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Midterm Report.

- Address G. M. as "Dear Editor"
- Does abstract describe the work.
- Equation → ~~Appendix~~
- Figures (Scaling!)
- Do not be ambiguous. → Give clear arguments.
- Be precise → Don't spend an extra 6 months re-formulate the question.
- point 8 + 9 → normally do not appear on a referee report.
- Normally, 1st reference should be enough.

Today

Moment description of plasmas.

- less detailed than the kinetic description
- a.k.a. Fluid description.

1. To answer a question in detail → need to use kinetic description.

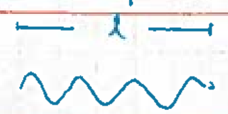
2. Idea of moments.

- Ideas is to project from the kinetic equation.

$$\boxed{\text{"}1\text{"},\ \text{"}\underline{v}\text{"},\ \text{"}\underline{v}\,\underline{v}\text{"},\ \ldots\ldots} \rightarrow \infty \text{ \# of moments.}$$

- Normally, moments are truncated.
- When does it work?

  - when $\frac{\ell}{\tau} \gg v_e$



live for a time τ

i.e., when the disturbance cannot resolve the particles.

- kinetic equation.

$$\boxed{\frac{\partial}{\partial t} f^{(s)} + \underline{v}\cdot\nabla_x f^{(s)} + \underline{a}\cdot\nabla_v f^{(s)} = 0 \left(\frac{\partial f^{(s)}}{\partial t}\right)_{\text{COLL}}}$$

(I) (II) (III) (IV)

(I) $\frac{\partial}{\partial t}\int d^3v\, f^{(s)}(\vec{x},\vec{v};t) = \frac{\partial}{\partial t} n^{(s)}(\vec{x};t)$

(II) $\int d^3v\, v\cdot\nabla_x f^{(s)}(\vec{x},\vec{v};t) =$

$$\nabla\cdot(v f^{(s)}) = f_{(s)}\underbrace{(\nabla\cdot\vec{v})}_{=0,\ \text{e.g.}\ \boxed{\frac{\partial}{\partial x}v_x = 0}} + \vec{v}\cdot\nabla f^{(s)}$$

(e.g. H, p_j ← cannonical momentum). ⟶ Vlasov Eq. is degenerate !!

· Example of Vlasov Equation.

· Consider.

$$H = \tfrac{1}{2} m v^2 - q\phi$$

← self-consistent ϕ.

· let.

$$f = f(H)$$

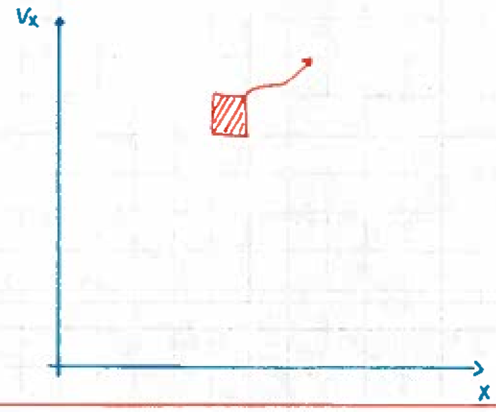
$$\cdot\ v \frac{\partial f}{\partial x} = v \frac{\partial f}{\partial H} \frac{\partial H}{\partial x} = v \cdot \left(-q \frac{\partial \phi}{\partial x}\right) \frac{\partial f}{\partial H}$$

$$\cdot\ \underline{a} \cdot \frac{\partial f}{\partial \underline{v}} = \frac{-qE}{m} \frac{\partial f}{\partial H} \frac{\partial H}{\partial v} = \frac{-qE}{m} \cdot (mv) \frac{\partial f}{\partial H}$$

$$\therefore \boxed{\begin{aligned} v \frac{\partial f}{\partial x} + a \frac{\partial f}{\partial v} &= v \left(-q \frac{\partial \phi}{\partial x}\right) \frac{\partial f}{\partial H} - \frac{qE}{m} (mv) \frac{\partial f}{\partial H} \\ &= (-qE + qE)\, v \frac{\partial f}{\partial H} = 0 \end{aligned}}$$

← regardless of what $f(H)$ is.

Sidenote: In phase space. ($f^{(s)}(\underline{x}, \underline{v}; t)$ description)



$\underline{v}$ and $\underline{x}$ are independent variables in a phase space, therefore, in the $f^{(s)}(\underline{x}, \underline{v}; t)$ description of plasma, then,

$$\boxed{\frac{\partial}{\partial x} v_x = 0}$$

$\therefore$ (I) becomes

$$\int d^3v \, \nabla_x \cdot \left[\vec{v} f_s \right] = \nabla_x \int d^3v \; \vec{v} \, f(\vec{v}; t)$$

- define.

$$\boxed{\langle \vec{v}_s \rangle = \frac{\int d^3v \; \vec{v} \; f_{(s)}(\vec{x}, \vec{v}; t)}{\int d^3v \; f_{(s)}(\vec{x}, \vec{v}; t)}}$$

hence.

$$\langle \vec{j}_{(s)} \rangle = \int \vec{v} \; f_{(s)}(\vec{x}, \vec{v}; t) \, d^3v = n(\vec{x}) \langle \vec{v}_s \rangle (\vec{x})$$

$$\boxed{(\mathrm{I}) = \nabla_x \cdot \left(n(\vec{x}) \; \langle \vec{v}_s \rangle (\vec{x}) \right)}$$

$$(\mathrm{II}) = \int d^3v \left[\frac{q}{m} \vec{E}(\vec{x}; t) + \frac{q}{mc} \underline{v} \times \vec{B}(\vec{r}; t) \right] \nabla_v f_s$$

$$- \frac{q}{m} \vec{E}(\vec{x}; t) \nabla_v f_s = \nabla_v \left(\frac{q}{m} \vec{E}(\vec{x}; t) f_s(\vec{x}, \vec{v}; t) \right)$$

$$- \underbrace{\cancel{\nabla_v \left(\frac{q}{m} \vec{E}(\vec{x}; t) \right)}}_{=0} \; f_s(\vec{x}, \vec{v}; t)$$

$f_s(\vec{x}, \vec{v}; t)$ vanishes as $\vec{v} \longrightarrow 0$, hence.

$$\boxed{\int d^3v \left[\frac{q}{m} \vec{E}(\vec{x}; t) \, \nabla_v f(\vec{x}, \vec{v}; t) \right] = 0}$$

- Consider $\nabla_v \cdot \left[(\vec{v} \times B(\vec{x}; t)) \, f_s(\vec{x}, \vec{v}; t) \right] = f_s \cdot \nabla (\vec{v} \times \vec{B}) + (\vec{v} \times \vec{B}) \cdot \nabla_v f(\vec{x}, \vec{v}$

$$\nabla_v [\,\underline{v} \times \underline{B}\,] = \underline{B} \cdot (\underbrace{\cancel{\nabla_v \times \vec{v}}}_{=0 \text{ (NO Rotation)}}) - \vec{v} \cdot (\underbrace{\cancel{\nabla_v \times \vec{B}}}_{=0 \text{ (NO } \vec{v} \text{ dependence)}})$$

$$\boxed{\begin{vmatrix} \hat{v}_x & \hat{v}_y & \hat{v}_z \\ \partial_{v_x} & \partial_{v_y} & \partial_{v_z} \\ v_x & v_y & v_z \end{vmatrix} = 0}$$

$$(\text{III}) : \int d^3v \left(\frac{\partial f_s}{\partial t}\right)_{\text{COLL}} = \left(\frac{\partial}{\partial t} n_s(\vec{x};t)\right)_{\text{COLL.}}$$

COLLISION INCLUDES:

- e^- - neutral collision
- ion - neutral collision
- other creation + destruction processes.

DOES NOT INCLUDE

- charge - particle collisions.
- coulomb scatterings.

"1" moment gives.

$$\boxed{\frac{\partial}{\partial t} n_s + \nabla \cdot \left[n_s \langle v_s \rangle (\vec{x}) \right] = \left(\frac{\partial}{\partial t} \langle n_s \rangle \right)_{\text{COLL}}} \quad \text{— } (*)$$

0 if no neutral present.

(*) is also known as the "continuity equation". for species s

Let's calculate the "$\vec{v}$" moment.

$$\int d^3v\ \vec{v} \Big[\underbrace{\frac{\partial}{\partial t} f_{(s)}}_{(\text{I})} + \underbrace{\vec{v} \cdot \nabla_x f_{(s)}}_{(\text{II})} + \underbrace{\vec{a} \cdot \nabla_v f_{(s)}}_{(\text{III})} \Big] = \underbrace{\int d^3v\ \vec{v} \left[\frac{\partial f}{\partial t} \right]_{\text{COLL.}}}_{\text{IV}}$$

$$(\text{I}) : \frac{\partial}{\partial t} \left[\int d^3v\ \vec{v}\ f_s(\vec{x},\vec{v};t) \right] = \frac{\partial}{\partial t} \langle \vec{v}_s \rangle n_s(\vec{x})$$

(II) $\longrightarrow$ this term gives great confusion, so well be careful here.

$$\int d^3v\ \vec{v}\ \vec{v} \cdot \nabla_x f_s(\vec{x},\vec{v};t) = \nabla \cdot \underbrace{\left[\int d^3v\ \underline{v}\,\underline{v}\ f_s(\vec{x},\vec{v};t) \right]}_{\text{Term having to do w/pressure.}}$$

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Introduce CM velocity $\langle \underline{v}_0 \rangle_{CM}$

$$\boxed{\langle \underline{v}_0 \rangle \equiv \frac{\sum_s m_s n_s \langle \underline{v}_s \rangle}{\sum_s m_s n_s}}$$

Define : a "CM" stress tensor $\overleftrightarrow{P}_s^{(0)}$, i.e.

$$\overleftrightarrow{P}_s^{(0)} \equiv \int d^3v \left[\vec{v} - \langle \vec{v}_0 \rangle \right] \left[\vec{v} - \langle \vec{v}_0 \rangle \right] f_s$$

- keeps track of random K.E. relative to the C.M.
- introducing CM frame gets rid of convective terms.

$$\overleftrightarrow{P}_s^{(0)} = \int d^3v \left| \underline{v}\,\underline{v}\, f_s \partial - n_s \langle \underline{v}_s \rangle \langle \underline{v}_0 \rangle - n_s \langle \underline{v}_0 \rangle \langle \underline{v}_s \rangle + n_s \langle \underline{v}_0 \rangle \langle \underline{v}_0 \rangle \right|$$

$$(\mathrm{II}) = \nabla \cdot \left\{ \overleftrightarrow{P}_s^{(0)} + n_s \left[\langle \vec{v}_s \rangle \langle \vec{v}_0 \rangle + \langle \vec{v}_0 \rangle \langle \vec{v}_s \rangle - \langle \vec{v}_0 \rangle \langle \vec{v}_0 \rangle \right] \right.$$

$$= \begin{bmatrix} v_{0x}^2 & & \\ & v_{0y}^2 & \\ & & v_{0z}^2 \end{bmatrix}$$

$$(\mathrm{III}) = \boxed{\int d^3v \; \underline{v}\,\underline{a} \cdot \nabla_v f_s = -\int d^3v \; \underline{a}\, f_s(\underline{x}, \underline{v}; t)} \longrightarrow \text{HW}$$

$$(\mathrm{IV}) = \int d^3v \; \vec{v} \left(\frac{\partial f}{\partial t} \right)_{COLL} =$$

- multiply by m_s

$$\frac{\partial}{\partial t} \underbrace{\left[m_s n_s \langle \vec{v}_s \rangle \right]}_{(\mathrm{I})} + \nabla_x \cdot \underbrace{\left\{ m_s n_s \left[\langle \underline{v}_s \rangle \langle \underline{v}_0 \rangle + \langle \underline{v}_0 \rangle \langle \underline{v}_s \rangle - \langle \underline{v}_0 \rangle \langle \underline{v}_0 \rangle \right] \right\}}_{(\mathrm{II})}$$

$$= -\nabla \cdot \underbrace{\left[m_s \overleftrightarrow{P}_s^{0} \right]}_{(\mathrm{III})} + \underbrace{\int d^3v \; \vec{a}\, f_s(\vec{x}, \vec{v}; t)}_{(\mathrm{IV})} + \underbrace{\int d^3v \; \vec{v} \left(\frac{\partial f}{\partial t} \right)_{COLL.}}_{(\mathrm{V})}$$

(I) momentum density (~~divergence of~~) ($\frac{\partial}{\partial t}$ of)

(II) (momentum density flow relative to CM)

(III) Thermal forces acting on C.M. (G.M. calls $S \rightarrow$ species S)

(IV) average force density acting on species S

(V) momentum loss due to collisions. $\longrightarrow$ collisions cannot be by the same species.

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$$\langle \vec{F}_s \rangle = \left[q_s \, \underline{E}(\underline{x}, t) + \frac{q_s}{m_s} \langle \underline{v}_s \rangle \times \underline{B}(\underline{x}, t) \right]$$

- sum over species. (MHD)
- higher moment equations.

Today
Fluid Equation.
Time - Evolution of Plasma CM's.
Waves in plasmas

1 Recall. from last time....
2nd moment eq.

$$\boxed{\begin{aligned} &\frac{\partial}{\partial t}\left[m_s n_s \langle \underline{v}_s \rangle \right] + \nabla \cdot \left\{ m_s n_s \left[2 \langle v_s \rangle \langle v_0 \rangle - \langle v_0 \rangle \langle v_0 \rangle \right] \right\} \\ &= n_s \langle \underline{F}_s \rangle + \langle \dot{\underline{P}}_s \rangle_{COLL} - \nabla \cdot \overset{\leftrightarrow}{P}{}^{0}_{s} \end{aligned}}$$

rel. to C.M.

1st moment :

$$\frac{\partial}{\partial t} \langle n_s \rangle + \nabla \cdot (n_s \langle \underline{v}_s \rangle) = 0$$

no ionization (or recombination)

- We want to sum over species....

(i) DEF :

$$\rho_m \equiv \sum_s m_s n_s$$

$$+ \quad \boxed{\langle v_0 \rangle = \frac{\sum_s m_s n_s \langle \vec{v}_s \rangle}{\sum m_s n_s}} \longrightarrow \text{CM velocity.}$$

$$\frac{\partial}{\partial t} \left(\sum_s m_s n_s \right) + \nabla \cdot \left(\sum_s m_s n_s \langle \vec{v}_s \rangle \right) = 0$$

$$\longrightarrow \boxed{\frac{\partial}{\partial t} \rho_m + \nabla \cdot (\rho_m \langle \underline{v}_0 \rangle) = 0}$$

sum the 2-nd moment equation over species.

$$\frac{\partial}{\partial t}\left[\sum_s m_s n_s \langle \underline{v}_s \rangle \right] + \nabla \cdot \left[\sum_s m_s n_s \langle \underline{v}_0 \rangle \langle \underline{v}_0 \rangle \right]$$

$$= \sum_s q_s n_s \vec{E} + \sum_s q_s \frac{n_s \langle \underline{v}_s \rangle}{c} \times \underline{B} + \underbrace{\sum_s (\dot{\underline{P}}_s)_{COLL}}_{(*)} - \nabla \cdot \sum_s \overset{\leftrightarrow}{P}{}^{0}_{s}$$

(*) $\sum_s (\dot{\underline{P}}_s)_{COLL} = 0 \longrightarrow$ system is closed.

- no bremstahlung. (wrong regime)
- no neutral collisions

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Let's call.

- $\sum_s n_s q_s = \rho_q$ → Macro-scopic charge of the plasma

- $\sum_s \frac{q_s n_s \langle \underline{v}_s \rangle}{c} = \vec{J}_q / c$ → Macro-scopic current density.

- $\nabla \cdot \sum_s \overleftrightarrow{P}_s^{\,0}$ → Pressure due to the species.
  - under thermal equilibrium, all $\overleftrightarrow{P}_s^{\,0}$ look the same...

hence, the 2-nd moment equation becomes

$$\frac{\partial}{\partial t}\left[\rho_m \langle \underline{v}_0 \rangle\right] + \nabla \cdot \left[\rho_m \langle \underline{v}_0 \rangle \langle \underline{v}_0 \rangle\right] = \rho_q \vec{E} + \frac{\underline{j} \times \underline{B}}{c} - \nabla \cdot \overleftrightarrow{P}_0$$

- $\overleftrightarrow{P}_0 = \sum_s \overleftrightarrow{P}_0^{\,s}$

- $\frac{\partial}{\partial t}\left[\rho_m \langle \underline{v}_0 \rangle\right] = \rho_m \frac{\partial}{\partial t} \langle \underline{v}_0 \rangle + \langle \underline{v}_0 \rangle \underbrace{\frac{\partial}{\partial t} \rho_m}_{\text{cont. equation}}$

$$= \rho_m \frac{\partial}{\partial t} \langle \underline{v}_0 \rangle + \cancel{\langle \underline{v}_0 \rangle \left[- \nabla \cdot \rho_m \langle \underline{v}_0 \rangle \right]}$$

- $\nabla \cdot \left[\rho_m \langle \underline{v}_0 \rangle \langle \underline{v}_0 \rangle\right] = \rho_m \langle \underline{v}_0 \rangle \cdot \underbrace{\nabla \langle \underline{v}_0 \rangle}_{\text{2nd rank.}} + \cancel{\underline{v}_0 \nabla \cdot \left[\rho_m \langle \underline{v}_0 \rangle\right]}$

→ i.e., a lower conservation law (density), makes the higher order conservation law non-symmetric., i.e.,

$$\boxed{\rho_m \underbrace{\left[\frac{\partial}{\partial t} \langle \underline{v}_0 \rangle + \langle \underline{v}_0 \rangle \cdot \nabla \langle \underline{v}_0 \rangle\right]}_{\text{Convective derivative.}} = \rho_q \underline{E} + \frac{\underline{j} \times \underline{B}}{c} - \nabla \cdot \overleftrightarrow{P}_0}$$

- In summary, we've derived the single C.M. equation of motion of a plasma.

$$\boxed{\rho_m \frac{d}{dt} \langle \underline{v}_0 \rangle = \rho_q \underline{E} + \frac{\underline{j} \times \underline{B}}{c} - \nabla \overleftrightarrow{P}_0}$$

we need to find a suitable pressure tensor...

e.g. if turbulence develops high freq. waves.

$$\rho_q \sim \rho_0 e^{-i\omega t}$$

$$\underline{E} \sim \underline{E}_0 e^{-i\omega t}$$

$$\underline{j} \sim \underline{j} e^{-i\omega t}.$$

Side-note.

$\boxed{\text{high frequency terms (after time-averaging), will manifest themselves as low-frequency forces and accelerate the plasma C.M.}}$

- ponderomotive force.
- laser heating of pellets.

Magnetic Confinement of a Plasma

$$\boxed{\frac{\underline{j} \times \underline{B}}{c} = \nabla \cdot \overleftrightarrow{P}^{\circ}}$$

i.e., one needs to develop a current in order to confine a plasma.

- Now, let's back-track and consider conservation laws for species s

$\longrightarrow$ Redefine stress-tensor.

- 2nd order moment equation

$$\frac{\partial}{\partial t}\left[m_s n_s \langle \underline{v}_s \rangle \right] + \nabla \cdot \left[m_s \int d^3v \, \underline{v}\,\underline{v}\, f_s \right] = n_s \langle \underline{F} \rangle + \langle \dot{\underline{P}}_s \rangle_{COLL}$$

let's define.

$$\overleftrightarrow{P}_s = \int d^3v \; m_s \left[\underline{v} - \langle \underline{v}_s \rangle \right]\left[\underline{v} - \langle \underline{v}_s \rangle \right] f_s$$

$$\overleftrightarrow{P}_s = \int d^3v \underset{m_s}{\wedge} \underline{v}\,\underline{v}\, f_s - m_s n_s \langle \underline{v}_s \rangle \langle \underline{v}_s \rangle - \cancel{m_s n_s \langle \underline{v}_s \rangle \langle \underline{v}_s \rangle}$$

$$+ \cancel{m_s n_s \langle \underline{v}_s \rangle \langle \underline{v}_s \rangle}$$

- $$\boxed{\left[\int m_s \, \underline{v}\,\underline{v}\, f_s \, d^3v \right] = \overleftrightarrow{P}_s + m_s n_s \langle \underline{v}_s \rangle \langle \underline{v}_s \rangle}$$

$$\frac{\partial}{\partial t}\left[m_s n_s \langle \underline{v}_s \rangle \right] + \nabla \cdot \left[\overleftrightarrow{P}_s + m_s n_s \langle \underline{v}_s \rangle \langle \underline{v}_s \rangle \right] = n_s \langle \underline{F}_s \rangle + \langle \dot{\underline{P}} \rangle_{COLL}$$

- Same prescription as before.

$$\frac{\partial}{\partial t}\left[m_s n_s \langle \underline{v}_s \rangle \right] + \nabla \cdot \left[m_s n_s \langle \underline{v}_s \rangle \langle \underline{v}_s \rangle \right]$$

$$= n_s \langle \underline{F}_s \rangle + \langle \dot{\underline{P}} \rangle$$

Use a lower conservation law, (as before)

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$$m_s n_s \frac{d}{dt} \langle \underline{v}_s \rangle = q_s n_s \underline{E} + q_s n_s \frac{\langle \underline{v}_s \rangle \times \underline{B}}{c} + \langle \dot{\underline{P}}_s \rangle_{coll} - \nabla \cdot \overleftrightarrow{P}_s$$

$$\left[\frac{\partial}{\partial t} + \langle \underline{v}_s \rangle \cdot \nabla \right]$$

2 main differences
- $\overleftrightarrow{P}_s$ replaces $\overleftrightarrow{P}_0$
- $q_s n_s$ is not trivial ($\rho_q = \sum_s q_s n_s \approx 0$)

Fluid Description of a Plasma (Magnetized)

- what information is hidden in the higher order equations?

— Use first 2 moments, i.e., no transport.

— Can only look at $\tau < \tau_{transport}$.

— characteristic phase velocity

$$\boxed{\frac{\omega}{k} \gg \langle v_s \rangle}$$

i.e., the wave does not sample the distribution function.

- far from boundaries. (no sheaths, no probes....)

Eq.'s to ~~describe~~ describe plasma. (no brackets)

- Continuity:

$$\frac{\partial}{\partial t} n_s + \nabla \cdot (n_s \underline{v}_s) = 0$$ no ionization

no brackets.

- Eq. of motion:

$$m_s n_s \left[\frac{\partial}{\partial t} \underline{v}_s + \underline{v}_s \cdot \nabla \underline{v}_s \right] = n_s q_s \underline{E} + \langle \dot{\underline{P}}_s \rangle_{coll} - \nabla \cdot \overleftrightarrow{P}_s$$

(dropping $\underline{j} \times \underline{B}$ term)

- Maxwell's Equation.

$$\nabla \cdot \underline{E} = 4\pi \sum_s n_s q_s$$

$$\nabla \times \underline{E} = -\frac{1}{c} \frac{\partial}{\partial t} \underline{B}$$

$$\nabla \cdot \underline{B} = 0$$

$$\nabla \times \underline{B} = \frac{4\pi}{c} \sum n_s q_s \langle \underline{v}_s \rangle + \frac{1}{c} \frac{\partial}{\partial t} \underline{E}$$

- however, we don't have

$$\boxed{\nabla \cdot \overleftrightarrow{P}_s} \quad !!!$$

(i) Assume $\overleftrightarrow{P}_s$ is a scalar.

$$\overleftrightarrow{P}_s = P_s \overleftrightarrow{I} \longrightarrow \nabla \cdot \overleftrightarrow{P}_s = \nabla P_s$$

(later we'll derive a relation between n, T, and P)

→ study of linear behaviors of.

- infinite (no boundaries)
- uniform. (0th order has no gradients)
- no 0-th order flow. ($\langle \underline{v}_s \rangle_0 = 0$)
- perturbed quantities are "small".

Difficulties

- Maxwell's Eq's are linear → Good.
- Cont. Eq. and the equation of motion has non-linear terms,

i.e.,

$$\boxed{\begin{array}{l} - n_s \dfrac{\partial}{\partial t} \langle \underline{v}_s \rangle \\ - n_s \langle \underline{v}_s \rangle \cdot \nabla \langle \underline{v}_s \rangle \\ - n_s \langle \underline{v}_s \rangle \end{array}}$$

In linear description, we'll let.

$$n_s = n_{0s} + \tilde{n}_s , \qquad \frac{|\tilde{n}_s|}{n_{0s}} \ll 1$$

$$\underline{v}_s = \cancel{\langle \underline{v}_s \rangle_0} + \tilde{\underline{v}}_s$$

linearize NL. terms, we get ...

$$n_s \frac{\partial}{\partial t} \underline{v}_s \simeq n_{0s} \frac{\partial}{\partial t} \tilde{\underline{v}}_s ,$$

i.e.,

$$\tilde{n}_s \frac{\partial}{\partial t} \tilde{\underline{v}}_s$$

is neglected.

$$n_s \langle \underline{V}_s \rangle \cdot \nabla \langle \underline{V}_s \rangle = 0$$

i.e., this term is intrincitly non-linear.

$$\boxed{n_s \underline{V}_s \approx n_{0s} \tilde{\underline{V}}_s}$$