I. Introduction

A. Definition of a Plasma; The Scope of Modern Plasma Physics

By a "plasma" we mean a many body system whose dynamical behavior, both equilibrium and non-equilibrium, is dominated by electromagnetic forces (i.e., Coulomb and, to a lesser extent, Lorentz forces) between charged particles rather than nuclear, atomic or molecular forces; in addition, collective or cooperative effects generally dominate binary collisional phenomena. Most of the work in this field has been concerned with gaseous plasmas, with density ranging up to 10^{17} cm^{-3} and thermal energies of 0.1 to 10^5 eV; however some attention has been given to solid state plasmas, both classical and quantum-mechanical. The plasma is almost always assumed to be neutral (equal electron and ion charge densities); but there has also been some study of non-neutral or even single component plasmas. The majority of effort has been devoted to non-relativistic plasmas; however relativistic plasmas (electron kinetic energy comparable to the rest energy, mc^2) are also of interest in view of applications to pulsars, quasars, relativistic electron beam machines (with energies of order 1 to 10 MeV, currents of order 10 to 1000 ka), and devices, like the electron ring accelerator, which may use collective plasma effects to accelerate baryons to relativistic energies.

The study of plasmas forms an exciting and challenging branch of classical physics, characterized by an unusual richness of phenomena in which both nonlinear and collective effects play a major role. Important areas of application include controlled fusion, ionospheric phenomena, space physics (the magnetosphere, solar wind, etc.), and astrophysics. In addition, it provides an excellent vehicle for training students in modern classical physics, thanks to its combination of basic and applied work; the close relation between theory and experiment; and the "relevance" of some of its applications.

As a distinct field of physics, plasma physics is still relatively young, and many approaches to an exposition of the theory have been used. Ours differs from most previous ones in two respects:

1) We emphasize first the collective effects, which are most easily explained for a plasma without an external magnetic field (an "unmagnetized" plasma) rather than the phenomena associated with single particle motion in a magnetic field (guiding center approximation, drifts, etc.). The latter are certainly important, but we feel that it is the cooperative phenomena which are the most essential aspect of the physics of plasmas.

2) We follow a deductive approach, starting with the most fundamental, microscopic formulation and deriving all others from it. While the inductive method, starting with the simplest, single fluid formulation and going on to successively more sophisticated representations, culminating in kinetic theory and a microscopic formalism, has undeniable pedagogic virtues, we believe this field has reached a state of maturity which justifies an exposition which puts in clear view the logical basis of the theory, illustrated, but not obscured, by specific applications.

Although no specific familiarity with plasma physics is assumed, many texts at the introductory level now exist, and the reader should consult these, as needed. However, two concepts in plasma physics are so fundamental that we shall briefly discuss the elementary theory of these before proceeding to the deductive exposition.

B. The Debye Length

The most important phenomenon in a plasma is Debye shielding; the most important parameter is the Debye length associated with this. We therefore begin with the simplest possible description of Debye shielding.

If a point "test charge", Q, is placed in a homogeneous plasma at temperature T, the equilibrium density of field particles having charge q will change from its undisturbed value, n, to

$$\tilde{n} = n \exp(-q\phi/T) ,$$

(1)

where ϕ is the self-consistent potential due to the charges (both test and plasma) and T is the temperature. (We shall always use energy units for T in this book, so the Boltzmann constant k will never appear.) Poisson's equation gives (assuming Q is at the origin)

$$\nabla^2\phi = -[4\pi Q\delta(\underline{r}) + 4\pi\Sigma\tilde{n}q]$$

(2)

where the summation is over the charge species of the plasma. If we can make the assumption, to be justified a posteriori, that

$$|q\phi/T| \ll 1$$

(3)

then we have

$$\nabla^2\phi - 4\pi\Sigma\left(\frac{nq^2}{T}\right)\phi = -4\pi(Q\delta(\underline{r}) + \Sigma nq)$$

(4)

For a neutral plasma, the last term on the right side vanishes and we have

$$\nabla^2\phi - K_D^2\phi = 4\pi Q\delta(\underline{r})$$

(5)

or

$$\phi = Qe^{-K_D r}/r$$

(6)

with
$$K_D{}^2 = \sum \frac{4\pi n q^2}{T}$$

In place of the Coulomb potential Q/r which the test charge Q would produce in vacuum, we have a Debye potential which drops off in a distance $K_D{}^{-1}$. The physics involved is clear. The test charge repels (attracts) plasma particles of like (unlike) sign, resulting in a neutralizing charge "cloud," whose dimension increases with T since thermal effects tend to keep the density uniform. Both ions and electrons contribute equally to the effect if $T_e = T_i$. However, we shall define the Debye wavenumber on a single species basis:

$$k_D \equiv (4\pi n_0 q^2/T)^{1/2} \tag{7}$$

Its inverse

$$L_D = k_D{}^{-1} \tag{8}$$

is the Debye length. (Both r_D and λ_D are used by some authors in place of L_D, but the second of these is ambiguous, since λ connotes wavelength and so λ_D might reasonably be assumed to denote $2\pi/k_D$ rather than $k_D{}^{-1}$.)

It remains, of course, to justify our approximation (3). We see from (6) that in a literal sense (3) cannot hold, since $\phi \to \infty$ as $r \to 0$. However, the region in which (3) fails can be expected to make a small contribution to the charge density provided that region is small compared to the Debye cloud, i.e. provided $|q\phi/T| \ll 1$ for $r \sim L_D$. This will be true provided

$$q^2 k_D/T = (k_D{}^3/4\pi n) = (4\pi n L_D{}^3)^{-1} \ll 1$$

More generally, we can say that the whole Debye shielding picture makes physical sense only if there are many particles in the Debye cloud, i.e. if

$$n L_D{}^3 \gg 1 \quad . \tag{9}$$

This dimensionless number is of transcendental importance in plasma physics. As in any many-body problem, development of a coherent theory is possible only if there is some small parameter in terms of which a perturbation expansion

can be made. The plasma parameter

$$\varepsilon_p = (nL_D^3)^{-1} \tag{10}$$

plays this role, and all of modern plasma theory is based on the assumption $\varepsilon_p \ll 1$. We note that the ratio of average potential energy to average kinetic energy in a plasma is of order ε_p:

$$\overline{V}/\overline{K} = \varepsilon_p/16\pi \tag{11}$$

It is also significant that ε_p orders the three basic scale lengths in an unmagnetized plasma, namely L_D; $L_T = e^2/T$, the "distance of closest approach"; and $L_n = n^{-1/3}$, the mean interparticle spacing:

$$L_T : L_n : L_D = \varepsilon_p/4\pi : \varepsilon_p^{1/3} : 1 \tag{12}$$

Before leaving the subject of Debye shielding, we note that it is, of course, not restricted to the simplest case -- point test charge, neutral plasma -- discussed here. In fact, so long as the basic condition $\varepsilon_p \ll 1$ which justifies (3) is satisfied, we can generalize (4) to

$$\nabla^2\phi - K_D^2\phi = S(\underline{r})$$

where S is a general source term, i.e. a superposition of external test charges plus the terms due to possible lack of plasma neutrality. The particular solution of this equation,

$$\phi(\underline{r}) = \int dr'[\exp(-K_D R)/4\pi R]S(\underline{r}')$$

$$R = |\underline{r} - \underline{r}'|$$

shows how Debye shielding manifests itself for an arbitrary source term.

C. Plasma Oscillations

Debye shielding nicely illustrates the collective character of plasma phenomena, i.e. the simultaneous interaction of many particles, but it involves thermal effects in an essential way. Plasma oscillations are the simplest example of a collective phenomenon which can occur even when thermal effects are neglected, although, as we shall see later, their effect can be very important here also. We treat electrons as a simple fluid characterized by density $n(\underline{x},t)$ and velocity $\underline{v}(\underline{x},t)$, and linearize the equations in $\underline{v}$ and $n_1 = n - n_0$, where n_0 is the uniform density of background ions, whose motion we may neglect. Then the usual continuity equation,

$$\partial n/\partial t + \nabla \cdot (n\underline{v}) = 0$$

becomes

$$\partial n_1/\partial t + n_0 \nabla \cdot \underline{v} = 0 \tag{13}$$

while the momentum equation is just

$$\partial \underline{v}/\partial t = -(e/m)\underline{E} = +(e/m)\nabla\phi \tag{14}$$

Finally, Poisson's equation gives

$$\nabla^2 \phi = 4\pi e n_1 \tag{15}$$

By taking the divergence of (14) and the time derivative of (13) we can eliminate $\underline{v}$, leaving

$$\partial^2 n_1/\partial t^2 = -(n_0 e/m)\nabla^2 \phi = -(4\pi n_0 e^2/m)n_1 \tag{16}$$

wo that n_1, $\underline{v}$ and ϕ all oscillate at the plasma frequency,

$$\omega_p = (4\pi n_0 e^2/m)^{1/2}$$

Again, this is a collective effect, no hint of which can be gleaned from a single particle description. If we define the thermal velocity by a:

$$ma^2/2 = T \qquad (17)$$

then we see that ω_p, k_D and a are related by

$$\omega_p = k_D a/2^{1/2} \qquad (18)$$

D. The Saha Equation

We shall concentrate on the physics of either fully ionized plasmas or those for which the processes of interest occur on a time scale short compared to ionization and recombination times, but it is useful to know what combination of density and temperature give a substantial degree of ionization. Conventional equilibrium statistical mechanics leads to the Saha euqation, which we quote without proof[1]: The densities of electrons, ions and neutral atoms for a monatomic system in equilibrium are related by

$$n_e n_i / n_0 = g \exp(-I/T)/\lambda_e^3 \qquad (19)$$

where I is the first ionization potential and λ_e is the electron deBroglie wavelength,

$$\lambda_e = (2\pi\hbar^2/mT)^{1/2} \qquad (20)$$

(We have assumed T large enough so that the molecule is dissociated if it is diatomic, like H_2; and small enough so that multiple ionization can be neglected for heavier atoms, like the rare gases.) The statistical factor, g, is

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[1]A simple but rigorous derivation is given in Chapter 3 of "Plasma Physics in Theory and Application", loc. cit.

defined by

$$g = 2\,Z_i/Z_0$$

where Z_i is the partition function for the ion,

$$Z_i = \sum_j \exp(-E_j/T)$$

where the E_j are energy levels of the ions, and

$$Z_0 = \sum_i \exp(W_i/T)$$

where W_i are the excitation energy levels of the atom measured from its ground state. For order of magnitude arguments, the principal dependence on density and temperature occurs through the λ_e and $\exp(-I/T)$ and we may take $g \approx 1$. While (19) can be written in many forms, one of the most useful is in terms of the degree of ionization,

$$\alpha = n_e(n_e+n_0)^{-1} = n_e/n$$

For a neutral plasma we have

$$\alpha = \frac{(\eta^2+4\eta)^{1/2} - \eta}{2} = \begin{cases} \eta^{1/2} & \eta \ll 1 \\ 1 - \eta^{-1} & \eta \gg 1 \end{cases}$$

where

$$\eta = g\,\exp(-I/T)/n\lambda_e^{\,3}$$

depends only on density and temperature. In particular, we note that the ionization can be substantial at low densities ($n\lambda_e^{\,3} \ll 1$) even if $T < I$.