1. The rank of the matrix is <= the no. rows

2. If I can prove linear dependence for 2<= rows then rows can be eliminated, meaning there will be more variables than equations.

   In a homogenous system $Ax = 0$ with a non-trivial solution, there must be a nonzero vector x that multiplies with A to result in the zero vector.

   $Ax = 0$ is taking a lin. com. of the rows of A with x, and there must be a nontrivial x.

   This implies that there are rows in A that produce the zero vector when multiplied by x.

   Given that the solution is the zero vector, there must be linearly dependent equations for the different combination of rows and x to produce 0.

3. The linearly dependent rows can be eliminated.

4. Thus, if we started with an $m \times n$ matrix (min. size as rank <= no. rows) we will now get a $(m-n) \times n$ matrix where n is the no. eliminated rows.

B: Invertible $n \times n$ $\quad$ g & h: n-dimensional columns

- Unique solution
- Full rank
- $A^{T^{-1}} = A^{-1^{T}}$
- $AA^{-1} = A^{-1}A = I$

Satisfies $1 + h^T B^{-1} g \neq 0$

$C = B + gh^T \qquad D = B^{-1} - \dfrac{B^{-1} g h^T B^{-1}}{1 + h^T B^{-1} g}$

$$CD = (B + gh^T)\left(B^{-1} - \frac{B^{-1} g h^T B^{-1}}{1 + h^T B^{-1} g}\right)$$

$$= BB^{-1} - \frac{\overset{I}{\cancel{BB^{-1}}}\, g h B^{-1}}{1 + h^T B^{-1} g} + gh^T B^{-1} - \frac{gh^T B^{-1} g h^T B^{-1}}{1 + h^T B^{-1} g}$$

$$= I - \frac{gh^T B^{-1}}{1 + h^T B^{-1} g} + gh^T B^{-1} - \frac{gh^T B^{-1} g h^T B^{-1}}{1 + h^T B^{-1} g}$$

$$= I + gh^T B^{-1} - \frac{ghB^{-1} + gh^T B^{-1} g h^T B^{-1}}{1 + h^T B^{-1} g}$$

$$= I + gh^T B^{-1} - \frac{g\cancel{(1 + h^T B^{-1} g)}\, h^T B^{-1}}{\cancel{1 + h^T B^{-1} g}}$$

$$= I + gh^T B^{-1} - gh^T B^{-1}$$

// Just like algebra very fun.

$$= I$$

$$CD = I$$

So C is invertible & the inverse is D.