# Velocity Relaxation in a Maxwellian Plasma

Frank S. Tsung

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## 1 Introduction

The problem of velocity relaxation in a thermal plasma has been extensively studied in 1-D[4][3], and 2- and 3-D as well[5]. The basic problem is to theoretically and emperically calculate the coefficients in the Fokker-Planck equation.

$$\frac{\partial f(t,\underline{v})}{\partial t} + \frac{\partial}{\partial v_i}A_i(\underline{v})f(t,\underline{v}) - \frac{1}{2}\frac{\partial}{\partial v_i}\frac{\partial}{\partial v_j}B_{ij}f(t,\underline{v}) = 0$$

where

$$A_i(\underline{v}) = \frac{\overline{\Delta v_i}}{\Delta t}$$

$$B_{ij}(\underline{v}) = \frac{\overline{\Delta v_i \Delta v_j}}{\Delta t}$$

Okuda[5] calculated the self-consistent drag coefficient for a test particle in a plasma,

$$A_{||} = \frac{2qq_t}{\pi}\int_0^{\pi/2} d\theta \int_0^{\infty} dk \frac{kS^2(k) sin\theta cos\theta Im\epsilon(k,\theta)}{|\epsilon(\underline{k}\cdot\underline{v},\underline{k})|^2}$$

$$A_{\perp} = 0$$

$$B_{||} = \frac{8\pi q^2 q_t^2 n}{(2\pi)^{1/2} v_t}\int_0^{\pi/2} d\theta \int_0^{\infty} dk \frac{S^4(k) sin\theta cos^2\theta}{k|\epsilon(\underline{k}\cdot\underline{v},\underline{k})|^2} e^{(\frac{-v^2 cos\theta}{2v_t^2})}$$

$$B_{\perp} = \frac{8\pi q^2 q_t^2 n}{(2\pi)^{1/2} v_t}\int_0^{\pi/2} d\theta \int_0^{\infty} dk \frac{S^4(k) sin^3\theta}{k|\epsilon(\underline{k}\cdot\underline{v},\underline{k})|^2} e^{(\frac{-v^2 cos\theta}{2v_t^2})}$$

where $||$ and $\perp$ refer to parallel and perpendicular directions of the velocity of a test cloud.

Dawson[4][3] calculated the drag coefficients for one-dimensional sheets ($|S(k)| = 1$), for fast sheets, it was assumed that the electrons are stationary, and the averaged acceleration is due to uniformly placed sheets, so the averaged drag is

$$\frac{d < V >}{dt} = A(v) = -\frac{{\omega_p}^2 \delta}{2}$$

Dawson[4] also calculated the drag coefficient and the diffusion coefficient for slow particles. The drag coefficient, with correction introduced by Feix and Eldridge, is:

$$A(v) = -\frac{\omega_p \delta}{(8\pi)^{1/2} \lambda_D} v$$

The diffusion coefficient for slow particles ($v = 0$) can be estimated by assuming that the Maxwellian $e^{\frac{v^2}{2\overline{v}^2}}$ is the time-independent solution to the Fokker-Planck equation, and assume that $B(v)$ is symmetric about v=0,

$$A(v)exp(\frac{-v^2}{2\overline{v}}) - \frac{1}{2}\frac{\partial}{\partial v} B(v)exp(\frac{-v^2}{2\overline{v}}) = 0$$

Assuming that $B(v)$ is symmetric about 0, then

$$B(0) = \{\frac{\delta \overline{v}}{(2\pi)^{1/2}}\}\omega_p^2$$ ← B is velocity-independent.

Following the notations from the Brownian particle Fokker-Planck equation, i.e.:

$$A(v) = -\frac{v}{\tau_o}$$

$$B(v) = \frac{2\overline{v}^2}{\tau_o}$$

The stopping time for the one dimentional plasma sheet model is:

$$\frac{1}{\tau_0} = \frac{\omega_p \delta}{(8\pi)^{1/2} \lambda_D}$$

## 2 Simulation Parameters

The numerical experiments were performed on **beps1**, a 1-D electrostatic code. The system is identical to those described in the midterm assignment, with the following changes.

- INDX = 10 (Number of grids = 1,024)
- NPX = 80,000
- AX $\in$ [0.0,0.866667]
- No beam particles
- DELTAV = 0.2 (half width of the tagged-particle)

The length and the number of particles are quadrupled so the sample size of particles can be larger, but the averaged separation between particles remain the same, hence the diffusion and drag coefficients remain the same. The parameter **ax** is reduced 0.86 to 0.1 to allow the simulation particles would "look" more like sheets. PIC codes are different than the sheet models used in Dawson[4] because PIC codes uses grids to reduce the number of calculations necessary to calculate particle interactions. The basic plasma parameters are:

- $\omega_p = 1.0$
- $\lambda_D = 2.0$
- $\delta = 0.0128$

And the above plasma parameters gives the stopping time $1/\tau_0$ of

$$\frac{1}{\tau_0} = 0.001276$$

$\boxed{\tau_0 \sim 784}$

# 3 Experimental Data

## Experimental Considerations

- Finite-Sized Particle Effects
- "Knee" in the diffusion data
- Noise in the drag data
- Comparason between diffusion and drag data
- Verify the steady-state distribution function experimentally

### 3.1 Finite-Sized Particle Effects

The code **beps1** uses a quadratic spline convoluted with a gaussian of width **ax** for the shape of its particles. Consequently, the results are slightly different than those obtained using the sheet model. A series of runs were made using $ax \in [0.0, 0.866667]$ to understand the effects of finite sized particles.

Runs were made to measure the diffusion and the drag coefficients for **vsamp** = 0.6667, and **deltavs**=0.2. The slopes are all taken for early time $t \in [0, 100]$. The following are the results of the run

| Diffusion Coefficient vs. AX | |
|---|---|
| AX | B(v) |
| Theory | 0.0102 |
| 0.000 | 0.00704 |
| 0.100 | 0.00699 |
| 0.300 | 0.00674 |
| 0.86667 | 0.00585 |

| Drag Coefficient vs. AX | |
|---|---|
| AX | A(v) |
| Theory | -0.00085004 |
| 0.000 | -0.00069970 |
| 0.100 | -0.00068636 |
| 0.300 | -0.00059188 |
| 0.86667 | -0.00030118 |

The results obtained using finite-sized particles are different than those obtained with sheets, however, we should be able to use the following model for our drag and diffusion coefficients $A(v)$ and $B(v)$.

$$A(v) = -S\frac{\omega_p \delta}{(2\pi)^{1/2}\lambda_D}v$$

$$B(v) = S \cdot \{\frac{\delta \overline{v}}{(2\pi)^{1/2}}\}\omega_p^2$$

$$\tau = \frac{1}{S}\tau_0$$

Where $S$ is a constant determined by the particle's shape function. The above model is chosen because the Maxwellian with thermal velocity $\overline{v}$ still satisfies the time-independent Fokker-Planck equation.

## 3.2 Preliminary (Zeroth-Iteration) Data

The following are results obtained using $ax = 0.0$

| Experimental Results | | | |
|---|---|---|---|
| VSAMP | A(v) | B(v) ($t \in [0, 50]$) | B(v) ($t \in [50, 300]$) |
| 0.0 | N/A | 0.0070039 | 0.00309743 |
| 0.66667 | -0.0006997 | 0.00704202 | 0.00305529 |
| 1.33333 | -0.0011057 | 0.00652118 | 0.00417278 |
| 2.00000 | -0.00232239 | 0.00747389 | 0.00412493 |
| 3.00000 | -0.00379532 | 0.00656611 | 0.00397373 |
| 4.00000 | -0.00467478 | 0.00695468 | 0.00501127 |

Recall, from the Brownian particle problem, that a Maxwellian of the form $exp(-\frac{v^2}{2\overline{v}^2})$ is a time-independent solution of the equation:

$$\frac{\partial}{\partial v}\frac{v}{\tau}f(v) + \frac{\partial^2}{\partial v^2}\frac{\overline{v}^2}{\tau}f(v) = 0$$

And since $B(v)$ is independent of the initial velocity of the tagged particles, then we expect our Fokker-Planck equation to be of the same form as above. Hence, assuming the Fokker-Planck equation of the Brownian form, then our job is to simply to find $1/\tau$, the inverse damping time.

## 3.3 Stopping Time via Diffusion Data

Using B(v) in the early time ($t \in [0, 50]$), the simulation gives the stopping time $1/\tau$:

$$1/\tau_{B(v)} = 0.000863947 \pm 0.00008542162$$

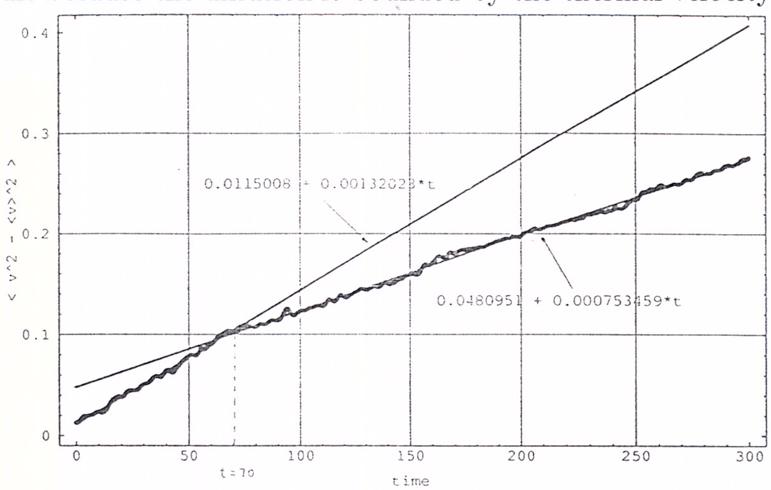
Repeat the same calculation for the diffusion data for later time ($t \in [50, 300]$), we find:

$$1/\tau_{B(v)} = 0.00048823 \pm 0.0000997764$$ → not used (see below)

In the following calculations, we will use the data in the early time only. Recall that the diffusion coefficient:

$$B(v) = \frac{\overline{v}^2}{\tau}$$

does not take into account that the standard deviation of velocity will asymptotically reach the thermal velocity $\overline{v}$ Consequently, the diffusion rate should approach zero asymptotically. I believe that the experimental value for $1/\tau$ evaluated for large time is smaller than the theoretical result because the diffusion is bounded by the thermal velocity of the plasma.




- Time History of $\overline{v^2} - \overline{v}^2$ for VSAMP=0.0, DELTAVS=0.2
- There are 2 slopes
- Early Time: (t between 0 and 70) slope = 0.00132023
- Later Time: (t between 70 and 300) slope = 0.000753459

## 3.4 Stopping Time via Drag Data

Similarly, we can obtain an experimental value of $1/\tau$ by finding the slope of the $A(v)$ curve. Using the drag data, we find another value for for the stopping time $\tau$ of the plasma,

$$1/\tau_{A(v)} = 0.001150 \pm 0.0001607$$

So, the two data do not agree with one another. In the next section, we will try to understand the source of the error and modify the diagnostics such that the error will be minimized.

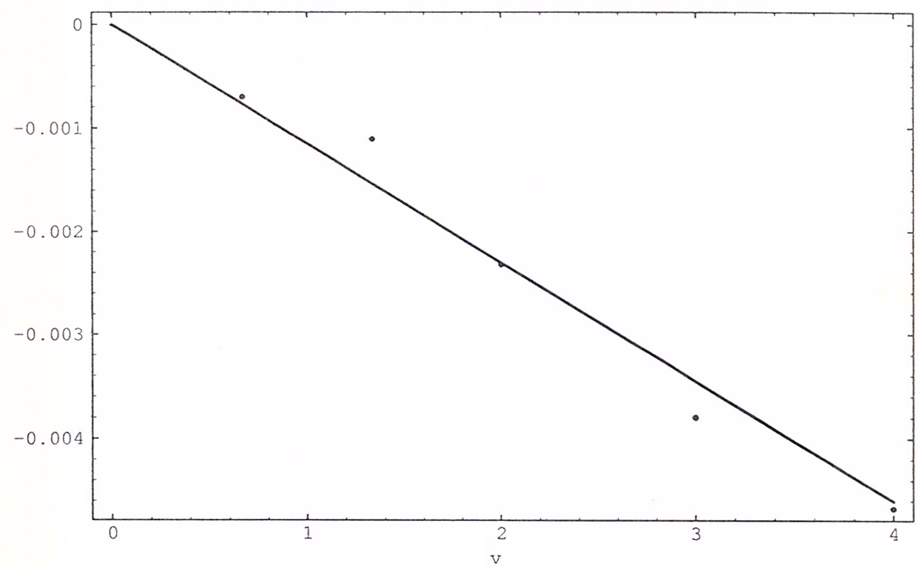



Fig 1: Drag coefficient A( v ) as a function of the initial velocity. The straight line is the least-square fit of the experimental data. The stopping time obtained using the drag data is approximately 25% larger than those obtained using the diffusion data.

### 3.5 Improved Drag Measurements

As we have seen from the previous section, the $1/\tau$ calculated using the drag data does not agree with data obtained using the diffusion data. One of the reason for it is that the error in the drag measurements is comparable to the velocity deviation in the "window" of our measurement.

Assuming the diffusion coefficient $B(v)$ is correct, we can estimate the velocity error and the systematic deviation as followed. Using central-limit theorem, the error in velocity for N particles is:

$$\epsilon v = \frac{1}{\sqrt{N}}\sqrt{B(v)\Delta t}$$

where $\Delta t$ is the width of our "window". The deviation in velocity due to the systematic drag is simply:

$$\Delta v = A(v)\Delta t$$

Recall, $A(v) = B(v)/(2\overline{v}^2)v$, for short time intervals, we can assume that the velocity is simply the initial velocity of the tagged particles, or

$$\Delta v = \frac{B(v)}{2\overline{v}^2}(vsamp)\Delta t$$

or

$$(\frac{\overline{v}}{vsamp}) \cdot \sqrt{(\frac{2\tau}{\Delta t})} << N$$

We postulate that the drag data is inadequate in our simulation, this assumption is made based on the fact that the drag data are much more noisy than the diffusion data and we believe that the majority of the error comes from the drag data. As a result, a new set of runs were made where the sample size is increased by a factor of 100, corresponding to reducing the random deviation of the drag data by a factor of 10.

The following table was made using:

- $B(v) = 0.0008$
- $\Delta t = 100.0$
- $\overline{v} = 2.0$

| Statistics for the New Drag Runs | | | |
|---|---|---|---|
| VSAMP | N | $\epsilon v$ (error) | $\Delta v$ systematic deviation. |
| 0.666 | 603,000 | 0.000364 | 0.0100 |
| 1.333 | 510,000 | 0.000396 | 0.0200 |
| 2.000 | 387,000 | 0.000455 | 0.0300 |
| 3.000 | 207,000 | 0.000622 | 0.0450 |
| 4.000 | 86,000 | 0.000964 | 0.0600 |

The maximum relative random deviation occurs at **VSAMP** = 0.666, where the random deviation is about 3.7% of the systematic error. With 100 times less particle, the relative error was approximately 40%, which explains why the two data do not agree with each other.

Using the new code, we obtain the following result.

| Drag Coefficient vs. VSAMP | |
|---|---|
| VSAMP | A(v) |
| 0.6667 | -0.000596943 |
| 1.3333 | -0.00127315 |
| 2.0000 | -0.00163001 |
| 3.0000 | -0.00260031 |
| 4.0000 | -0.0035776 |

The new data produces the stopping time of

$$1/\tau_{A(v)} = 0.00088529 \pm 0.000020306$$

This agrees very well with the stopping time obtained using the diffusion data.

$$1/\tau_{B(v)} = 0.000863947 \pm 0.00008542162$$

Therefore, the Maxwellian with thermal velocity $\overline{v} = 2.0$ indeed satisfies the time-independent Fokker-Planck equation. Furthermore, the shape-factor constant $S$ is:

$$S = 0.6938$$

And the stopping time of the plasma is:

$$\tau = 1132.6(1/\omega_p)$$

## 4 Landau Damping

Using the potential post-processor, we can look at the Landau damping of the waves from resonance particle motion. Using linear theory, we can find the real and the imaginary (damping) part of the frequency:

Real part:

$$\omega \approx \sqrt{\omega_p^2 + 3 \cdot (kv)^2}$$

Imaginary part:

$$\gamma = \frac{\pi}{2} \omega \frac{{\omega_p}^2}{k^2} f_0'(\frac{\omega}{k})$$

| Real Frequencies for Modes 6-12 | | | |
|---|---|---|---|
| Mode | K | $\omega_{th}$ | $\omega_{sim}$ |
| 6 | 0.1473 | 1.12 | 1.14 |
| 7 | 0.1718 | 1.16 | 1.15 |
| 8 | 0.1963 | 1.21 | 1.21 |
| 9 | 0.2209 | 1.26 | 1.37 |
| 10 | 0.2454 | 1.31 | 1.37 |
| 11 | 0.2700 | 1.37 | 1.37 |
| 12 | 0.2945 | 1.43 | 1.40 |

| Phase Velocities for Modes 6-12 | | | |
|---|---|---|---|
| Mode | K | $v_{th}$ | $v_{sim}$ |
| 6 | 0.1473 | 7.60 | 7.74 |
| 7 | 0.1718 | 6.75 | 6.69 |
| 8 | 0.1963 | 6.16 | 6.16 |
| 9 | 0.2209 | 5.70 | 6.29 |
| 10 | 0.2454 | 5.34 | 5.58 |
| 11 | 0.2700 | 5.07 | 5.07 |
| 12 | 0.2945 | 4.86 | 4.75 |

Using the measured real frequency, we will compare the theoretical damping rate with simulation

| Damping Rates for Modes 6-12 | | | |
|---|---|---|---|
| Mode | K | $\gamma_{th}$ | $\gamma_{sim}$ |
| 6 | 0.1473 | 0.0224 | 0.0275 |
| 7 | 0.1718 | 0.0349287 | 0.0266 |
| 8 | 0.1963 | 0.0656 | 0.0428 |
| 9 | 0.2209 | 0.099308 | 0.119 |
| 10 | 0.2454 | 0.12811 | 0.09318 |
| 11 | 0.2700 | 0.15013 | 0.1493 |
| 12 | 0.2945 | 0.16386 | 0.1798 |

The measured damping rate agrees acceptably well with the theoretical value. The possible sources of error in the Landau damping measurements are:

- For the smaller modes (modes 6, 7, and 8) the number of resonant particles (i.e., particles whose velocities are close to the phase velocity of the wave) are small and the data is subjected to large random fluctuation.

- For the larger modes, the wave disappears very quickly because of the Landau damping, therefore it is difficult to measure the damping rate accurately with the given step-size (**dt**=0.1).

# 5 Tagged-Particle Diagnostics

For sufficiently large number of particles, it is possible to eliminate the use of test particles (beam particles) by "tagging" particles in the background plasma. This method has the advantage of not inducing two-stream instability in the plasma. Also, if the sampled velocity is low enough, the number of tagged particles in the background is significantly larger than the number of test particles, improving the noise properties of the data.

The new input parameters for particle tagging are:

- **DELTAVS: DELTAVS** is the half-width for particle tagging.
- **VSAMP: VSAMP**(sampling velocity) is the mid-point in velocity space for particle tagging. All particles with initial velocity in the range [**VSAMP-DELTAVS, VSAMP+DELTAVS**] are tagged.

The averaged velocity of the tagged particles are saved in a file called *vavg* and the standard deviation are saved in a file called *sigmav*. Each line in the files has the time, and the averaged velocity (or the standard deviation). This will allow the data to be read in by **Mathematica**, using the command:

**ReadList["vavg",{Real,Real}]**

or

**ReadList["sigmav",{Real,Real}]**

**Mathematica** has the facility to curve-fit the averaged velocity (or the standard deviation) as a function of time, using the command **Fit[...]**.

Hence, a typical **Mathmatica** session would look like:

**In[1] = ReadList["vavg",{Real,Real}]**

**Out[1]={{0.00,...},{0.20,...},{0.40,...} ,...}**

**In[2] = Fit[Out[1],{1,x},x]**

## 6 Statistics of the Tagged Particles

Because the tagged particles are picked from the background plasma distribution, therefore, its distribution function is simply a slice of the background Maxwellian. This is different than the test-particle method that is suggested in the mid-term assignment, so it is useful to find out some basic properties of the Maxwellian "slice". For the background Maxwellian distribution:

$$f(v) = \frac{NPX}{\sqrt{2\pi\overline{v}}} e^{(-\frac{v^2}{2\overline{v}^2})}$$

We can calculate the number of tagged particles and their standard deviation in velocity, as followed, define $v_1$ =**VSAMP-DELTAVS**, $v_2$ =**VSAMP+DELTAVS**, then the number of particles is:

$$N = \frac{NPX}{\sqrt{2\pi\overline{v}}} \int_{v_1}^{v_2} dv e^{(-\frac{v^2}{2\overline{v}^2})}$$

Use the change of variable $t = \frac{v^2}{2\overline{v}^2}$, we can write the above integral in terms of incomplete gamma functions:

$$N = \frac{NPX}{2\sqrt{\pi}} (\gamma(1/2, \frac{{v_2}^2}{2\overline{v}^2}) - \gamma(1/2, \frac{{v_1}^2}{2\overline{v}^2}))$$

(See p.940 of Gradsheyn and Ryzhik for the definition of $\gamma$ [1].)

For small **DELTAVS**, we can use the mid-point rule to approximate the integral, or:

$$N \approx \frac{2 \cdot DELTAV \cdot NPX}{\sqrt{2\pi\overline{v}^2}} e^{(-\frac{VSAMP^2}{2\overline{v}^2})}$$

The averaged velocity $< v >$ is simply

$$< v > = \frac{(NPX) \cdot \overline{v}}{N\sqrt{2\pi}} (e^{(-\frac{{v_1}^2}{2\overline{v}^2})} - e^{(-\frac{{v_2}^2}{2\overline{v}^2})})$$

For small **DELTAVS**, we can assume the velocity distribution is flat around **VSAMP**, then

$$< v > \approx VSAMP$$

[1]Mathmatica does not have the function $\gamma$ so if you want to evaluate the above expression, you have to use the complement of the $\gamma$ function, and switch $v_1$ and $v_2$

The averaged velocity square $< v^2 >$ can be evaluated using the same change of variable as before.

$$< v^2 >= \frac{\overline{v}^2 \cdot NPX}{N \cdot \sqrt{\pi}}(\gamma(3/2, \frac{{v_2}^2}{2\overline{v}^2}) - \gamma(3/2, \frac{{v_1}^2}{2\overline{v}^2}))$$

For small **DELTAVS**, we assume the distribution is flat around **VSAMP**, then

$$< v^2 >= VSAMP^2 + \frac{1}{3}DELTAVS^2$$

and the variance ${\sigma_v}^2 =< v^2 > - < v >^2$ is

$${\sigma_v}^2 \approx \frac{1}{3}DELTAV^2$$

For $NPX = 100\,000$, **DELTAVS**=0.2, $\overline{v}$=2.0, the sampling size (N) as a function of **VSAMP** is as followed:

| Sample Size vs. VSAMP | |
|---|---|
| VSAMP | N |
| 0.000 | 7,966 |
| 0.666 | 7,536 |
| 1.333 | 6,383 |
| 2.000 | 4,839 |
| 3.000 | 2,596 |
| 4.000 | 1,085 |

## 6.1 More Input Parameters: Multiple Ensembles

As shown earlier, the drag data needs a significantly larger number of samples in order for the statistical error to be significantly smaller than the systematic deviation of the velocity. As a result, new diagnostic varibles are needed to take multiple number of the samples in one simulation and take average over multiple sets of particles. The newest set of variables are:

- **NENS**: **NENS** is the number of ensembles (sets of tagged particles) in the run. The total number of particles in the sample is $NENS \cdot N$, where $N$ is the sample size calculated in the last section.

- **NSAMP**: **NSAMP** describes the number of data points per each set of tagged particles.

- **NDELAY**: **NDELAY** describes the number of time-steps between tagging a new set of particles. **NDELAY** should be much greater than one in order to avoid tagging the same particles over and over.

## References


[1] W. H. Press, B. P. Flannery, S. A. Teukolsky, W. T. Vetterling in *Numerical Recipes*, (Cambridge University Press, Cambridge), 1986.

[2] C. K. Birdsall, A. B. Langdon, *Plasma Physics via Computer Simulation*, (McGraw-Hill Book Company, New York), 1985.

[3] J. M. Dawson, *Thermal Relaxation in a One-Species, One-Dimensional Plasma* **Ph. Fl.** 7, p. 419.(1964)

[4] J. M. Dawson, *One-Dimensional Plasma Model* **Ph. Fl.** 5, p. 445.(1962)

[5] J. M. Dawson, *Collisions in a Plasma of Finite-Size Particles* **Ph. Fl.** 5, p. 445.(1962)

Mammoth

Guest Name: Frank Tsung
1079 Gelnhaven Drive
Pacific Palisades, CA 90272 USA

Room #: V2222
Folio#: R1F0405F
Group #: 33911
Guests: 1
Clerk: JRIX

Arrive: 06/18/23 Time: 19:22 Depart: 06/23/23 Time: 10:38 Stat: HIST

| Date | Description | Reference | Comment | Charges | Credits |
|---|---|---|---|---|---|
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| 06/18/23 | ROOM CHARGE | V2222 | | $235.00 | |
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| 06/18/23 | CIVIC FEE | Recur 207t | Recurring: Tsung V2222 | $2.60 | |
| 06/18/23 | TOURISM ASSMNT | Recur 207t | Recurring: Tsung V2222 | $2.60 | |
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| 06/18/23 | MMCF DONATION | Recur 307 | Recurring: Tsung V2222 | $1.00 | |
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| 06/20/23 | OCCUPANCY TAX | Recur 207t | Recurring: Tsung V2222 | $33.80 | |
| 06/20/23 | MMCF DONATION | Recur 307 | Recurring: Tsung V2222 | $1.00 | |
| 06/21/23 | ROOM CHARGE | V2222 | | $235.00 | |
| 06/21/23 | RESORT FEE | Recur 207 | Recurring: Tsung V2222 | $25.00 | |
| 06/21/23 | CIVIC FEE | Recur 207t | Recurring: Tsung V2222 | $2.60 | |
| 06/21/23 | TOURISM ASSMNT | Recur 207t | Recurring: Tsung V2222 | $2.60 | |
| 06/21/23 | OCCUPANCY TAX | Recur 207t | Recurring: Tsung V2222 | $33.80 | |
| 06/21/23 | MMCF DONATION | Recur 307 | Recurring: Tsung V2222 | $1.00 | |
| 06/22/23 | ROOM CHARGE | V2222 | | $235.00 | |
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| 06/22/23 | CIVIC FEE | Recur 207t | Recurring: Tsung V2222 | $2.60 | |
| 06/22/23 | TOURISM ASSMNT | Recur 207t | Recurring: Tsung V2222 | $2.60 | |
| 06/22/23 | OCCUPANCY TAX | Recur 207t | Recurring: Tsung V2222 | $33.80 | |
| 06/22/23 | MMCF DONATION | Recur 307 | Recurring: Tsung V2222 | $1.00 | |
| 06/23/23 | PAY VISA | 06239537362 | ************4104 052941 | $0.00 | ($1,201.00) |
| | | | Sub Totals: | $1,500.00 | ($1,500.00) |

Folio Balance: $0.00

Guest Signature: ______________________