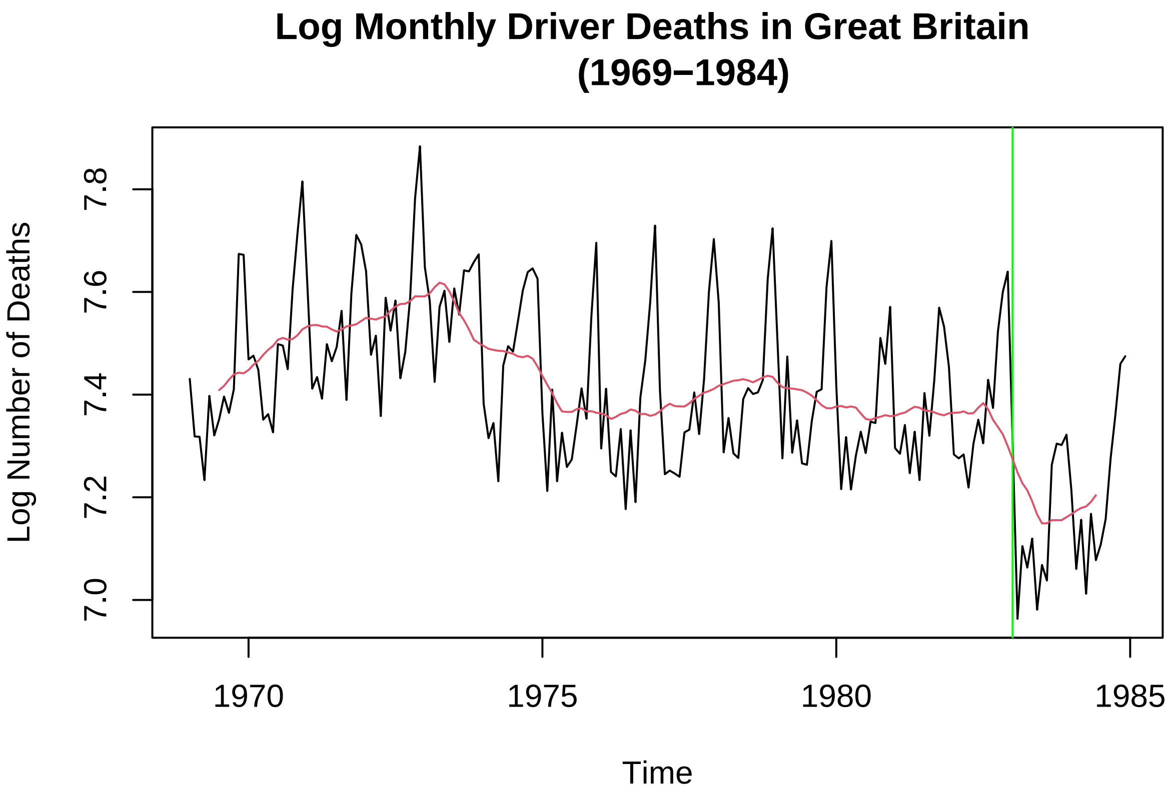
1. Summary

We consider the monthly time series UKDriverDeaths containing the well-known data from Harvey and Durbin (1986) on car drivers killed or seriously injured in the United Kingdom from January 1969 through December 1984. These are also known as the "seatbelt data", as they were used by Harvey and Durbin (1986) for evaluating the effectiveness of compulsory wearing of seatbelts introduced on 1983-01-31. Intervention analysis format will be used in the proceeding sections.



The seatbelt data reveal clear trends and seasonality, necessitating the removal of these elements to achieve stationarity for accurate forecasting of future driver fatalities. A log transformation is advisable to stabilize potential non-constant variance and improve the normality of the data. The red line on the plot smooths out the data, elucidating underlying patterns by minimizing the impact of seasonal spikes. The green line marks the seat belt legislation, after which a significant decrease in the data's level suggests the law's effectiveness in lowering fatalities and injuries. This observed downward trend underscores the need for differencing the log-transformed data to achieve stationarity, a critical step for subsequent forecasting.



Our final SARIMA model: $\Delta\Delta_{12}Z_t = (1 - 0.6967B)(1 - 0.8985B^{12})a_t - 0.2278 \cdot X_t$

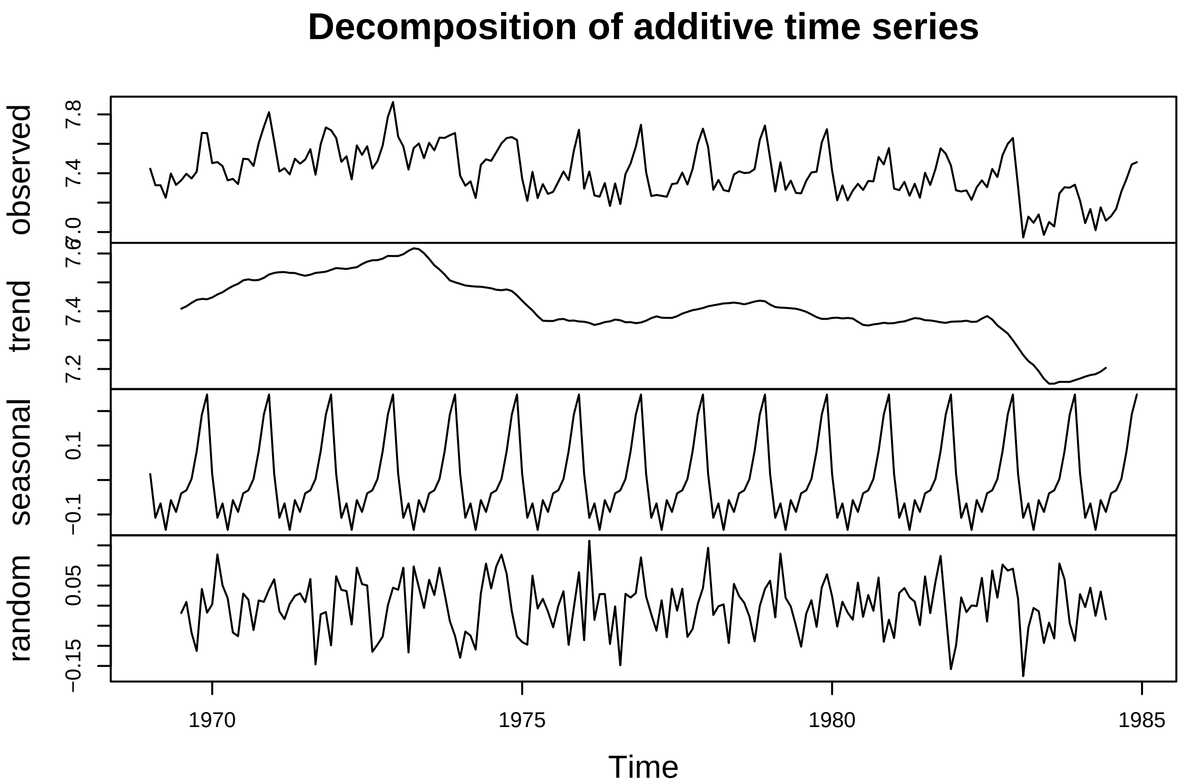
where $\Delta\Delta_{12}Z_t$ is the differenced time series at time t, a_t is the white noise at time t and the intervention variable X_t represents the effect of the seat belt law implementation at time t.

This model is also famously known as the airline model due to its application to a series of airline passengers in the classical text by Box and Jenkins (1970).

2. Analysis

2.1 EDA

By observing the data, we apply the additive decomposition since the seasonal effect remains constant over time and does not vary with the level of the time series.The trend component indicates there may be a long-term decrease in driver deaths over the period shown. The seasonal component captures regular fluctuations within a year, possibly related to seasonal factors affecting driving conditions, such as ice or fog in winter months.



The t-test statistic is 8.5523 with a p-value much smaller than 0.05, indicating a very significant difference in means before and after the law. The 95% confidence interval for the difference in means ranges from 302.9403 491.8350, suggesting the reduction in the number of deaths is statistically significant and not due to random chance.

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   7.177   7.323   7.411   7.438   7.565   7.884
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   6.963   7.066   7.156   7.176   7.288   7.475
```

```
##
##  Welch Two Sample t-test
##
## data:  exp(pre_law) and exp(post_law)
## t = 8.5523, df = 33.828, p-value = 5.682e-10
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  302.9403 491.8350
## sample estimates:
## mean of x mean of y
##  1719.083  1321.696
```

2.2 Stationarity Testing

ADF Test: Indicates the series is stationary, suggesting no unit root is present. This is a good sign for further analysis as many time series models require stationarity as a prerequisite.

KPSS Test for Level Stationarity: Suggests the series is not stationary when considering level stationarity. This could indicate the presence of a trend or changing variance over time.

KPSS Test for Trend Stationarity: Indicates that once a trend is considered, the series does not appear to have a unit root, suggesting it is trend stationary.

Given these results, the UKDriverDeaths series seems to be trend stationary. This means that the underlying process generating the data could have a deterministic trend component, and once that trend is accounted for, the series behaves in a stationary manner.

```
##
##  Augmented Dickey-Fuller Test
##
## data:  pre_law
## Dickey-Fuller = -5.9991, Lag order = 5, p-value = 0.01
## alternative hypothesis: stationary

##
##  KPSS Test for Level Stationarity
##
## data:  pre_law
## KPSS Level = 0.74941, Truncation lag parameter = 4, p-value = 0.01

##
##  KPSS Test for Trend Stationarity
##
## data:  pre_law
## KPSS Trend = 0.11661, Truncation lag parameter = 4, p-value = 0.1
```

After the first difference is taken, the tests suggest that the time series does not have a unit root, thereby indicating its stationarity. From the plot, the log-transformed seatbelt data also appears to be stationary after the first differencing.

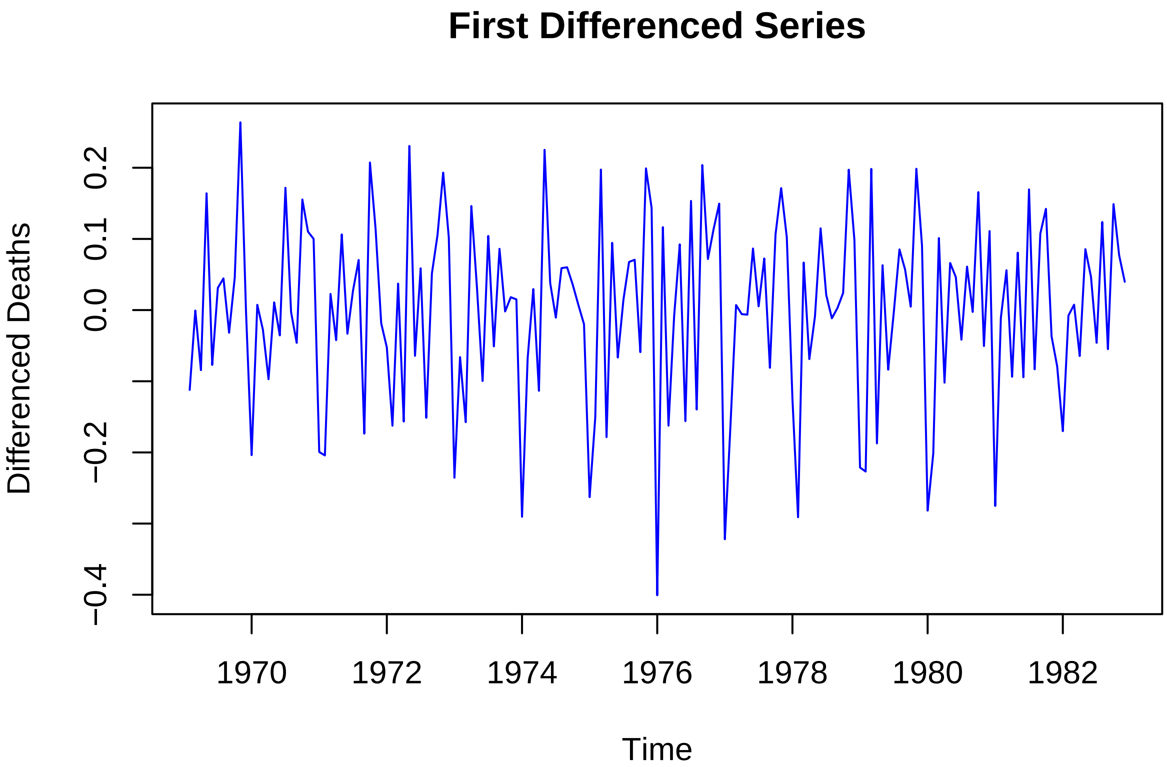
```
## Number of differences required to achieve stationarity: 1

## Number of differences required to achieve seasonally stationarity: 1

##
##  Augmented Dickey-Fuller Test
##
## data:  diff(pre_law)
## Dickey-Fuller = -7.6595, Lag order = 5, p-value = 0.01
## alternative hypothesis: stationary

##
##  KPSS Test for Level Stationarity
##
## data:  diff(pre_law)
## KPSS Level = 0.026658, Truncation lag parameter = 4, p-value = 0.1

##
##  KPSS Test for Trend Stationarity
##
## data:  diff(pre_law)
## KPSS Trend = 0.022174, Truncation lag parameter = 4, p-value = 0.1
```

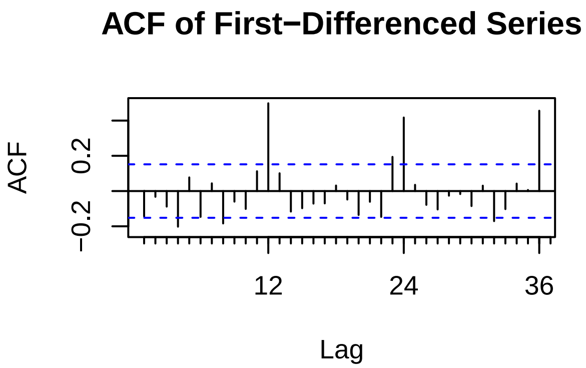


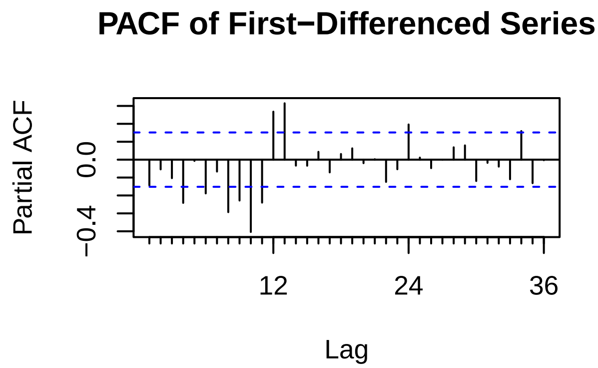
2.3 Model Specification

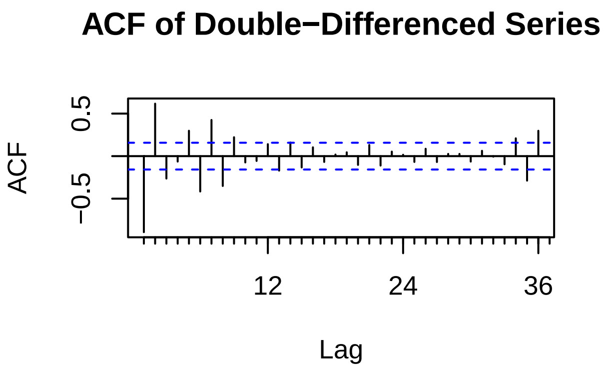
The use of Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) correlograms helped choose the SARIMA model parameters (p, q) and (P,Q) for the data. The observations within the 95% significance bounds generally indicate a whitenoise process.

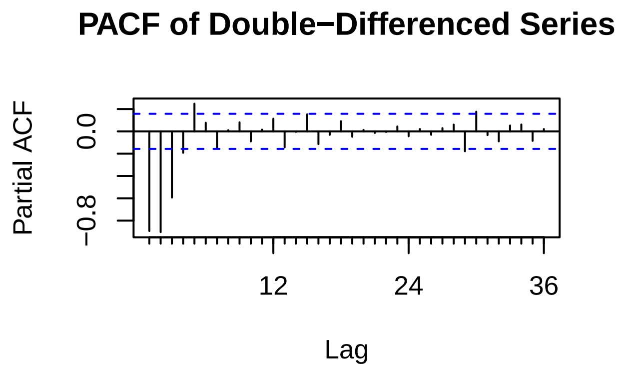
The first-differenced ACF plot reveals significant autocorrelations at regular intervals, specifically at lags 12, 24, and 36, suggesting a potential annual seasonality in the monthly data. Meanwhile, the first-differenced PACF plot shows significant partial autocorrelations at the initial 13 lags, indicating a possible combination of autoregressive process and seasonality.

After double differencing, which includes a seasonal difference, the ACF and PACF plots still show significant spikes at first 3 lags. These spikes could be random or signify an important feature. Therefore, we assume that the optimal P and Q are around 3.









2.4 Pre-Intervention Modelling

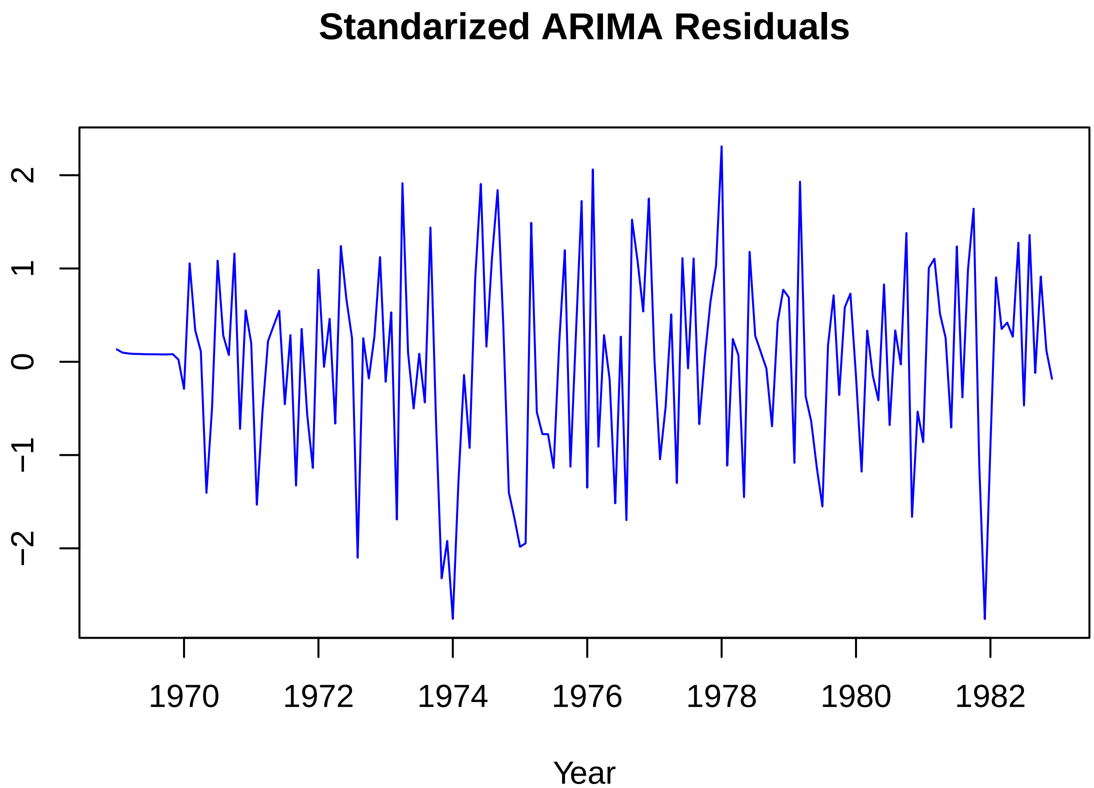
In order to select an appropriate SARIMA model that best fits the seatbelt data, a matrix of BIC values was used. BIC tends to favor simpler models than AIC, which can be an advantage for model interpretation and when the goal is to identify the true model. The BIC values suggested the $SARIMA(0,1,1)(0,1,1)_{12}$ model appeared to most adequately fit the seatbelt data.

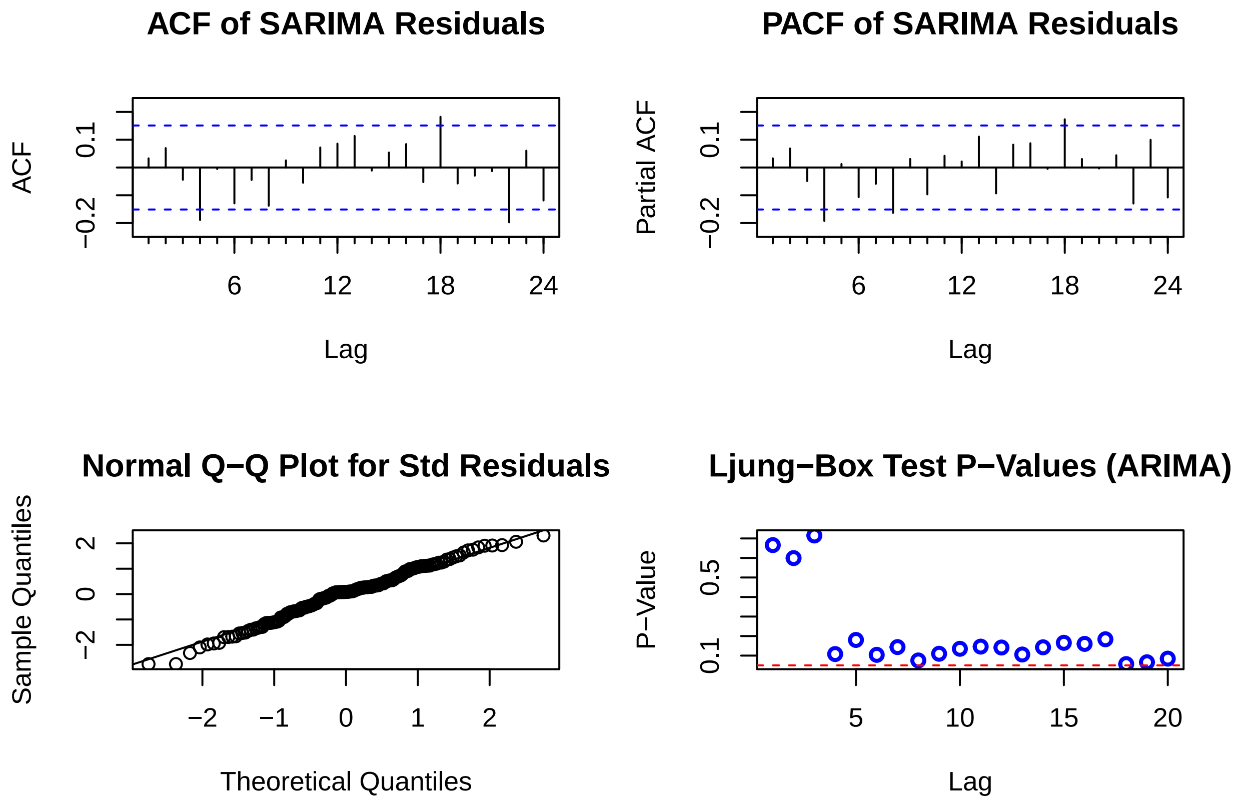
```
##    ar diff ma sar sdiff sma
## 31  0    1  1   0     1   1
```

```
## Series: pre_law
## ARIMA(0,1,1)(0,1,1)[12]
##
## Coefficients:
##           ma1     sma1
##       -0.6737  -1.0000
## s.e.   0.0795   0.1538
##
## sigma^2 = 0.005573:  log likelihood = 167.24
## AIC=-328.47   AICc=-328.31   BIC=-319.34
##
## Training set error measures:
##                        ME       RMSE       MAE         MPE      MAPE      MASE
## Training set -0.005170345 0.07124433 0.0549913 -0.07527299 0.7392861 0.6683346
##                    ACF1
## Training set 0.03300836
```

Residual Analysis

The standardized residuals plot for the SARIMA model indicates the residuals are homoscedastic and center around zero mean. The ACF and PACF correlograms show that the residuals appear to look like white noise. All of the p-values for the Ljung-Box statistic are above 0.05 at lags 1-20,suggesting the residuals do not show significant autocorrelation, and our model has adequately captured the autocorrelations and seasonality in the data. Lastly, the normal Q-Q plot only have small deviations from the 45 degrees line,overall, the $SARIMA(0,1,1)(0,1,1)_{12}$ model seem to have taken care of the significant spikes at lag 1-3 and the seasonality.





2.5 Intervention Analysis

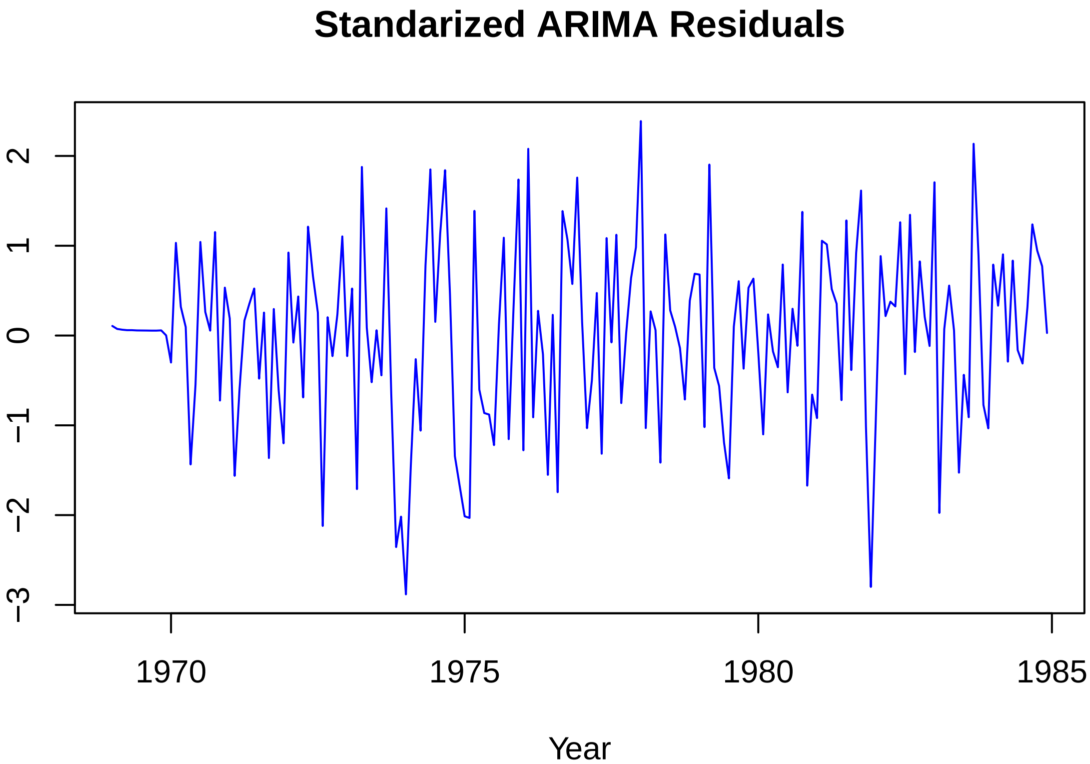
With an added intervention component representing the enactment of the seat belt law in 1983, the model indicates a statistically significant impact of the law on reducing driver deaths. The intervention coefficient of -0.2278, with a very low p-value (approximately 0.000057), strongly suggests that the introduction of the seat belt law was associated with a decrease in fatalities. This result, given the robustness of the ARIMA modeling framework, underscores the effectiveness of the seat belt legislation in improving road safety.

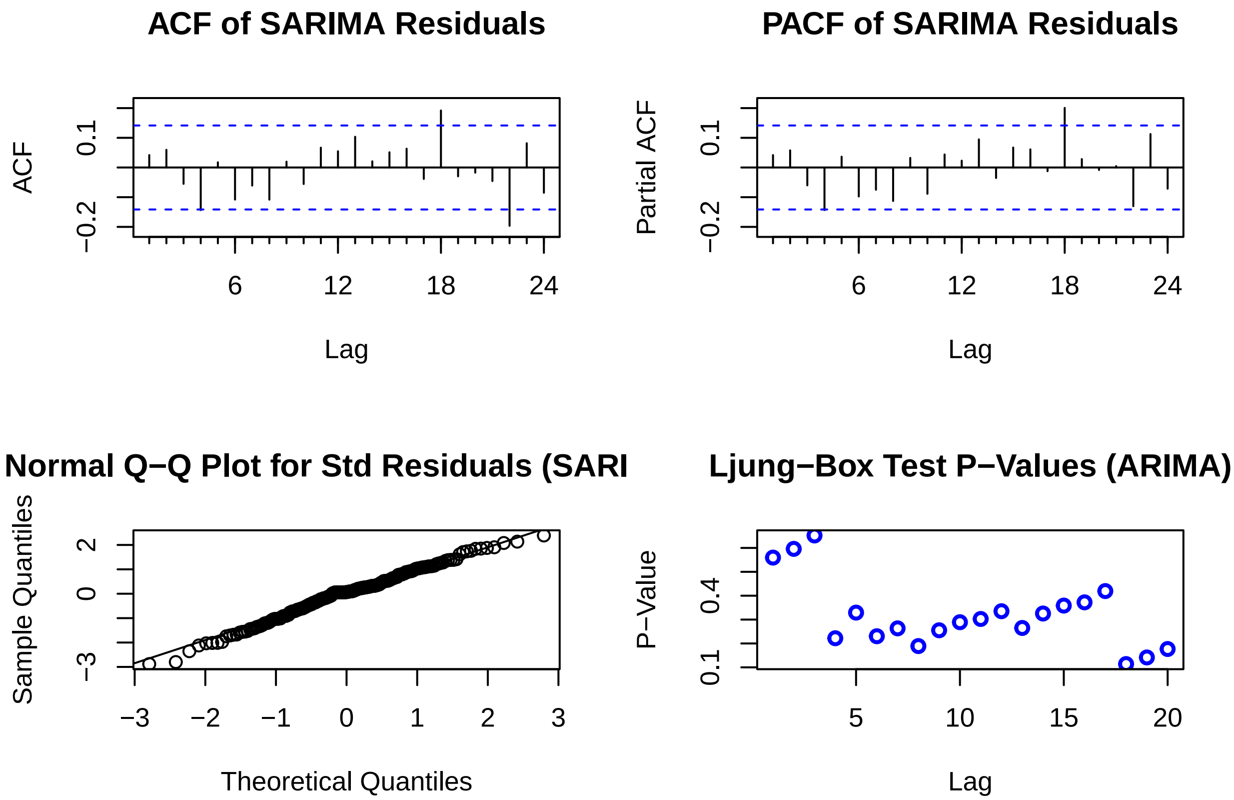
```
## Series: UKDriverDeaths
## Regression with ARIMA(0,1,1)(0,1,1)[12] errors
##
## Coefficients:
##           ma1     sma1     xreg
##       -0.6967  -0.8985  -0.2278
## s.e.   0.0698   0.0916   0.0553
##
## sigma^2 = 0.005981:  log likelihood = 195.73
## AIC=-383.47   AICc=-383.24   BIC=-370.72
##
## Training set error measures:
##                        ME       RMSE        MAE         MPE     MAPE      MASE
## Training set -0.003608681 0.07404209 0.05732185 -0.05449295 0.774091 0.6209186
##                    ACF1
## Training set 0.04177936
```

```
## [1] 5.677266e-05
```

Residual Analysis

These plots show the similar results to fitting with pre-law data. The $SARIMA(0,1,1)(0,1,1)_{12}$ model with intervention seems to have taken care of the autocorrelation and seasonality.

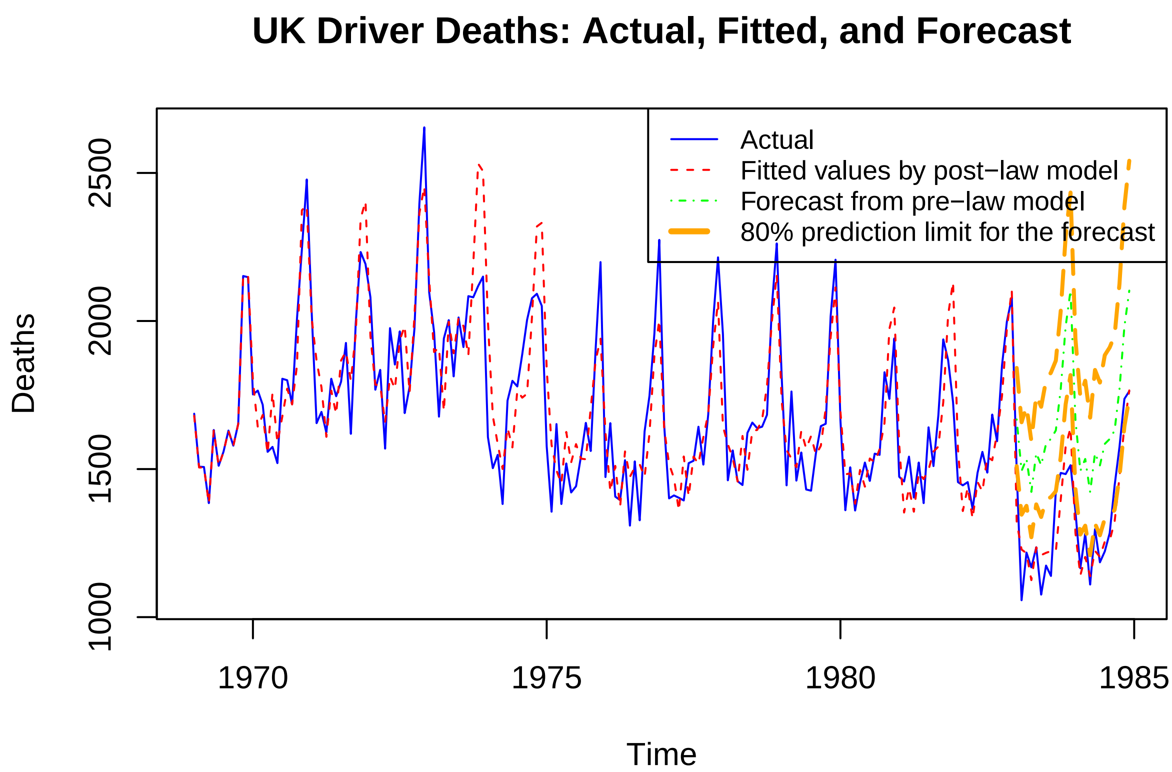




2.6 Forcasting

The plot presents actual historical data in blue solid line, with fitted values from apost-law implementation model in red dashed line and forecasts from a pre-law model in green dotdash line. Accompanying this forecast, the orange dashed lines depict an 80% prediction interval, indicating the range within which future values are expected to fall with 80% probability.

The close tracking of the fitted values to the actual data illustrates the post-law model's accuracy in capturing the impact of the seat belt legislation. In contrast, the forecast, extending beyond the observed data, projects what might have occurred without the law's influence, showing a clear deviation from the fitted values. This divergence, where the forecast predicts higher fatalities than the observed and fitted data, suggests that the introduction of the seat belt law had a tangible, beneficial effect in reducing the number of driver deaths.


UK Driver Deaths: Actual, Fitted, and Forecast
Actual
Fitted values by post−law model
Forecast from pre−law model
80% prediction limit for the forecast
Deaths
1000
1500
2000
2500
1970
1975
1980
1985
Time