Unit 5. Karnaugh Maps

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Objectives – To Learn

Topics introduced in this chapter

- ⇨ Given a function (completely or incompletely specified) of three to five variable, plot it on a Karnaugh map. This function may be given in minterm, maxterm, or algebraic form.
- ⇨ Obtain the minimum sum-of-products or minimum product-of-sums form of a function from the map.
- ⇨ Understand the relation between operations performed using the map and the corresponding algebraic operation.

Minimum Form of Switching Functions

1. Combine terms by using $XY' + XY = X$

   Do this repeatedly to eliminate as many literals as possible.

   A given term may be used more than once because $X + X = X$

2. Eliminate redundant terms by using the consensus theorems.

Minimum Forms of Switching Functions

Example: Find a minimum sum-of-products

$$F(a,b,c) = \sum m(0,1,2,5,6,7)$$

$$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$$

$$= a'b' + b'c + bc' + ab$$

$$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$$

$$= a'b' + bc' + ac$$

Minimum Forms of Switching Functions

Example: Find a minimum product-of-sums

$$(A+B'+C+D')(A+B'+C'+D')(A+B'+C'+D)(A'+B'+C'+D)(A+B+C'+D)(A'+B+C'+D)$$

$$= (A+B'+D') \quad (A+B'+C') \quad (B'+C'+D) \quad (B+C'+D)$$

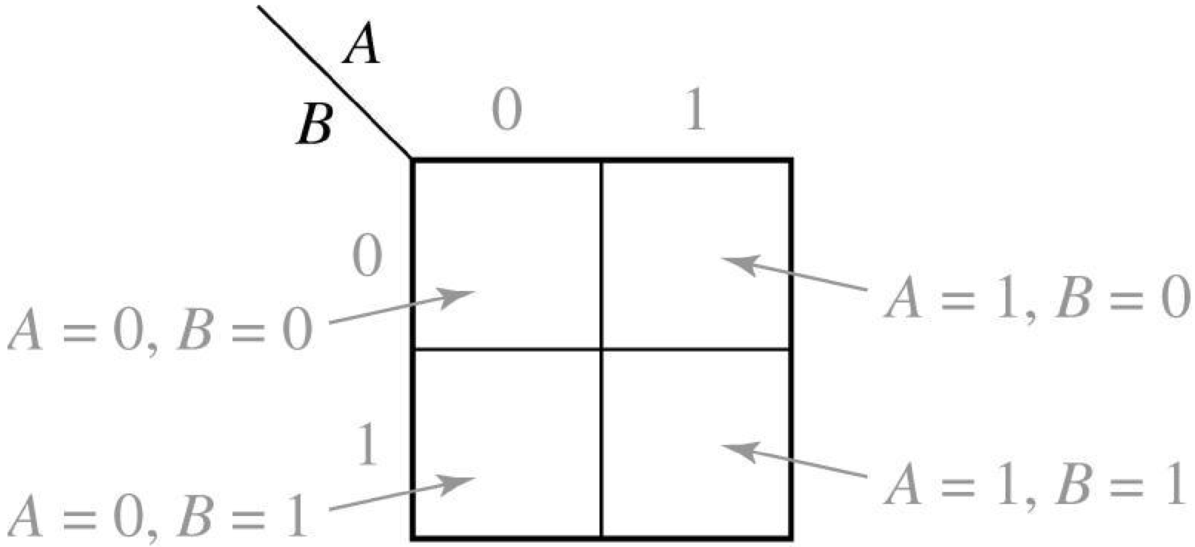
$$= (A+B'+D') \quad \underbrace{(A+B'+C')} \quad (C'+D)$$

$$= (A+B'+D')(C'+D)$$

Eliminate by consensus

Two-Variable Karnaugh Maps

A 2-variable Karnaugh Map

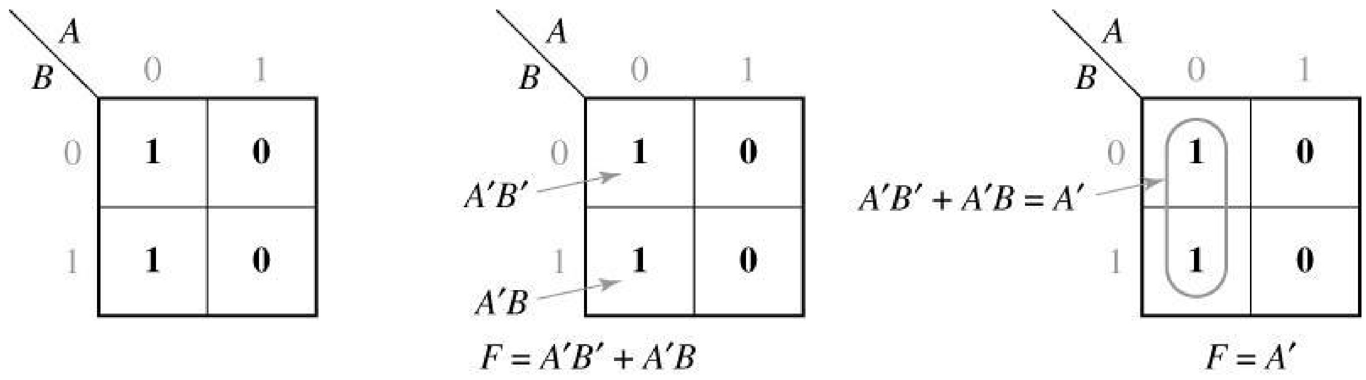


Two-Variable Karnaugh Maps

Truth Table for a function F

A	B	F
0	0	1
0	1	1
1	0	0
1	1	0

(a)



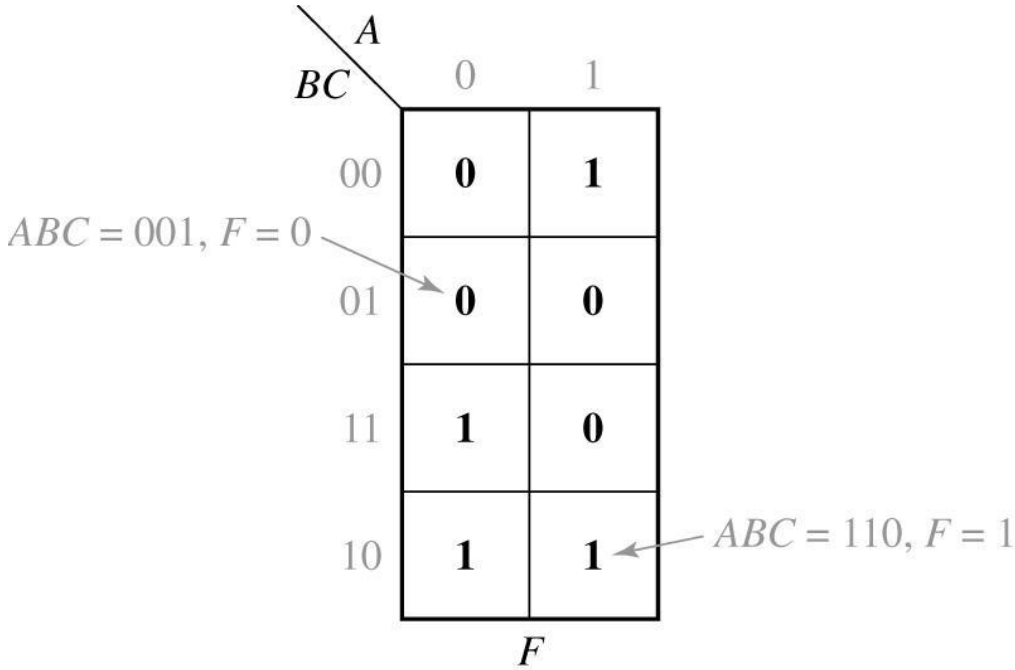
(b) (c) (d)

Three-Variable Karnaugh Maps

Truth Table and Karnaugh Map for Three-Variable Function

A B C	F
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	1
1 0 1	0
1 1 0	1
1 1 1	0

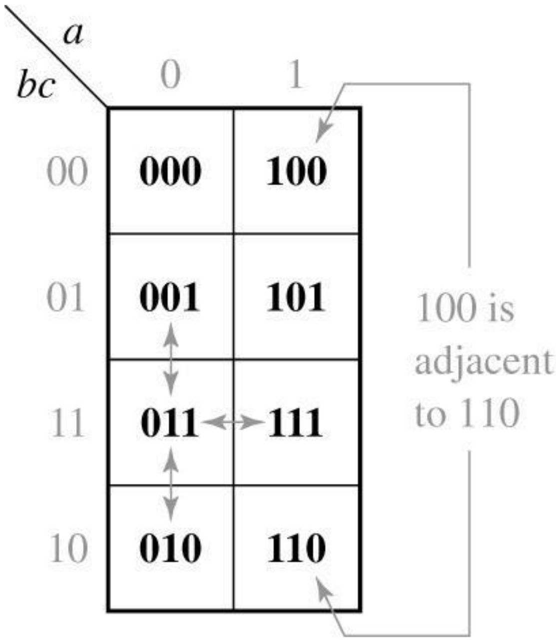
(a)



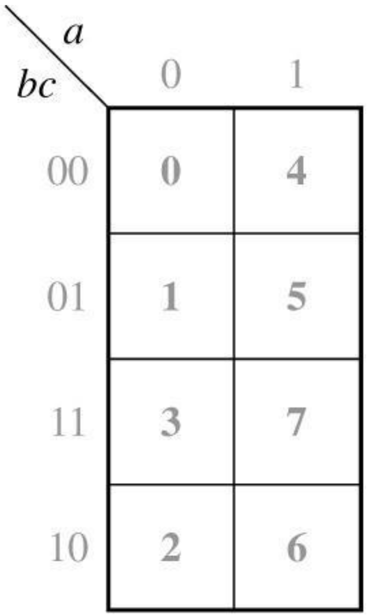
(b)

Three-Variable Karnaugh Maps

Location of Minterms on a Three-Variable Karnaugh Map



(a) Binary notation



(b) Decimal notation

Three-Variable Karnaugh Maps

Karnaugh Map of F(*a, b, c*) = ∑m(1, 3, 5) = ∏(0, 2, 4, 6, 7)

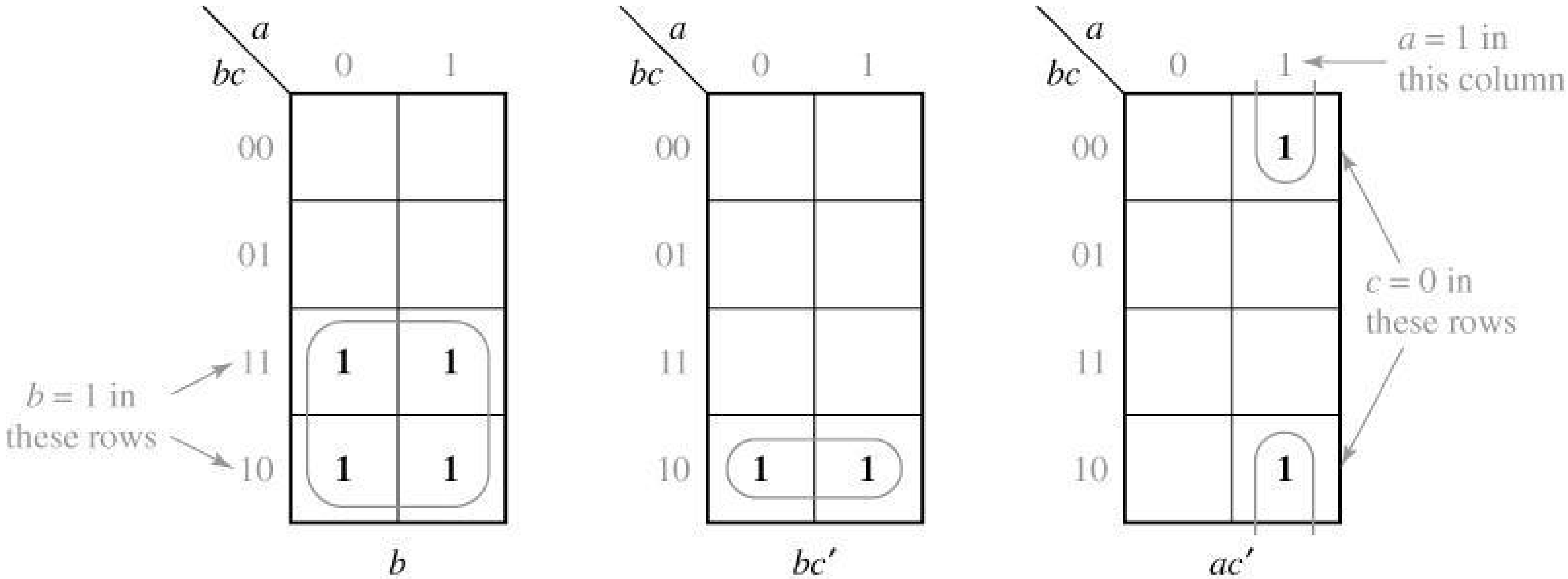
bc \ *a*	0	1
00	**0** (0)	**0** (4)
01	**1** (1)	**1** (5)
11	**1** (3)	**0** (7)
10	**0** (2)	**0** (6)

$$F(a,b,c) = m_1 + m_3 + m_5$$

$$= M_0 M_2 M_4 M_6 M_7$$

Three-Variable Karnaugh Maps

Karnaugh Maps for Product Terms

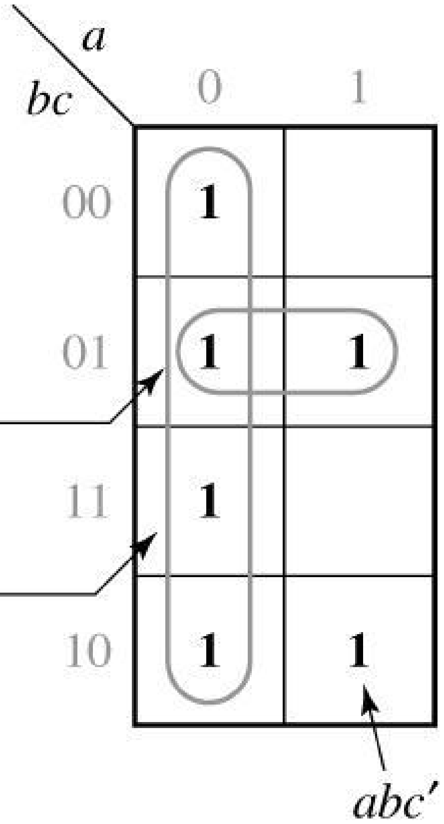


Three-Variable Karnaugh Maps

Given Function

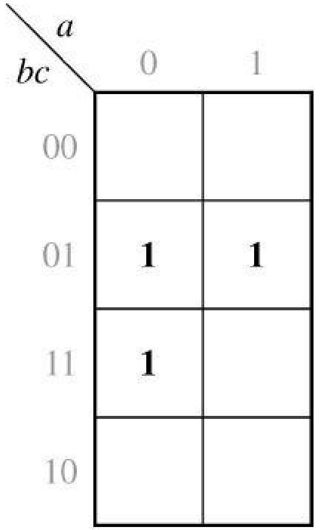
$$f(a,b,c) = abc' + b'c + a'$$

1. The term abc' is 1 when $a = 1$ and $bc = 10$, so we place a 1 in the square which corresponds to the $a = 1$ column and the $bc = 10$ row of the map.
2. The term $b'c$ is 1 when $bc = 01$, so we place 1's in both squares of the $bc = 01$ row of the map.
3. The term a' is 1 when $a = 0$, so we place 1's in all the squares of the $a = 0$ column of the map. (Note: Since there already is a 1 in the $abc = 001$ square, we do not have to place a second 1 there because $x + x = x$.)

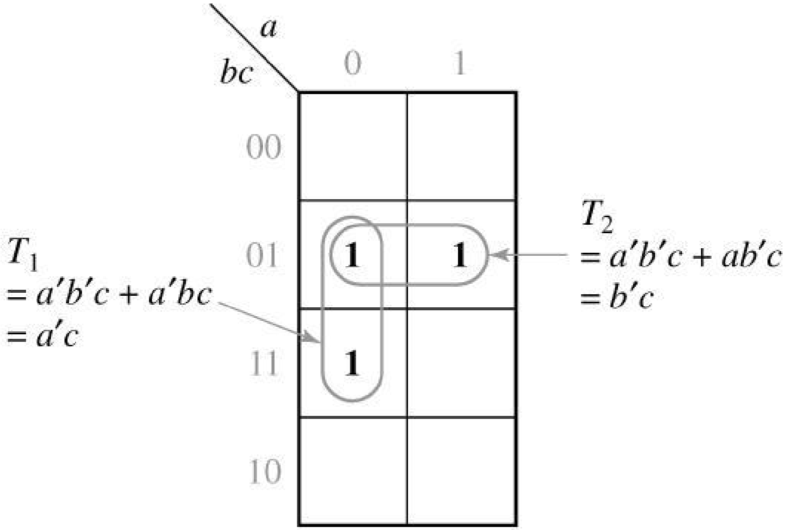


Three-Variable Karnaugh Maps

Simplification of a Three-Variable Function



(a) Plot of minterms

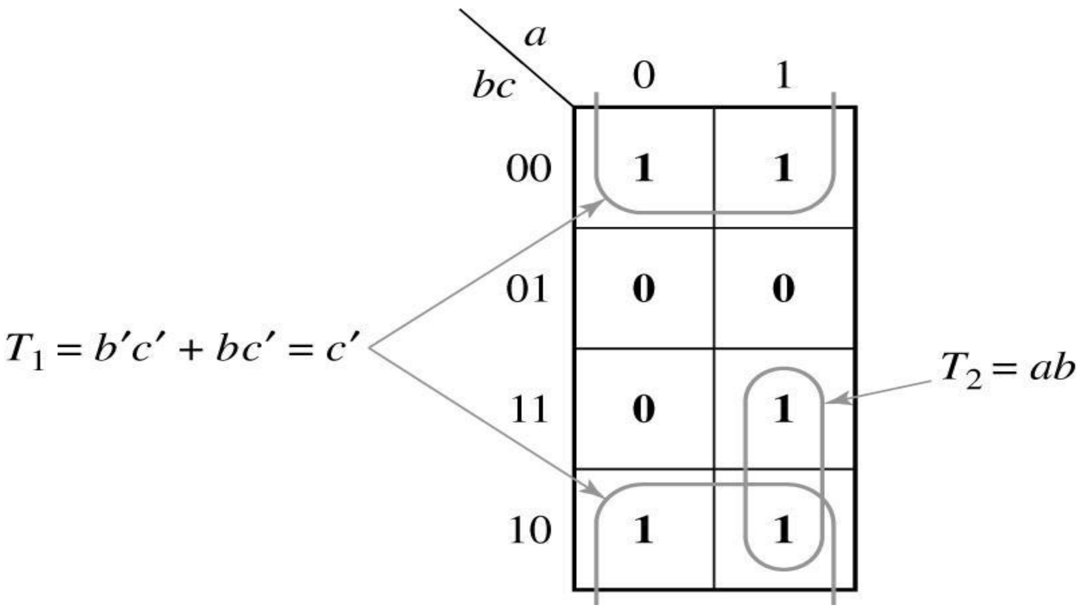


(b) Simplified form of F

$$F = T_1 + T_2 = a'c + b'c$$

Three-Variable Karnaugh Maps

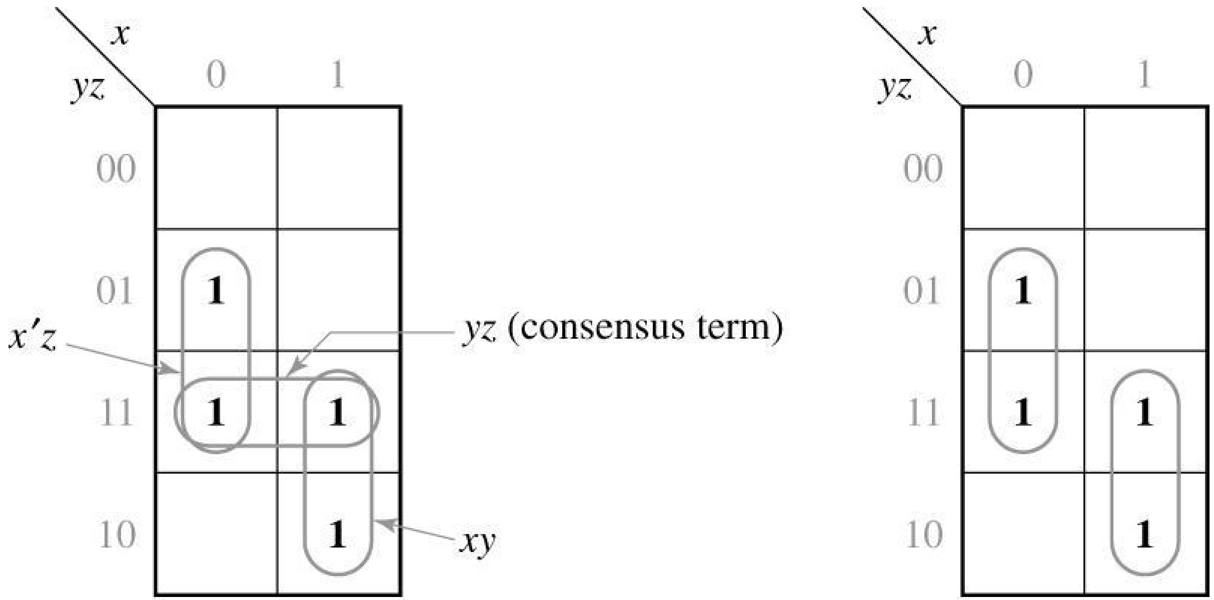
Simplification of a Three-Variable Function



$$F' = T_1 + T_2 = c' + ab$$

Three-Variable Karnaugh Maps

Karnaugh Maps Which Illustrate the Consensus Theorem



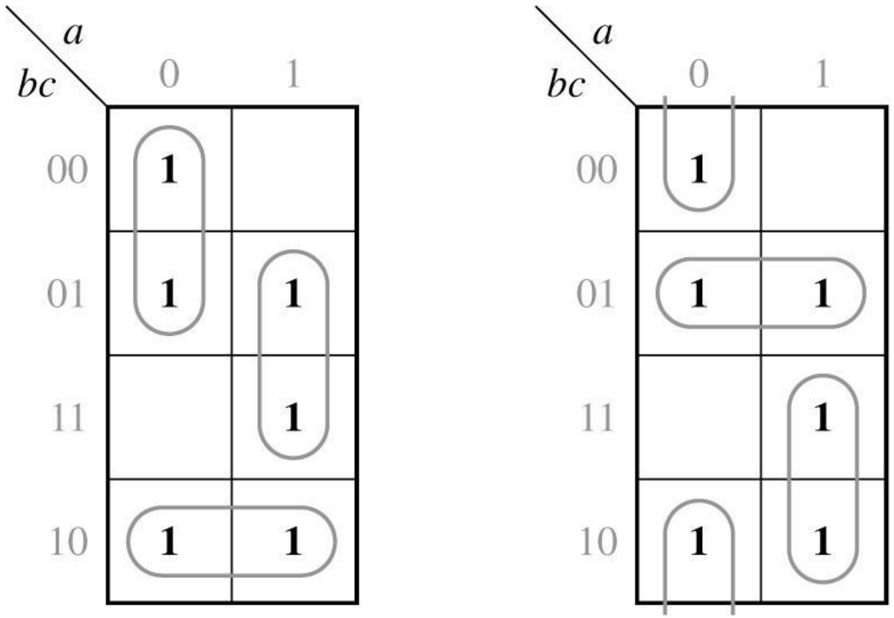
$$xy + x'z + yz = xy + x'z$$

Consensus term is redundant

Three-Variable Karnaugh Maps

Function with Two Minimal Forms

$$F = \sum m(0,1,2,5,6,7)$$



$F = a'b' + bc' + ac$ $\qquad$ $F = a'c' + b'c + ab$

Four-Variable Karnaugh Maps

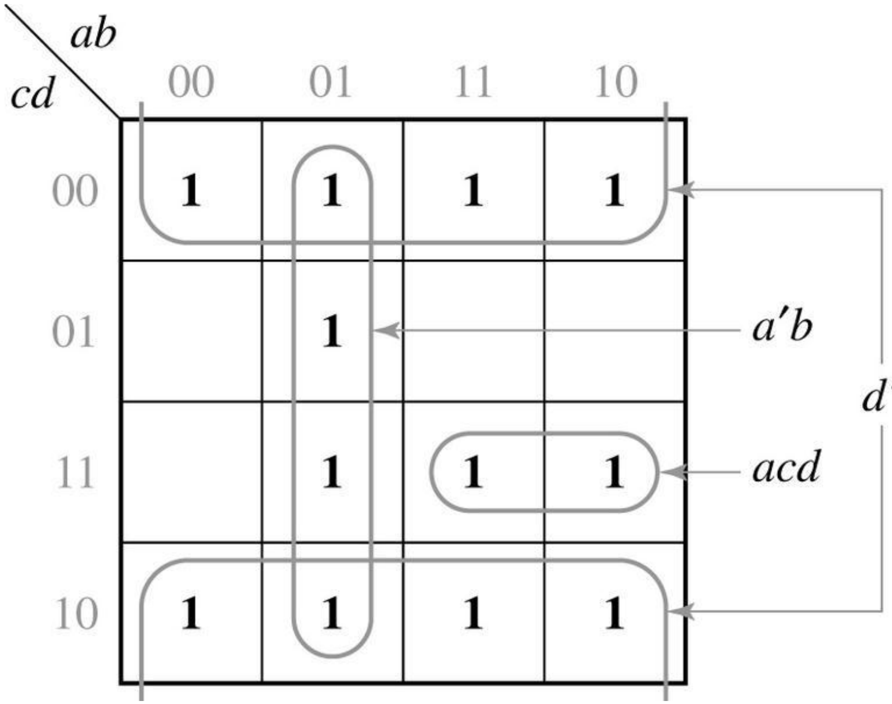
Location of Minterms on Four-Variable Karnaugh Map

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

Four-Variable Karnaugh Maps

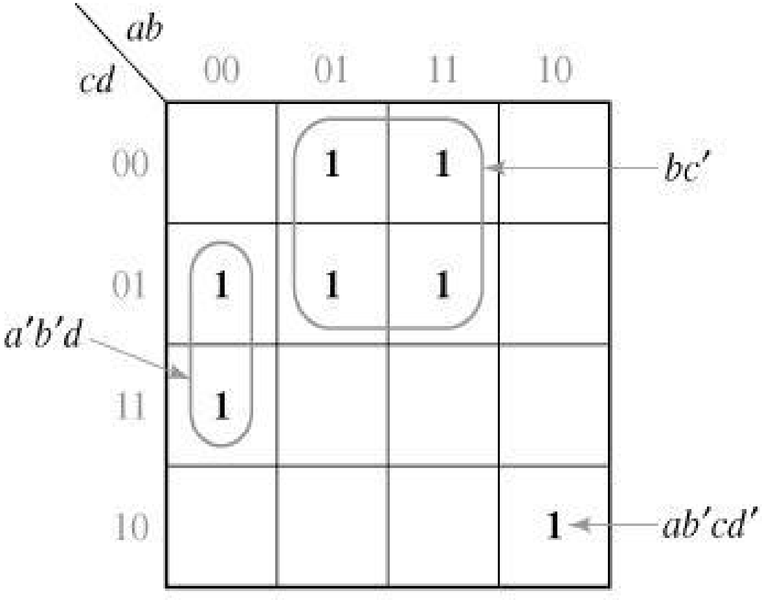
Plot of ***acd + a'b + d'***

$$f(a,b,c,d) = acd + a'b + d'$$



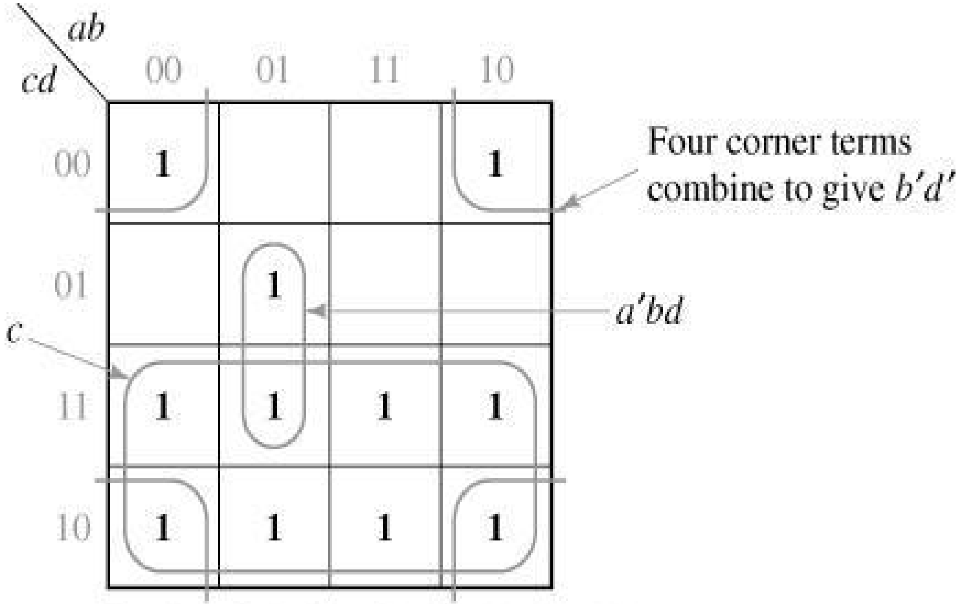
Four-Variable Karnaugh Maps

Simplification of Four-Variable Functions



$f_1 = \Sigma m(1, 3, 4, 5, 10, 12, 13)$
$= bc' + a'b'd + ab'cd'$

(a)

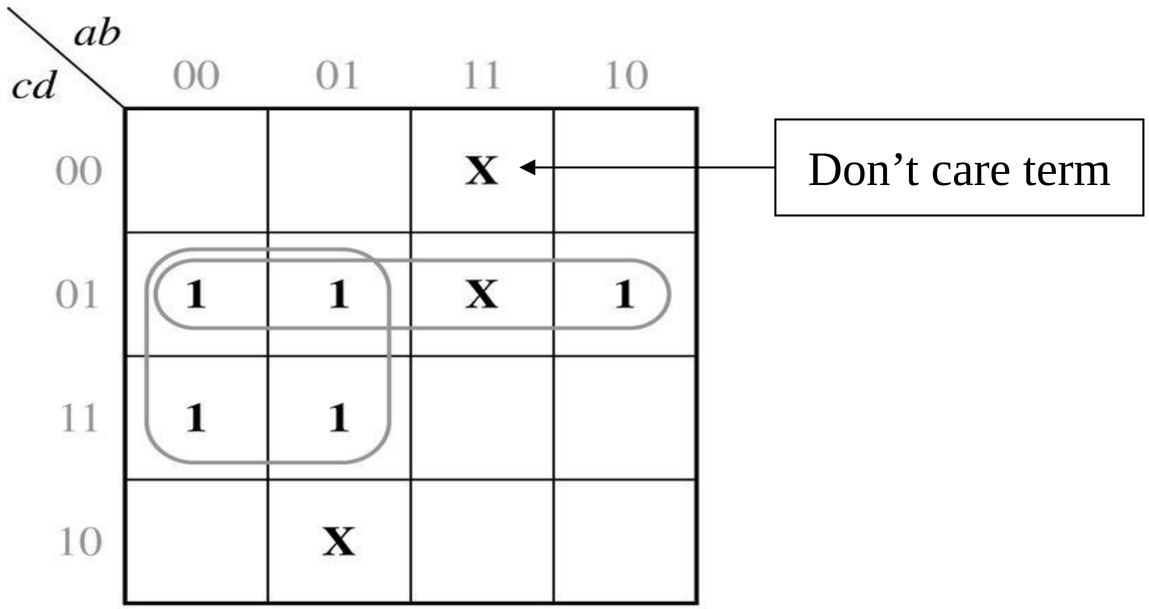


$f_2 = \Sigma m(0, 2, 3, 5, 6, 7, 8, 10, 11, 14, 15)$
$= c + b'd' + a'bd$

(b)

Four-Variable Karnaugh Maps

Simplification of an Incompletely Specified Function



$$f = \Sigma m(1, 3, 5, 7, 9) + \Sigma d(6, 12, 13)$$
$$= a'd + c'd$$

Four-Variable Karnaugh Maps

Figure 5-14

1's of f

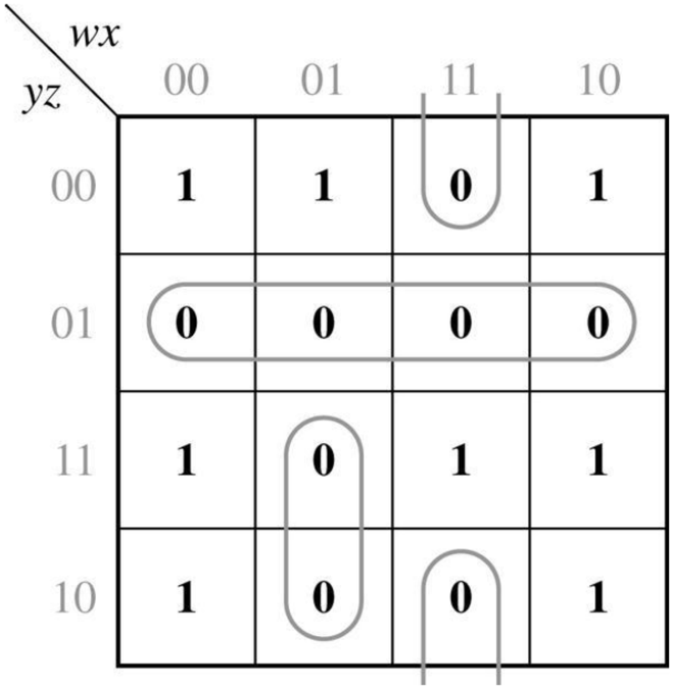
$$f = x'z' + wyz + w'y'z' + x'y$$

0's of f

$$f' = y'z + wxz' + w'xy$$

$$f = (y + z')(w' + x' + z)(w + x' + y')$$

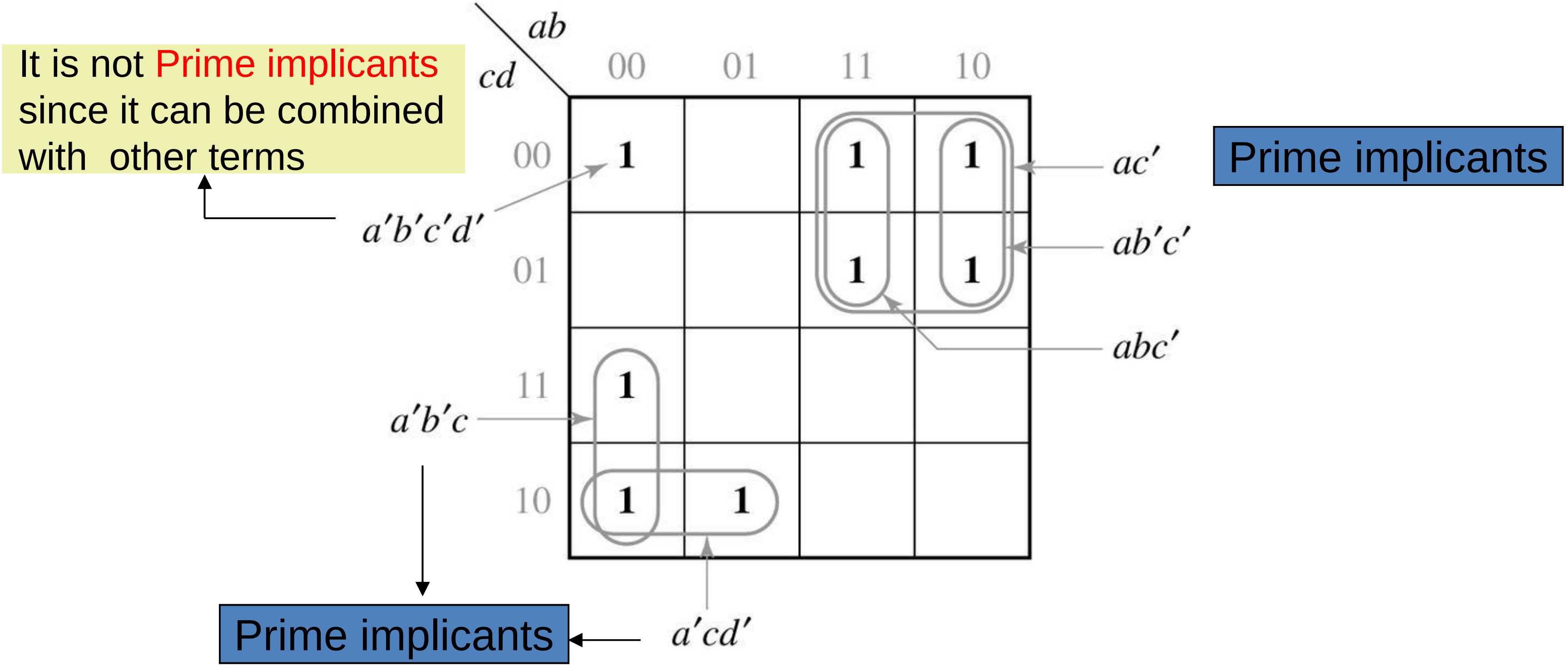
minimum product of sum for f



Determination of Minimum Expressions Using Essential Prime Implicants (EPI)

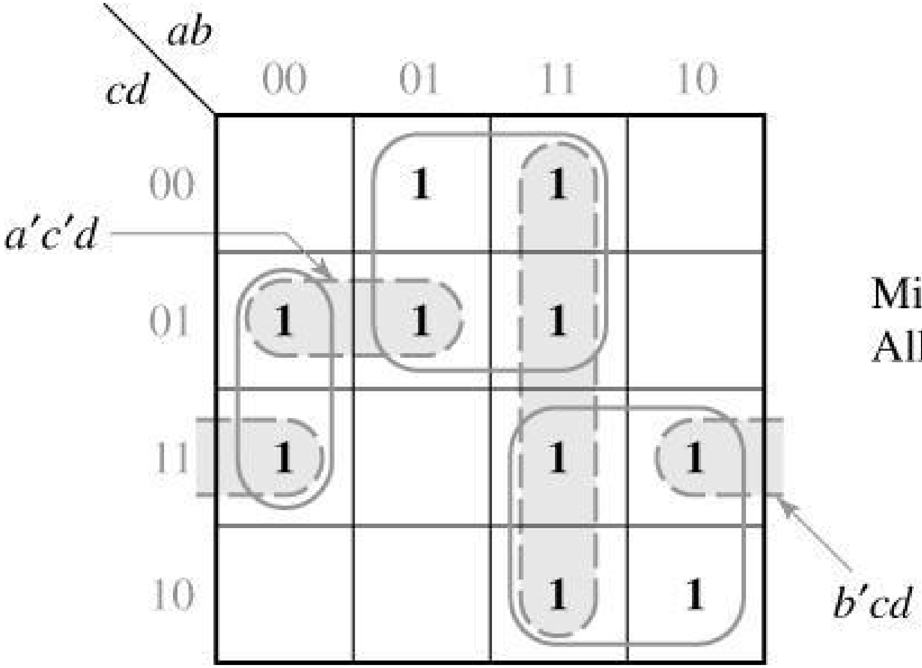
- ***Implicants of F*** : Any single '1' or any group of "1's which can be combined together on a Map

- ***prime implicants of F*** : A product term if it can not be combined with other terms to eliminate variable → ac′, a′b′c, a′cd′



Determination of Minimum Expressions Using Essential Prime Implicants (EPI)

Determination of All Prime Implicants

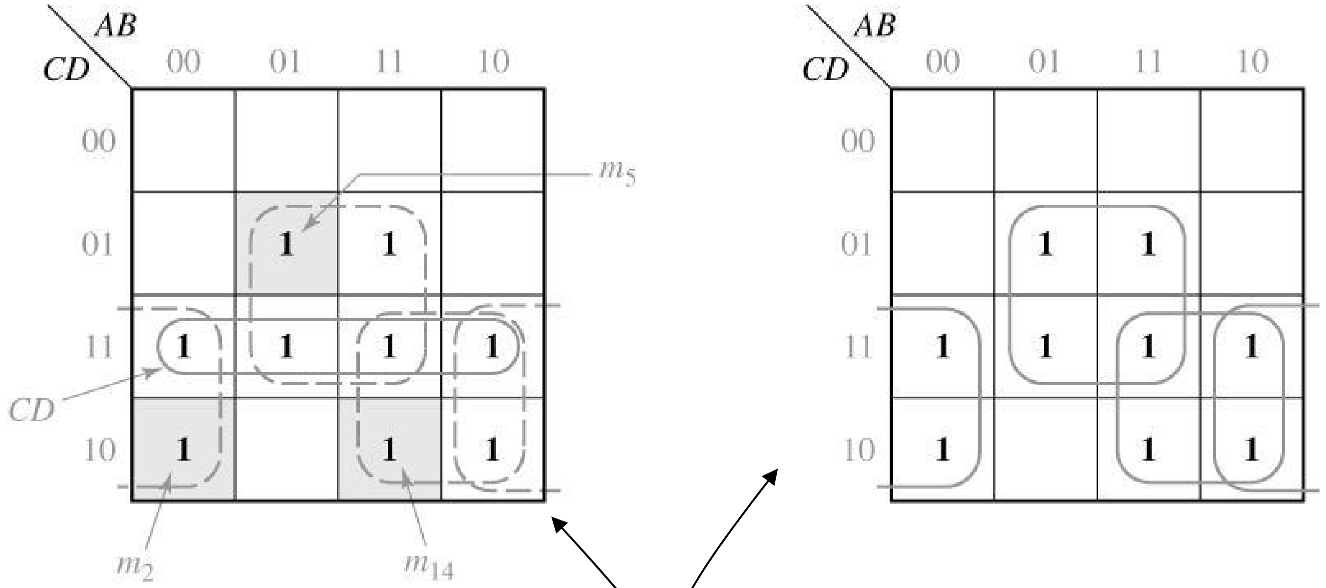


Minimum solution: $F = a'b'd + bc' + ac$

All prime implicants: $a'b'd$, bc', ac, $a'c'd$, ab, $b'cd$

Determination of Minimum Expressions Using Essential Prime Implicants (EPI)

Because all of the prime implicants of a function are generally not needed in forming the minimum sum of products, selecting prime implicants is needed.



$f = CD + BD + B'C + AC$

(a)

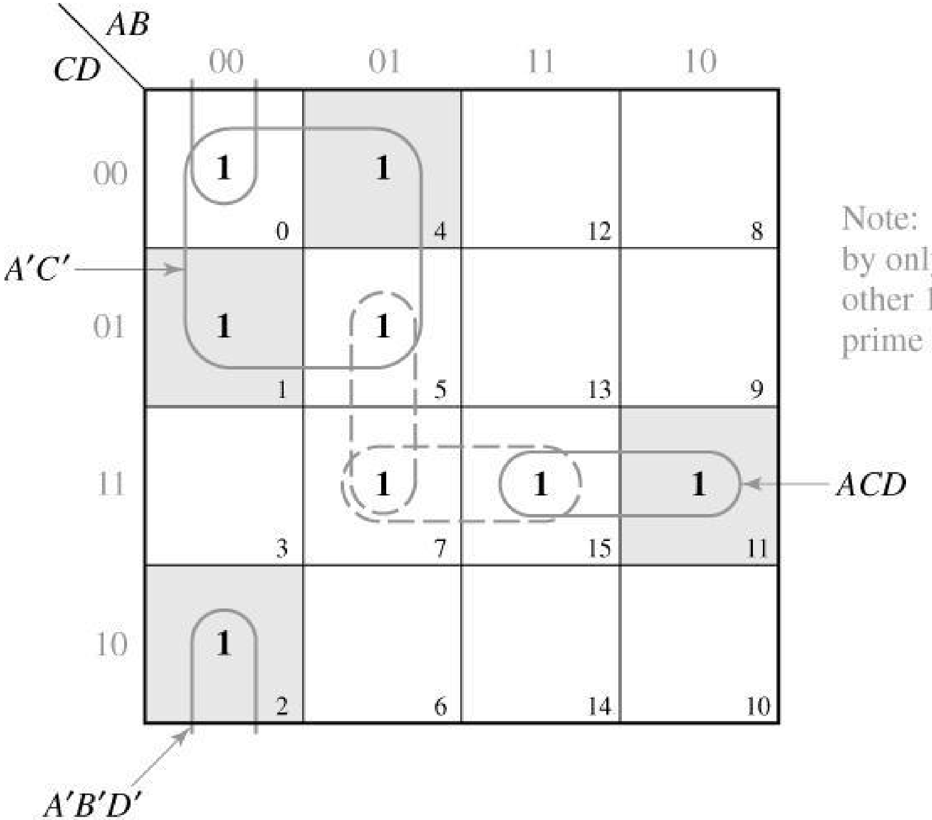
$f = BD + B'C + AC$

(b)

- ***CD*** **is not needed to cover for minimum expression**
- ***B'C*** **,** ***AC*** **,** ***BD*** **are "essential" prime implicants**
- ***CD*** **is not an "essential " prime implicants**

Determination of Minimum Expressions Using Essential Prime Implicants (EPI)

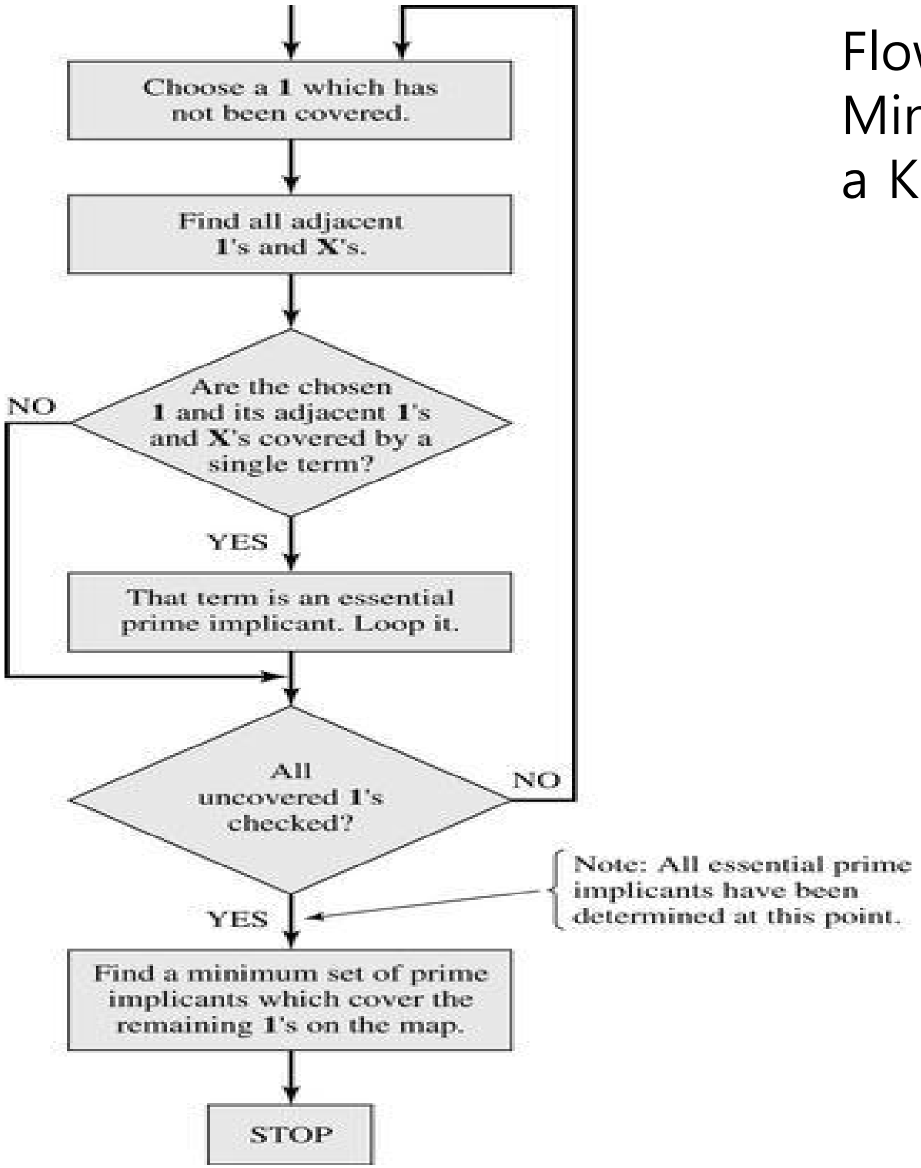
1. **First, find essential prime implicants**
2. **If minterms are not covered by essential prime implicants only, more prime implicants must be added to form minimum expression.**



Note: 1's shaded in blue are covered by only one prime implicant. All other 1's are covered by at least two prime implicants.

$$A'C' + A'B'D' + ACD + \begin{cases} A'BD \\ \text{or} \\ BCD \end{cases}$$

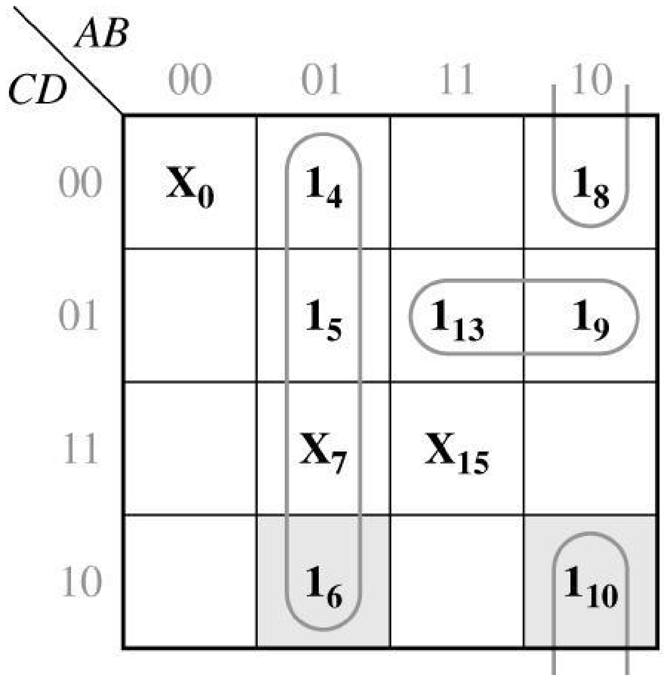
Determination of Minimum Expressions Using Essential Prime Implicants (EPI)



Flowchart for Determining a Minimum Sum of Products Using a Karnaugh Map

Determination of Minimum Expressions Using Essential Prime Implicants (EPI)

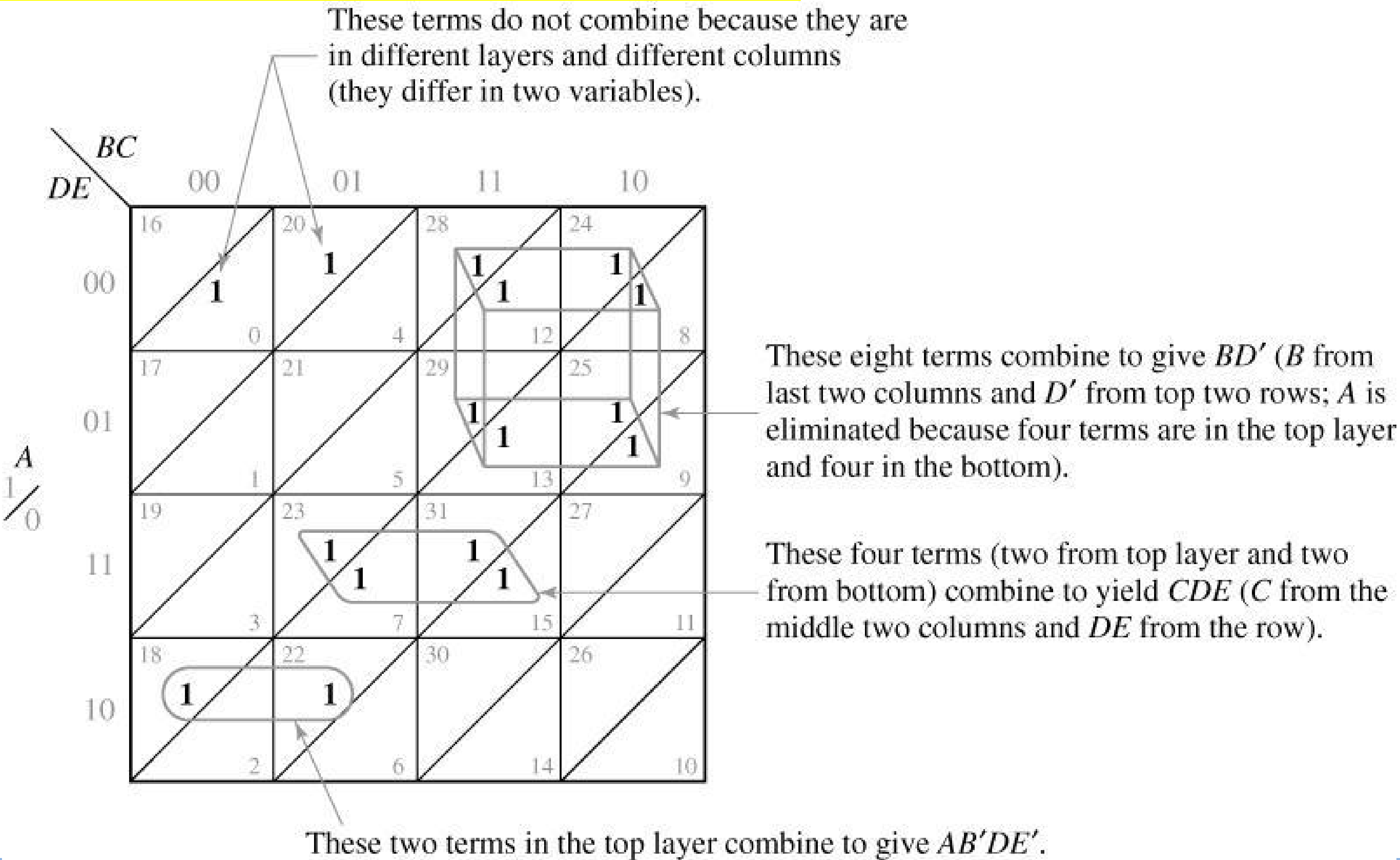
1) *A'B* covers I_6 and its adjacent → essential PI
2) *AB'D'* covers I_{10} and its adjacent → essential PI
3) *AC'D* is chosen for minimal cover → *AC'D* is not an essential PI



Shaded 1's are covered by only one prime implicant.

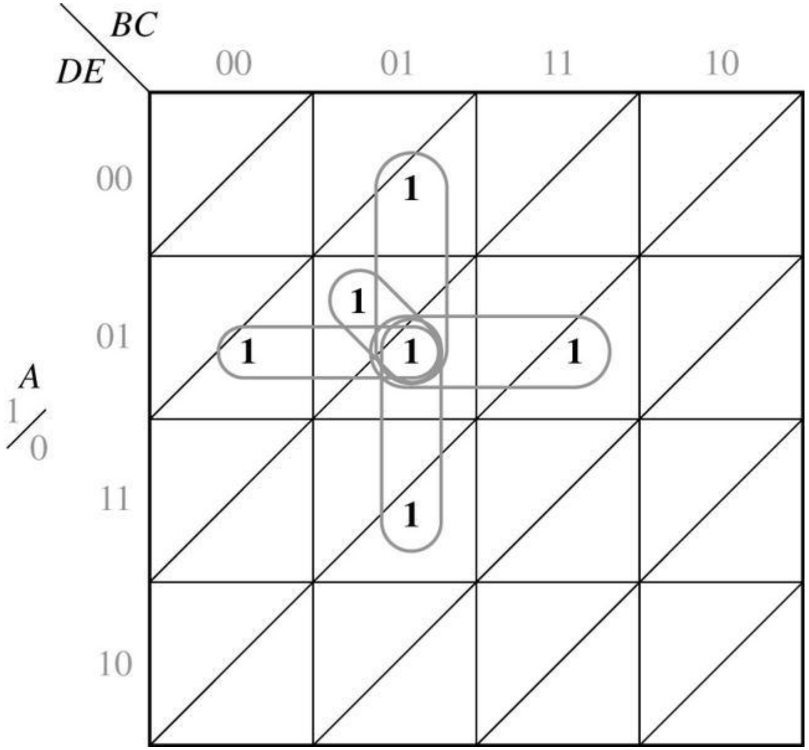
Five-Variable Karnaugh Maps

Five-Variable Karnaugh Map



Five-Variable Karnaugh Maps

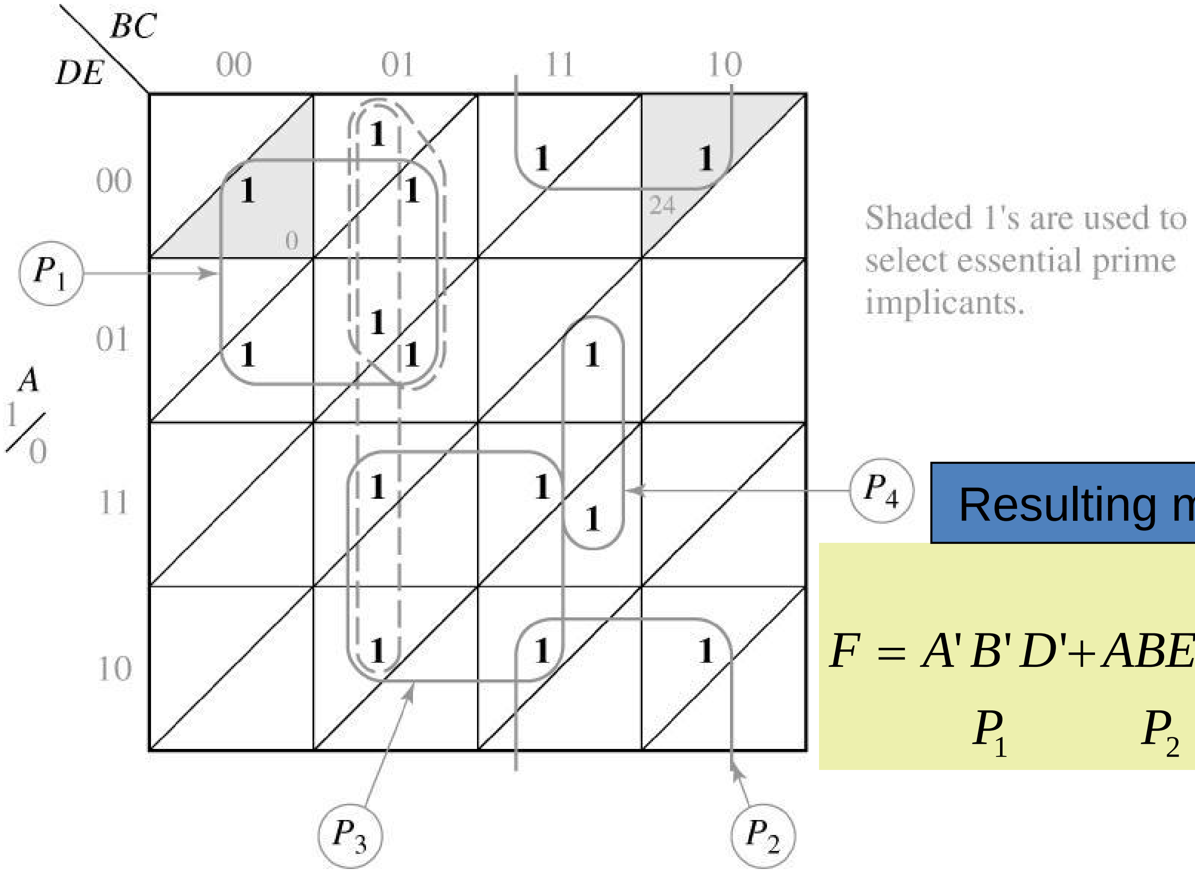
Figure 5-22



- Each term can be adjacent to exactly five other terms:
  - four in the same layer
  - one in the other layer

Five-Variable Karnaugh Maps

Figure 5-23 $F(A,B,C,D,E) = \sum m(0,1,4,5,13,15,20,21,22,23,24,26,28,30,31)$



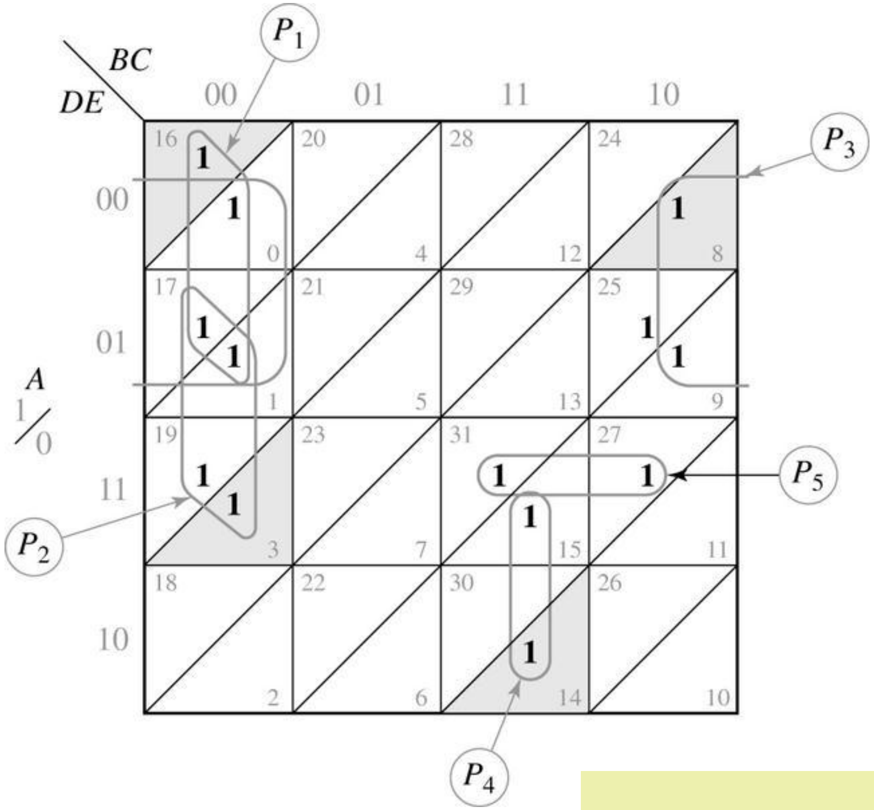
Resulting minimum solution

$$F = \underset{P_1}{A'B'D'} + \underset{P_2}{ABE'} + \underset{P_3}{ACD} + \underset{P_4}{A'BCE} + \left\{ \begin{matrix} AB'C \\ \text{or} \\ B'CD' \end{matrix} \right\}$$

Five-Variable Karnaugh Maps

Figure 5-24

$$F(A,B,C,D,E)=\sum m(0,1,3,8,9,14,15,16,17,19,25,27,31)$$



Final solution

$$F=\underbrace{B'C'D'}_{P_1}+\underbrace{B'C'E}_{P_2}+\underbrace{A'C'D'}_{P_3}+\underbrace{A'BCD}_{P_4}+\underbrace{ABDE}_{P_5}+\begin{array}{c}C'D'E\\ \text{or}\\ AC'E\end{array}$$

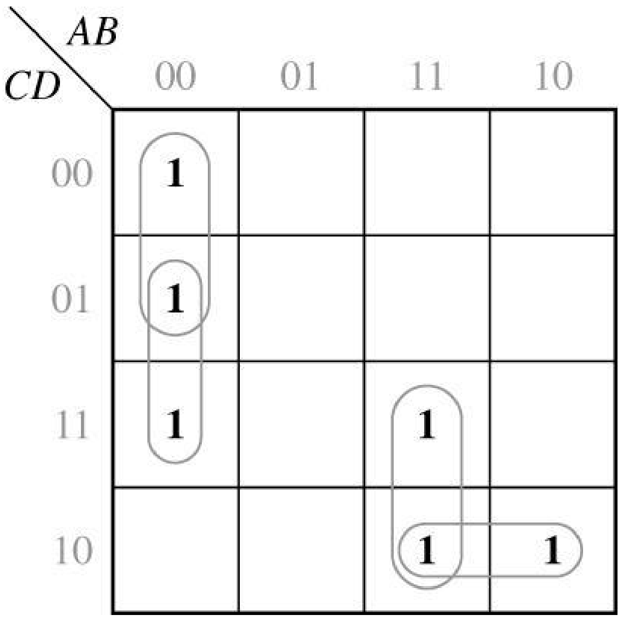
Other Uses of K-Maps

- Prove two functions are equal (Fig 5-14)

$$\text{minturm expansion of } f \text{ is } \quad f = \sum m(0,2,3,4,8,10,11,15)$$
$$\text{maxterm expansion of } f \text{ is } \quad f = \prod M(1,5,6,7,9,12,16,14)$$

same

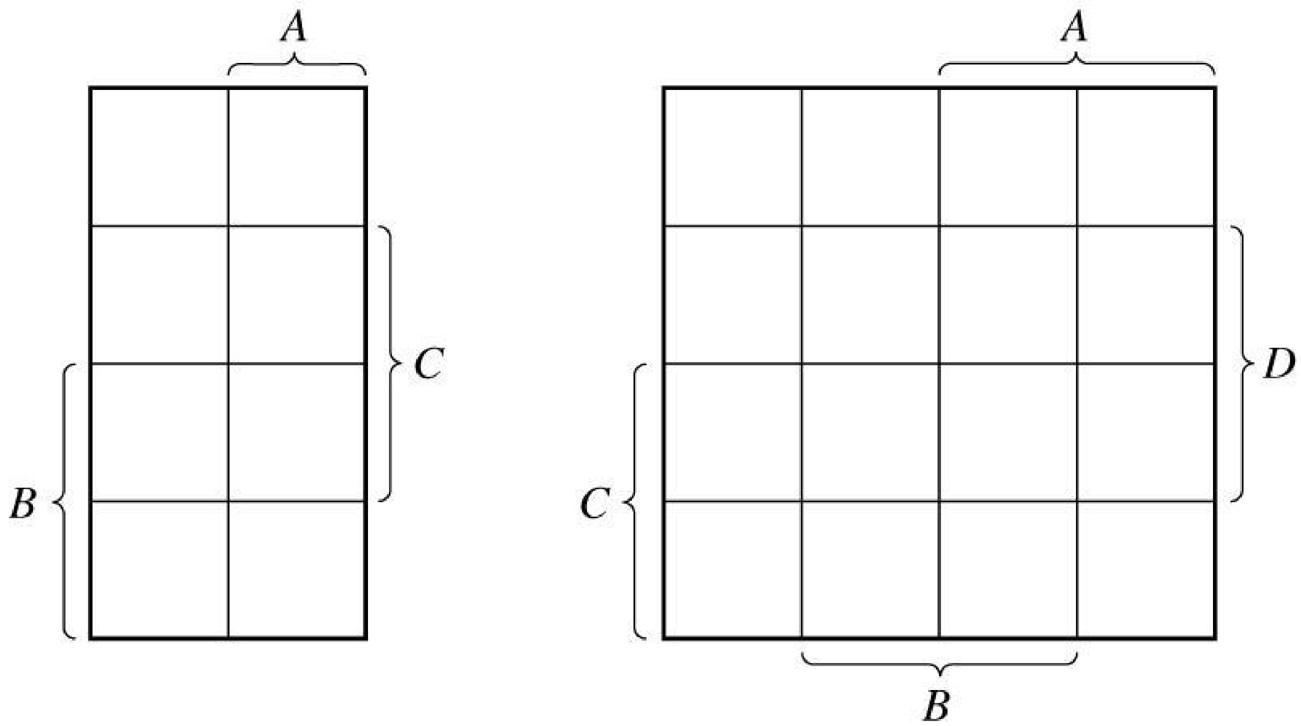
- Factoring an expression



$$F = A'B'(C' + D) + AC(B + D')$$

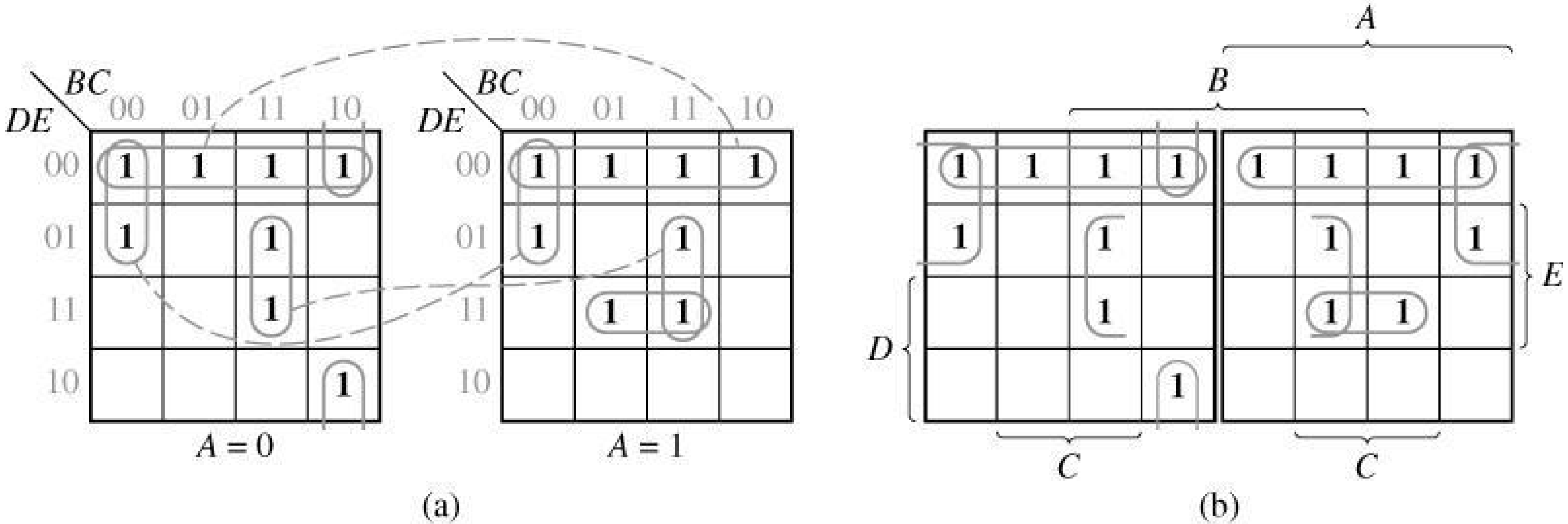
Other Forms of Karnaugh Maps

◆ Veitch Diagrams : useful for plotting functions given in algebraic form



Two Alternative Forms of 5-Variable K-Maps

(Figure 5-28) Simple Two Maps **Mirror Image Maps**



$$F = D'E' + B'C'D' + BCE + A'BC'E' + ACDE$$