# Unit 4. Application of Boolean Algebra Minterm and Maxterm Expansions

Spring 2015

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# Objectives – To Learn

## Topics introduced in this chapter

- ⇨ Conversion of English Sentences to Boolean Equations
- ⇨ Combinational Logic Design Using a Truth Table
- ⇨ Minterm and Maxterm Expansions
- ⇨ General Minterm and Maxterm Expansions
- ⇨ Incompletely Specified Functions (Don’t care term)
- ⇨ Examples of Truth Table Construction
- ⇨ Design of Binary Adders (Full Adder) and Subtracters

# Conversion of English Sentences to Boolean Eq.

**Steps in designing a single-output combinational switching circuit**

- ⇨ Find switching function which specifies the desired behavior of the circuit
- ⇨ Find a simplified algebraic expression for the function
- ⇨ Realize the simplified function using available logic elements

1. F is ‘true’ if A and B are both ‘true’ → F=AB

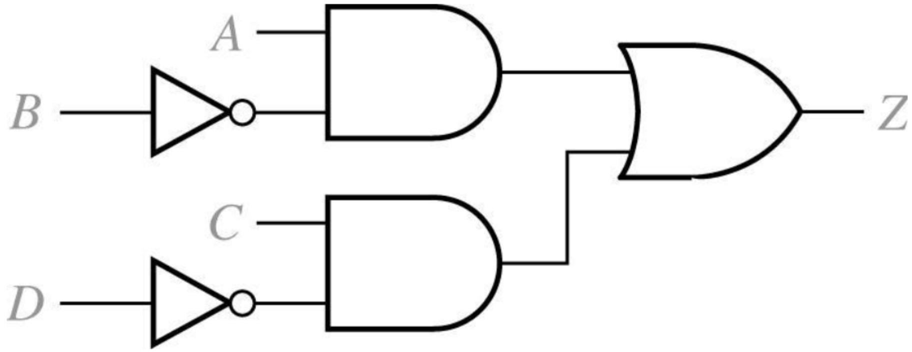
# Conversion of English Sentences to Boolean Eq.

1. The alarm will ring(Z) iff the alarm switch is turned on(A) ***and*** the door is not closed(B'), ***or*** it is after 6PM(C) and window is not closed(D')

2. Boolean Equation

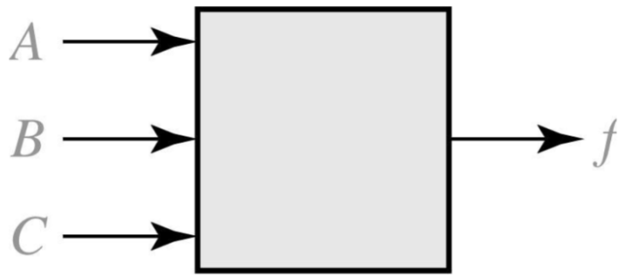
$$Z = AB' + CD'$$

3. Circuit realization



# Combinational Logic Design Using a Truth Table

**- Combinational Circuit with Truth Table**




(a)

| A | B | C | f | f' |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 |

(b)

**When expression for *f=1* →**

$$f = A'BC + AB'C' + AB'C + ABC' + ABC$$

# Combinational Logic Design Using a Truth Table

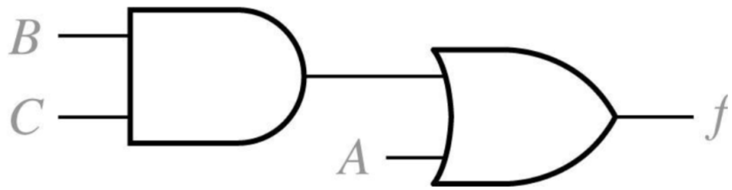
Original equation →

$$f = A'BC + AB'C' + AB'C + ABC' + ABC$$

Simplified equation →

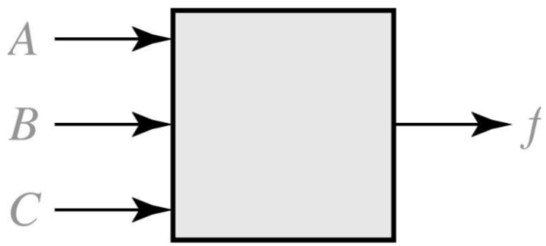
$$f = A'BC + AB' + AB = A'BC + A = A + BC$$

Circuit realization →



# Combinational Logic Design Using a Truth Table

**- Combinational Circuit with Truth Table**




(a)

| A | B | C | f | f′ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 |

(b)

**When expression for *f=0* →**

$$f = (A+B+C)(A+B+C')(A+B'+C)$$

$$f = (A+B)(A+B'+C) = A+BC$$

**When expression for *f ’=1* →**

**and take the complement of *f***

$$f' = A'B'C' + A'B'C + A'BC'$$

$$f = (A+B+C)(A+B+C')(A+B'+C)$$

# Minterm and Maxterm Expansions

- *Minterm* of *n* variables is a product of *n* literals in which each variable appears exactly once in either true (*A*) or complemented form(*A'*), but not both. (→ $m_0$)

-Minterm expansion,

-Standard Sum of Product →

$$f = A'BC + AB'C' + AB'C + ABC' + ABC$$

$$f(A,B,C) = m_3 + m_4 + m_5 + m_6 + m_7$$

$$f(A,B,C) = \sum m(3,4,5,6,7)$$

| Row No. | A B C | Minterms | Maxterms |
|---|---|---|---|
| 0 | 0 0 0 | $A'B'C' = m_0$ | $A + B + C = M_0$ |
| 1 | 0 0 1 | $A'B'C = m_1$ | $A + B + C' = M_1$ |
| 2 | 0 1 0 | $A'BC' = m_2$ | $A + B' + C = M_2$ |
| 3 | 0 1 1 | $A'BC = m_3$ | $A + B' + C' = M_3$ |
| 4 | 1 0 0 | $AB'C' = m_4$ | $A' + B + C = M_4$ |
| 5 | 1 0 1 | $AB'C = m_5$ | $A' + B + C' = M_5$ |
| 6 | 1 1 0 | $ABC' = m_6$ | $A' + B' + C = M_6$ |
| 7 | 1 1 1 | $ABC = m_7$ | $A' + B' + C' = M_7$ |

# Minterm and Maxterm Expansions

- *Maxterm* of *n* variables is a sum of *n* literals in which each variable appears exactly once in either true (*A*) or complemented form(*A'*) , but not both.(→ $M_0$)

- Maxterm expansion,
- Standard Product of Sum →

$$f = (A+B+C)(A+B+C')(A+B'+C)$$

$$f(A,B,C) = M_0 M_1 M_2$$

$$f(A,B,C) = \prod M(0,1,2)$$

# Minterm and Maxterm Expansions

$$f(A,B,C) = m_3 + m_4 + m_5 + m_6 + m_7$$

$$f' = m_0 + m_1 + m_2 = \sum m(0,1,2)$$

$$f(A,B,C) = M_0 M_1 M_2 \longrightarrow f' = \prod M(3,4,5,6,7) = M_3 M_4 M_5 M_6 M_7$$

- *Minterm* and *Maxterm* expansions are complement each other

$$f' = (m_3 + m_4 + m_5 + m_6 + m_7)' = m'_3 \, m'_4 \, m'_5 \, m'_6 \, m'_7 = M_3 M_4 M_5 M_6 M_7$$

$$f' = (M_0 M_1 M_2)' = M'_0 + M'_1 + M'_2 = m_0 + m_1 + m_2$$

P89 Example

# General Minterm and Maxterm Expansions

| A B C | F |
|---|---|
| 0 0 0 | $a_0$ |
| 0 0 1 | $a_1$ |
| 0 1 0 | $a_2$ |
| 0 1 1 | $a_3$ |
| 1 0 0 | $a_4$ |
| 1 0 1 | $a_5$ |
| 1 1 0 | $a_6$ |
| 1 1 1 | $a_7$ |

-General truth table

for 3 variables

- $a_i$ is either '0' or '1'

- Minterm expansion for general function

$$F = a_0 m_0 + a_1 m_1 + a_2 m_2 + \ldots + a_7 m_7 = \sum_{i=0}^{7} a_i m_i$$

$a_i$ =1, minterm $m_i$ is present

$a_i$ =0, minterm $m_i$ is not present

- Maxterm expansion for general function

$$F = (a_0 + M_0)(a_1 + M_1)(a_2 + M_2)\ldots(a_7 + M_7) = \prod_{i=0}^{7}(a_i + M_i)$$

$a_i$ =1, $a_i + M_i$ =1 , Maxterm $M_i$ is not present

$a_i$ =0, Maxterm is present

# General Minterm and Maxterm Expansions

$$F' = [\prod_{i=0}^{7} (a_i + M_i)]' = \sum_{i=0}^{7} a'_i M'_i = \sum_{i=0}^{7} a'_i m_i$$

→ All minterm which are not present in *F* are present in *F'*

$$F' = [\sum_{i=0}^{7} a_i m_i]' = \prod_{i=0}^{7} (a'_i + m'_i) = \prod_{i=0}^{7} (a'_i + M_i)$$

→ All maxterm which are not present in *F* are present in *F'*

$$F = \sum_{i=0}^{2^n-1} a_i m_i = \prod_{i=0}^{2^n-1} (a_i + M_i)$$

$$F' = \sum_{i=0}^{2^n-1} a'_i m_i = \prod_{i=0}^{2^n-1} (a'_i + M_i)$$

# General Minterm and Maxterm Expansions

*If i and j are different,* $m_i m_j = 0$

$$f_1 = \sum_{i=0}^{2^n-1} a_i m_i \qquad f_2 = \sum_{j=0}^{2^n-1} b_j m_j$$

$$f_1 f_2 = \left(\sum_{i=0}^{2^n-1} a_i m_i\right)\left(\sum_{j=0}^{2^n-1} b_j m_j\right) = \sum_{i=0}^{2^n-1}\sum_{j=0}^{2^n-1} a_i b_j m_i m_j = \sum_{i=0}^{2^n-1} a_i b_i m_i$$

Example

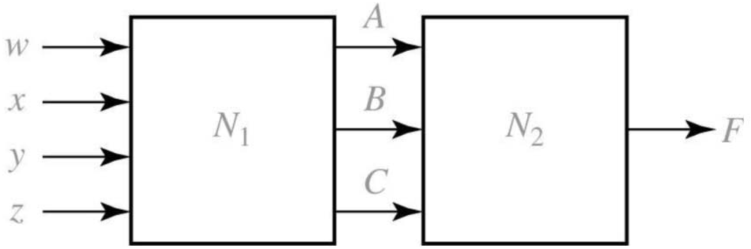
$$f_1 = \sum m(0,2,3,5,9,11) \text{ and } f_2 = m(0,3,9,11,13,14)$$

$$f_1 f_2 = \sum m(0,3,9,11)$$

# Conversion between Minterm and Maxterm Expansions of F and F'

| | Minterm Expansion of f | Maxterm Expansion of f | Minterm Expansion of f' | Maxterm Expansion of f' |
|---|---|---|---|---|
| f= $\sum m(3,4,5,6,7)$ | | $\prod M(0,1,2)$ | $\sum m(0,1,2)$ | $\prod M(3,4,5,6,7)$ |
| f= $\prod M(0,1,2)$ | $\sum m(3,4,5,6,7)$ | | $\sum m(0,1,2)$ | $\prod M(3,4,5,6,7)$ |

# Incompletely Specified Functions




If $N_1$ output does not generate all possible combination of A,B,C, the output of N2(F) has 'don't care' values.

Truth Table with Don't Cares

| A B C | F |
|---|---|
| 0 0 0 | 1 |
| 0 0 1 | X |
| 0 1 0 | 0 |
| 0 1 1 | 1 |
| 1 0 0 | 0 |
| 1 0 1 | 0 |
| 1 1 0 | X |
| 1 1 1 | 1 |

# Incompletely Specified Functions

Finding Function:

Case 1: assign '0' on X's

$$F = A'B'C' + A'BC + ABC = A'B'C' + BC$$

Case 2: assign '1' to the first X and '0' to the second 'X'

$$F = A'B'C' + A'B'C + A'BC + ABC = A'B' + BC$$

Case 3: assign '1' on X's

$$F = A'B'C' + A'B'C + A'BC + ABC' + ABC = A'B' + BC + AB$$

→ The case 2 leads to the simplest function

# Incompletely Specified Functions

- Minterm expansion for incompletely specified function

$$F = \sum m(0,3,7) + \sum d(1,6)$$

Don't Cares

- Maxterm expansion for incompletely specified function

$$F = \prod M(2,4,5) \prod D(1,6)$$

| A B C | F |
|---|---|
| 0 0 0 | 1 |
| 0 0 1 | X |
| 0 1 0 | 0 |
| 0 1 1 | 1 |
| 1 0 0 | 0 |
| 1 0 1 | 0 |
| 1 1 0 | X |
| 1 1 1 | 1 |

# Examples of Truth Table Construction

## Example 1 : Binary Adder

| a | b | Sum | |
|---|---|---|---|
| 0 | 0 | 0 0 | 0+0=0 |
| 0 | 1 | 0 1 | 0+1=1 |
| 1 | 0 | 0 1 | 1+0=1 |
| 1 | 1 | 1 0 | 1+1=2 |

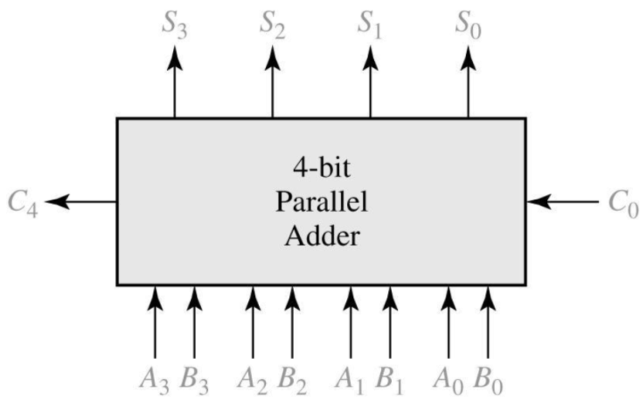
→

| A | B | X | Y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

$$X = AB\,, Y = A'B + AB' = A \oplus B$$

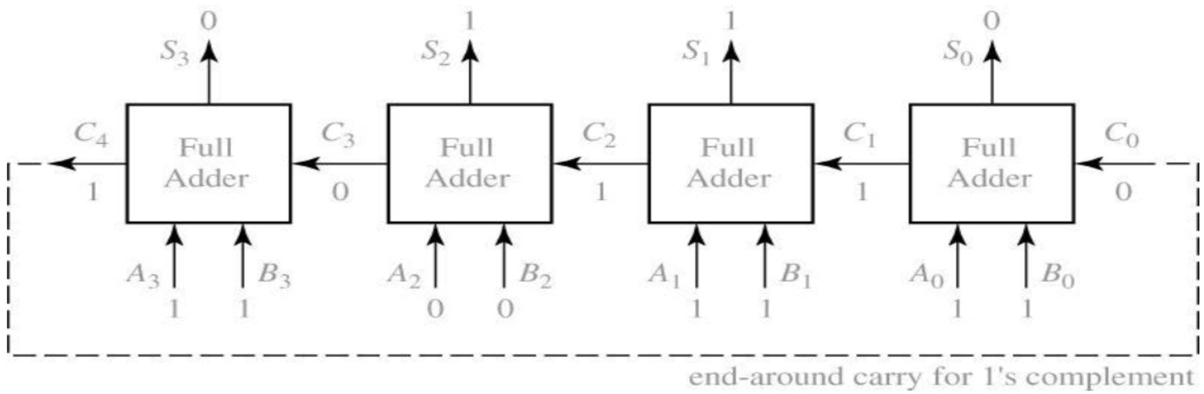
# Design of Binary Adders and Subtracters

## Parallel Adder for 4 bit Binary Numbers

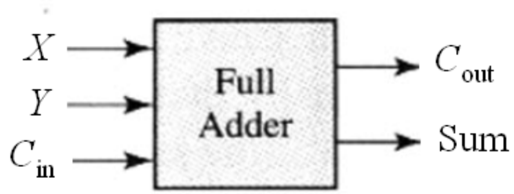



$$
\begin{array}{r}
10110 \text{ (carries)} \\
1011 \\
+1011 \\
\hline
10110
\end{array}
$$

## Parallel adder composed of four full adders ← Carry Ripple Adder (slow!)



# Design of Binary Adders and Subtracters

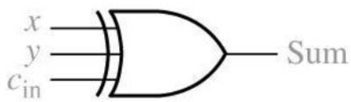


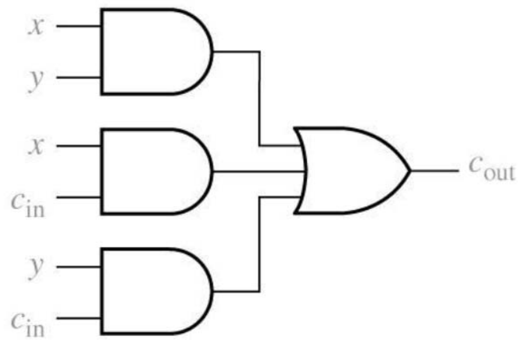

**Truth Table for a Full**

| X | Y | $C_{in}$ | $C_{out}$ | Sum |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

$$\begin{aligned} Sum &= X'Y'C_{in} + X'YC'_{in} + XY'C'_{in} + XYC_{in} \\ &= X'(Y'C_{in} + YC'_{in}) + X(Y'C'_{in} + YC_{in}) \\ &= X'(Y \oplus C_{in}) + X(Y \oplus C_{in})' = X \oplus Y \oplus C_{in} \end{aligned}$$

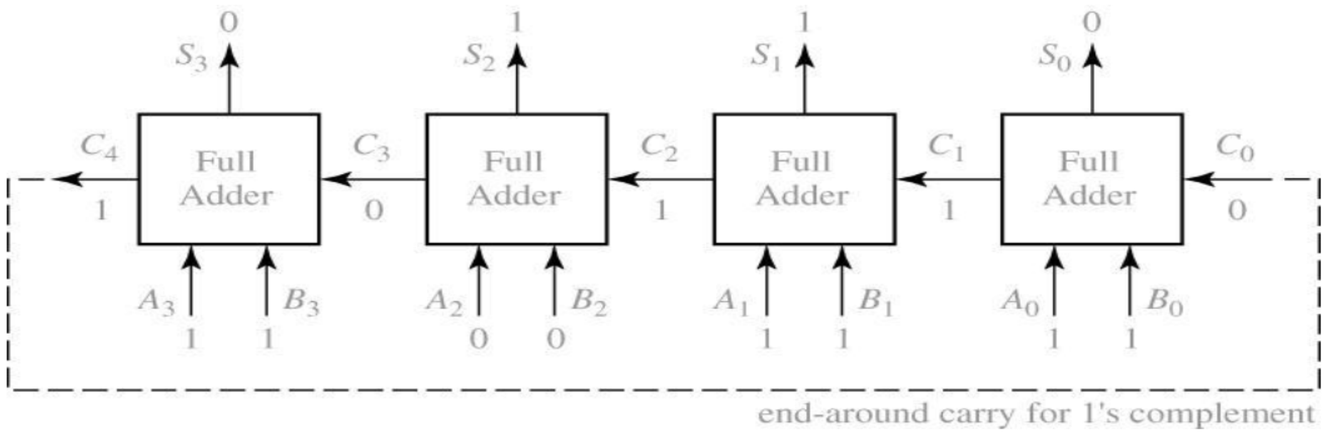
$$\begin{aligned} C_{out} &= X'YC_{in} + XY'C_{in} + XYC'_{in} + XYC_{in} \\ &= (X'YC_{in} + XYC_{in}) + (XY'C_{in} + XYC_{in}) + (XYC'_{in} + XYC_{in}) \\ &= YC_{in} + XC_{in} + XY \end{aligned}$$



# Design of Binary Adders and Subtracters

When 1's complement is used, the end-around carry is accomplished by connecting C4 to C0 input.




Overflow(V) when adding two signed binary number

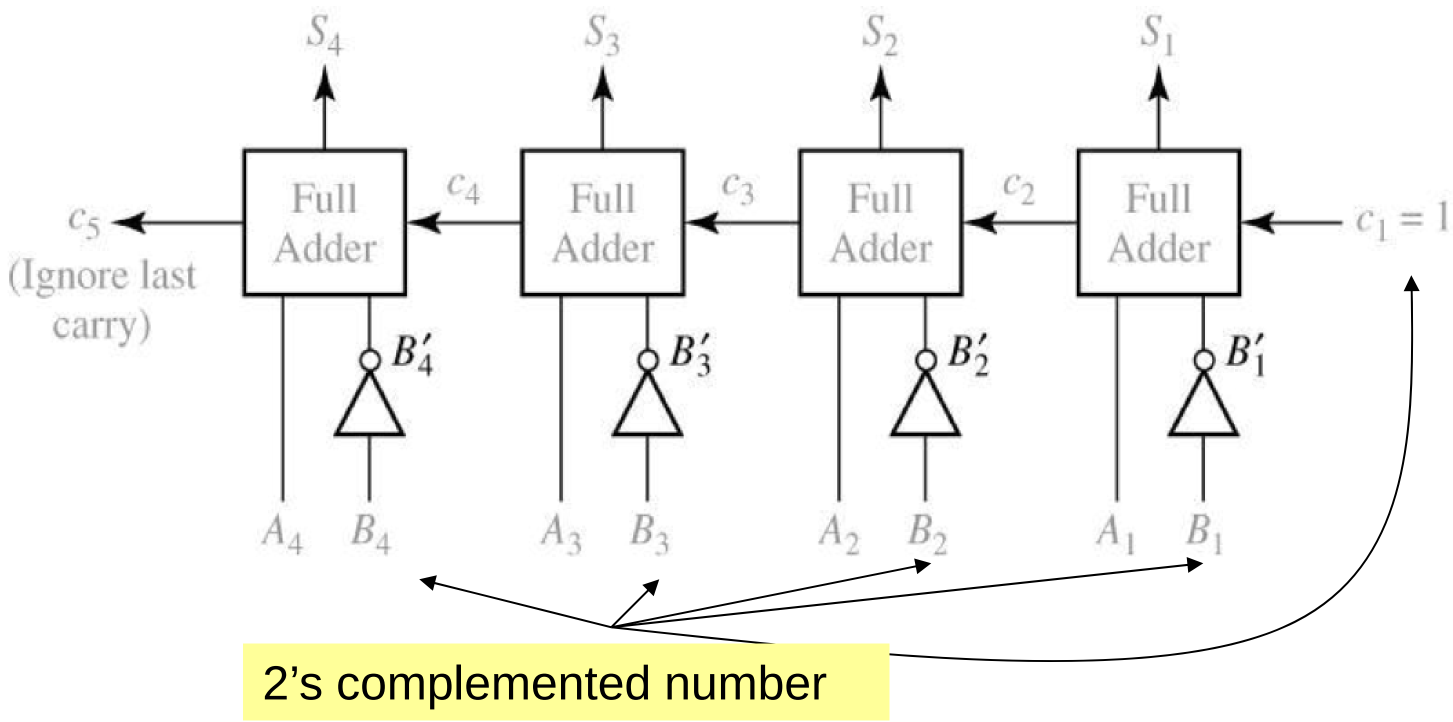
$$V = A'_3 B'_3 S_3 + A_3 B_3 S'_3$$

# Subtracters

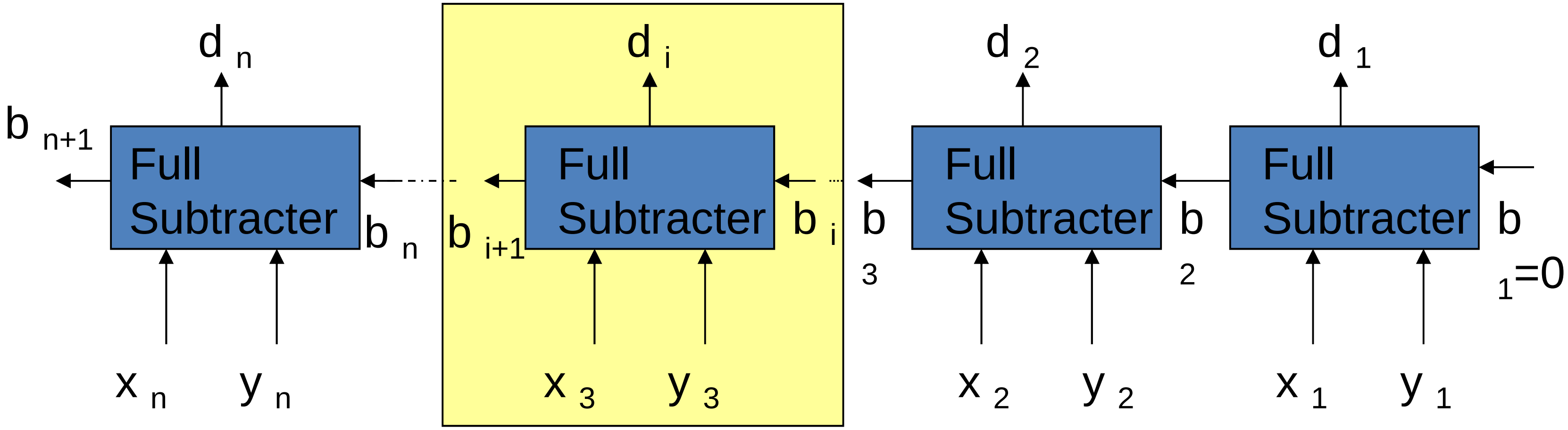
Binary Subtracter using full adder

-Subtraction is done by adding the 2's complemented number to be subtracted

-Example: A - B



# Parallel Subtractors using Full Subtracter




**Truth Table for a Full Subtracter**

| xi | yi | bi | bi+1 | di |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |