$$\frac{\partial (P_{\perp o} + P_{\perp 1})}{\partial t} = 2\,\frac{P_{\perp o} + P_{\perp 1}}{n_o + n_1}\,\frac{\partial (n_o + n_1)}{\partial t} \tag{98}$$

which can be written

$$\cancelto{0}{\left[\frac{\partial P_{\perp o}}{\partial t} - 2\,\frac{P_{\perp o}}{n_o + n_1}\,\frac{\partial n_o}{\partial t}\right]}$$
$$-2\left[\cancelto{0}{\frac{P_{\perp 1}}{n_o + n_1}\,\frac{\partial n_o}{\partial t}} + \frac{P_{\perp 1}}{n_o + n_1}\,\frac{\partial n_1}{\partial t}\right] + \left[\frac{\partial P_{\perp 1}}{\partial t} - 2\,\frac{P_{\perp o}}{n_o + n_1}\,\frac{\partial n_1}{\partial t}\right] = 0. \tag{99}$$

The first bracket is zero from Eq. (97) (or since P_o and n_o are not functions of time). The next term is zero, since n_o is not a function of time, while the following term is neglected since it involves the product of two small quantities $P_{\perp 1}$ and n_1. So there remains, dropping n_1 as compared with n_o in the denominator,

$$\frac{\partial P_{\perp 1}}{\partial t} - 2\,\frac{P_{\perp o}}{n_o}\,\frac{\partial n_1}{\partial t} = 0\,. \tag{100}$$

A similar process carried out on Eqs. (93), (94), and (95) yields

$$\frac{\partial n_1}{\partial t} = n_o\ \vec{\nabla}\cdot\vec{V}_1 \tag{101}$$

$$\frac{\partial \vec{B}_1}{\partial t} = \vec{B}_o\ \vec{\nabla}\cdot\vec{V}_1 \tag{102}$$

$$\rho_o\,\frac{\partial \vec{V}_1}{\partial t} = -\vec{\nabla}\left[P_\perp + B^2/8\pi\right]_1 \tag{103}$$

Since the magnetic field behaves like a gas with $\gamma = 2$ as well as $P_\perp$, we can also write

$$\frac{\partial}{\partial t}\left[P_\perp + \frac{B^2}{8\pi}\right]_1 = \frac{2}{n_o}\left[P_{\perp o} + \frac{B_o^2}{8\pi}\right]\frac{\partial n_1}{\partial t}\,. \tag{104}$$

Eq. (104) can be immediately integrated to give

$$\left[p_\perp + \frac{B^2}{8\pi}\right]_1 = \frac{2}{n_0}\left[p_{\perp 0} + \frac{B_0^2}{8\pi}\right] n_1 + \tilde{n}(x, y). \tag{105}$$

Here $\tilde{n}$ is an arbitrary function of x and y . However, since the perturbed pressure and magnetic field must be zero when n_1 is zero, $\tilde{n}$ must be zero.

Substituting Eq. (105) in Eq. (103) gives

$$\rho_0 \frac{\partial \vec{v}_1}{\partial t} = -\frac{2}{n_0}\left[p_{\perp 0} + \frac{B_0^2}{8\pi}\right] \vec{\nabla} n_1. \tag{106}$$

Taking the time derivative of Eq. (101) and the divergence of Eq. (106) gives

$$\frac{\partial^2 n_1}{\partial t^2} = \frac{2}{\rho_0}\left[p_{\perp 0} + \frac{B_0^2}{8\pi}\right] \nabla^2 n_1. \tag{107}$$

Eq. (107) is a wave equation for waves propagating with velocity

$$V^2 = \frac{2}{\rho_0}\left[p_{\perp 0} + \frac{B_0^2}{8\pi}\right]. \tag{108}$$

These waves are called magnetoacoustic waves. For small magnetic fields they reduce to ordinary acoustic waves propagating at the acoustic velocity

$$V^2 = 2\frac{p_{\perp 0}}{\rho_0}. \tag{109}$$

The magnetic field increases the effective pressure in the gas. These waves show no dispersion and propagate at a constant velocity. The waves are longitudinal because $\vec{v}_1$ is parallel to $\vec{\nabla} n_1$ by Eq. (106), and hence will be in the direction of $\vec{k}$ or the direction of wave propagation if we Fourier-analyze Eq. (107).

Problem: (1) Do the wave solutions of Eq. (107) include all possible motions of the plasma?

(2) How is it that we have lost the inertia of the magnetic field (or equivalently the energy which goes into the electric field) in this calculation? Can you include it? What effect does this have on the energy conservation calculation given on page 103?

XI. The Rayleigh-Taylor Instability

As a second example of the use of our two-dimensional hydromagnetic equations we will consider the problem of the Rayleigh-Taylor instability. The situation here is shown in Fig. 39.

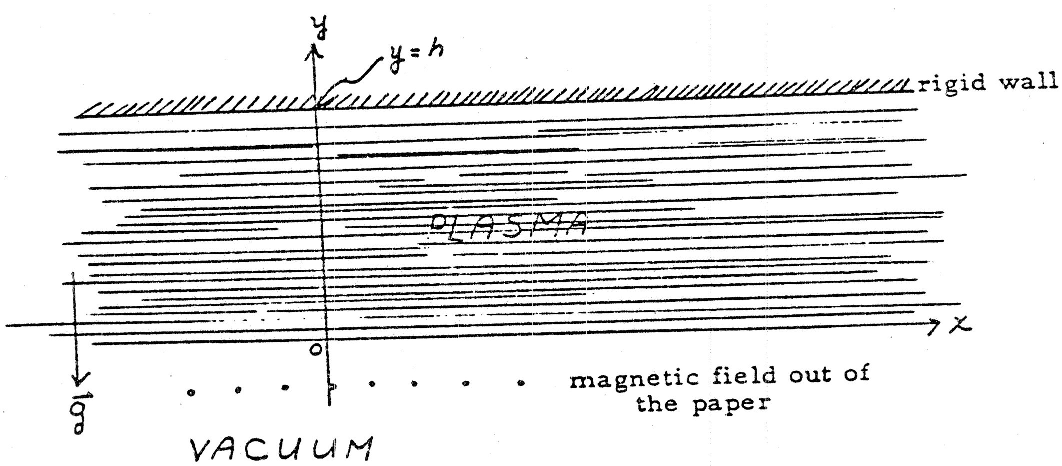

Figure 39

We have a slab of plasma of thickness h supported against a gravitational field $\vec{g}$ by a magnetic field. We take the plasma to lie entirely above the x axis and to be separated from a vacuum region below the x axis by a sharp boundary. The equilibrium conditions are obtained by setting all time derivatives and velocities equal to zero in Eqs. (93) and (96).

$$0 = -\rho \vec{g} - \vec{\nabla}\left[p_\perp + \frac{B^2}{8\pi}\right]. \tag{110}$$

Rather than treat the most general problem, we will treat the specific situation in which the equilibrium plasma density and pressure are constant within the slab. Eq. (110) then reduces to

$$\rho_0 g - \frac{1}{8\pi}\frac{\partial B^2(y)}{\partial y} = 0 \quad (y > 0) \tag{111}$$

or

$$\frac{B_\rho^2(y)}{8\pi} = \frac{B_\rho^2(0)}{8\pi} - \rho_0 g y \quad (y > 0). \tag{112}$$

At the boundary (y = 0) the magnetic field strength must jump so as to balance the plasma pressure. The vacuum field is given by

$$B_v^2 = B_\rho^2(0) + 8\pi p_{\perp 0}. \tag{113}$$

The general procedure now would be to linearize Eqs. (93) to (96) for small departures from this equilibrium and to look for wave solutions to the resultant equations. Before doing this we will, however, make one further approximation. Here we are interested in the gravitational instability. For this mode the velocities and speeds of propagation are in general small compared to the velocity of a magnetoacoustic wave. This suggests that the plasma's compressibility can play only a small role in the motion and so we look for a solution assuming an incompressible plasma. We may justify this assumption a posteriori. Our set of equations then reduces to

$$\rho_0 \frac{d\vec{V}}{dt} = -\hat{y}\rho_0 g - \vec{\nabla}\left[P_{\perp 0} + \frac{B_0^2}{8\pi}\right] \tag{114}$$

and

$$\vec{\nabla}\cdot\vec{V} = 0. \tag{115}$$

The pressure equations (96) and (92) cannot be used here because ρ does not change. The incompressibility assumption assumes that the pressure is a very strong function of ρ, and hence we can have any pressure for the same density. The pressure must be determined by making it self-consistent with the motion.

In addition to Eqs. (114) and (115) we have the boundary condition that at the wall $y = h$, the normal velocity to the wall is zero

$$v_y = 0 \qquad y = h$$

The other boundary condition at the plasma vacuum interface is given in Eq. (113).

We now linearize Eqs. (114) and (115) to obtain

$$\rho_0 \frac{\partial \vec{V}}{\partial t} = -\vec{\nabla}\,\Pi_1 \tag{116}$$

and

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0 \tag{117}$$

where

$$\Pi = \left[P_{\perp 0} + \frac{B_0^2}{8\pi}\right], \tag{118}$$

We now look for solutions which go like

$$e^{i\omega t}. \tag{119}$$

From Eq. (116) we get

$$\vec{V} = \frac{i}{\rho_0\omega}\,\vec{\nabla}\,\Pi_1 \tag{120}$$

and from Eqs. (117) and (120) we have

$$\nabla^2 \Pi_1 = 0. \tag{121}$$

The solutions to this equation of interest here are

$$\Pi_1 = \Pi_{1\pm} \, e^{ikx \pm ky} \tag{122}$$

or

$$\Pi_1 = (\Pi_{1+} e^{ky} + \Pi_{1-} e^{-ky}) e^{ikx}. \tag{123}$$

From Eqs. (120) and (123) we obtain for $\vec{v}$

$$\vec{v} = \frac{i}{\rho_0 \omega} \{ \hat{y} k e^{ikx} (\Pi_{1+} e^{+ky} - \Pi_{1-} e^{-ky})$$

$$+ \hat{x} \, ik \, e^{ikx} (\Pi_{1+} e^{+ky} + \Pi_{1-} e^{-ky}) \} \tag{124}$$

The boundary condition that $v_y = 0$ at $y = h$ gives

$$\Pi_{1+} e^{kh} - \Pi_{1-} e^{-kh} = 0 \tag{125}$$

or

$$\Pi_{1-} = \Pi_{1+} e^{2kh}. \tag{126}$$

Hence

$$\Pi_1 = \Pi_{1+} (e^{ky} + e^{k(2h-y)}) e^{ikx}. \tag{127}$$

Now at the plasma vacuum interface the boundary condition requires that

$$\Pi_\rho = \Pi_v = \frac{B_v^2}{8\pi} \tag{128}$$

since the magnetic pressure must be constant throughout this region. Before we can satisfy this condition we must find where the new boundary is.

This is obtained from the equation of motion for the boundary

$$\dot{y} = V_y \tag{129}$$

From this equation and Eqs. (124) and (126), substituting $y = 0$ to obtain the lowest order change in y, we get

$$\delta y = \frac{1}{\rho_0 \omega^2} k\, e^{ikx} \Pi_{1+} [1 - e^{2kh}]. \tag{130}$$

Now to first order $\pi = \pi_0 + \pi_1$ at the new boundary is given by

$$\Pi_v = \Pi_p = \Pi_0(y=0) + \Pi_1(y=0) + \left.\frac{\partial \Pi_0}{\partial y}\right|_{y=0} \delta y. \tag{131}$$

From Eqs. (128) and (131), and since $\pi_0 = \pi_p$, we have

$$\Pi_1(y=0) + \left.\frac{\partial \Pi_0}{\partial y}\right|_{y=0} \delta y = 0, \tag{132}$$

but

$$\left.\frac{\partial \Pi_0}{\partial y}\right|_{y=0} \delta y = -\rho_0 g \tag{133}$$

hence

$$\Pi_1(y=0) - \rho_0 g\, \delta y = 0. \tag{134}$$

Substituting the solutions we have found for π_1 and δy [Eqs. (127) and (130)] in Eq. (134) gives

$$\Pi_{1+}(1 + e^{2kh}) - \frac{g}{\omega^2} k\, \Pi_{1+}(1 - e^{2kh}) 0 \tag{135}$$

or

$$\omega^2 = g \frac{k(1 - e^{2kh})}{(1 + e^{2kh})} < 0 \quad \text{for all } k. \tag{136}$$

For large h this reduces to

$$\omega^2 = -gk. \tag{137}$$

If g is positive ($\vec{g}$ acting in the negative y direction), this gives an instability with growth rate $1/\tau = \omega$

or
$$\tau = \frac{1}{\sqrt{gk}}. \tag{138}$$

If g is negative ($\vec{g}$ acting in the positive y direction), then one gets only stable oscillations.

We may form the following physical picture of how this instability comes about. Consider the rippled plasma surface as is shown in Fig. 40.

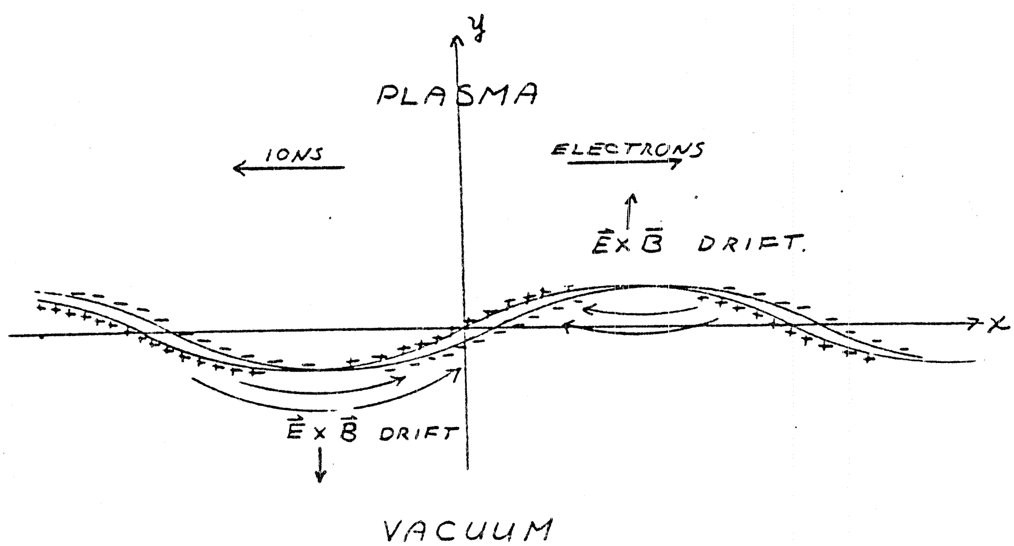

Figure 40

The gravitational field acts in the negative y direction; the magnetic field is out of the paper. Because of the gravitational field the ions drift to the left and the electrons drift to the right. If the surface is flat, no electric field develops, but if the surface is rippled the two charges are separated along the boundary by these drifts, as illustrated by the solid and dashed curves. The resultant $E \times B$ drift is in the negative y direction in the regions where the plasma has been displaced downward and is in the positive y direction where it has been displaced upward. The electric field causes the perturbation to grow.

Problem: Show that the incompressibility approximation was a good one by using π_1 to compute ρ_1 and $\dot{\rho}_1$. What is the phase relationship of all the quantities associated with the wave? Is there an $\vec{E}$ field?

XII. Alfvén Waves

As another example of the use of orbit theory we now calculate the propagation of waves parallel to $\vec{B}$ (Alfvén waves). The equilibrium situation is the same as for the case of magnetoacoustic waves; however, the propagation vector is parallel to $\vec{B}$. In this case the wave magnetic field is perpendicular to $\vec{B}$, so that we cannot use our two-dimensional hydromagnetic equation but must calculate the currents and fields directly. We assume that all quantities go like

$$e^{i(kz-\omega t)} \tag{139}$$

and that there is no variation in the x, y directions. We shall linearize about the equilibrium state, writing all quantities as $A_o + A_1$, where A_o is the equilibrium value and A_1 is the perturbation.

From $\vec{\nabla} \cdot \vec{B} = 0$ we immediately get

$$\vec{k} \cdot \vec{B}_1 = 0 \tag{140}$$

or

$$\hat{z} \cdot \vec{B}_1 = 0 \tag{141}$$

(this also implies that the magnitude of $\vec{B}$ is unchanged to first order $|B| = \sqrt{B_o^2 + B_1 \cdot B_1}$). Thus $\vec{B}_1$ has only x and y components. From Eq. (6) in section V we have, for the perturbed magnetization current,

$$\vec{j}_{M_1} = c\, i\, \vec{k} \times \vec{M}_1 = -c\, i\, \vec{k} \times \left[\left(\frac{p_\perp}{B^2}\right)_1 \vec{B}_o + \frac{p_\perp}{B^2} \vec{B}_1 \right] \qquad (142)$$

or

$$j_{M_1} = -i c \, \frac{p_{\perp o}}{B_o^2} \, \vec{k} \times \vec{B}_1 . \qquad (143)$$

The lines will be bent in the motion, so there will arise a first order curvature current. This is obtained from Eq. (8) in section V and is given by

$$\vec{j}_R = \frac{c\, p_{\parallel o}}{B_o^2} \hat{z} \times \left[(\hat{z} \cdot i \vec{k}) \frac{\vec{B}_1}{B_o} \right] \qquad (144)$$

or

$$\vec{j}_R = \frac{c\, p_{\parallel o}}{B_o^2} \, i \, \vec{k} \times \vec{B}_1 . \qquad (145)$$

Next, the first order gradient current is zero since the magnitude of $\vec{B}$ is not changed to first order

$$\vec{j}_{G_1} = 0 . \qquad (146)$$

And finally, we have the polarization current which is obtained from Eq. (7) in section V and is given by

$$\vec{j}_{P_1} = \frac{\rho_o c^2}{B_o^2} \left(- i \omega \vec{E}_{\perp 1} \right) . \qquad (147)$$

There is no $E_\parallel$ since none of the currents above is along $\vec{B}$, and further, any such $\vec{E}$ would be canceled out by motion of charges along $\vec{B}$ if it were to develop.

We now substitute in Maxwell's equations to obtain

$$i\,\vec{k}\times\vec{B}_1 = -\frac{i\omega}{c}\vec{E}_1 + \frac{4\pi}{c}\left[\frac{-ic\,P_{\perp o}}{B_o^2}\,\vec{k}\times\vec{B}_1\right] + \frac{4\pi}{c}\left[\frac{ic\,P_{\parallel o}}{B_o^2}\,\vec{k}\times\vec{B}_1 - \frac{i\,\omega\rho_o c^2}{B_o^2}\vec{E}_1\right], \tag{148}$$

$$i\,\vec{k}\times\vec{E}_1 = i\frac{\omega}{c}\vec{B}_1, \tag{149}$$

or

$$\left[1 + \frac{4\pi}{B_o^2}(P_{\perp o} - P_{\parallel o})\right]\vec{k}\times\vec{B}_1 = -\frac{\omega}{c}\left[1 + \frac{4\pi\rho_o c^2}{B_o^2}\right]\vec{E}_1. \tag{150}$$

Crossing with $\vec{k}$ and substituting $\vec{k}\times\vec{E}_1$ gives

$$\left[1 + \frac{4\pi}{B_o^2}(P_{\perp o} - P_{\parallel o})\right]k^2\vec{B}_1 = \frac{\omega^2}{c^2}\left[1 + \frac{4\pi\rho_o c^2}{B_o^2}\right]\vec{B}_1. \tag{151}$$

This gives the dispersion relation

$$\omega^2 = c^2k^2\frac{\left[B_o^2/4\pi + (P_{\perp o} - P_{\parallel o})\right]}{\left[B_o^2/4\pi + \rho_o c^2\right]}. \tag{152}$$

For negligible ρ_o and P this equation simply gives

$$\omega^2 = k^2c^2. \tag{153}$$

which is a light wave traveling along the magnetic lines of force.

For isotropic pressure, $P_{\parallel o} = P_{\perp o}$ and $\rho_o c^2 >> B^2/4\pi$, this equation reduces to

$$\omega^2 = k^2\frac{B_o^2}{4\pi\rho_o c^2}. \tag{154}$$

These waves are known as Alfvén waves. They are transverse waves propagating along $\vec{B}$.

Finally, we note that if

$$\frac{B_o^2}{4\pi} + P_{\perp o} - P_{\parallel o} < 0 \tag{155}$$

then

$$\omega^2 < 0 \tag{156}$$

and we have an instability. The instability arises because of the motion of the particles along the curved lines of force.

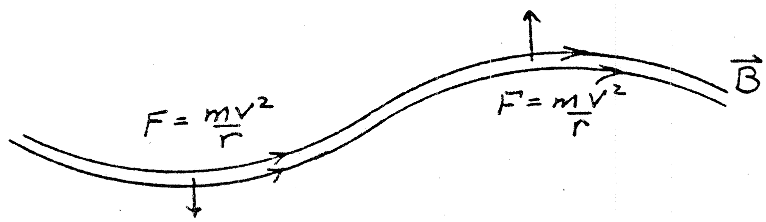

Figure 41

The centrifugal force which the particle exerts on the lines of force tends to distort them further. If this force can overcome the tension in the lines of force $(B_o^2/4\pi)$ the system is unstable.

The perpendicular pressure also supplies a restoring force. According to Eq. (143), $\vec{J}_m$, which is the source of this $P_\perp$ term, is in the direction $\vec{B}_1 \times \vec{k}$. This produces a $\vec{J} \times \vec{B}_o$ force which is in the direction $\vec{B}_1 \times \vec{k} \times \vec{B}_o$ which is opposite to $\vec{B}_1$.

Another argument demonstrating this effect is that work must be done against the perpendicular pressure to set up $\vec{B}_1$. We have from the constancy of μ that

$$\frac{W_\perp}{B} = \frac{W_{\perp o}}{B_o} . \tag{157}$$

Thus

$$\Delta W_\perp = \frac{W_{\perp 0}}{B_0} \Delta B. \tag{158}$$

But B is given by

$$B = \sqrt{B_0^2 + B_1^2} \cong B_0 \left(1 + \frac{B_1^2}{2B_0^2}\right) \tag{159}$$

(there can be no second order change to $B_z = B_0$ because $\vec{\nabla} \cdot \vec{B} = \frac{\partial B}{\partial z} = 0$).

Thus ΔB and $\Delta W_\perp$ are given by

$$\Delta B = B_1^2 / 2 B_0 \tag{160}$$

and

$$\Delta W_\perp = \frac{W_{\perp 0}}{2} \left(\frac{B_1}{B_0}\right)^2. \tag{161}$$

XIII. The Boltzmann Equation Approach

Up to this point we have been working with individual particle orbits. We have been able to combine the orbit calculations with Maxwell's equations for some simple cases to obtain gross plasma motions. However, we have not looked into the problem associated with the particles having a distribution of velocities or into the effects of collisions. We should like to investigate these effects. Our starting point will be the Boltzmann equation. Both limits of small and large collision rates can be treated from this equation (see appendix of Spitzer's book).

We start by defining a phase space for the particles. This space is a six-dimensional space; three of the coordinates are the position coordinates for a particle and the other three are the velocity coordinates. Given the position and velocity of a particle, we know its position in phase space, and vice versa, if we know the position of a particle in phase space we know its position and velocity. There is a point in phase space associated with every particle in the system. This set of points forms a dust or gas in phase space. As the particles move around and change their velocities, the dust moves around in the phase space. If the system contains a great many particles, then the dust will be very dense and we may treat it as a fluid. We may then define a density of points in phase space by the relation

$$f(\vec{r},\vec{v})\,\Delta^3 r\,\Delta^3 v = \text{Number of particles in } \Delta^3 r\,\Delta^3 v \text{ centered at } r \text{ and } v \tag{162}$$

where $\Delta^3 r$ is one element of volume in ordinary space and $\Delta^3 v$ an element of volume in velocity space. In order for this to be meaningful we must be able to choose $\Delta^3 r$ and $\Delta^3 v$ sufficiently large so that there are many particles in $\Delta^3 r\ \Delta^3 v$ and yet sufficiently small so that f does not change appreciably from one cell to the next. We assume that this is so and that f can be treated as a continuous function. Now as the particles move around, f may change with time. Since the number of particles is conserved, the changes in f must be such as to give this conservation. Let us first look at how f changes if there are no collisions between particles.

If particles flow out of a volume in phase space the density must decrease — that is, we may treat the flow of phase points like the flow of a fluid, and may write a continuity equation in this six-dimensional space. The continuity equation is

$$\frac{\partial f}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0 \tag{163}$$

where $\vec{J}$ is the current of phase points and $\vec{\nabla}$ is the six-dimensional gradient operator in phase space. $\vec{\nabla}$ is given by

$$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z} + \hat{u}\frac{\partial}{\partial u} + \hat{v}\frac{\partial}{\partial v} + \hat{w}\frac{\partial}{\partial w} \qquad (164)$$

where u, v, and w are the velocities in the x, y, and z directions, respectively. The current $\vec{J}$ is given by

$$\vec{J} = \vec{V}f$$
$$= \hat{x}u + \hat{y}v + \hat{z}w + \hat{u}a_x + \hat{v}a_y + \hat{w}a_z \qquad (165)$$

where $\vec{V}$ is the six-dimensional velocity of the phase points and a_x, a_y, and a_z are the accelerations (velocity in the u, v, w directions) in the x, y, and z directions.

If we have velocity-independent forces, then substituting these relations in Eq. (163) gives

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} + w\frac{\partial f}{\partial z} + a_x\frac{\partial f}{\partial u} + a_y\frac{\partial f}{\partial v} + a_z\frac{\partial f}{\partial w} = 0 \qquad (166)$$

or, vectorially,

$$\frac{\partial f}{\partial t} + \vec{V}\cdot\vec{\nabla}_r f + \vec{a}\cdot\vec{\nabla}_v f = 0 \qquad (167)$$

where

$$\nabla_r = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z} \qquad (168)$$

and

$$\nabla_v = \hat{u}\frac{\partial}{\partial u} + \hat{v}\frac{\partial}{\partial v} + \hat{w}\frac{\partial}{\partial w}. \qquad (169)$$

In general, if the forces are velocity-dependent, Eq. (163) cannot be written in the form of Eq. (166), but the accelerations must be left inside the differentiation. However, for the important case of magnetic forces one again obtains Eq. (166), even though the a's are functions of the velocity. To see this, consider

$$\vec{\nabla}_v \cdot \left[(\vec{V} \times \vec{B}) f \right] = \vec{\nabla}_v \cdot \left[\hat{u} (v B_z - w B_y) f \right.$$

$$\left. + \hat{v} (w B_x - u B_z) f + \hat{w} (u B_y - v B_x) f \right]. \tag{170}$$

For the $\hat{u}$ term the acceleration $[v B_z - w B_y]$ is independent of u and hence it can be carried to the other side of $\vec{\nabla}_v \cdot$. Thus

$$\vec{\nabla}_v \cdot \left[(\vec{V} \times \vec{B}) f \right] = (\vec{V} \times \vec{B}) \cdot \vec{\nabla}_v f. \tag{171}$$

Hence for electromagnetic forces, Eq. (163) becomes

$$\frac{\partial f}{\partial t} + \vec{V} \cdot \vec{\nabla}_r f + \frac{q}{m} (\vec{E} + \vec{V} \times \vec{B}) \cdot \vec{\nabla}_v f = 0. \tag{172}$$

When the particles collide with each other we must add to Eq. (172) a term $\left[\frac{\partial f}{\partial t} \right]_{coll}$ which gives the change of f due to collisions. If we also add external forces $\vec{F}$ which are not electromagnetic, then Eq. (172) becomes

$$\frac{\partial f}{\partial t} + \vec{V} \cdot \vec{\nabla}_r f + \left[\frac{q}{m} (\vec{E} + \vec{V} \times \vec{B}) + \frac{\vec{F}}{m} \right] \cdot \vec{\nabla}_r f = \left[\frac{\partial f}{\partial t} \right]_{coll} \tag{173}$$

To find the change in f due to collisions, $\left[\frac{\partial f}{\partial t} \right]_{coll}$, we must look at the details of the collisional processes. We will do this later and for the time being simply carry these terms along symbolically as $\left[\frac{\partial f}{\partial t} \right]_{coll}$.

If we have more than one species of particle then there is one