The First Practice

April, 11, 2018

You need to complete Problem 1 and Problem 2. Problem 3 is bonus.

Problem 1. *Use Taylor series to approximate the exact value of e^2. The based point can be arbitrarily chosen. Show the convergence by the comparison with numpy.exp.*

[Hint:]
Let c_N be the N-th order Taylor series approximation, please calculate the error $= |e^2 - c_N|$ for each N until error $< 10^{-10}$.

Problem 2. *Let $f(x) = \sin(x)$ for $x \in (0, \pi)$, $h = \frac{\pi}{N}$, where N is a positive integer. (i) Please find all the values $f''(x_j)$ by second order central difference method, where $x_j = jh$, $j = 1, \ldots, N-1$. (ii) Find the convergence rate with respect to h. (You may compare the approximated results with numpy.)*

[Hint:]

- *You can try to let $x = 0$ and try more N, $N = 4$ or $N = 8$ or ...*
- *Please refer to the file given on 3/31 for the second order central difference method and the method of finding the numerical error.*

Problem 3. *As the same setting in Problem 3, you can try higher-order methods (We restrict ourselves to central difference methods.) and show the convergence rate of your applied methods.*

[Hint:]
For example, if the function f is smooth,

$$f''(x) = \frac{-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h)}{12h^2} + O(h^4).$$