Hessian-Based Norm Penalty for Weighted Least-square CBCT Reconstruction

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Purpose: To develop a Hessian-based norm penalty for cone-beam CT (CBCT) reconstruction that has a similar ability in suppressing noise as the total variation (TV) penalty while avoiding the staircase effect and better preserving low-contrast objects.

Methods: We extended the TV penalty to a Hessian-based norm penalty based on the Frobenius norm of the Hessian matrix of an image for CBCT reconstruction. The objective function was constructed using the penalized weighted least-square (PWLS) principle. An effective algorithm was developed to minimize the objective function using a majorization-minimization (MM) approach. We evaluated and compared the proposed penalty with the TV penalty on a CatPhan® 600 phantom and an anthropomorphic head phantom, each acquired at a low-dose protocol (10mA/10ms) and a high-dose protocol (80mA/12ms). For both penalties, contrast-to-noise (CNR) in four low-contrast region-of-interest (ROI) and the full-width-at-half-maximum (FWHM) of two point-like objects in constructed images were calculated and compared.

Results: In the experiment of CatPhan® 600 phantom, the Hessian-based norm penalty has slightly higher CNRs and approximately equivalent FWHM values compared with the TV penalty. In the experiment of the anthropomorphic head phantom at the low-dose protocol, the TV penalty result has several staircase effects while in the Hessian-based norm penalty the image appears smoother and more similar to that of the FDK result using the high-dose protocol.

Conclusion: The proposed Hessian-based norm penalty has a similar performance in suppressing noise to the TV penalty, but has a potential advantage in suppressing the staircase effect and preserving low-contrast objects.

Innovation/Impact: We developed a Hessian-based norm penalty for cone-beam CT (CBCT) reconstruction for the first time. The proposed penalty outperforms the TV penalty, particularly in suppressing the staircase effect.

Methods: The objective function is $\Phi(\mu)=(\hat{p}-A\mu)^T\Sigma^{-1}(\hat{p}-A\mu)+\beta R(\mu)$, where the first term is a penalized data-fidelity term and the second is a penalty term. The TV penalty is defined as $R_{TV}=\sum_{x,y,z}\|\nabla\mu(\mathrm{x,y,z})\|_2$. As a natural extension of the TV penalty, we proposed to use a Hessian-based norm penalty $R_{Hessian}=\sum_{x,y,z}\|\mathrm{H}_\mu(\mathrm{x,y,z})\|_F$, with each term being the summary of the Frobenius norm of the Hessian matrix at a specific pixel in the image. This Hessian-based norm penalty has some excellent properties such as rotation invariance and convexity, desired for many image processing applications[1]. We used a majorization-minimization (MM) strategy to effectively minimize the objective function.

Key Results: Fig.1 shows the reconstruction results for the CatPhan® 600 phantom at the low-dose protocol. Noise level was characterized by the standard derivation of a square uniform area. Four regions-of-interest (ROI) of low contrast were chosen to calculate contrast-to-noise (CNR). CNRs of the two penalties at different noise levels are listed in Table 1. It can be observed that the Hessian-based norm penalty have slightly higher CNR values than those of the TV penalty at a matched noise level, showing better reconstructed image quality of the low-contrast objects.

Table 1: CNRs of the low-contrast ROIs indicated in Fig 1

ROI	Noise Level ($\times10^{-4}$)	5.22	4.43	3.22	2.28	1.51	1.20
ROI1	TV	5.99	7.11	9.45	12.61	18.12	22.33
	Hessian	6.39	7.71	10.79	15.47	23.20	28.17
ROI2	TV	3.97	4.69	6.35	8.91	13.17	15.67
	Hessian	3.96	4.66	6.53	9.63	15.07	18.43
ROI3	TV	3.54	4.29	6.02	8.58	12.80	15.72
	Hessian	3.57	4.28	5.91	8.41	12.94	16.70
ROI4	TV	4.28	5.05	6.66	8.73	11.61	13.47
	Hessian	4.24	4.98	6.87	9.49	13.27	15.85

We plot the profiles through two point-like objects and fitted the profiles to a Gaussian function. The full-width-at-half-maximum (FWHM) of the brighter source is 3.46 pixels for the TV penalty and 3.41 pixels for the Hessian-based norm penalty. The FWHM of the darker source is 3.13 pixels for the TV penalty and 3.09 pixels for the Hessian-based norm penalty. This indicates that the two penalties have a similar ability in preserving edge.

Fig.2 shows the results for the anthropomorphic head phantom. One specific region was enlarged for clear details. At the low-dose protocol, several piece-wise constant areas known as the staircase effect can be observed in the TV penalty, while in the Hessian-based norm penalty, the image appears smoother and more similar to that of the FDK result using the high-dose protocol (80mA/12ms).

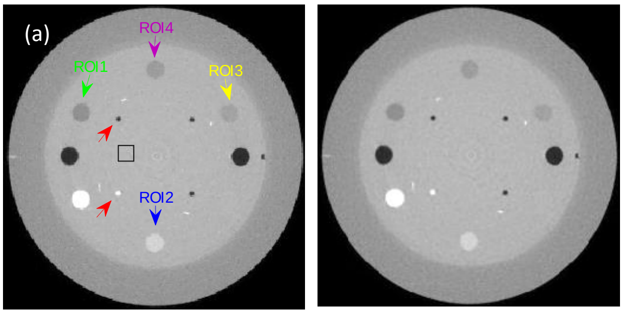

Fig 1: Iterative reconstruction images of CatPhan 600 (10mA/10ms) using (a) TV penalt and (b) Hessian-based

Reference: [1] Lefkimmiatis, S., Bourquard, A., Unser, M.: Hessian-based norm regularization for image restoration with biomedical applications. Image Processing, IEEE Transactions on 21(3), 983-995 (2012).

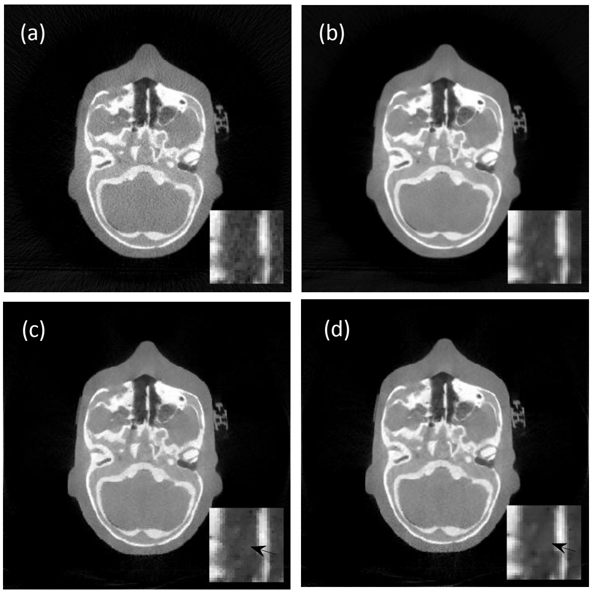

Fig 2: FDK reconstruction images of the head phantom at (a) low-dose (10mA/10ms) and (b) high-dose (80mA/12ms); iterative reconstruction images of the head phantom (10mA/10ms) using (c) TV penalt and (d)