Lecture 10: Functions and Recursion – Part II

Class page: https://github.com/tsung-wei-huang/cs1410-40

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Recap: Function Definition

- **The format of a function definition is as follows:**

```
return-value-type  function-name(  parameter-list  )
{
     declarations and statements
}
```

- **The *function-name* is any valid identifier.**
- **The *return-value-type* is the data type of the returned result to the caller.**
  - The type `void` indicates that a function does not return a value.
- **All variables defined in a function are local variables—they're known only in the function in which they're defined.**
- **Most functions have a list of parameters that provide the means for communicating information between functions.**
  - A function's parameters are also local variables of that function.

Example

```
// Creating and using a programmer-defined function.
#include <iostream>
using namespace std;

int square( int ); // function prototype

int main()
{
   // loop 10 times and calculate and output the
   // square of x each time
   for ( int x = 1; x <= 10; x++ )
      cout << square( x ) << "  "; // function call

   cout << endl;
} // end main

// square function definition returns square of an integer
int square( int y ) // y is a copy of argument to function
{
   return y * y;     // returns square of y as an int
} // end function square
```

Practice 1: Lower Bound of Sum

Write a function named ”boundary" that takes one integer argument, call it "goal" and returns as its value the smallest positive integer n for which 1+2+3+. . . +n is at least equal or larger than “goal”

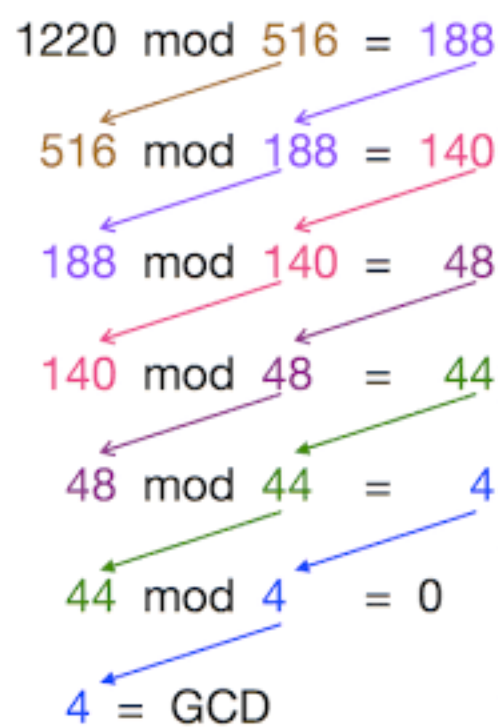
e.g. goal = 9, function returns 4, as 1+2+3+4 >= 9 but 1+2+3<9

e.g., goal=21, function return 6, as 1+2+3+4+5+6=21 but 1+2+3+4+5<21

Practice 2: GCD

Write a function named ”gcd" that takes two positive integers and return their greatest common divisor (GCD)

e.g. gcd(1220, 516) = 4



Call by Value vs Call by Reference

- **Call by value**
  - copies the actual **value** of an argument into the formal parameter of the function
- **Call by reference**
  - copies the address of an argument into the formal parameter

Call by Value vs Call by Reference

```
// Comparing pass-by-value and pass-by-reference with references.
#include <iostream>
using namespace std;

int squareByValue( int ); // function prototype (value pass)
void squareByReference( int & ); // function prototype (reference pass)

int main()
{
   int x = 2; // value to square using squareByValue
   int z = 4; // value to square using squareByReference

   // demonstrate squareByValue
   cout << "x = " << x << " before squareByValue\n";
   cout << "Value returned by squareByValue: "
      << squareByValue( x ) << endl;
   cout << "x = " << x << " after squareByValue\n" << endl;

   // demonstrate squareByReference
   cout << "z = " << z << " before squareByReference" << endl;
   squareByReference( z );
   cout << "z = " << z << " after squareByReference" << endl;
} // end main
```

Call by Value vs Call by Reference

```
// squareByValue multiplies number by itself, stores the
// result in number and returns the new value of number
int squareByValue( int number )
{
   return number *= number; // caller's argument not modified
} // end function squareByValue

// squareByReference multiplies numberRef by itself and stores the result
// in the variable to which numberRef refers in function main
void squareByReference( int &numberRef )
{
   numberRef *= numberRef; // caller's argument modified
} // end function squareByReference
```

```
x = 2 before squareByValue
Value returned by squareByValue: 4
x = 2 after squareByValue

z = 4 before squareByReference
z = 16 after squareByReference
```

Reference is "Alias"

- References can also be used as aliases for other variables within a function.
- For example, the code

```
int count = 1; // declare integer variable count
int &cRef = count; // create cRef as an alias for count
cRef++; // increment count (using its alias cRef)
```

  increments variable `count` by using its alias `cRef`.
- Reference variables must be initialized in their declarations and cannot be reassigned as aliases to other variables.
- Once a reference is declared as an alias for another variable, all operations performed on the alias are actually performed on the original variable.

Reference is “Alias”

```

// Initializing and using a reference.
#include <iostream>
using namespace std;

int main()
{
    int x = 3;
    int &y = x; // y refers to (is an alias for) x

    cout << "x = " << x << endl << "y = " << y << endl;
    y = 7; // actually modifies x
    cout << "x = " << x << endl << "y = " << y << endl;
} // end main
```

```
x = 3
y = 3
x = 7
y = 7
```

Reference must be Initialized!

```

// References must be initialized.
#include <iostream>
using namespace std;

int main()
{
   int x = 3;
   int &y; // Error: y must be initialized

   cout << "x = " << x << endl << "y = " << y << endl;
   y = 7;
   cout << "x = " << x << endl << "y = " << y << endl;
} // end main
```

Recursive Function

- **A recursive function is a function that calls itself, either directly, or indirectly (through another function).**
- **The function only knows how to solve the simplest case(s), or so-called base case(s).**
  - If the function is called with a base case, the function simply returns a result
  - For complex problem, the function divides a problem into
    - What it can do (base case) → return the result
    - What it cannot do → resemble the original problem, but be a slightly simpler or smaller version
    - The function calls a new copy of itself (recursion step) to solve the smaller problem
  - Eventually base case gets solved
    - Gets plugged in, works its way up and solves whole problem

Practice 3: Fibonacci Number (iterative)

Write a function named ”fib" that takes a positive integer N and returns the N-th Fibonacci number

e.g. N=6, returns 8

e.g. N=9, returns 34

1,1,2,3,5,8,13,21,34,55,89,144,233,377…

1+1=2	**13+21=34**
1+2=3	**21+34=55**
2+3=5	**34+55=89**
3+5=8	**55+89=144**
5+8=13	**89+144=233**
8+13=21	**144+233=377**

Practice 4: Fibonacci Number (Recursive)

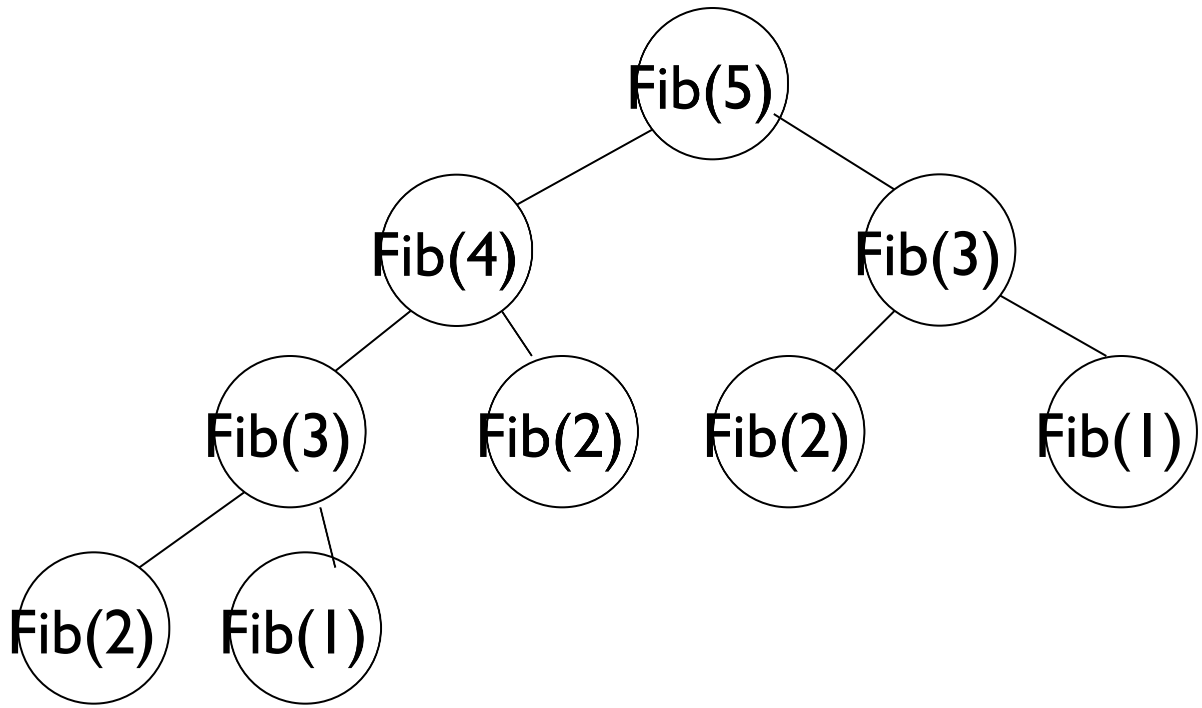
Write a function named ”fib" that takes a positive integer N and returns the N-th Fibonacci number

e.g. N=6, returns 8

e.g. N=9, returns 34

1,1,2,3,5,8,13,21,34,55,89,144,233,377…

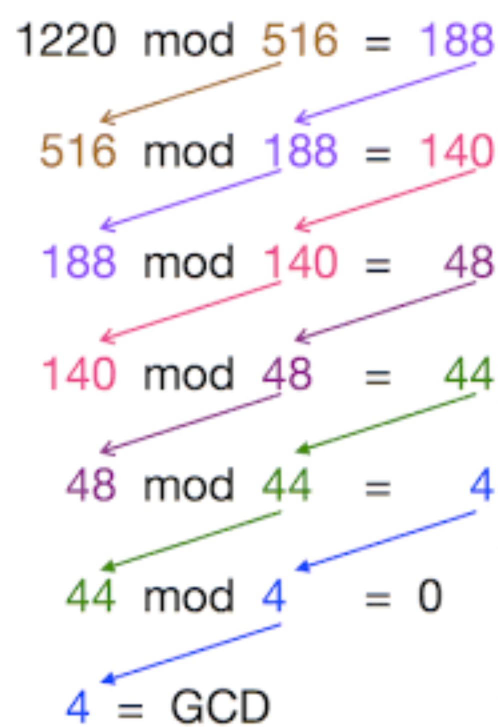
Duplicate Computations …



Practice 5: GCD (Recursive)

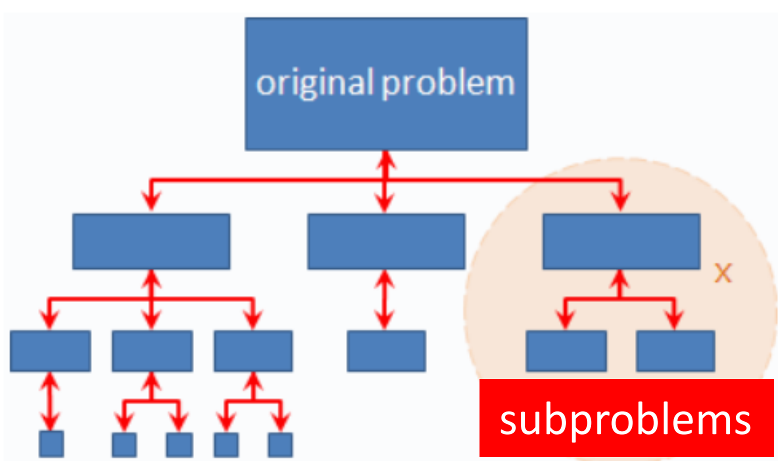
Write a function named ”gcd" that takes two positive integers and return their greatest common divisor (GCD)

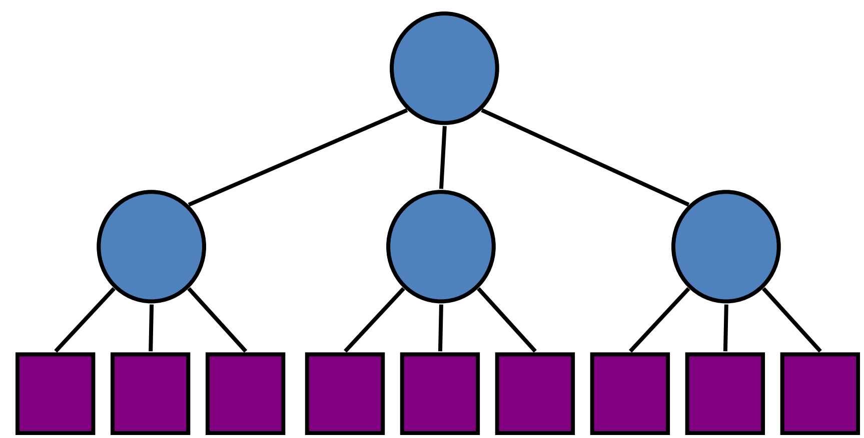
e.g. gcd(1220, 516) = 4



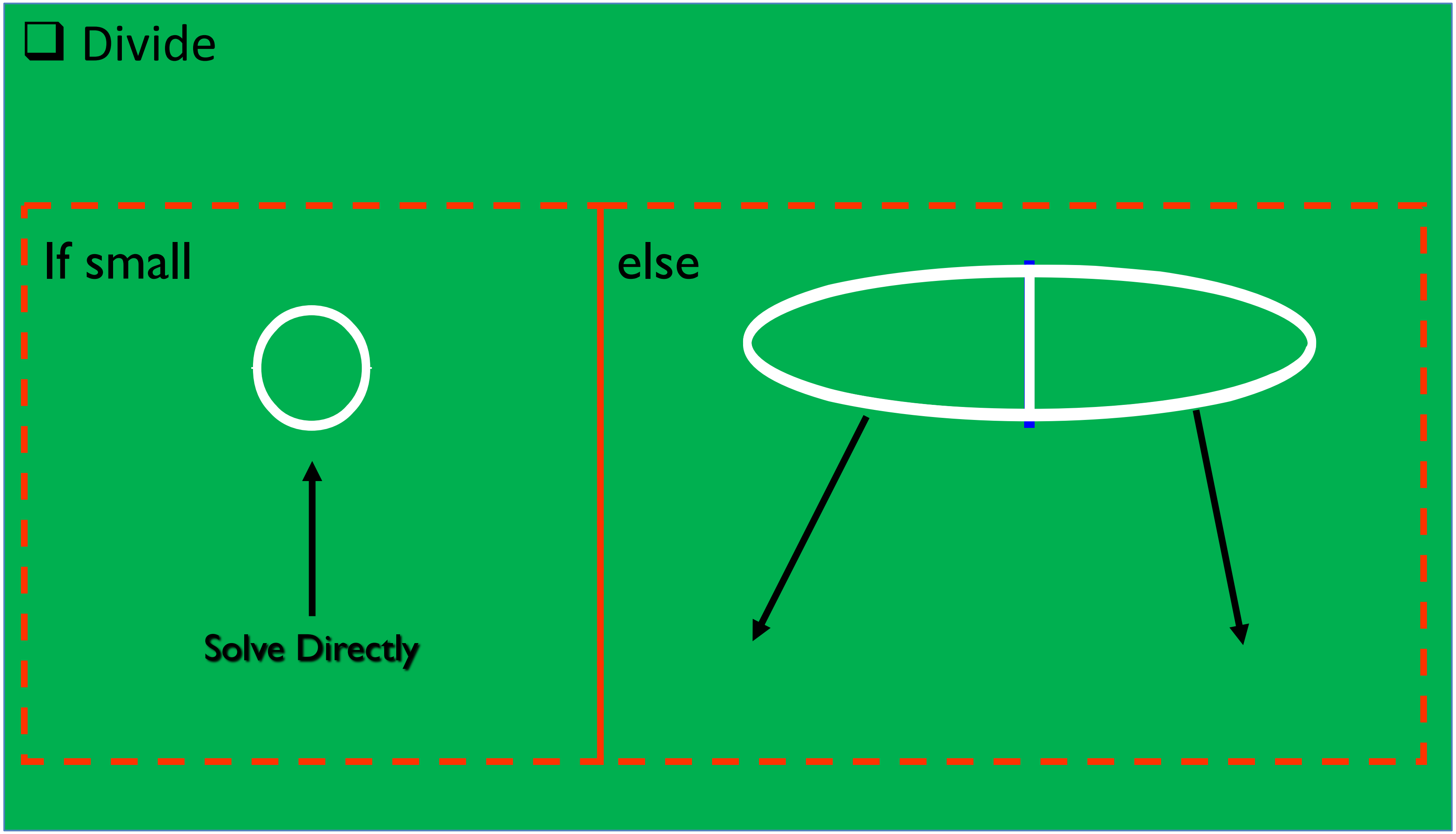
Divide and Conquer

- **Divide & Conquer**
  - Divide: Divide the original problem into smaller subproblems
  - Recurse: Solve each small subproblem recursively
  - Conquer: Combine these subproblems all the way to the top
- **Illustration**



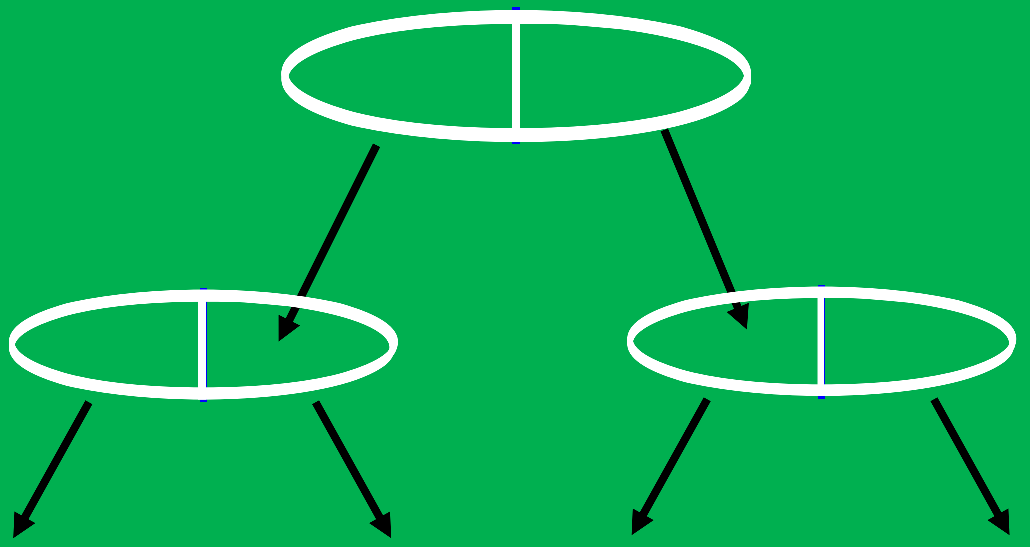


Visualization of Divide and Conquer



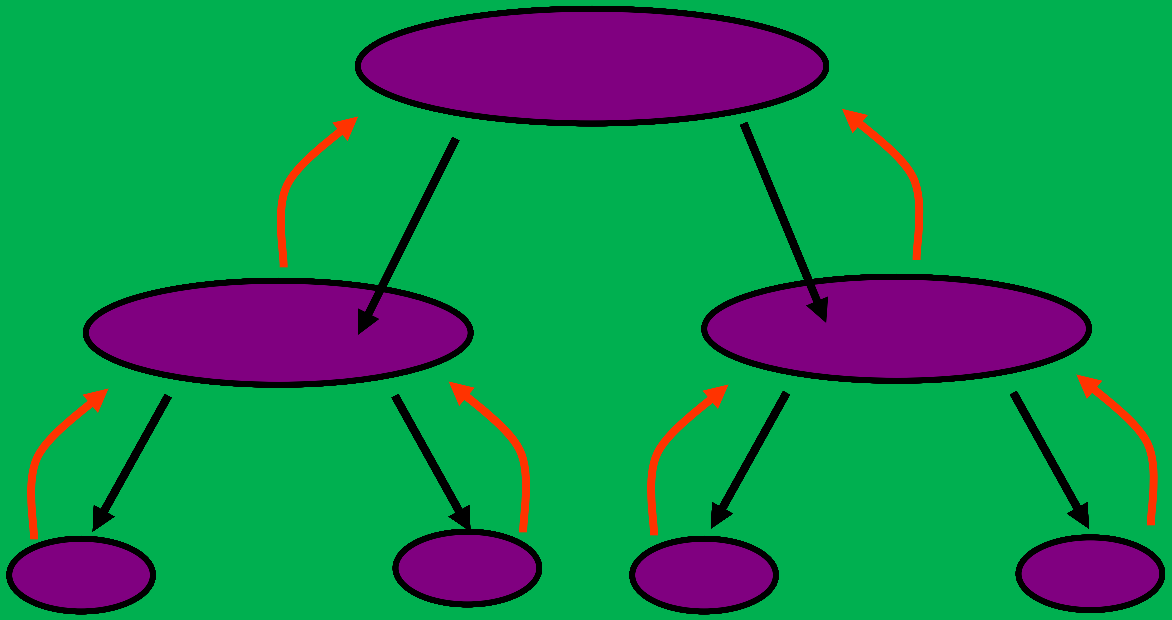
Visualization of Divide and Conquer

- Recursive



Visualization of Divide and Conquer

- Solve and Conquer



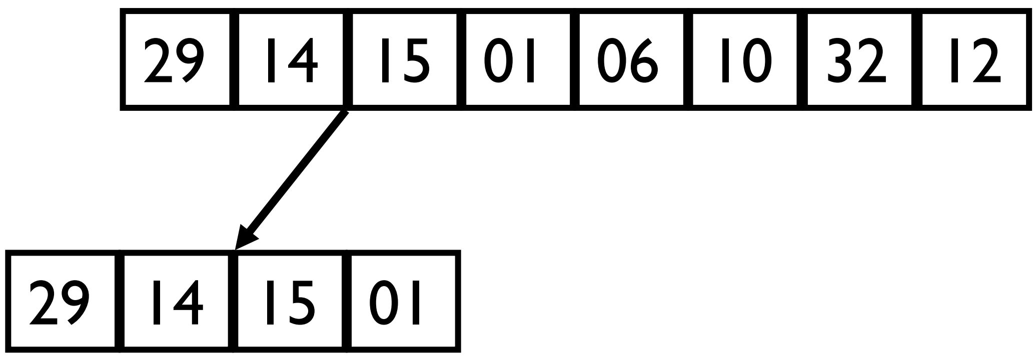
Example: Find the Maximum Value

- Divide

29	14	15	01	06	10	32	12

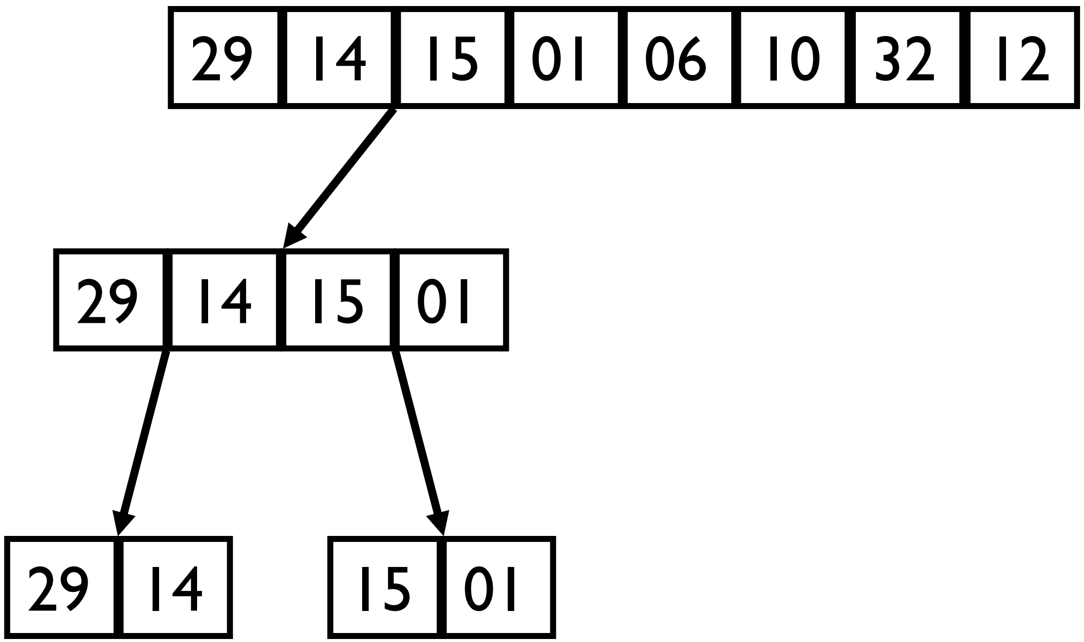
Example: Find the Maximum Value

- Divide



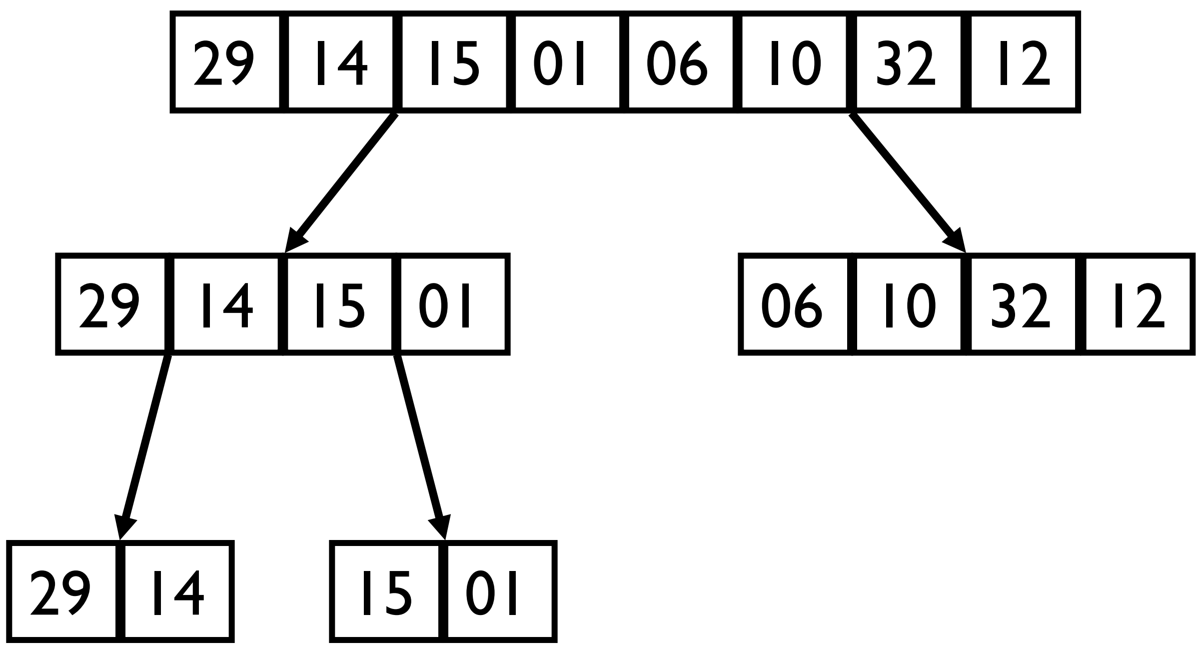
Example: Find the Maximum Value

- Divide



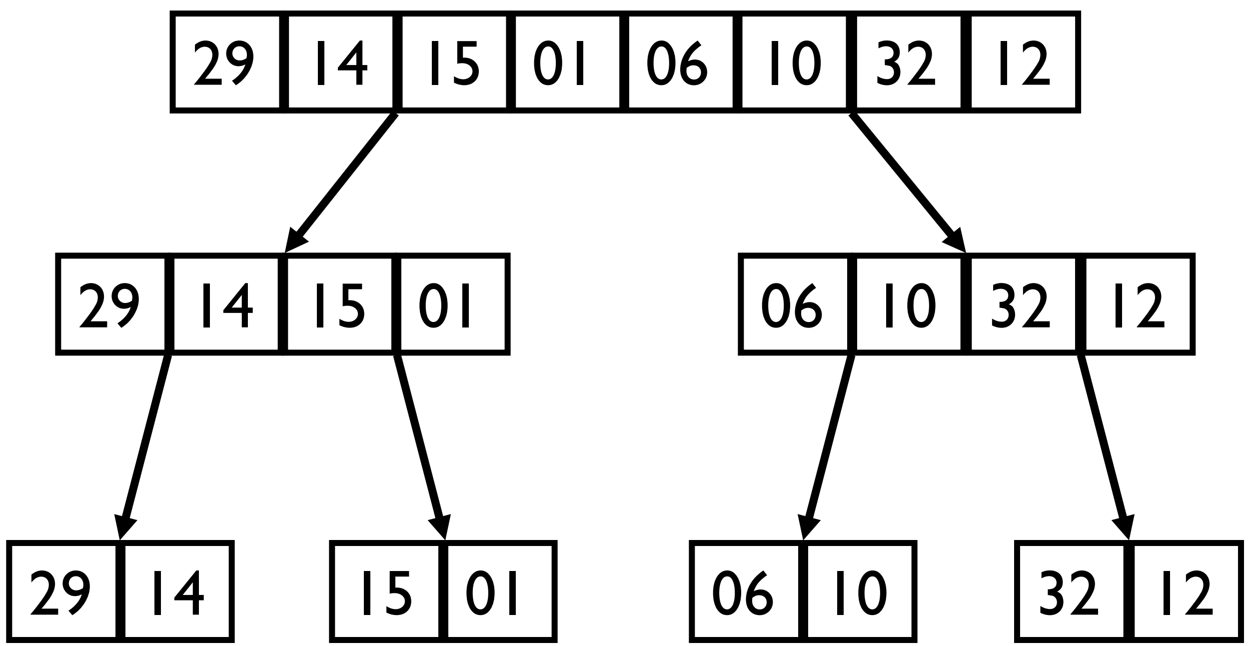
Example: Find the Maximum Value

- Divide



Example: Find the Maximum Value

❑ Divide



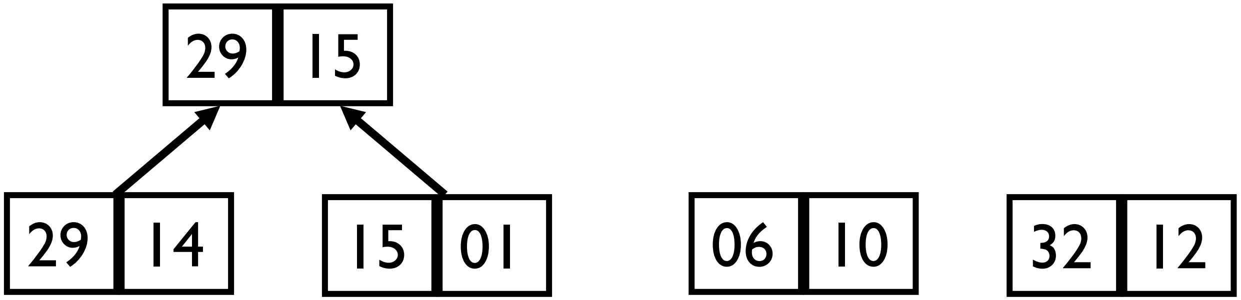
Example: Find the Maximum Value

- Conquer

29	14		15	01		06	10		32	12

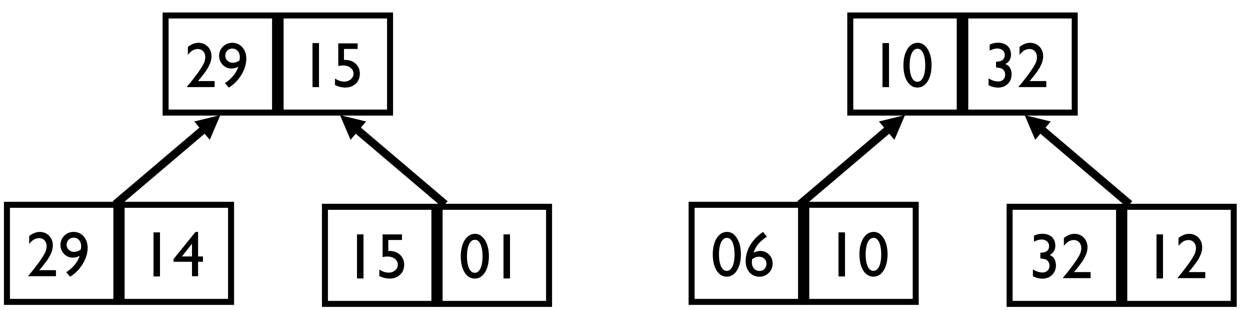
Example: Find the Maximum Value

- Conquer



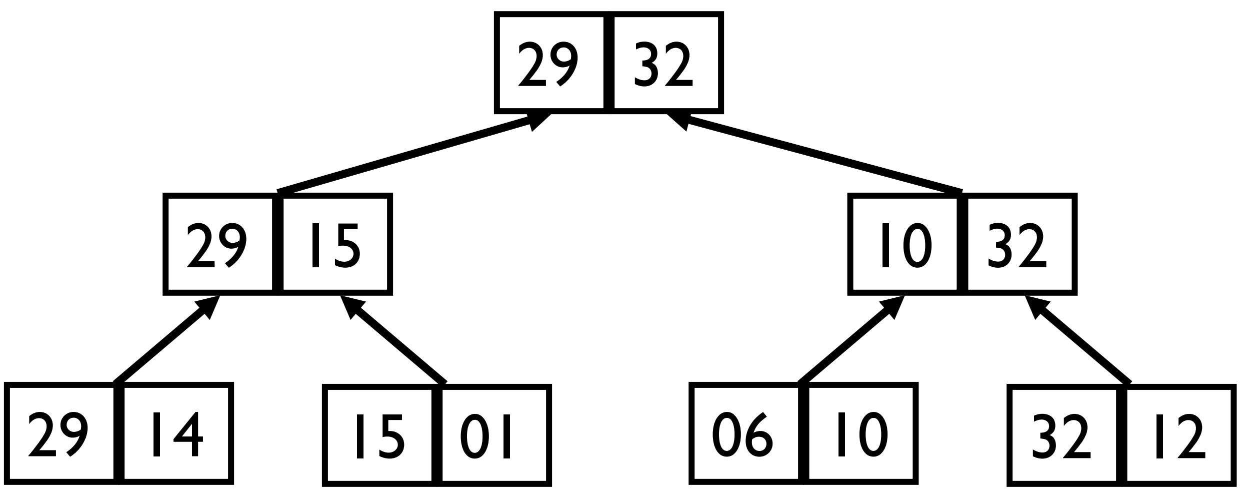
Example: Find the Maximum Value

- Conquer



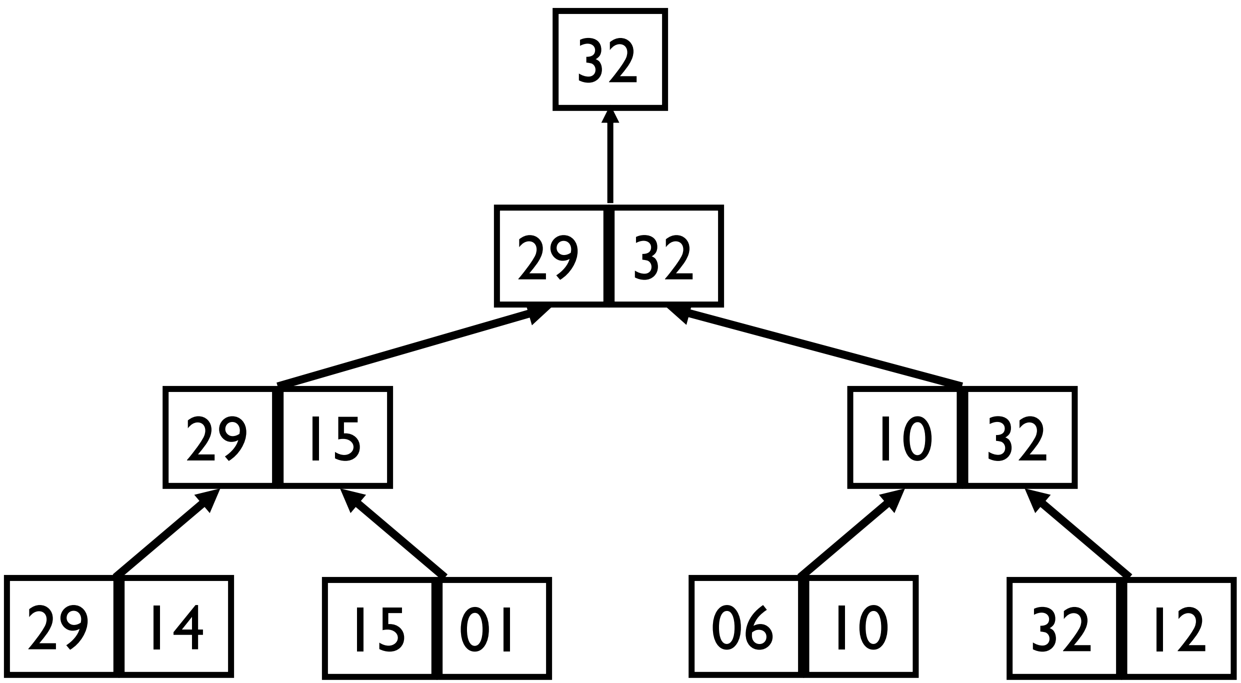
Example: Find the Maximum Value

- Conquer



Example: Find the Maximum Value

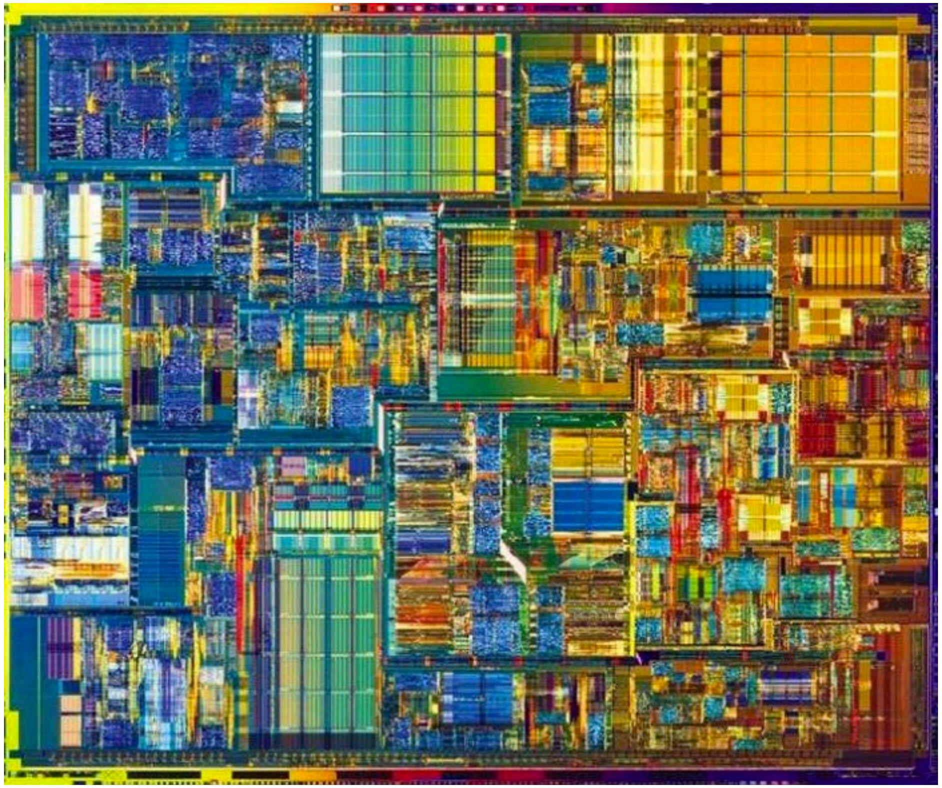
- Conquer



Divide and Conquer is Heavily used in CAD

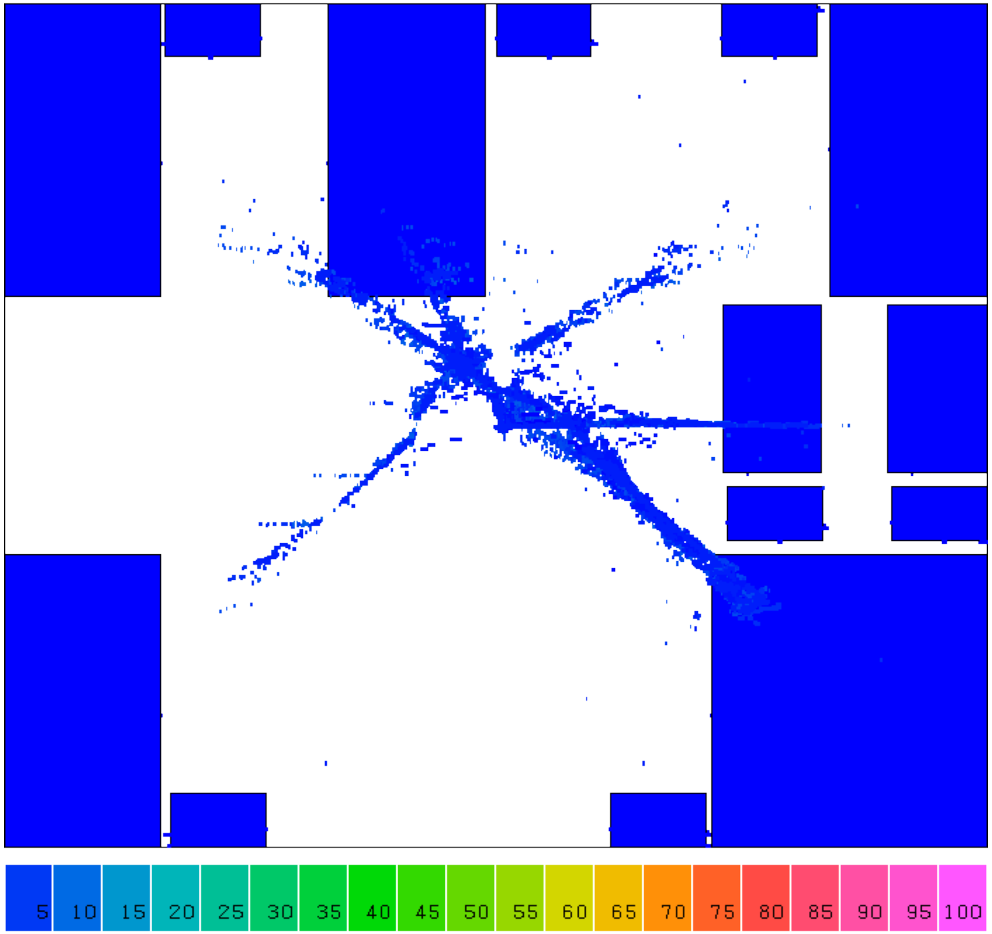
- ❑ **Modern circuits sizes are too large to handle in flat**

2000 – Intel Pentium4
42M transistors
1.5MHz; 224 mm^2



Directly solving the original problem takes forever to finish …

Example: Placement



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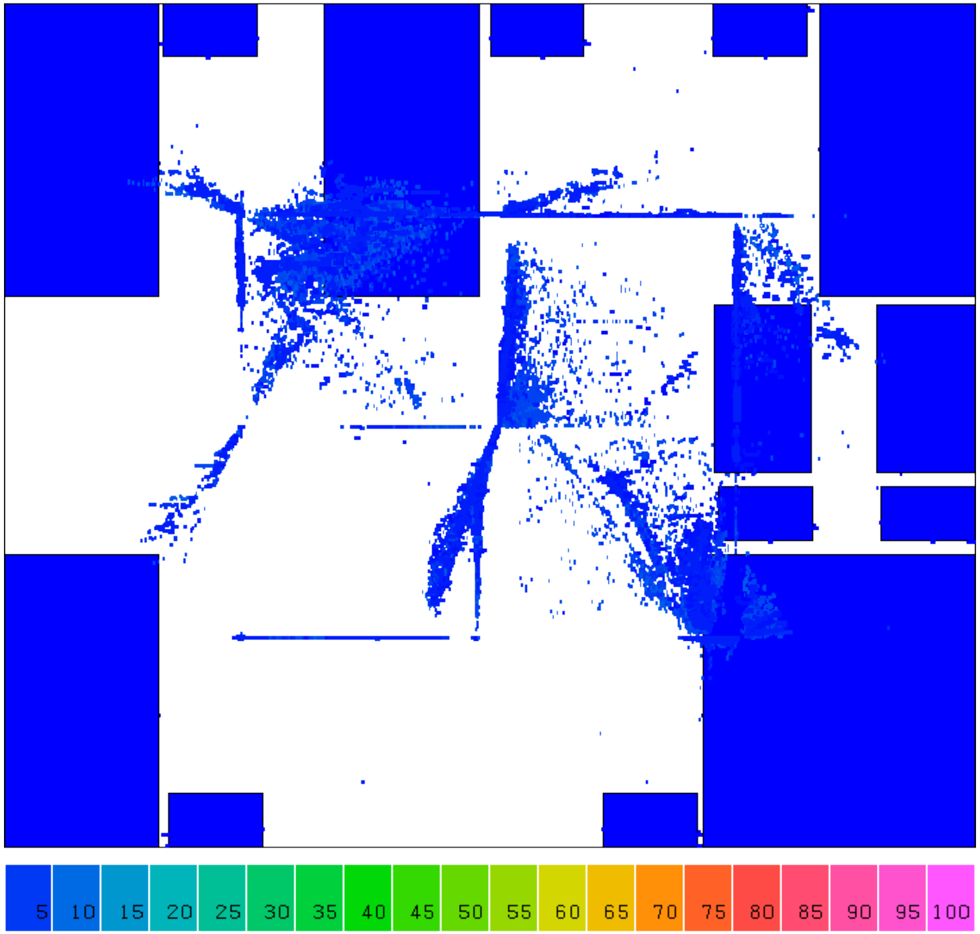

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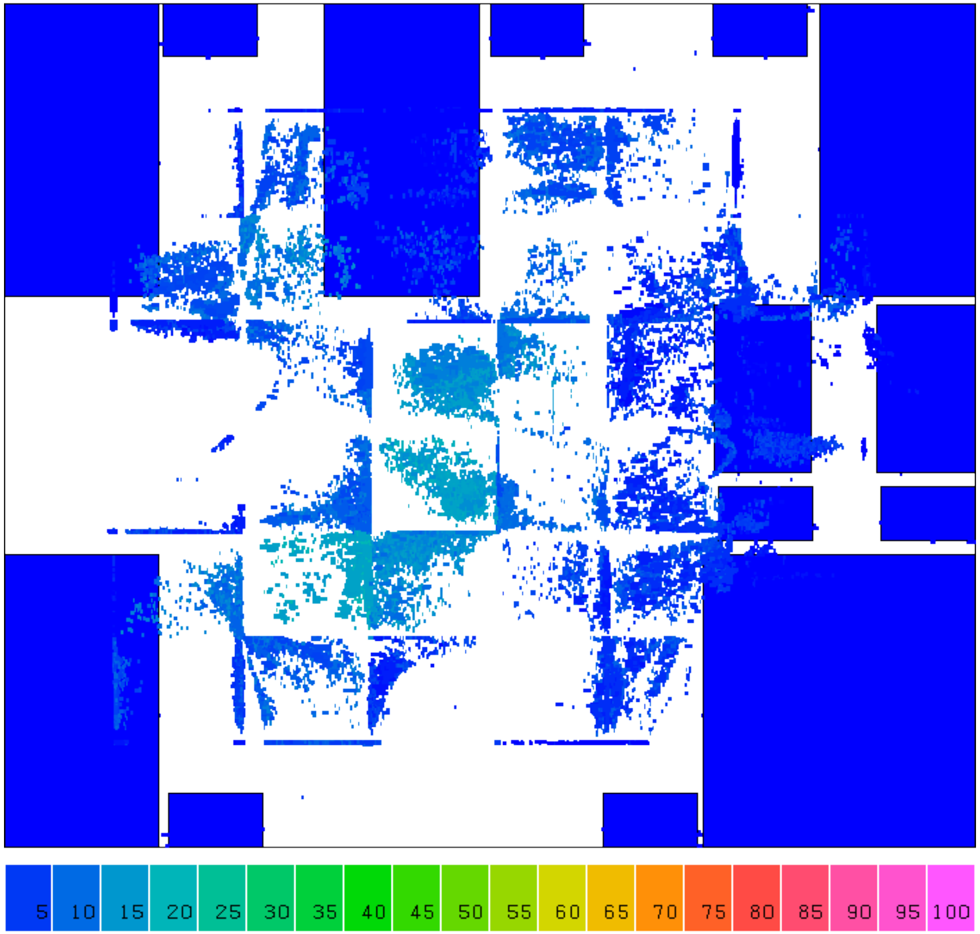

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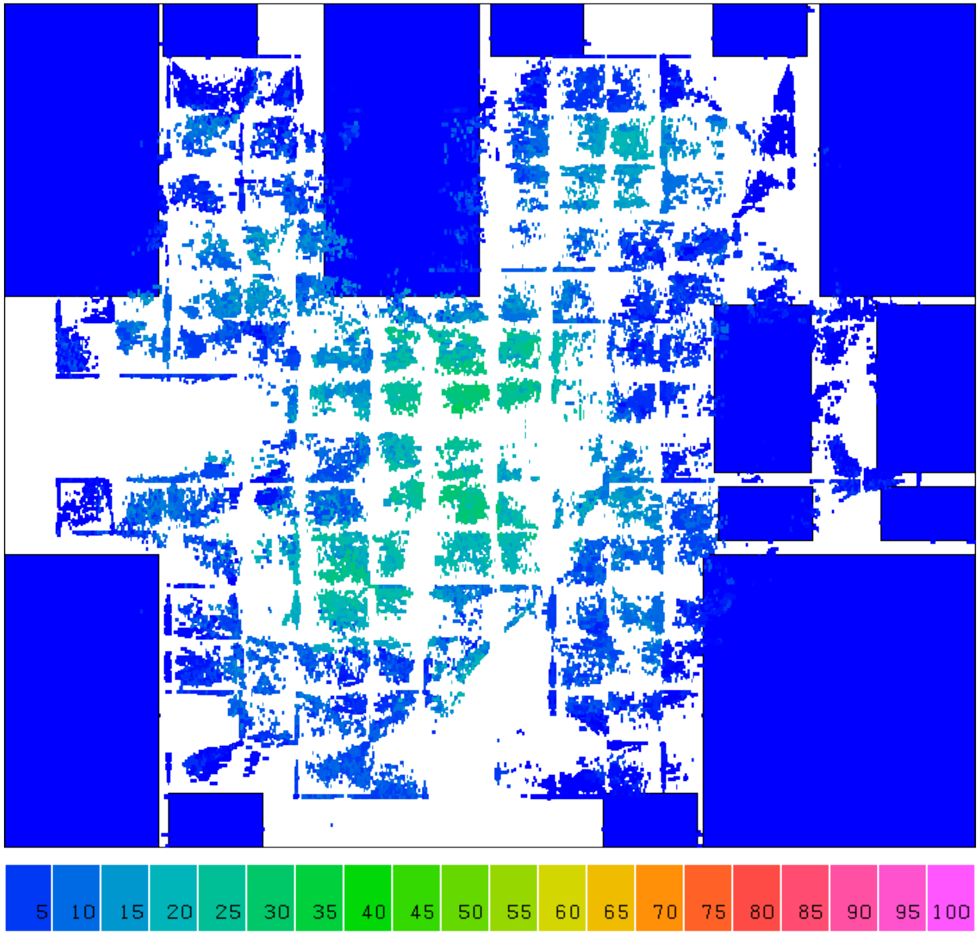

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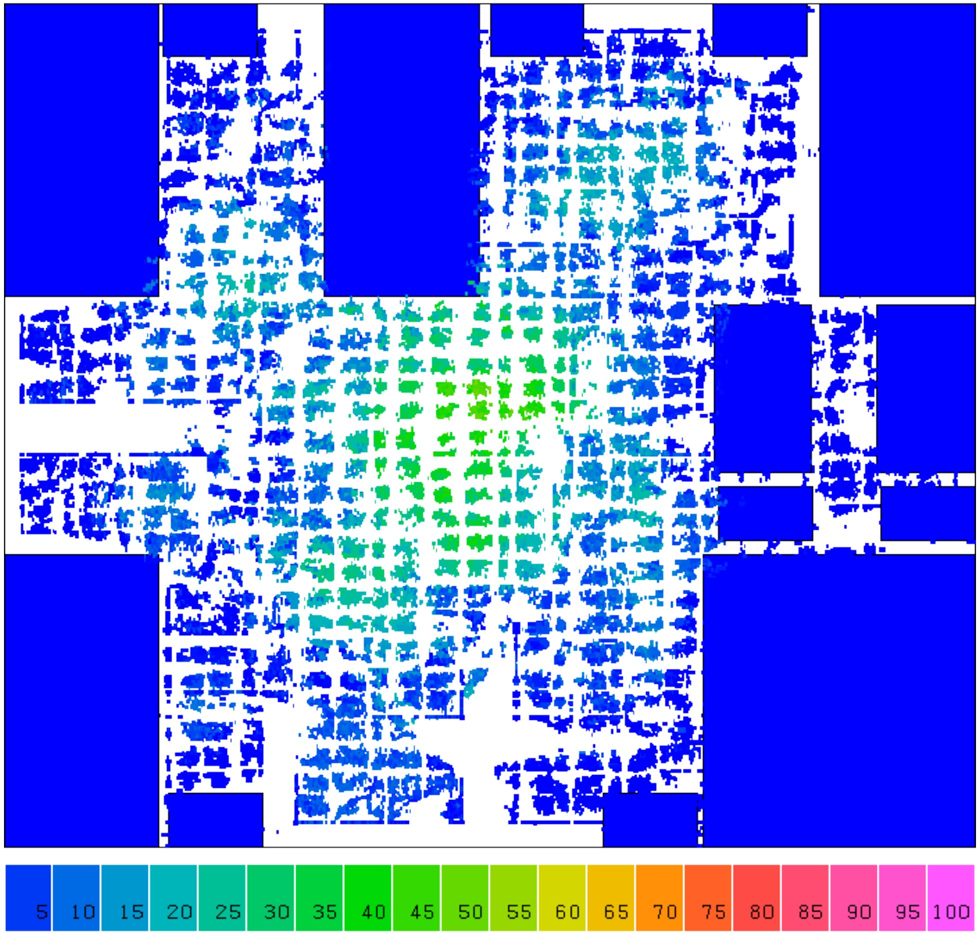

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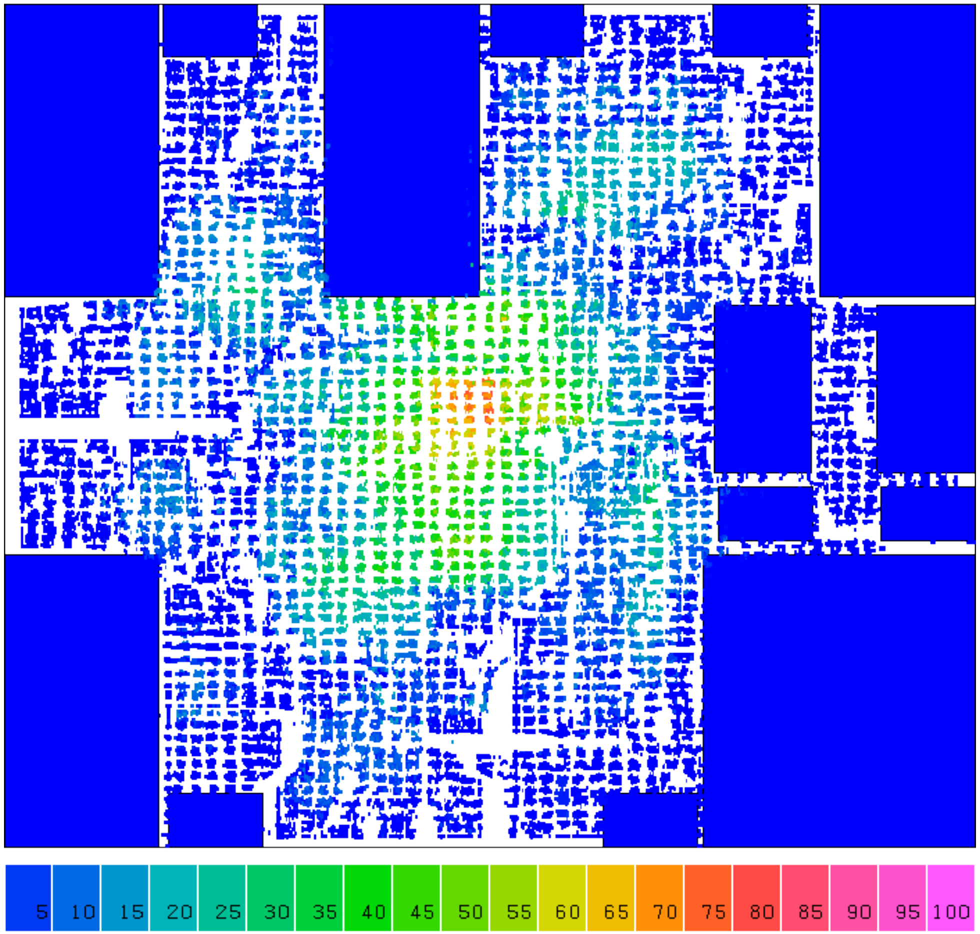

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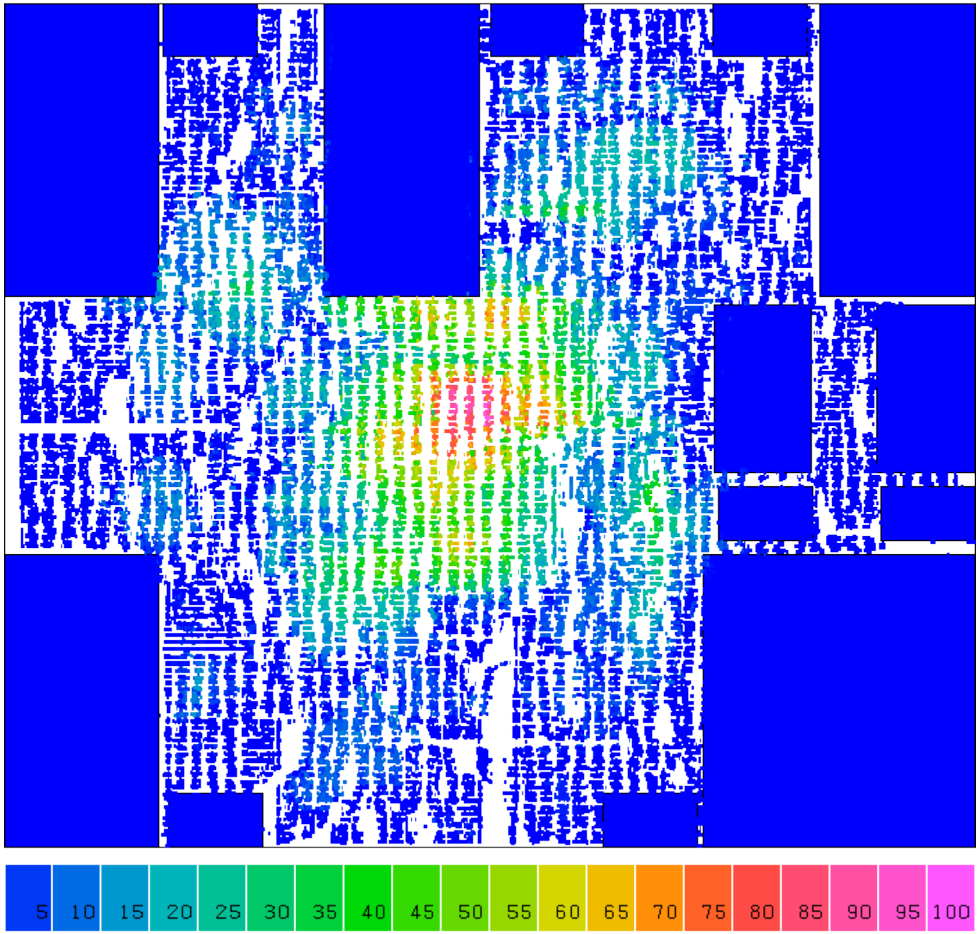

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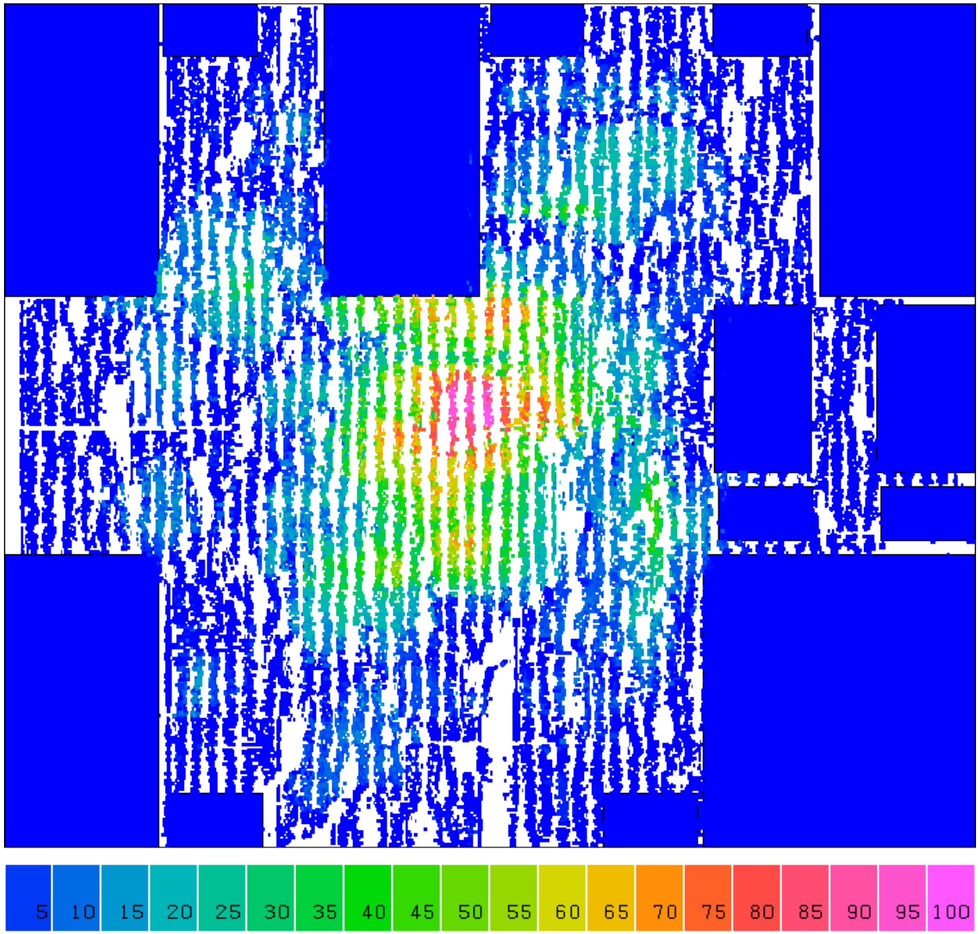

Summary

- **Function**
- **Recursive Function**
- **Divide and Conquer**
  - Used to solve 90% of the computer science problems
  - Break a large problem into smaller pieces
  - Solve each smaller piece
  - Merge the solutions