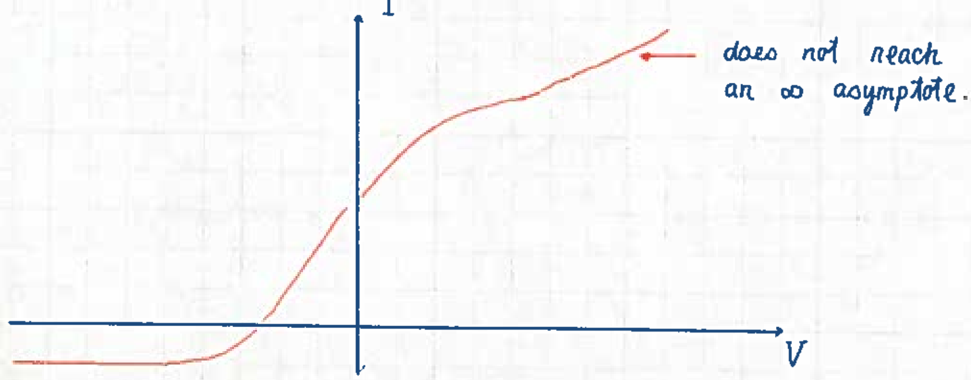
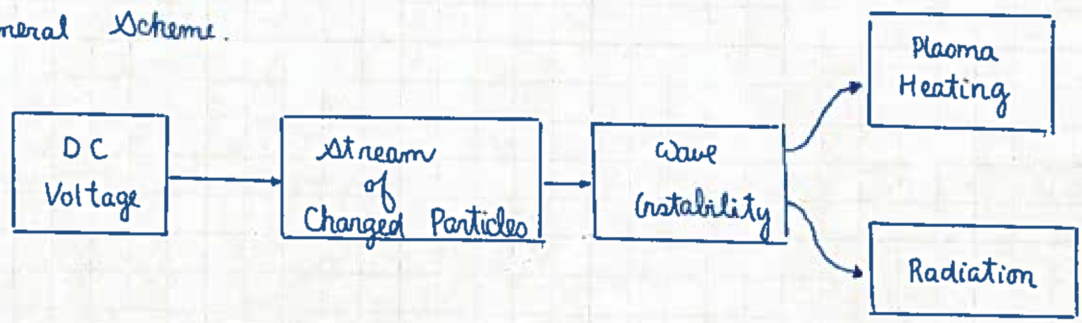
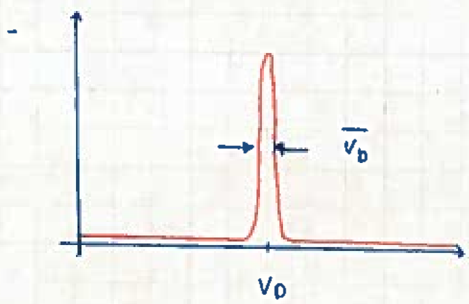
- Geometry effects the saturation current



- <u>~~Hydrogen~~ dynamic Streaming Instabilities</u>
  - μ-wave instability : klystrons. traveling wave tubes (TWT) backward wave oscillators, magnetrons.
  - modern : free electron lasers. gyrotrons, CARM's
  - General Scheme.

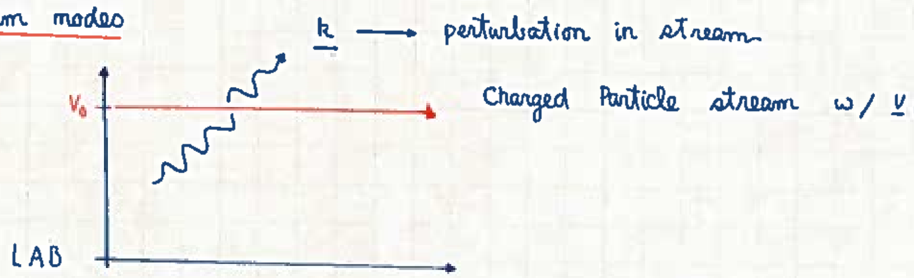


- Hydrodynamic → Moment description is OK



$$\boxed{\begin{array}{l} \cdot \text{ if } \quad V_D \gg v_b \\ f \longrightarrow \delta^3(\underline{v} - \underline{V_D}) \end{array}}$$

- <u>Beam modes</u>



- linearized eq. of motion.

$$\boxed{\frac{\partial}{\partial t} \tilde{\underline{v}} + \underline{v_0} \cdot \nabla_x \tilde{\underline{v}} = -q \nabla \phi}$$

- Cont. Equation

$$\boxed{\frac{\partial}{\partial t}\tilde{n} + \nabla\cdot\left[n_0 \underline{\tilde{v}} + \underline{v_0}\,\tilde{n} \right] = 0}$$

Now, let $\underline{\tilde{v}}, \tilde{n}, \phi \sim e^{i[\underline{k}\cdot\underline{x} - \omega t]}$

- $i\left[\underline{k}\cdot\underline{v}_0 - \omega\right]\underline{\tilde{v}} = -\frac{q i \underline{k}}{m}\phi$ (force eq.)

$$\boxed{\underline{\tilde{v}} = \frac{q\,\underline{k}\,\phi}{m\left[\omega - \underline{k}\cdot\underline{v}_0\right]}}$$

- Cont. Eq.

$$i\left[\underline{k}\cdot\underline{v}_0 - \omega\right]\tilde{n} = n_0\, i\,\underline{k}\cdot\underline{\tilde{v}}$$

$$\tilde{n} = \frac{n_0 q k^2 \phi}{m(\omega - \underline{k}\cdot\underline{v}_0)^2}$$

- Poisson's Eq.

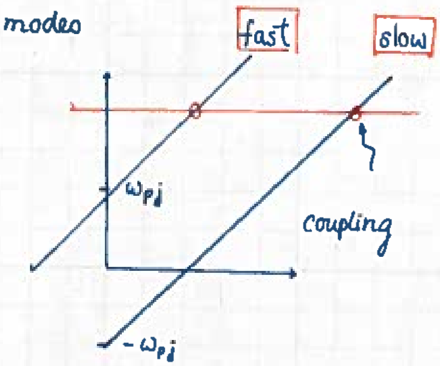
$$k^2\phi = 4\pi q\tilde{n} = \frac{4\pi n_0 q^2 k^2 \phi}{m(\omega - \underline{k}\cdot\underline{V}_0)^2}$$

$$\boxed{\left[1 - \frac{\omega_p^2}{(\omega - \underline{k}\cdot\underline{V}_0)^2}\right]k^2\phi = 0} \cdots$$

$$(\omega - \underline{k}\cdot\underline{V}_0)^2 = \omega_p^2$$

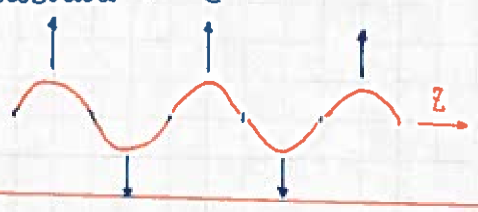
$$\boxed{\omega = \underline{k}\cdot\underline{v} \pm \omega_p} \longrightarrow \text{2 beam roots.}$$

- Last time.
    - Beam modes



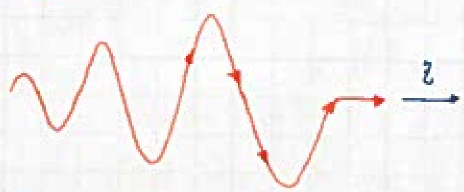
- $\boxed{\omega = \underline{k} \cdot \underline{v} \pm \omega_{pj}}$
- —— beam mode
- —— collective mode

- Instabilities.
    - Absolute : $e^{\gamma t}$

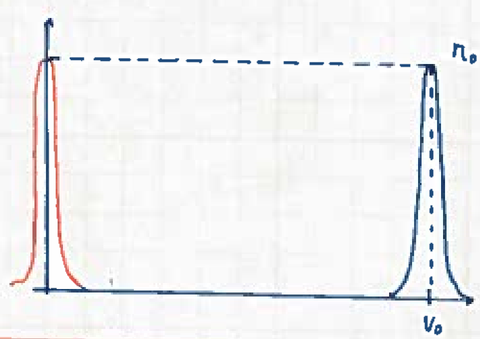


@ fixed $\underline{k}$ ← We'll be discussing instabilities of this type.

- Convective :



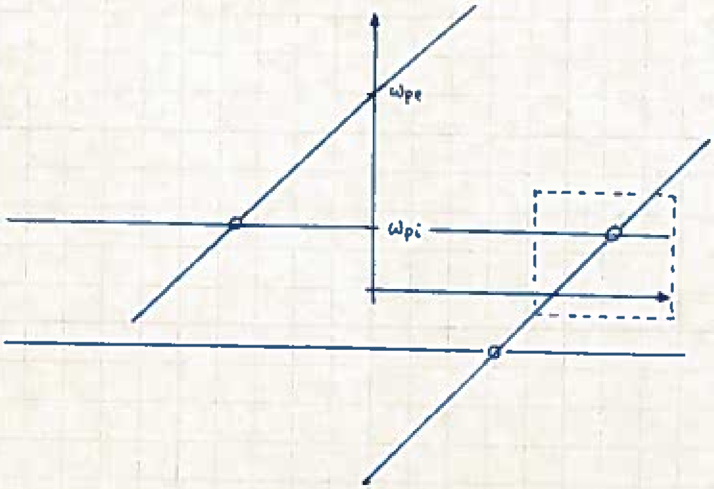
Ex : Electron - Ion 2 stream instability or "Buneman Instability"
- Phys. Rev. 115, 503 (1959)



- Assume electron stream pass ions
- Assume thermal spread is negligible.

$$\boxed{\epsilon(\omega) = 1 - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pe}^2}{(\omega - \underline{k} \cdot \underline{v}_0)^2}}$$

• (Buneman)



• without electron.

$\omega \approx \pm\, \omega_{pi}$.

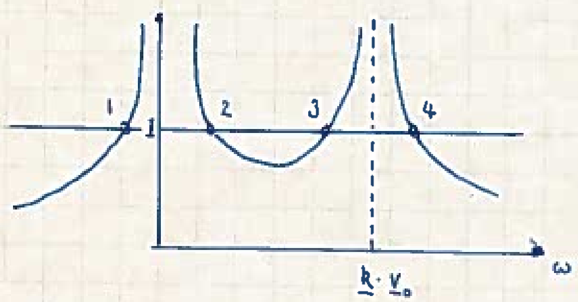
• with electrons

$\omega \approx \pm\, \omega_{pe} + \underline{k} \cdot \underline{v}_0$

• Mode coupling implies.

$$\boxed{1 = \frac{\omega_{pi}^2}{\omega^2} + \frac{\omega_{pe}^2}{(\omega - \underline{k} \cdot \underline{v}_0)^2}}$$

(i) Case 1



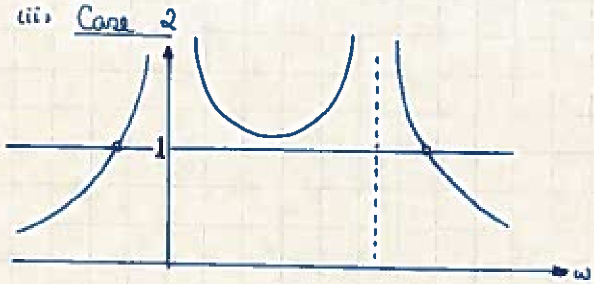
• 1, 2, 3, 4 $\longrightarrow$ 4 real roots.

• singularities @

$\omega = 0$

$\omega = \underline{k} \cdot \underline{v}_0$

(ii) Case 2



• 2 real roots, 2 imaginaries roots.

• Algebra

$$1 - \frac{\omega_{pe}^2}{(\omega - \underline{k} \cdot \underline{v}_0)^2} = \frac{\omega_{pi}^2}{\omega^2}$$

• $$\left[(\omega - \underline{k} \cdot \underline{v}_0)^2 - \omega_{pe}^2 \right] = \frac{\omega_{pi}^2}{\omega^2} (\omega - \underline{k} \cdot \underline{v}_0)^2$$

(small k)

• $$\boxed{\left[(\omega - \underline{k} \cdot \underline{v}_0) + \omega_{pe} \right] \underbrace{\left[(\omega - \underline{k} \cdot \underline{v}_0) - \omega_{pe} \right]}_{2\omega_e} = \frac{\omega_{pi}^2}{\omega^2} \left[\overset{\omega_{pe}}{(\omega - \underline{k} \cdot \underline{v}_0)^2} \right]}$$

$$\boxed{\omega^2\left[\omega - (\underline{k}\cdot\underline{v}_0 - \omega_{pe})\right] = -\frac{\omega_{pi}^2\,\omega_{pe}}{2}}$$

- Resonant mode dispersion Relation.
- Reduced a 4th-order relation to a 3rd order relation

- Introduce scaling freq.

$$\boxed{\omega_s = \left[\frac{\omega_{pi}^2\,\omega_{pe}}{2}\right]^{1/3}}$$

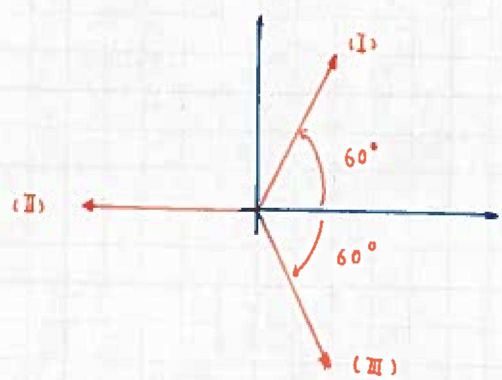
- $W = \dfrac{\omega}{\omega_s}$
- $y = \dfrac{\underline{k}\cdot\underline{V}_0 - \omega_p}{\omega_s}$

$\longrightarrow$ $\boxed{W^2\left[W - \overset{\text{detuning ?}}{y}\right] = -1}$

* Consider $y = 0 \longrightarrow \underline{k}\cdot\underline{V}_0 = \omega_{pe}$

$W^3 = -1$

$\longrightarrow \boxed{W = e^{i(2n+1)/3}}$



- (I) has $\boxed{\mathrm{Im}(\omega) > 0}$ $\longrightarrow$ i.e., it is unstable.
- (II) has $\mathrm{Im}(\omega) = 0$ $\longrightarrow$ i.e., it oscillates
- (III) has $\mathrm{Im}(\omega) < 0$ $\longrightarrow$ i.e., its a damping root

- for $y = 0$
  - $\mathrm{Re}(\omega) = \left(\dfrac{\omega_{pi}\,\omega_{pe}^2}{2}\right)^{1/3}\cos\left(\dfrac{\pi}{3}\right)$
  - $\mathrm{Im}(\omega) = \left(\dfrac{\omega_{pi}\,\omega_{pe}^2}{2}\right)^{1/3}.\sin\left(\dfrac{\pi}{3}\right)$

  i.e., $\boxed{\mathrm{Im}(\omega) > \mathrm{Re}(\omega)}$ $\longrightarrow$ purely growing.

- find fastest growing root.

$$W^2\left[W - y\right] = -1$$

apply. $\dfrac{d}{dy}$

$$2W\frac{dW}{dy}\left[W - y\right] + W^2\left[\frac{dW}{dy} - \overset{1}{\cancel{W}}{}^2\right] = 0$$

$$\frac{dW}{dy}\left[2W^2 - 2Wy + W^2\right] = W^2$$

$$\boxed{\frac{dW}{dy} = \frac{W}{3W - 2y}}$$

- Max growth rate $\longrightarrow$ $\boxed{\frac{d}{dy}(\mathrm{Im}(W)) = 0}$

$$\frac{dW}{dy} = \frac{W_r + iW_i}{(3W_r - 2y) + i3W_i}$$

$$\frac{dW_i}{dy} = \frac{\mathrm{Im}\left[(W_r + iW_i)(3W_r - 2y - 3iW_i)\right]}{(3\omega_r - 2y)^2 + (3\omega_i)^2}$$

$$0 = -\cancel{3\omega_r\omega_i} + \cancel{3\omega_r\omega_i} - 2y\omega_i$$

$$y = 0 \longrightarrow \boxed{\frac{d(\mathrm{Im}(W))}{dy} = 0}$$

- What is the phase velocity of fastest growth?

  - $v_p = \frac{\mathrm{Re}(\omega)}{k}$

$$\boxed{\begin{aligned} &- \mathrm{Re}(\omega) = \omega_s \cos(\pi/3) \\ &- k = \omega_{pe}/V_0 \end{aligned}}$$

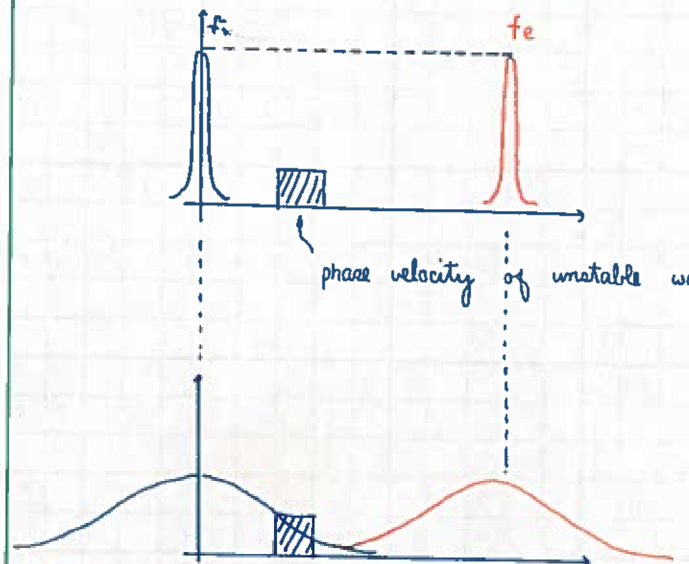
$$V_p = \frac{\omega_s \cos(\pi/3)}{\omega_{pe}} V_0$$

$$= \left[\frac{\omega_{pi}^2 \, \omega_{pe}}{2}\right]^{1/3} \frac{\cos(\pi/3)}{\omega_{pe}} V_0$$

- $$V_p = \omega_{pi}\left[\frac{1}{2}\left(\frac{M}{m}\right)^{1/2}\right]^{1/3} \frac{\cos(\pi/3)}{\omega_{pe}} V_0$$

$$V_p = \left(\frac{m}{M}\right)^{1/2}\left(\frac{M}{m}\right)^{1/6} \frac{(1/2)}{(2)^{1/3}} V_0$$

$$V_p = \underbrace{\left(\frac{m}{M}\right)^{1/3}}_{\sim 10} \frac{(1/2)}{(2^{1/3})} V_0$$



• configuration @ $t=0$.

• @ $t>0$, the electrons heats up the ions, inducing a "anonymous resistivity"

$\boxed{\text{• Ex : TTT} \longrightarrow \text{Texas Turbulent Torus}}$

• Width (in k-space) of instability.

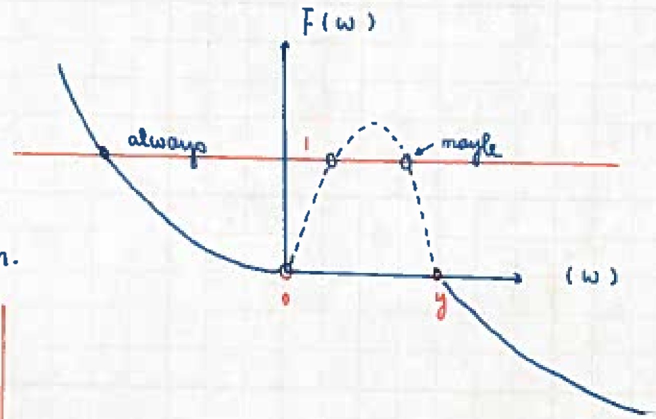
$$W^2(W - y) = -1.$$

• $F(W) = W^2(y - W)$

- Case 1 : $y > 0$.

  • 2 zero crossings

$$\boxed{\begin{aligned} W &= 0 \\ W &= y \end{aligned}}$$



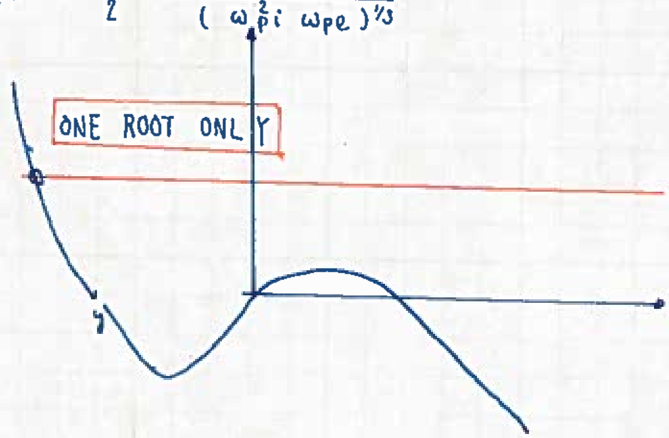
  • stability condition.

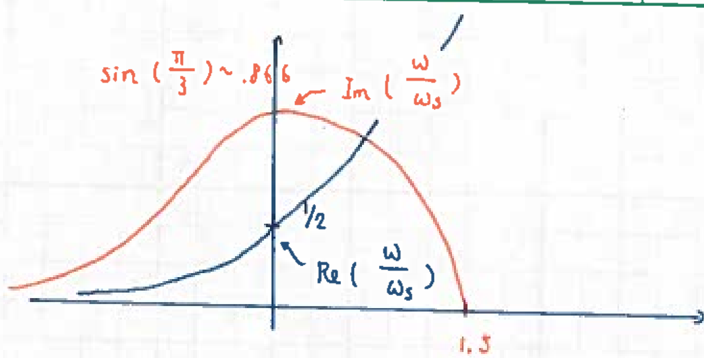
$$\boxed{\begin{aligned} \frac{dF}{dW} &= 0 \\ F &\geq 1 \end{aligned}}$$

$$\longrightarrow (\underline{k} \cdot \underline{v}_0 - \omega_{pe}) < \frac{3}{2} \frac{\underline{k} \cdot \underline{V} - \omega_{pe}}{(\omega_{pi}^2 \omega_{pe})^{1/3}}$$

- Case 2 : $y < 0$

  • Always unstable.




sin(π/3) ~ .866
Im(ω/ωs)
1/2
Re(ω/ωs)
1.5