$$\boxed{\frac{\partial}{\partial t}\tilde{n}(\vec{r};t) = -i\omega_p\,\tilde{n}(\vec{r};t) + \frac{\partial}{\partial t}A(\vec{r};t)\,e^{-i\omega_p t}}$$

- $$\frac{\partial^2}{\partial t^2}\tilde{n} = -i\omega_p\left[-i\omega_p\,\tilde{n}(\vec{r};t) + \frac{\partial}{\partial t}A(\vec{r};t)\,\boxed{e^{-i\omega_p t}}\right] + \frac{\partial^2}{\partial t^2}A(\vec{r};t)\,e^{-i\omega_p t}$$
$$-i\omega_p\frac{\partial}{\partial t}A(\vec{r};t)\,e^{-i\omega_p t}$$

- $-i\omega_p$ term dominates

$$\boxed{-\omega_p^2\,\tilde{n}(\vec{r};t) - 2i\omega_p\frac{\partial}{\partial t}A(\vec{r};t)\,e^{-i\omega_p t} - 3\bar{v}_e^2\,\nabla^2 A(\vec{r};t) + \omega_p^2\,\tilde{n}(\vec{r};t) = 0}$$

$$-2i\omega_p\frac{\partial}{\partial t}A(\vec{r};t) - 3\bar{v}_e^2\,\nabla^2 A(\vec{r};t) = 0$$

is Schrödinger's Eq.

- Simplest NL. correction is.

$$\boxed{\omega_p^2 \longrightarrow (\omega_p^2)_0 + \alpha|A|^2}$$

which gives interesting effects such as solitons.

Low freq waves from moment description. (Recall $\omega = 0$ solution)

- electron response is.

$$m_e\frac{d}{dt}\underline{v}_e = e\nabla\phi - \frac{1}{n_e}\nabla P_e$$

as $\omega \to 0$, $m\frac{d}{dt}$ is small $\longrightarrow$ $0 \approx e\nabla\phi - \frac{1}{n_e}\nabla P_e$

for low freq. waves, our model for pressure is.

$$\boxed{P_e = n_e T_e \quad [\gamma = 1]}$$

$$\boxed{0 \approx e\nabla\phi - \nabla\left[T_e \ln n_e\right]}$$

$$\longrightarrow n_e = n_e^0 \exp\left[\frac{e\phi}{T_e}\right]$$

i.e., Boltzmann factor.

Q: When is this approximation valid?

i.e., we said.

$$\boxed{m\,\omega_{pe}\,\tilde{v}_e \ll \frac{T_e}{n_0}\,k\,\tilde{n}_e}$$

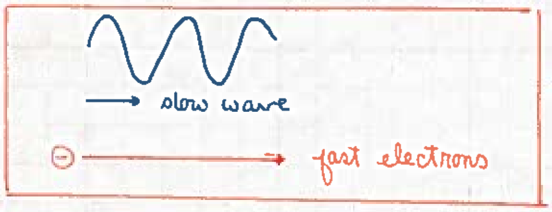
but continuity said.

$$\omega \tilde{n}_e = n_0 k \tilde{v}_e$$

$$\longrightarrow \quad m\omega \cancel{\tilde{v}_e} \ll \frac{T_e}{n_0}\,k\,\frac{n_0 k}{\omega}\,\cancel{\tilde{v}_e}$$

i.e., our approximation requires that.

$$\boxed{\left[\frac{\omega}{k}\right]^2 \ll \frac{T_e}{m} \equiv \bar{v}_e^{\,2}}$$



$\longrightarrow$ electron response.

Ion response.

- $M\dfrac{\partial}{\partial t}\underline{v_i} = -q\nabla\phi - \dfrac{1}{n_i}\nabla p_i$
- large $M \longrightarrow \nabla p_i$ is negligible (??)
- linearized ion equation.

$$\boxed{\frac{\partial}{\partial t}\underline{v_i} = \frac{q}{m}\nabla\phi}$$

- Our approximation requires.

$$\boxed{\left[\frac{\omega}{k}\right]^2 \gg \bar{v}_i^{\,2}}$$

which follows line-for-line of the electron equation

Ion cont. equation

$$\frac{\partial}{\partial t}\tilde{n}_i + \nabla\cdot(n_{0i}\,\underline{\tilde{v}}_i) = 0$$

- apply $\dfrac{\partial}{\partial t}$.

$$\frac{\partial^2}{\partial t^2}\tilde{n}_i + \nabla\cdot\left[n_{0i}\cdot\frac{q}{m}\nabla\phi\right] = 0$$

- Poisson's Eq.

$$\nabla^2\phi = 4\pi\left[e\tilde{n}_e - q\tilde{n}_i\right]$$

$$\tilde{n}_i = -\frac{1}{4\pi q}\nabla^2\phi + \frac{e}{q}\tilde{n}_e$$

$$\boxed{\frac{\partial^2}{\partial t^2}\left\{-\frac{1}{4\pi q}\nabla^2\phi + \frac{e}{q}\tilde{n}_e\right\} - \nabla\cdot\left[\frac{n_{oi}\,q}{M}\nabla\phi\right] = 0}$$

- NOTE: The above eq. is valid for non-uniform plasma.

for uniform background n_{oi}

$$\frac{\partial^2}{\partial t^2}\left[\frac{e}{q}\tilde{n}_e\right] - \left\{\frac{1}{4\pi q}\frac{\partial^2}{\partial t^2} + \frac{n_{oi}\,q}{M}\right\}\nabla^2\phi = 0$$

$$\longrightarrow \boxed{\frac{\partial^2}{\partial t^2}\left[4\pi e\tilde{n}_e\right] - \left\{\frac{\partial^2}{\partial t^2} + \omega_{pi}^2\right\}\nabla^2\phi = 0}$$

- Recall that we made.
  - $\tilde{n}_e = n_{eo}\exp\left[\frac{e\phi}{kT_e}\right]$ $\longrightarrow$ non-linear PDE
  - Good for
    - shock formation
    - solitons

- Linearized response.

$$\tilde{n}_e = n_{oe}\, e^{\frac{e\phi}{kT}}$$

$$\approx n_e^0\left[1 + \frac{e\phi}{kT} + \dots\right]$$

$$\longrightarrow \frac{\partial^2}{\partial t^2}\left[\frac{4\pi e^2 n_e^0}{T_e}\phi\right] - \left\{\cancel{\frac{\partial^2}{\partial t^2}}^{\approx 0} + \omega_{pi}^2\right\}\nabla^2\phi = 0.$$

Assume $\boxed{\frac{\partial^2}{\partial t^2} << \omega_{pi}^2, \quad \text{i.e.,} \quad \omega << \omega_{pi}}$

$$\frac{\partial^2}{\partial t^2}\phi - \left[\frac{\cancel{4\pi} n_i^0 q^2}{M}\,\frac{T_e}{\cancel{4\pi} e^2 n_e^0}\right]\nabla^2\phi = 0$$

- $e n_e^0 = q n_i^0 \longrightarrow$ quasi-neutrality.
- let $q = Ze$

$$\boxed{\frac{\partial^2}{\partial t^2}\phi - \left[\frac{Z\,T_e}{M}\right]\nabla^2\phi = 0}$$

- Define $\boxed{C_s = \sqrt{\frac{Z\,T_e}{M}}}$ Ion acoustic speed.

- Rewrite

$$C_s = \sqrt{\frac{Z\,T_e}{T_i}}\;\bar{v}_i$$

However, recall that $\boxed{\omega/k \gg \bar{V}_i}$ → our assumption

- Ion acoustic requires.

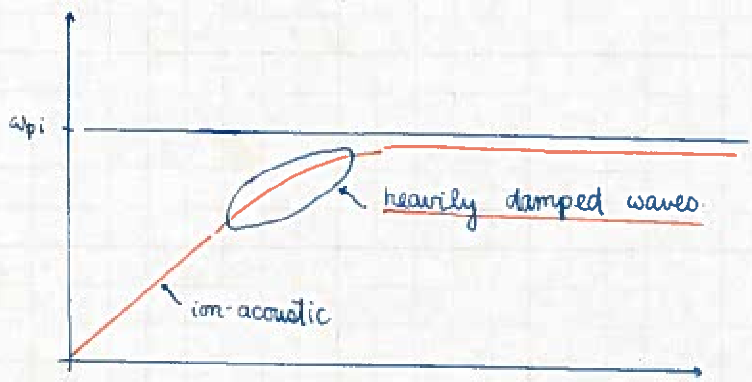
$$C_s \gg \bar{v}_i \longrightarrow \boxed{T_e \gg T_i} \rightarrow$$ Ion acoustic regime.

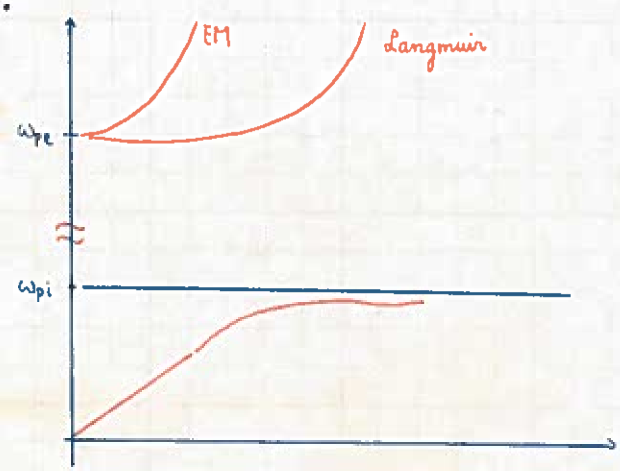
- for plane waves

$$\phi \sim e^{i(\underline{k}\cdot\underline{r} - \omega t)}$$

$$\omega^2 = k^2 C_s^2 \longrightarrow \boxed{v_g = V_\theta = C_s}$$

→ non-dispersive (linear relation between ω and k)

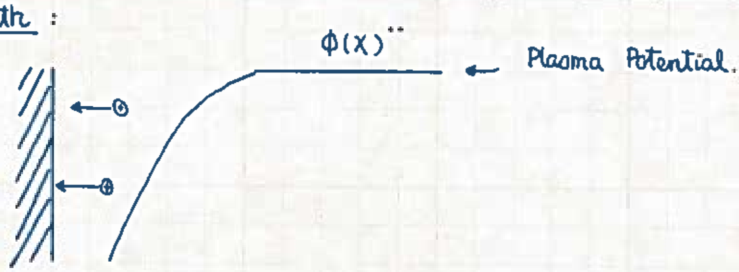


- 

- Mode coupling is allowed between EM waves + Langmuir waves e.g., a density gradient will cause such a coupling

- Only NL processes (e.g., ω^2, ω^3 effects) can cause mode transfer between Langmuir + ion - acoustic modes

- Sheath :



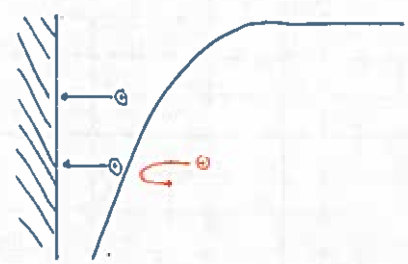
- Setting $\Gamma_e = \Gamma_i$.

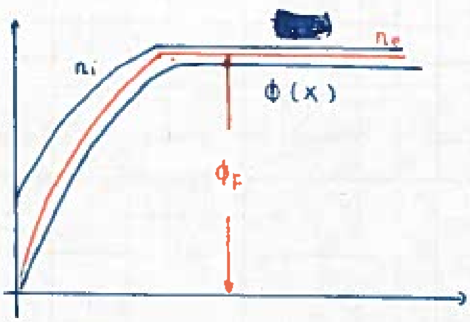
$$\phi_F = \frac{T_e}{2e}\left[\ln\left(\frac{M}{m}\right) + \ln\left(\frac{T_e}{T_i}\right)\right]$$

- for H.

$$\phi_F \sim 3.8\,\frac{T_e}{e}$$

- Sheath





- floating potential

$$\boxed{\phi_F = \frac{e T_e}{2e}\left[\ln\left(\frac{M}{m}\right) + \frac{1}{2}\ln\left(\frac{T_e}{T_i}\right)\right]}$$

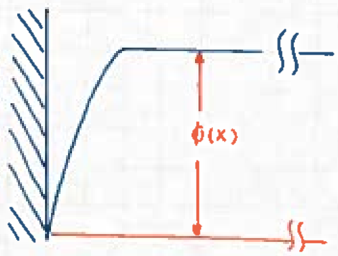
for H

$$\phi_F \approx 3.8 \cdot e T_e / e$$

Phy 222 A. 11/18

Today
Sheaths.

- Planar Sheath
  - 1-D
  - Steady state, $T_e >> T_i$
    (Ions will pick up sound speed in this case)
  - Resistive sheath vs. Reactive sheath.



- $n_e(\underline{x}) = n_e^0 \exp\left[\frac{e\phi}{T_e}\right]$
- $n_i = n_0$ for ions

- $M n_i \left[\cancel{\frac{\partial}{\partial t}\underline{v}_i}^{\,0} + \underline{v}_i \cdot \nabla \underline{v}_i \right] = -q n_i \nabla \phi - \cancel{\nabla P_i}^{\,0}$

$$\rightarrow \frac{\partial}{\partial x}\left[\frac{m v_i^2}{2} + q\phi \right] = 0$$

$$\frac{m v_i(x)}{2} + q\phi(x) = \frac{m v_i^2(\infty)}{2} + q\,\cancel{\phi(x\to\infty)}$$

- $v_i(x) = \sqrt{v_i^2(\infty) - \frac{2q}{M}\phi(x)}$

- but cont. equation states.

$$\frac{\partial}{\partial t} n_{0i} + \nabla \cdot (n_i \underline{v}) = 0$$

$$\boxed{\frac{\partial}{\partial x}\left[n_i v_i \right] = 0}$$

- $\boxed{n_i(x) = \frac{n_i(\infty)\, v_i(\infty)}{v_i(x)} = \frac{n_i(\infty)\, v_i(\infty)}{\left[v_i^2(\infty) - \frac{2q}{M}\phi(x) \right]^{1/2}}}$

Use Poisson's Eq.

$$\Longrightarrow \quad \frac{\partial^2}{\partial x^2}\phi = 4\pi e \left[n_{e0}\, e^{-(e\phi/T_e)} - \frac{Z n(\infty)\, v_i(\infty)}{\left[v_i^2(\infty) - \frac{2q}{M}\phi(x) \right]^{1/2}} \right]$$

- Looking for a physical solution.
  - Bohm criterion
  - The famous Morales scaling.

The natural units are.

$$\boxed{\begin{aligned} \bullet\ \phi &= -\frac{e\Phi}{T} \\ \bullet\ \xi &= k_D\, x \\ \bullet\ r &\equiv \frac{2Z\, T_e}{M_i\, v_i^2(\infty)} \end{aligned}}$$

$$\boxed{\bullet\ n_e^0 = Z\, n_i^0(\infty)}$$

· Poisson's Eq. becomes

$$\boxed{\frac{d^2}{d\xi^2}\phi = \frac{1}{\left(1 + r\,\phi(\xi)\right)^{1/2}} - e^{-\Phi}}$$

@ a few λ_D away, we have

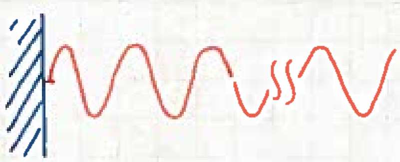
$$\left|\frac{e\Phi}{T}\right| \equiv \phi << 1$$

$$\frac{d^2}{d^2\xi}\Phi = \left[1 - \tfrac{1}{2} r\, \Phi(\xi)\right] - \left[1 - \phi\right]$$

$$= -\left(\frac{1}{2} r - 1\right)\phi$$

$$\bullet\ \boxed{\frac{d^2}{d^2\xi}\phi + \left[\frac{1}{2} r - 1\right]\Phi = 0}$$

· Case 1 : if $\frac{r}{2} > 1 \longrightarrow$ Solutions are sin, cos



$\longrightarrow$ Wall effects go all the way to the interior.

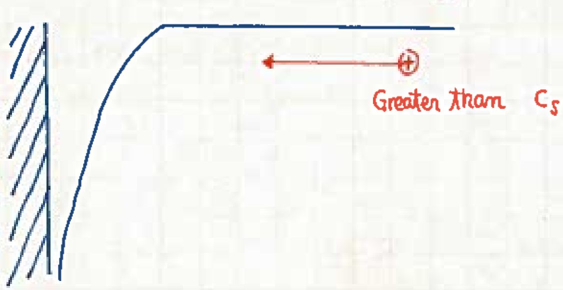
· Case 2 : $\boxed{\frac{r}{2} < 1} \longrightarrow$ Exponential Solutions.

$$\bullet\ \phi(x) \sim \exp\left[\pm\, \xi \sqrt{1 - \frac{r}{2}}\right]$$

* Stable sheath requires.

$$\frac{1}{2}\ \frac{2Z\, T_e}{M\, v_i^2(\infty)} < 1$$

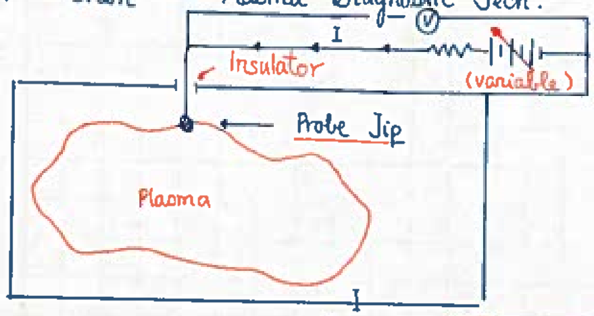
$$\longrightarrow \boxed{v_i(\infty) > \sqrt{\frac{Z\, T_e}{M}} = C_s} \longrightarrow \text{Bohm Criterion}$$

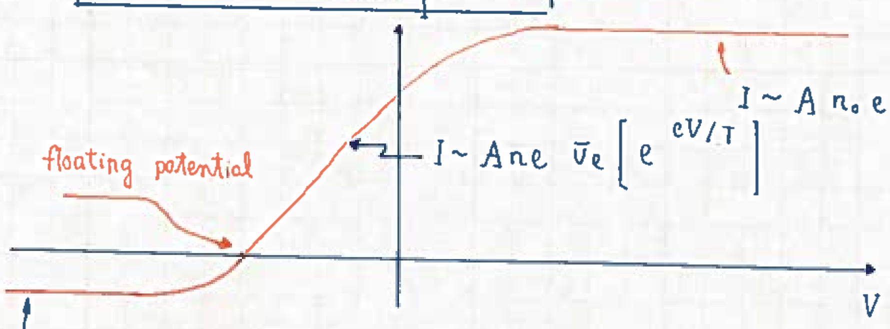


• $q\nabla\phi - \frac{\nabla\cdot P}{n_i}$: Ignoring ∇p_i ⟷ ions free-fall into the sheath.

Langmuir Probe

- Langmuir + Tonks
- F. Chen "Plasma Diagnostic Tech." p. 113 - 200





$I \sim A n_0 e \langle v_e \rangle \sim \sqrt{\frac{1}{m_e}}$

$I \sim A n e \bar{v}_e \left[e^{eV/T} \right]$

$I \sim A\, \underline{n_0 e C_s} \sim \sqrt{\frac{1}{M_i}}$

- Bohm velocity.

• $\boxed{\begin{array}{l} A n_0 e c_s \longrightarrow \text{ion saturation current.} \\ A n_0 e \langle v_e \rangle \longrightarrow \text{electron saturation current.} \end{array}}$ ⟶ Good for measuring density profile.

• In the electron (exponential) part.

- $\frac{\partial}{\partial V} \ln I = \frac{d}{\partial V} \frac{eV}{T} = \boxed{\frac{e}{T_e}}$
- $I \sim \exp\left[\frac{eV}{T_e} \right]$

• Problems.

- Noise.
- does not take care of Tail.