COMP90086: Assignment 1 Report

Tanzid Sultan (ID# 1430660)

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1. Laplacian Image Pyramid

In this section, we present some results from our code implementing the *Laplacian pyramid* algorithm from the assignment specifications.

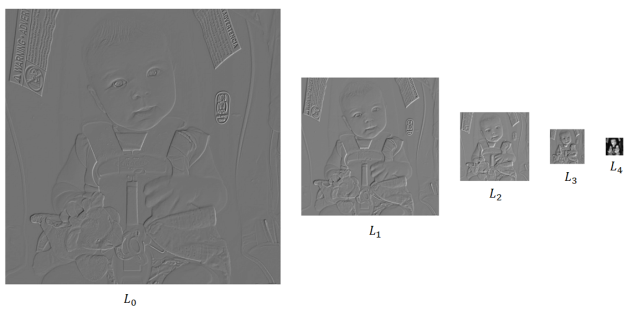

Figure 1: 5-level Laplacian Pyramid of example image

Figure 1. shows the difference images L_0, L_1, L_2, L_3 and L_4 obtained from the 5-level Laplacian Pyramid of our example image, which is a 1024 x 1024 pixel gray-scale image. Note that each higher level difference image in the pyramid is half the size of it's predecessor due to down-sampling.

Figure 2: (a) Original image, (b) Perfectly reconstructed image

The original image is reconstructed by up-sampling the highest level difference image L_5 by a factor of 2, convolving with a gaussian filter and adding with L_4 (the pixel values are also scaled by a factor of 4 to compensate for brightness loss due to the up-sampling and filtering). The resulting image is then up-sampled and filtered again and added to the difference image from the next level below and this process continues until level 0 is reached and we obtain the reconstructed image. This version of the algorithm does not quantize the difference images which allows for lossless/exact reconstruction of the original image from the Laplacian Pyramid, as shown in Figure 2.

2. Quantization and Compression

By quantizing the Laplacian pyramid difference images at each level, the size of the reconstructed image can be significantly compressed. The process of quantizing an image into b bits involves dividing the pixel values into 2^b bins and then replacing each pixel value in the image with the mean pixel value within the bin that the pixel occupies. This process inevitably leads to a loss of information from the image, called the *quantization error* (due to the difference between the actual pixel value and the mean value of the bin occupied by the pixel during quantization), which we perceive as a degradation in image quality. Quantizing an image into fewer bits leads to greater quantization error, as shown in Figure 3. It is remarkable to note that the degradation in image quality, in this particular example, only starts to become perceivable at $b = 4$ which is after a 50% reduction in image size.

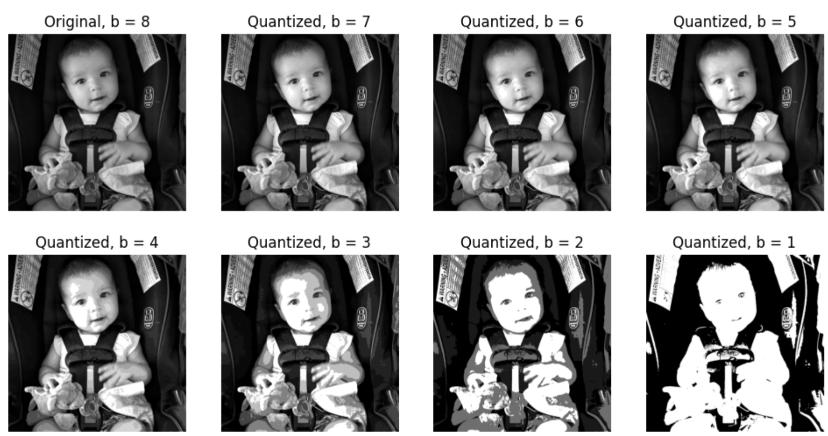

Figure 3: Original 8-bit grayscale image quantized at $b = 7$ down to $b = 1$

By appropriately choosing the number of quantization bits per level in the Laplacian pyramid, we can significantly compress the reconstructed image with very little perceived degradation in the image quality. Since the lower level difference images of a Laplacian pyramid (*e.g.* L_0, L_1) are more finely sampled, they contain much of the smaller-scale details of the image, whereas the smaller/coarsely sampled difference images at the higher levels of the pyramid (*e.g.* L_{n-2}, L_{n-1}) contain primarily the large-scale variations in the image. Because changes in the medium to larger-scale variations are perceived more easily by the human visual system, we are more sensitive to detecting image quality degradation resulting from quantization of the higher levels of the pyramid. Whereas, quantization of the lower levels will affect the smaller-scale details which will not be easily perceivable. This motivates us to apply more quantization/compression, i.e. fewer bits, to the lower level pyramid images and less quantization to higher level images if we want to preserve the quality of the reconstructed image.

Figure 4: (a) Original image, (b) Lowest level heavily compressed ($b = [1, 4, 5, 7, 8, 8, 8]$), (c) Multiple lower levels heavily compressed ($b = [2, 2, 2, 7, 7, 8, 8]$), (d) Higher levels heavily compressed ($b = [6, 5, 4, 2, 2, 1, 8]$)

Figure 4. shows the results from compressing our example image using a 7-level Laplacian pyramid with three different combinations of bits per level. For the compressed image in (b), the lowest level of the pyramid is most heavily quantized ($b = 1$), while moderate to high b-values are used for the higher levels. Therefore most of the quantization error for this combination resides in the small-scale features and are therefore not perceived as easily, yielding the best balance between good image quality and substantial compression, with a compression rate of 2.46-bits/pixel which is almost a 70% reduction in the image size.

For image (c), several of the lower levels are heavily quantized, which means that the quantization error affects both the small and medium-scale features. In particular, the errors in the medium-scale features can be perceived and we can see severe degradation in the image quality in the form of distortions in the brightness, including artifacts such as bright specs and an overall blurry appearance (especially noticeable around the region of the baby's face in the image). The compression rate for this image is 2.76-bits/pixel which is a 66% reduction in image size, however the trade-off is very low image quality. For image (d), we investigated the effect of heavily quantizing several of the higher levels, which again results in significant degradation in image quality in the form of artifacts such as brightness distortions and spurious extended bright patches along with a bright diffuse halo around sharp edges. This is clearly a consequence of higher quantization error in the large-scale image features which the human visual system is more sensitive to detecting. The compression rate is 7.52-bits/pixel which is only a modest 6% reduction in the image size. Therefore this combination of having lower bits per level at the higher levels of the pyramid leads to both poor compression rate and poor image quality.

So in general, to strike a balance between compression and image quality, it is better to apply lower b values for quantizing the lower levels of the Laplacian pyramid, higher b values for quantizing the higher levels and moderate b-values for levels in between. This will ensure that most of the quantization error will affect the small scale features in the image, leaving the medium to larger scale features intact which will result in both good image quality and good compression rate.

3. Evaluation in the Frequency Domain

Since the levels of the Laplacian pyramid make up a multi-scale/multi-resolution representation of an image, with the lower levels containing more of the higher spatial frequencies (i.e. small-scale variations) and the higher levels containing the lower spatial frequencies (i.e. large-scale variations), quantizing any particular level will primary introduce errors in the reconstructed/compressed image at the spatial frequencies associated with that level.

These effects become more evident when we analyze our compressed images in the *Fourier* domain. Figure 5. shows the Fourier transform (log) magnitude and phase of the **quantization error**[1] in each of the three compressed images from Figure 4. of the previous section. The top row shows the fourier transform of error for the compressed image which had lowest level of the pyramid heavily quantized and minimal quantization at the higher levels. Both the fourier amplitide and phase maps of the error are dark near the center region which contains the low spatial frequencies (corresponding to low/zero error) and becomes brighter away from the center at the higher spatial frequencies (corresponding to larger positive error). Thus, the quantization error in this compressed image is mostly in the high spatial

[1] Here, we define the quantization error as simply the absolute point-wise difference between the original image and compressed image.

frequency bands which is why we don't perceive any substantial degradation in the image quality after compression (except for some slight blurring of the small details in the image which become apparent only upon zooming in).

The middle row of Figure 5. shows the fourier transform amplitude and phase of the error for the second compressed image for which several of the lower levels have been heavily quantized, which is why we see a smaller dark region near the center as quantization error has creeped into the medium spatial frequency bands resulting in degradation in the perceived image quality. The bottom row of the figure shows the fourier transform of the error for the third compressed image for which several of the higher levels have been heavily quantized. In this case, we notice that the center region of the magnitiude and phase maps is the brighest indicating that the quantization error is mostly in the low spatial frequencies, once again resulting in degradation of the image quality after compression.

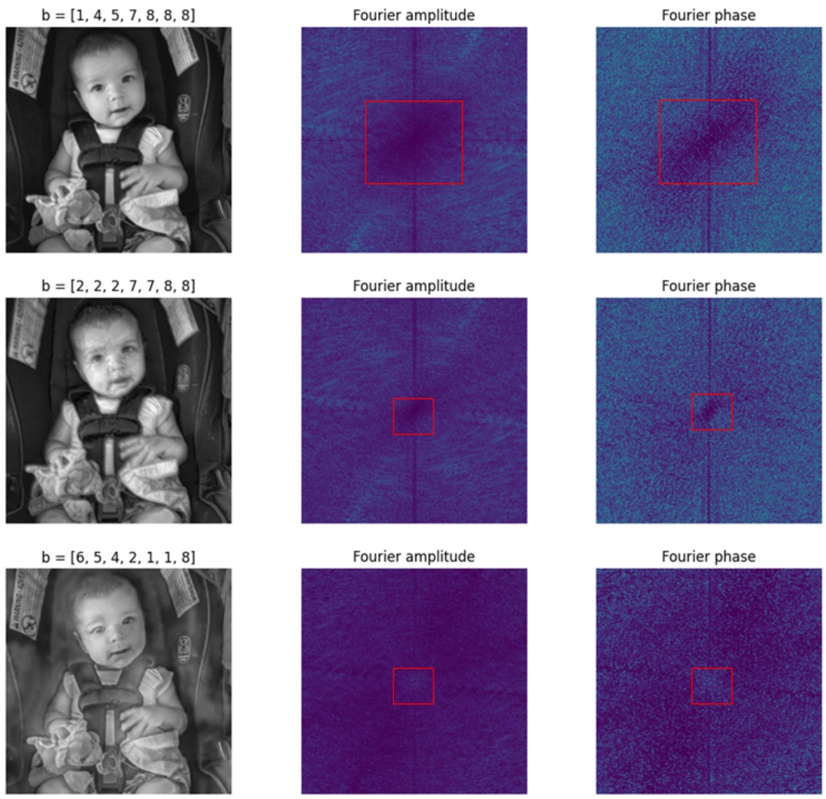

Figure 5: Fourier transforms of compressed images. Top Row: Lowest level heavily compressed ($b = [1, 4, 5, 7, 8, 8, 8]$) , Middle Row: Multiple lower levels heavily compressed ($b = [2, 2, 2, 7, 7, 8, 8]$), Bottom Row: Higher levels heavily compressed ($b = [6, 5, 4, 2, 2, 1, 8]$). For the top and middle rows, the red bounding box contains the region of low quantization error, however for the bottom row it contains region of high error.