



All of b is in a but only some a is in c, so there can be some a in c

$a \neq c$      $b \neq c$

$a \neq b$

Some a is not in b

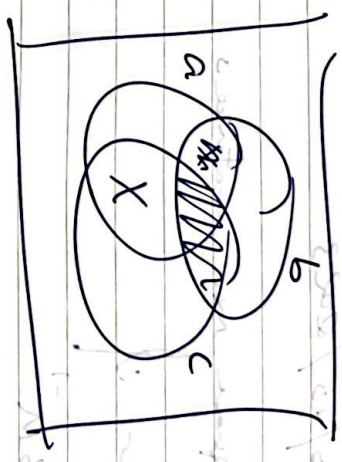
~~some a is not in b~~

$a \neq c$      $b \neq c$      $a \neq b$

So 1 + 2 is



4 + 3 is



So if b entails c and there is some a that's in c and some a that's not b, it concludes that there can be some a that is in c but not b.

~~C ⊆ A~~

1.  $a \not\subseteq c$        ~~$b \subseteq c$~~        ~~$c \subseteq b$~~   
 $a \not\subseteq b$

$a \not\subseteq b$

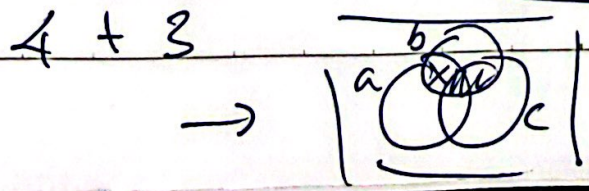
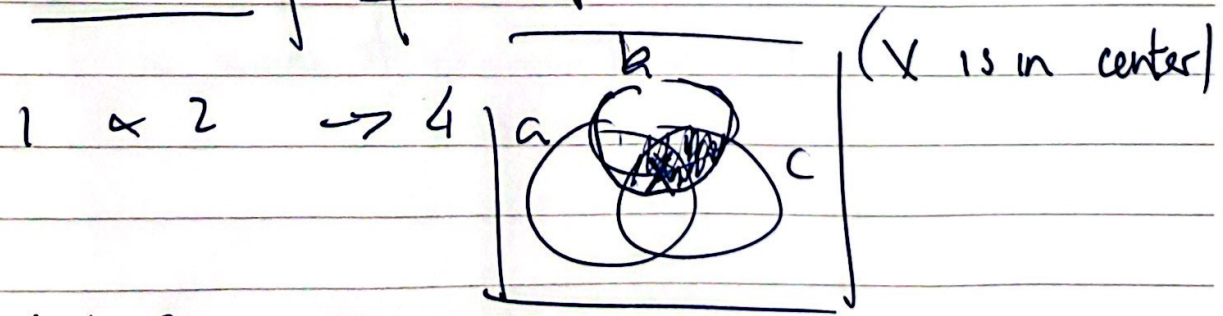
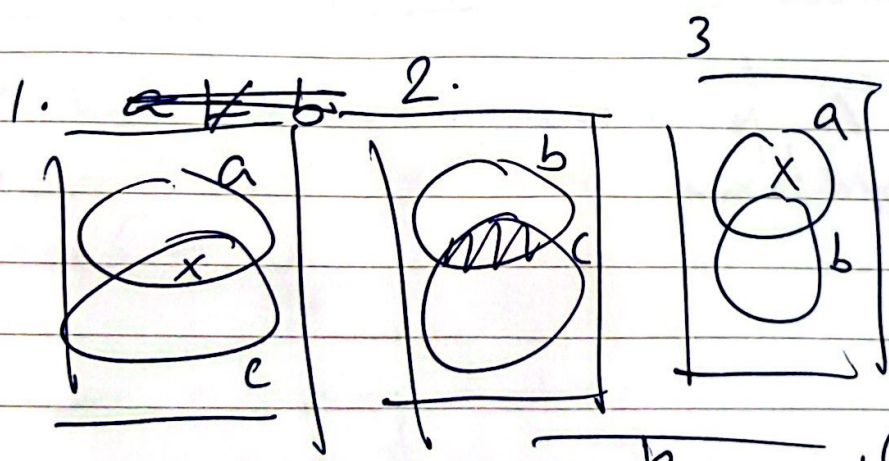
~~There is~~

There is some  $x$  in  $a$  that is not part of  $b$ . Doesn't entail all  $a$  is not  $b$ .

~~$c \subseteq b$~~        ~~$A$~~

~~$a \subseteq b$~~        ~~$b \subseteq a$~~

$a \not\subseteq c$        $c \subseteq b$   
 $a \not\subseteq b$





$$2. \frac{a \neq b \quad b \neq c}{a \neq c}$$

$$\frac{a \neq b \quad a \neq c}{b \neq c}$$

\* Contradiction  
Replacement  
\* Predicates  
\* Double negation

~~Contradiction~~

Substitute  $a$  for  $\neg a$

$$\frac{\neg a \neq b \quad \neg a \neq c}{b \neq c}$$

double negation x2

$$\frac{a \neq \neg b \quad \neg a \neq \neg c}{b \neq c}$$

Contrapositive Barbara (sequent)

$$\frac{a \neq \neg b \quad a \neq \neg c}{b \neq c}$$

Contrapositive + double negation

$$\frac{a \neq \neg b \quad a \neq \neg c}{c \neq b}$$

$$3. (p \wedge q) \vee \neg p \vee \neg q$$

AK

C I guess you can

$$\frac{\frac{(p \models p, q) \quad \neg q \models p, q}{p \wedge q \models p, q} \quad I}{p \wedge q \models p, q} \quad I$$

$$\frac{p, q \models p, q}{p, q \models p \wedge q} \quad \text{Immediate} \quad \wedge R$$

$$\frac{p, q \models p \wedge q}{\models (p \wedge q), \neg p, \neg q} \quad \neg R, \neg R$$

$$\frac{\models (p \wedge q), \neg p, \neg q}{\models (p \wedge q) \vee \neg p \vee \neg q} \quad \vee R, \vee R$$



$$4. \frac{\dots}{\Gamma, p \rightarrow q \vdash \Delta} \rightarrow L$$

$$\frac{\dots}{\Gamma \vdash p \rightarrow q, \Delta} \rightarrow R$$

$p \rightarrow q = \neg p \wedge q$

~~$$\frac{\Gamma \vdash \Delta \quad \Gamma, p \vee q \vdash \Delta}{\Gamma, p \vee q \vdash \Delta} \vee E$$

$$\frac{\Gamma, p \rightarrow q \vdash \Delta}{\Gamma, p \vee q \vdash \Delta} \vee E$$~~

$$\frac{\Gamma, q \vee \neg p \vdash \Delta}{\Gamma, p \rightarrow q \vdash \Delta} \rightarrow L$$

$$\frac{\Gamma \vdash q \vee \neg p, \Delta}{\Gamma \vdash p \rightarrow q, \Delta} \rightarrow R$$