CS 2420: Tree Traversal

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Outline

- This topic will cover tree traversals:
    - A means of visiting all the objects in a tree data structure
    - We will look at
        - Breadth-first traversals
        - Depth-first traversals
    - Applications
    - General guidelines

Background

- All the objects stored in an array or linked list can be accessed sequentially
- When discussing deques, we introduced iterators in C++:
  - These allow the user to step through all the objects in a container
- Question: how can we iterate through all the objects in a tree in a predictable and efficient manner
  - Requirements: $\Theta(n)$ run time and $o(n)$ memory

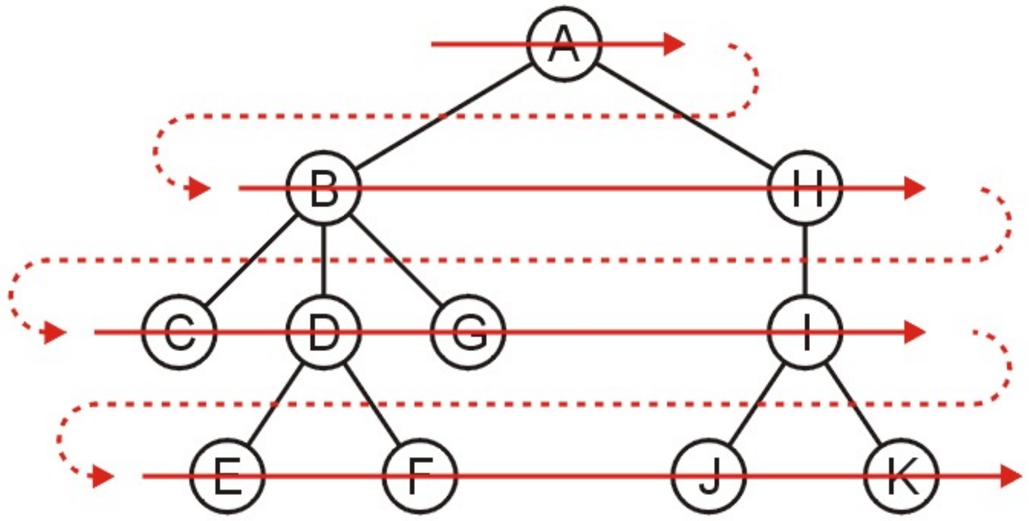
Types of Traversals

- We have already seen one traversal:
  - The breadth-first traversal visits all nodes at depth k before proceeding onto depth $k + 1$
  - Easy to implement using a queue

- Another approach is to visit always go as deep as possible before visiting other siblings: *depth-first traversals*

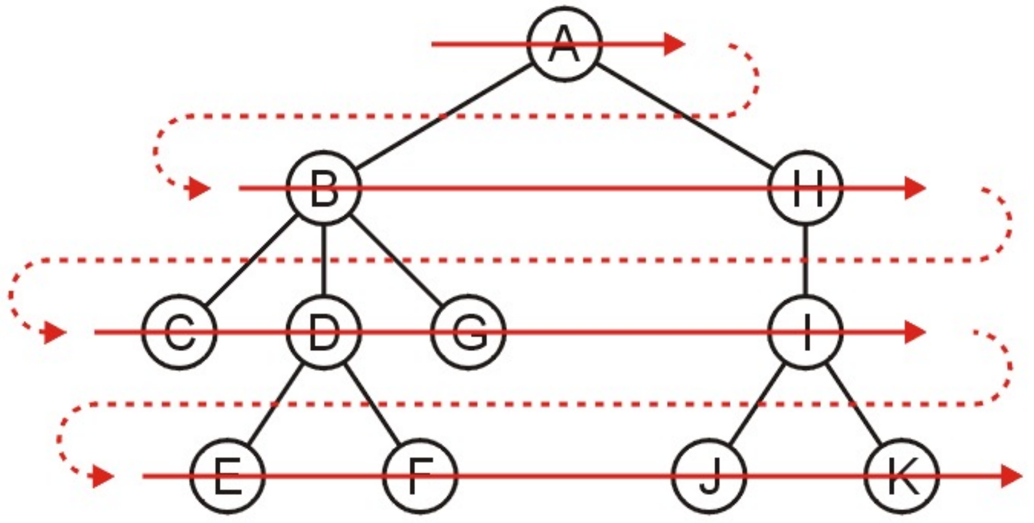
Breadth-First Traversal

- Breadth-first traversals visit all nodes at a given depth
  - Can be implemented using a queue
  - Run time is $\Theta(n)$
  - Memory is potentially expensive: maximum nodes at a given depth
  - Order: A B H C D G I E F J K



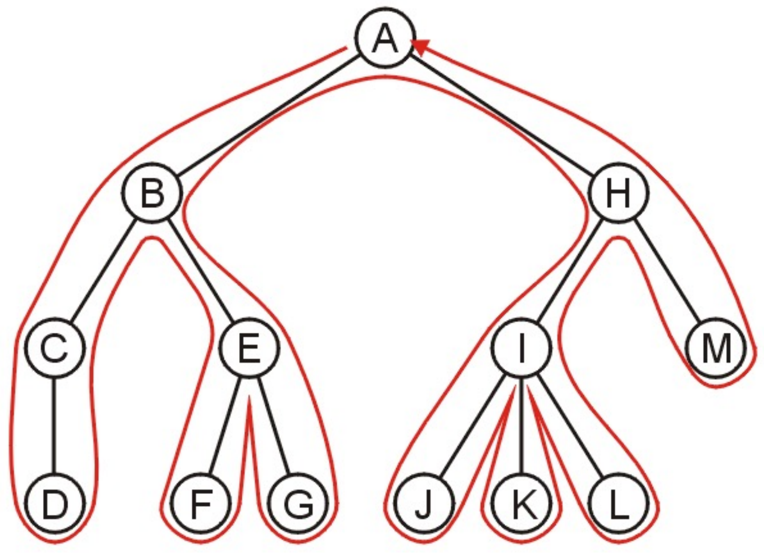
Breadth-First Traversal

- The implementation was already discussed:
  - Create a queue and push the root node onto the queue
  - While the queue is not empty:
    - Push all of its children of the front node onto the queue
    - Pop the front node



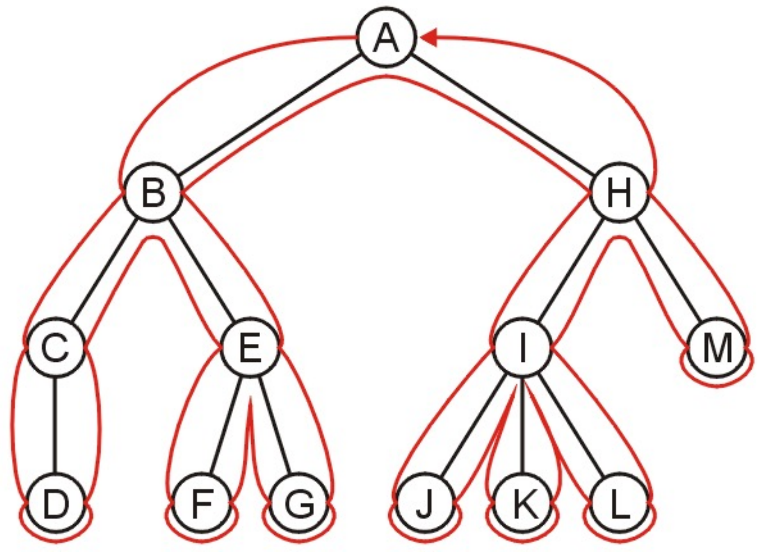
Backtracking

- To discuss depth-first traversals, we will define a backtracking algorithm for stepping through a tree:
  - At any node, we proceed to the first child that has not yet been visited
  - Or, if we have visited all the children (of which a leaf node is a special case), we backtrack to the parent and repeat this decision making process
- We end once all the children of the root are visited



Depth-first Traversal

- We define such a path as a *depth-first traversal*
- We note that each node could be visited twice in such a scheme
  - The first time the node is approached (before any children)
  - The last time it is approached (after all children)



Implementing depth-first traversals

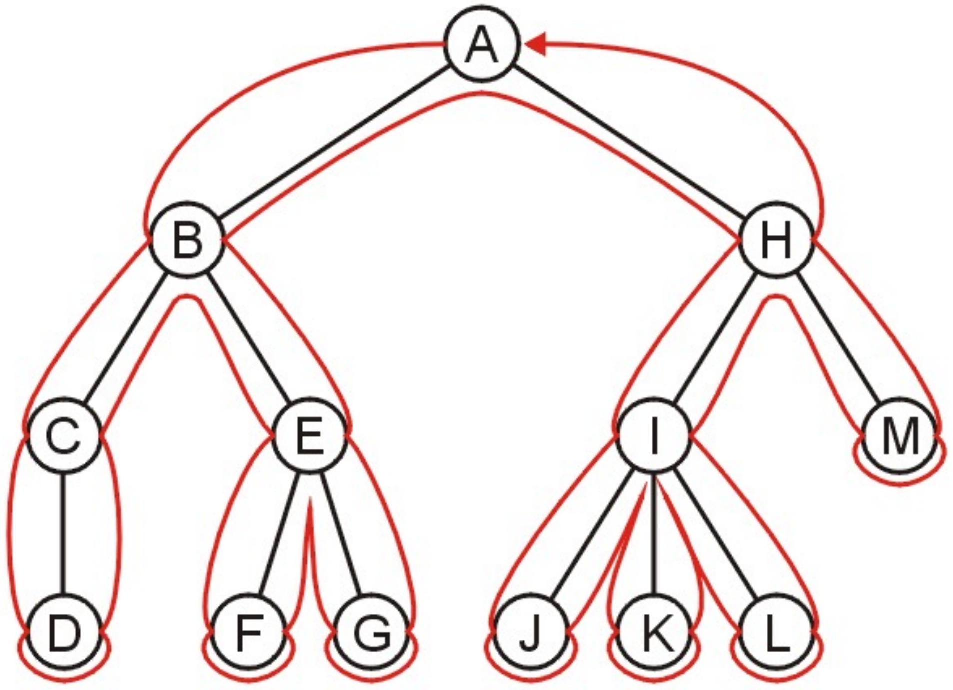
- Depth-first traversals can be implemented with recursion:

```
template <typename Type>
  void Simple_tree<Type>::depth_first_traversal() const {
     // Perform pre-visit operations on the value
     std::cout << "<" << node_value << ">";
     // Perform a depth-first traversal on each of the children
     for ( auto *child = children.head(); child != children.end();
           child = ptr->next() ) {
         child->value()->depth_first_traversal();
     }
     // Perform post-visit operations on the value
     std::cout << "</" << node_value << ">";
  }
```

Implementing depth-first traversals

- Performed on this tree, the output would be

```
<A><B><C><D></D></C><E><F></F><G></G></E></B><H><I><J></J><K></K><L></L></I><M></M></H></A>
```



Implementing depth-first traversals

- Alternatively, we can use a stack:
  - Create a stack and push the root node onto the stack
  - While the stack is not empty:
    - Pop the top node
    - Push all of the children of that node to the top of the stack in reverse order
  - Run time is $\Theta(n)$
  - The objects on the stack are all unvisited siblings from the root to the current node
    - If each node has a maximum of two children, the memory required is $\Theta(h)$: the height of the tree

- With the recursive implementation, the memory is $\Theta(h)$: recursion just hides the memory

Guidelines

- Depth-first traversals are used whenever:
  - The **parent** needs information about all its children or **descendants**, or
  - The **children** require information about all its parent or **ancestors**

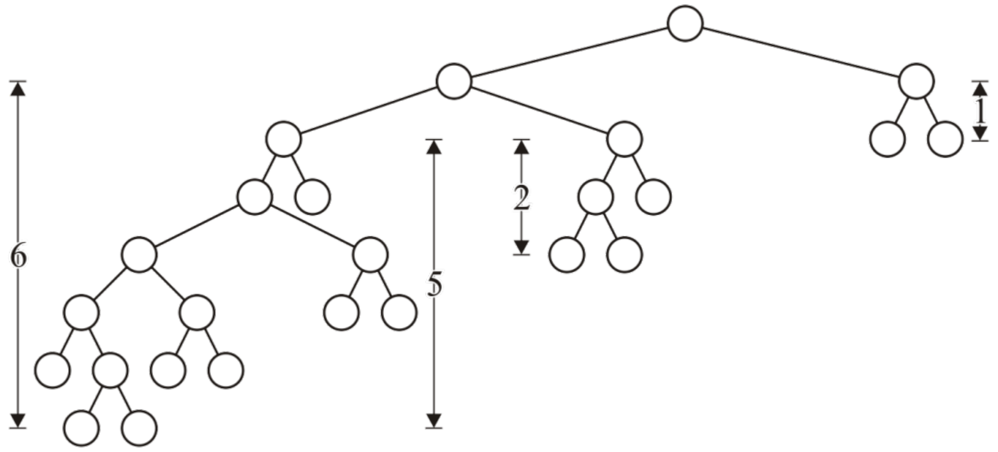
- In designing a depth-first traversal, it is necessary to consider:
  1. Before the children are traversed, what initializations, operations and calculations must be performed?
  2. In recursively traversing the children:
     a) What information must be passed to the children during the recursive call?
     b) What information must the children pass back, and how must this information be collated?
  3. Once all children have been traversed, what operations and calculations depend on information collated during the recursive traversals?
  4. What information must be passed back to the parent?

Applications

- Tree application: displaying information about directory structures and the files contained within
  - Finding the height of a tree
  - Printing a hierarchical structure
  - Determining memory usage

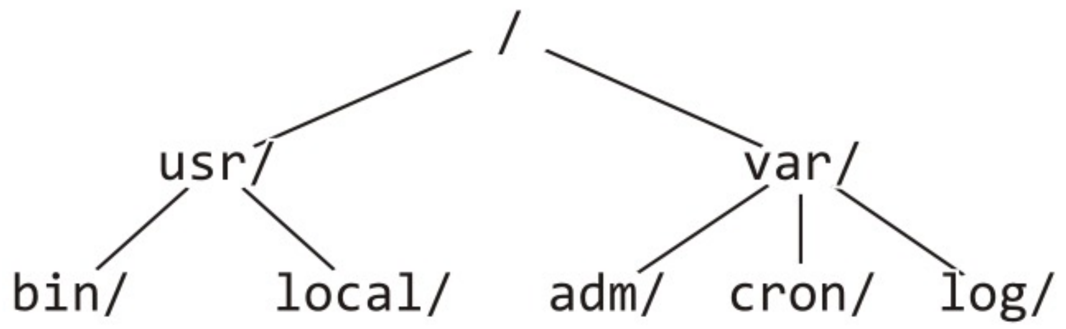
Height

- The `int height() const` function is recursive in nature:
  1. Before the children are traversed, we assume that the node has no children and we set the height to zero: $h_{\text{current}} = 0$
  2. In recursively traversing the children, each child returns its height h and we update the height if $1 + h > h_{\text{current}}$
  3. Once all children have been traversed, we return h_{current}
- When the root returns a value, that is the height of the tree



Printing a Hierarchy

- Consider the directory structure presented on the left—how do we display this in the format on the right?



```
/
    usr/
        bin/
        local/
    var/
        adm/
        cron/
        log/
```

- What do we do at each step?

Printing a Hierarchy

- For a directory, we initialize a tab level at the root to 0

- We then do:
  1. Before the children are traversed, we must:
     a) Indent an appropriate number of tabs, and
     b) Print the name of the directory followed by a `'/'`
  2. In recursively traversing the children:
     a) A value of one plus the current tab level must be passed to the children, and
     b) No information must be passed back
  3. Once all children have been traversed, we are finished

Printing a Hierarchy

- Assume the function `void print_tabs( int n )` prints *n* tabs

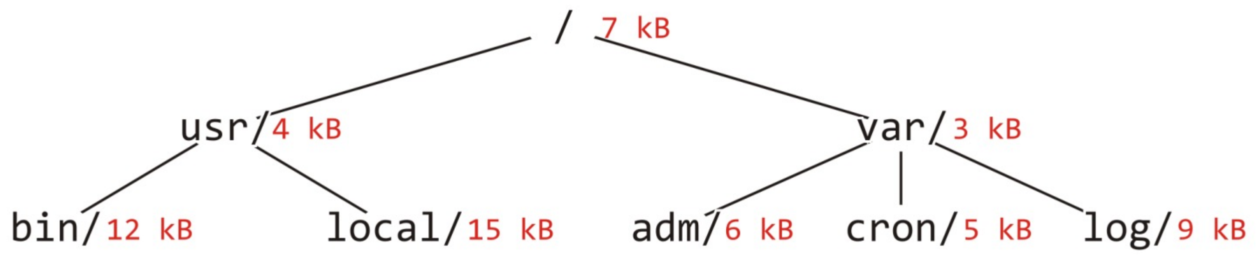
```
template <typename Type>
void Simple_tree<Type>::print( int depth ) const {
  print_tabs( depth );
  std::cout << value()->directory_name() << '/' << std::endl;

   for ( auto *child = children.head(); child != children.end();
         child = ptr->next() ) {
      child->value()->print( depth + 1 );
   }
}
```

Determining Memory Usage

- Suppose we need to determine the memory usage of a directory and all its subdirectories:
  - We must determine and print the memory usage of all subdirectories before we can determine the memory usage of the current directory

Determining Memory Usage

- Suppose we are printing the directory usage of this tree:



```
        bin/ 12
        local/ 15
    usr/ 31
        adm/ 6
        cron/ 5
        log/ 9
    var/ 23
/ 61
```

Determining Memory Usage

- For a directory, we initialize a tab level at the root to 0

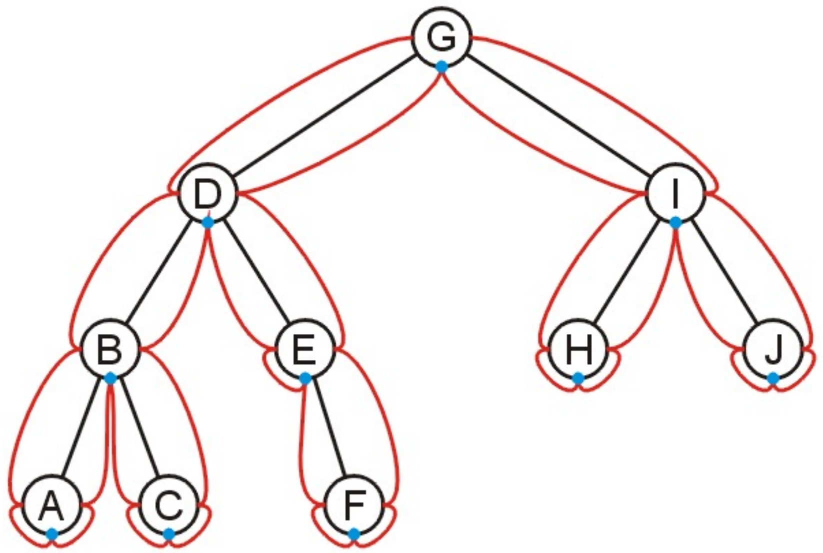
- We then do:
  1. Before the children are traversed, we must:
     a) Initialize the memory usage to that in the current directory.
  2. In recursively traversing the children:
     a) A value of one plus the current tab level must be passed to the children, and
     b) Each child will return the memory used within its directories and this must be added to the current memory usage.
  3. Once all children have been traversed, we must:
     a) Print the appropriate number of tabs,
     b) Print the name of the directory followed by a "/ ", and
     c) Print the memory used by this directory and its descendants

Printing a Hierarchy

```
template <typename Type>
int Simple_tree<Type>::du( int depth ) const {
  int usage = value()->memory_usage();

   for ( auto *child = children.head(); ptr != children.end();
         ptr = ptr->next() ) {
      usage += ptr->value()->du( depth + 1 );
   }

  print_tabs( depth );
  std::cout << value()->directory_name() << "/ " << usage << std::endl;

  return usage;
}
```

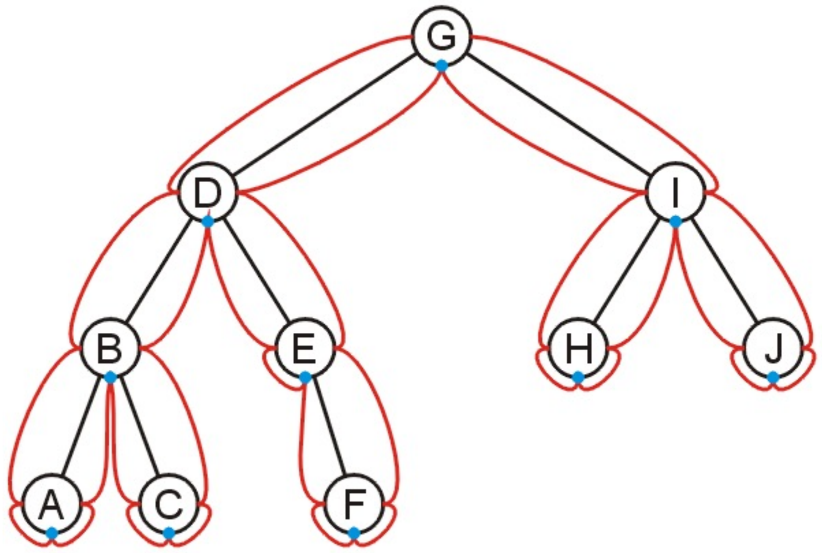
In-order Traversals

- For binary trees, there is a third intermediate visit
  - An *in-order* depth-first traversal



In-order Traversals

- This visits a binary search tree in order



A, B, C, D, E, F, G, H, I, J

In-order Traversal Implementation

- An implementation of an in-order traversal

```
template <typename Type>
void Binary_tree<Type>::in_order_traversal() const {
    if ( left() != nullptr ) {
        left()->in_order_traversal();
    }
    cout << value();
    if ( right() != nullptr ) {
        right()->in_order_traversal();
    }
}
```

Pre-order Traversal Implementation

- An implementation of an pre-order traversal

```
template <typename Type>
void Binary_tree<Type>::in_order_traversal() const {
    cout << value();
    if ( left() != nullptr ) {
        left()->in_order_traversal();
    }
    if ( right() != nullptr ) {
        right()->in_order_traversal();
    }
}
```

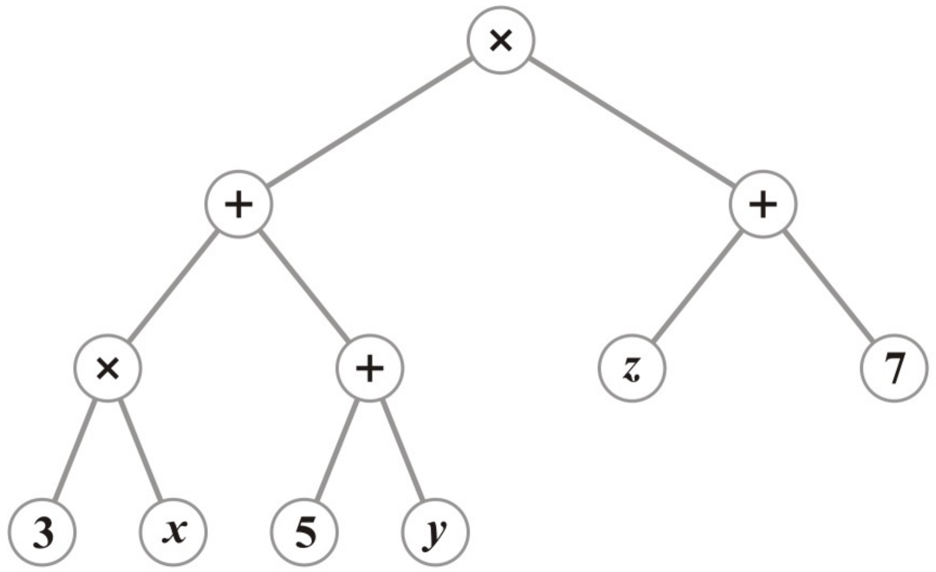
Post-order Traversal Implementation

- An implementation of an post-order traversal

```
template <typename Type>
void Binary_tree<Type>::in_order_traversal() const {
    if ( left() != nullptr ) {
        left()->in_order_traversal();
    }
    if ( right() != nullptr ) {
        right()->in_order_traversal();
    }
    cout << value();
}
```

In-order Traversals on Expression Tree

- Printing an expression tree (*pretty printing* or *human-readable printing*) using in-fix notation requires an in-order traversal



$$(3x + 5 + y)(z + 7)$$

Application

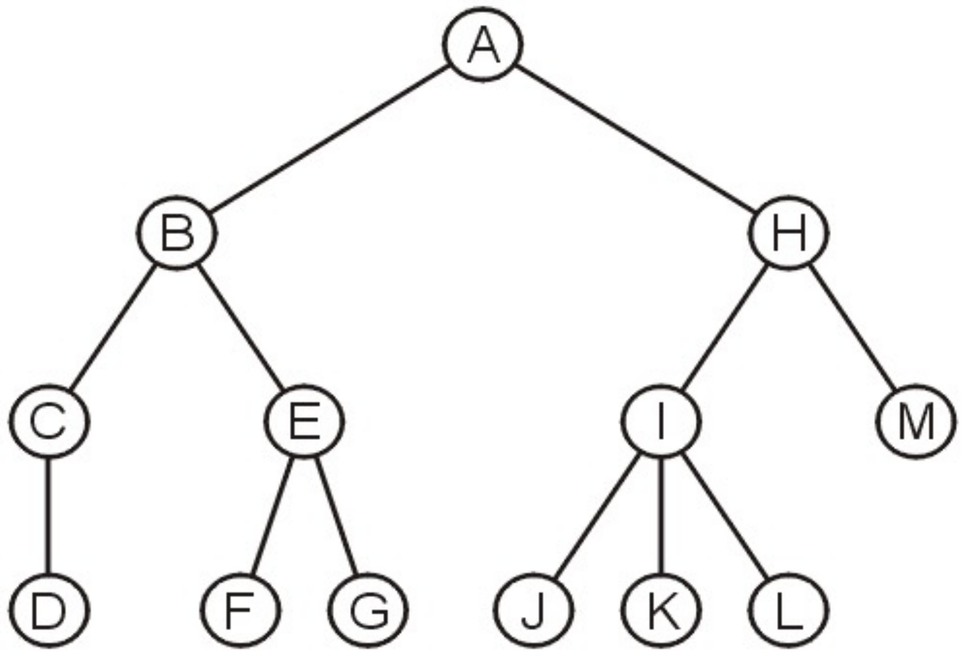
```
class Expression_node;

void Expression_node::pretty_print() {
    if ( left() != nullptr ) {
        // If the precedence of the parent is higher than that of the
        // current operator, we need to print an opening parenthesis

        if ( parent()->precedence() > precedence() ) {
            cout << "(";
        }  // pre-order visit

        left()->pretty_print(); // traverse left tree
    }

    // The in-order step:  print this object
    cout << this;  // print this object
```

Application

```
    if ( right() != nullptr() ) {
        right()->pretty_print(); // traverse right sub-tree

        // If the precedence of the parent is higher than that of the
        // current operator, we need to print a closing parenthesis

        if ( parent()->precedence() > precedence() ) {
            cout << ")";
        }  // post-order visit
    }
}
```

In-order Traversals on General Trees

- An *in-order* traversal does NOT make sense for either general trees or N-ary trees with $N > 2$



Summary

- This topic covered two types of traversals:
  - Breadth-first traversals
  - Depth-first traversals
  - Applications
  - Determination of how to structure a depth-first traversal
- Traversals of binary trees
  - In-order
  - Post-order
  - Pre-order