

The second practice

May 18, 2018

Consider the following partial differential equation:

$$\begin{cases} \frac{\partial u(t,x)}{\partial t} = \frac{\partial^2 u(t,x)}{\partial x^2} & (t,x) \in (0,T] \times (0,1), \\ u(t,0) = u(t,1) = 0 & t \in (0,T], \\ u(0,x) = \sin(\pi x) & x \in (0,1), \end{cases} \quad (1)$$

where T is a finite number. The exact solution of (1) is given by

$$u(t,x) = e^{-\pi^2 t} \sin(\pi x) \quad (2)$$

Please solve (1) numerically by the following two scheme:

- simple implicit scheme
- Crank-Nicolson scheme

The code shall reach the following requirements:

- Please compute the numerical results until $t = 0.1$.
- The mesh sizes h 's shall be uniform and the uniform time steps Δt 's are given correspondingly. Please compare the results among

$$(h, \Delta t) = (0.1, 0.01), (0.05, 0.005), (0.02, 0.002).$$

- There shall be a for loop including the numerical computation under all the required $(h, \Delta t)$ pairs.
- The L^2 error results with respect to the exact solution (2) at $t = 0.1$ shall be shown.
- The original numerical results shall be output to a text file. Please output your data in the form : (time, space, computed value)