I. Introduction

A. Definition of a Plasma; The Scope of Modern Plasma Physics

By a "plasma" we mean a many body system whose dynamical behavior, both equilibrium and non-equilibrium, is dominated by electromagnetic forces (i.e., Coulomb and, to a lesser extent, Lorentz forces) between charged particles rather than nuclear, atomic or molecular forces; in addition, collective or cooperative effects generally dominate binary collisional phenomena. Most of the work in this field has been concerned with gaseous plasmas, with density ranging up to 10^{17} cm^{-3} and thermal energies of 0.1 to 10^5 eV; however some attention has been given to solid state plasmas, both classical and quantum-mechanical. The plasma is almost always assumed to be neutral (equal electron and ion charge densities); but there has also been some study of non-neutral or even single component plasmas. The majority of effort has been devoted to non-relativistic plasmas; however relativistic plasmas (electron kinetic energy comparable to the rest energy, mc^2) are also of interest in view of applications to pulsars, quasars, relativistic electron beam machines (with energies of order 1 to 10 MeV, currents of order 10 to 1000 ka), and devices, like the electron ring accelerator, which may use collective plasma effects to accelerate baryons to relativistic energies.

The study of plasmas forms an exciting and challenging branch of classical physics, characterized by an unusual richness of phenomena in which both nonlinear and collective effects play a major role. Important areas of application include controlled fusion, ionospheric phenomena, space physics (the magnetosphere, solar wind, etc.), and astrophysics. In addition, it provides an excellent vehicle for training students in modern classical physics, thanks to its combination of basic and applied work; the close relation between theory and experiment; and the "relevance" of some of its applications.

As a distinct field of physics, plasma physics is still relatively young, and many approaches to an exposition of the theory have been used. Ours differs from most previous ones in two respects:

1) We emphasize first the collective effects, which are most easily explained for a plasma without an external magnetic field (an "unmagnetized" plasma) rather than the phenomena associated with single particle motion in a magnetic field (guiding center approximation, drifts, etc.). The latter are certainly important, but we feel that it is the cooperative phenomena which are the most essential aspect of the physics of plasmas.

2) We follow a deductive approach, starting with the most fundamental, microscopic formulation and deriving all others from it. While the inductive method, starting with the simplest, single fluid formulation and going on to successively more sophisticated representations, culminating in kinetic theory and a microscopic formalism, has undeniable pedagogic virtues, we believe this field has reached a state of maturity which justifies an exposition which puts in clear view the logical basis of the theory, illustrated, but not obscured, by specific applications.

Although no specific familiarity with plasma physics is assumed, many texts at the introductory level now exist, and the reader should consult these, as needed. However, two concepts in plasma physics are so fundamental that we shall briefly discuss the elementary theory of these before proceeding to the deductive exposition.

B. The Debye Length

The most important phenomenon in a plasma is Debye shielding; the most important parameter is the Debye length associated with this. We therefore begin with the simplest possible description of Debye shielding.

If a point "test charge", Q, is placed in a homogeneous plasma at temperature T, the equilibrium density of field particles having charge q will change from its undisturbed value, n, to

$$\tilde{n} = n \exp(-q\phi/T) , \tag{1}$$

where ϕ is the self-consistent potential due to the charges (both test and plasma) and T is the temperature. (We shall always use energy units for T in this book, so the Boltzmann constant k will never appear.) Poisson's equation gives (assuming Q is at the origin)

$$\nabla^2\phi = -[4\pi Q\delta(\underline{r}) + 4\pi\Sigma\tilde{n}q] \tag{2}$$

where the summation is over the charge species of the plasma. If we can make the assumption, to be justified a posteriori, that

$$|q\phi/T| << 1 \tag{3}$$

then we have

$$\nabla^2\phi - 4\pi\sum\left(\frac{nq^2}{T}\right)\phi = -4\pi(Q\delta(\underline{r}) + \Sigma nq) \tag{4}$$

For a neutral plasma, the last term on the right side vanishes and we have

$$\nabla^2\phi - K_D{}^2\phi = 4\pi Q\delta(\underline{r}) \tag{5}$$

or

$$\phi = Qe^{-K_D r}/r \tag{6}$$

with $$K_D{}^2 = \sum \frac{4\pi nq^2}{T}$$

In place of the Coulomb potential Q/r which the test charge Q would produce in vacuum, we have a Debye potential which drops off in a distance $K_D{}^{-1}$. The physics involved is clear. The test charge repels (attracts) plasma particles of like (unlike) sign, resulting in a neutralizing charge "cloud," whose dimension increases with T since thermal effects tend to keep the density uniform. Both ions and electrons contribute equally to the effect if $T_e = T_i$. However, we shall define the Debye wavenumber on a single species basis:

$$k_D \equiv (4\pi n_0 q^2/T)^{1/2} \qquad (7)$$

Its inverse

$$L_D = k_D{}^{-1} \qquad (8)$$

is the Debye length. (Both r_D and λ_D are used by some authors in place of L_D, but the second of these is ambiguous, since λ connotes wavelength and so λ_D might reasonably be assumed to denote $2\pi/k_D$ rather than $k_D{}^{-1}$.)

It remains, of course, to justify our approximation (3). We see from (6) that in a literal sense (3) cannot hold, since $\phi \to \infty$ as $r \to 0$. However, the region in which (3) fails can be expected to make a small contribution to the charge density provided that region is small compared to the Debye cloud, i.e. provided $|q\phi/T| \ll 1$ for $r \sim L_D$. This will be true provided

$$q^2 k_D/T = (k_D{}^3/4\pi n) = (4\pi n L_D{}^3)^{-1} \ll 1$$

More generally, we can say that the whole Debye shielding picture makes physical sense only if there are many particles in the Debye cloud, i.e. if

$$nL_D{}^3 \gg 1 \quad . \qquad (9)$$

This dimensionless number is of transcendental importance in plasma physics. As in any many-body problem, development of a coherent theory is possible only if there is some small parameter in terms of which a perturbation expansion can be made. The plasma parameter

$$\varepsilon_p = (nL_D{}^3)^{-1} \tag{10}$$

plays this role, and all of modern plasma theory is based on the assumption $\varepsilon_p \ll 1$. We note that the ratio of average potential energy to average kinetic energy in a plasma is of order ε_p:

$$\overline{V}/\overline{K} = \varepsilon_p/16\pi \tag{11}$$

It is also significant that ε_p orders the three basic scale lengths in an unmagnetized plasma, namely L_D; $L_T = e^2/T$, the "distance of closest approach"; and $L_n = n^{-1/3}$, the mean interparticle spacing:

$$L_T : L_n : L_D = \varepsilon_p/4\pi : \varepsilon_p{}^{1/3} : 1 \tag{12}$$

Before leaving the subject of Debye shielding, we note that it is, of course, not restricted to the simplest case -- point test charge, neutral plasma -- discussed here. In fact, so long as the basic condition $\varepsilon_p \ll 1$ which justifies (3) is satisfied, we can generalize (4) to

$$\nabla^2\phi - K_D{}^2\phi = S(\underline{r})$$

where S is a general source term, i.e. a superposition of external test charges plus the terms due to possible lack of plasma neutrality.
The particular solution of this equation,

$$\phi(\underline{r}) = \int d\underline{r}'[\exp(-K_D R)/4\pi R]S(\underline{r}')$$

$$R = |\underline{r} - \underline{r}'|$$

shows how Debye shielding manifests itself for an arbitrary source term.

C. Plasma Oscillations

Debye shielding nicely illustrates the collective character of plasma phenomena, i.e. the simultaneous interaction of many particles, but it involves thermal effects in an essential way. Plasma oscillations are the simplest example of a collective phenomenon which can occur even when thermal effects are neglected, although, as we shall see later, their effect can be very important here also. We treat electrons as a simple fluid characterized by density $n(\underline{x},t)$ and velocity $\underline{v}(\underline{x},t)$, and linearize the equations in $\underline{v}$ and $n_1 = n - n_0$, where n_0 is the uniform density of background ions, whose motion we may neglect. Then the usual continuity equation,

$$\partial n/\partial t + \nabla \cdot (n\underline{v}) = 0$$

becomes

$$\partial n_1/\partial t + n_0 \nabla \cdot \underline{v} = 0 \tag{13}$$

while the momentum equation is just

$$\partial \underline{v}/\partial t = -(e/m)\underline{E} = +(e/m)\nabla\phi \tag{14}$$

Finally, Poisson's equation gives

$$\nabla^2\phi = 4\pi e n_1 \tag{15}$$

By taking the divergence of (14) and the time derivative of (13) we can eliminate $\underline{v}$, leaving

$$\partial^2 n_1/\partial t^2 = -(n_0 e/m)\nabla^2\phi = -(4\pi n_0 e^2/m)n_1 \tag{16}$$

wo that n_1, $\underline{v}$ and ϕ all oscillate at the plasma frequency,

$$\omega_p = (4\pi n_0 e^2/m)^{1/2}$$

Again, this is a collective effect, no hint of which can be gleaned from a single particle description. If we define the thermal velocity by a:

$$ma^2/2 = T \tag{17}$$

then we see that ω_p, k_D and a are related by

$$\omega_p = k_D a/2^{1/2} \tag{18}$$

D. The Saha Equation

We shall concentrate on the physics of either fully ionized plasmas or those for which the processes of interest occur on a time scale short compared to ionization and recombination times, but it is useful to know what combination of density and temperature give a substantial degree of ionization. Conventional equilibrium statistical mechanics leads to the Saha euqation, which we quote without proof[1]: The densities of electrons, ions and neutral atoms for a monatomic system in equilibrium are related by

$$n_e n_i/n_0 = g \exp(-I/T)/\lambdabar_e^{\,3} \tag{19}$$

where I is the first ionization potential and $\lambdabar_e$ is the electron deBroglie wavelength,

$$\lambdabar_e = (2\pi\hbar^2/mT)^{1/2} \tag{20}$$

(We have assumed T large enough so that the molecule is dissociated if it is diatomic, like H_2; and small enough so that multiple ionization can be neglected for heavier atoms, like the rare gases.) The statistical factor, g, is

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[1]A simple but rigorous derivation is given in Chapter 3 of "Plasma Physics in Theory and Application", loc. cit.

defined by

$$g = 2\, Z_i/Z_0$$

where Z_i is the partition function for the ion,

$$Z_i = \sum_j \exp(-E_j/T)$$

where the E_j are energy levels of the ions, and

$$Z_0 = \sum_i \exp(W_i/T)$$

where W_i are the excitation energy levels of the atom measured from its ground state. For order of magnitude arguments, the principal dependence on density and temperature occurs through the $\lambda\!\!\!^{-}_e$ and $\exp(-I/T)$ and we may take $g \approx 1$. While (19) can be written in many forms, one of the most useful is in terms of the degree of ionization,

$$\alpha = n_e(n_e+n_0)^{-1} = n_e/n$$

For a neutral plasma we have

$$\alpha = \frac{(\eta^2+4\eta)^{1/2} - \eta}{2} = \begin{cases} \eta^{1/2} & \eta \ll 1 \\ 1 - \eta^{-1} & \eta \gg 1 \end{cases}$$

where

$$\eta = g \exp(-I/T)/n\lambda\!\!\!^{-}_e{}^3$$

depends only on density and temperature. In particular, we note that the ionization can be substantial at low densities ($n\lambda_e^3 \ll 1$) even if $T < I$.

II. Fundamental Theoretical Formulation; the Microscopic Kinetic Equations

A. Maxwell's and Newton's Equations

We start from the simplest and most basic description of a collection of charged particles: Maxwell's equations for the electromagnetic field and Newton's equation (or its quantum mechanical equivalent, Schrodinger's equation) for the particle motion. The sources of the electromagnetic field are any external charges and currents plus those due to the plasma particles.

We shall adopt a classical point of view here, using Newton's equation for the mechanical motion and treating the electrons and ions as point particles. So long as the deBroglie wavelength, $(2\pi\hbar^2/mT)^{\frac{1}{2}}$, for each particle is much smaller than any other length in the problem, this is a good approximation. In the high density/low temperature regime where quantum mechanical effects become important, the details of the formulation must be appropriately modified for the mechanical motion and, more importantly, other expansion parameters replace ϵ_p. This regime is outside the scope of this text.

Our basic equations are then

$$m_i\dot{\underline{v}}_i = m_i\ddot{\underline{x}}_i = q_i(\underline{\mathcal{E}} + \underline{v}_i\times\underline{B}/c) \qquad i = 1,2, \ldots N_0 \tag{2.1}$$

$$\begin{aligned} \nabla\cdot\underline{\mathcal{E}} &= 4\pi(\rho_p + \rho_e) & \nabla\cdot\underline{\mathcal{E}} + \dot{\mathcal{B}}/c &= 0 \\ \nabla\times\underline{\mathcal{B}} &= 4\,(\underline{j}_p + \underline{j}_e + \dot{\underline{\mathcal{E}}}/c) & \nabla\cdot\mathcal{B} &= 0 \end{aligned} \tag{2.2}$$

where the subscripts p,e denote plasma particle and external source terms, respectively, and N_0 is the total number of particles. Since this is a microsopic theory, we need only one electric field vector, $\underline{\mathcal{E}}$ and one magnetic field vector, which we designate by $\underline{\mathcal{B}}$. All "material" effects are in the charge and current densities,

$$\rho_p(\underline{x},t) = \sum_1^{N_0} \delta[\underline{x} - \underline{x}_i(t)]\,q_i$$

and

$$j_p(\underline{x},t) = \sum_1^{N_0} q_i\underline{v}_i(t)\delta[\underline{x} - \underline{x}_i(t)] \tag{2.3}$$

and since we shall be examining these in great detail, there is no particular advantage in introducing the auxiliary fields $\underline{D}$ and $\underline{H}$.

B. The Microscopic Distribution Function and Kinetic Equation

Our study of plasma physics is based entirely on the set of equations (2.1), (2.2), (2.3). They are exact, but, of course unsolvable, since the total number of particles, N_0, is of order 10^6 or larger. It is convenient to reformulate (2.1) and (2.3) by introducing the concept of the microscopic distribution function of Klimontovich[4]. For each species, α, we define

$$\mathcal{F}_\alpha(\underline{x},\underline{v},t) = \bar{n}_\alpha^{-1} \sum_{i=1}^{N_\alpha} \delta[\underline{x}-\underline{x}_i(t)]\delta[\underline{v}-\underline{v}_i(t)] \tag{2.4}$$

where N_α is the total number of particles of this species and n_α is their average density, $\bar{n}_\alpha = N_\alpha/V$, V being the volume of the system. In terms of the $\mathcal{F}_\alpha$, we have instead of (2.3)

$$\begin{aligned} \rho_p(\underline{x},t) &= ⨍ d\underline{v}\bar{n}\mathcal{F}(\underline{x},\underline{v},t)q \\ \underline{j}_p(\underline{x},t) &= ⨍ d\underline{v}\bar{n}\underline{v}\,\mathcal{F}(\underline{x},\underline{v},t)q \end{aligned} \tag{2.5}$$

where the symbol $⨍$ means integration over $\underline{v}$ and summation over the species index, α. (Note that in (5) we have suppressed the species index, as we shall generally do in order to minimize notational clutter.) Our choice of normalization, whose advantages will be apparent later, gives

$$\int d\underline{x}d\underline{v}\,\mathcal{F} = V \quad .$$

Using the equations of motion, (2.1) and the definition (2.4) of $\mathcal{F}$, we have

$$\frac{\partial \mathcal{f}}{\partial t}(\underline{x},\underline{v},t) = -\bar{n}^{-1}\sum_{1}^{N}[\underline{v}_i\cdot\nabla\delta(\underline{x}-\underline{x}_i)\delta(\underline{v}+\underline{v}_i) + \dot{\underline{v}}_i\cdot\nabla_v\delta(\underline{v}-\underline{v}_i)\delta(\underline{x}-\underline{x}_i)] =$$

$$= -\underline{v}\cdot\partial\mathcal{f}/\partial\underline{x} - (q/m)(\underline{\mathcal{E}} + \underline{v}\times\underline{\mathcal{B}}/c)\cdot\partial\mathcal{f}/\partial\underline{v}$$

where we take advantage of the delta functions to replace $\underline{v}_i$ by $\underline{v}$ in the first term and $\dot{\underline{v}}_i$ by $(q/m)(\underline{\mathcal{E}}+\underline{v}\times\underline{\mathcal{B}}/c)$ in the second, $\underline{\mathcal{E}}$ and $\underline{\mathcal{B}}$ being the fields at $\underline{x}$. (We have also suppressed the species indices.) Thus, we have replaced the equation of motion (2.1) by the <u>microscopic kinetic equation</u>

$$\boxed{\partial\mathcal{f}/\partial t + \underline{v}\cdot\partial\mathcal{f}/\partial\underline{x} + (q/m)(\underline{\mathcal{E}} + \underline{v}\times\underline{\mathcal{B}}/c)\cdot\partial\mathcal{f}/\partial\underline{v} = 0} \qquad (2.6)$$

This equation is <u>deceptively</u> simple in appearance, involving only 6 independent phase space variables plus the time. However, it is nonlinear and, even more important, $\mathcal{f}$ is stochastic, i.e. a wildly varying function of $\underline{x}$, $\underline{v}$, and t, due to the delta functions involved in its definition. Naturally, none of the many-body complexity of (2.1) has been eliminated by our reformulation.

C. Ensemble Averages and Fluctuations

As in all statistical mechanical problems, an exact formulation is not only insoluble, but, even if solved, would be of little value. Of principal physical interest are the average properties and the fluctuations about the mean values. We therefore introduce the usual statistical ensemble: many (conceptual) copies of the physical system, having the same observable properties but different microscopic parameters, e.g. the initial particle positions and velocities. We denote the ensemble averages by

$$f = \langle\mathcal{f}\rangle \qquad \underline{E} = \langle\underline{\mathcal{E}}\rangle \qquad \underline{B} = \langle\underline{\mathcal{B}}\rangle \qquad (2.7)$$

No special notation is required for the external sources, since these will be the same for each member of the ensemble. Thus

$$\begin{aligned} \langle\rho\rangle &= \rho_e + \int d\underline{v}\ \bar{n}f \\ \langle\underline{j}\rangle &= \underline{j}_e + \int d\underline{v}\ \bar{n}\underline{v}f \end{aligned} \tag{2.8}$$

The ensemble averages $\underline{E},\underline{B}$ are to be identified with the physical electromagnetic fields measured in an experiment. The averaged distribution function, f, has just the significance of the single particle distribution function of classical kinetic theory: $nf(\underline{x},\underline{v},t)$ is the number of particles, per unit phase space volume, which would be found at time t in the vicinity of the phase space point $(\underline{x},\underline{v})$. We shall sometimes use ξ to denote the set $(\underline{x},\underline{v})$. Thus, f, $\underline{E}$ and $\underline{B}$ are the principal physically significant dynamical variables. Next most important are fluctuations about these ensemble averages,

$$\delta\mathcal{F} = \mathcal{F} - f \qquad \delta\underline{\mathcal{E}} = \underline{\mathcal{E}} - \underline{E} \qquad \delta\underline{\mathcal{B}} = \underline{\mathcal{B}} - \underline{B} \tag{2.9}$$

where, by definition,

$$\langle\delta\mathcal{F}\rangle = \langle\delta\underline{\mathcal{E}}\rangle = \langle\delta\underline{\mathcal{B}}\rangle = 0 \tag{2.10}$$

It will often simplify the notation to denote the total electromagnetic force by

$$\underline{\tilde{\mathcal{E}}} \equiv \underline{\mathcal{E}} + \underline{v} \times \underline{\mathcal{B}}/c$$

with (2.11)

$$\tilde{E} = \langle\tilde{\mathcal{E}}\rangle, \quad \delta\tilde{E} = \underline{\tilde{\mathcal{E}}} - \underline{\tilde{E}}$$

Taking the ensemble average of (2.6) gives an equation for the single particle distribution function, f,

$$\partial f/\partial t + \underline{v}\cdot\nabla f + (q/m)\underline{\tilde{E}}\cdot\partial f/\partial\underline{v} = -(q/m)\ \langle\delta\underline{\tilde{\mathcal{E}}}\cdot\partial\delta\mathcal{F}/\partial\underline{v}\rangle \tag{2.12}$$

and subtracting this from (2.6) gives an equation for the fluctuations,

$$(\partial/\partial t + \underline{v}\cdot\nabla + (q/m)\underline{\tilde{E}}\cdot\nabla_v)\delta\mathcal{F} + (q/m)\delta\underline{\tilde{\mathcal{E}}}\cdot\nabla_v f = -(q/m)\nabla_v\cdot[\delta\underline{\tilde{\mathcal{E}}}\delta\mathcal{F} - \langle\delta\underline{\tilde{\mathcal{E}}}\ \delta\mathcal{F}\rangle] \tag{2.13}$$

Since Maxwell's equations are linear, their partition into average and fluctuation parts is trivial:

$$\nabla\cdot\underline{E} = 4\pi(\rho_e + \int d\underline{v}\bar{n}qf) \qquad \nabla\times\underline{E} + \dot{\underline{B}}/c = 0$$
$$\nabla\times\underline{B} = 4\pi c^{-1}\underline{j}_e + \int d\underline{v}\bar{n}q\underline{v}f) + \dot{E}/c \qquad \nabla\cdot\underline{B} = 0 \tag{2.14}$$

$$\nabla\cdot\delta\underline{\mathcal{E}} = 4\pi\int d\underline{v}\bar{n}q\delta\mathscr{f} \qquad \nabla\times\delta\,\underline{\mathcal{E}} + \delta\,\dot{\underline{\mathcal{B}}}/c = 0$$
$$\nabla\times\delta\underline{\mathcal{B}} = 4\pi c^{-1}(\int d\underline{v}\bar{n}q\underline{v}\delta\mathscr{f}) + \delta\,\dot{\underline{\mathcal{E}}}/c \qquad \nabla\cdot\delta\underline{\mathcal{B}} = 0 \tag{2.15}$$

Again, we are simply making definitions, and the set (2.12), through (2.15) is identical, as regards both contents and difficulty, with the original set (2.1) through (2.3). However, the problem is now formulated in a way to facilitate the approximations necessary if we are to make any progress. We see that (2.12) and (2.14) would constitute a closed set of equations for the ensemble averages f, $\underline{E}$, $\underline{B}$ if only we knew enough about the fluctuation to compute the average value $\langle\delta\mathscr{f}\delta\,\underline{\tilde{\mathcal{E}}}\rangle$ which occurs on the right side of (2.12) . On the other hand, if we knew f, $\underline{E}$, and $\underline{B}$ we need only solve the equations (2.13) and (2.15) for the fluctuations (a task made formidable, of course, by their nonlinear character). At the very least, some approximation scheme to decouple the fluctuations from the ensemble averages would be helpful.

D. The Expansion in Fluctuations

The rationale of the method of approximation which we shall use is very simple: we suppose the fluctuations to be, in some sense, "small" and therefore expand in the fluctuations. Specifically, to lowest order, we neglect terms of second order in the fluctuations, such as the r.h. side of (2.12); this completely decouples (2.12) and (2.14) from (2.13) and (2.15).

Thus, to lowest order (we shall call it first order, since the next order involves retention of terms quadratic in the fluctuations) we have the correlationless kinetic equation, plus the ensemble averaged Maxwell equations,

$$\mathcal{L}f \equiv (\partial/\partial t + \underline{v}\cdot\nabla + (q/m)\underline{\tilde{E}}\cdot\nabla_v)f = 0 \qquad (2.16)$$

$$\nabla\cdot\underline{E} = 4\pi(\fint d\underline{v}\bar{n}qf + \rho_e) \qquad \nabla\times\underline{E} + \dot{\underline{B}}/c = 0$$

$$\nabla\times\underline{B} = 4\pi c^{-1}(\fint d\underline{v}\bar{n}q\underline{v}f + \underline{\ \ }_e) + \dot{\underline{E}}/c \qquad \nabla\cdot\underline{B} = 0 \qquad (2.17)$$

This set of equations was first written down, on phenomenological grounds, by A. Vlasov and is generally referred to by his name, although the misnomer "collisionless Boltzmann equation" is sometimes used. We shall refer to this lowest order of the expansion as the <u>Vlasov approximation</u>.

In second order we retain the quadratic terms, $\langle\delta\mathcal{E}\delta\mathcal{F}\rangle$ on the right side of (2.12) but neglect terms of third order in the fluctuations. When (2.13) and (2.15) are solved for $\delta\mathcal{F}$ and $\delta\dot{\underline{\mathcal{E}}}$, the contributions of the right side of (2.13) will lead to terms of third order in (2.12), so in this order we can neglect the right hand side of (2.13) from the start. Thus to second order we have

$$\mathcal{L}f = -(q/m)\nabla_v \cdot \langle\delta\mathcal{F}\,\delta\,\underline{\tilde{\mathcal{E}}}\rangle \qquad (2.18)$$

$$\mathcal{L}\delta\mathcal{F} + (q/m)\delta\,\underline{\tilde{\mathcal{E}}}\cdot\nabla_v f = 0 \qquad (2.19)$$

plus the Maxwell equations (2.14) and (2.15) for the self-consistent determination of $\underline{\tilde{E}}$ and $\delta\,\underline{\tilde{\mathcal{E}}}$. We shall designate this as the <u>quasilinear approximation</u> since (2.19) is linear in $\delta\mathcal{F}$, albeit nonlinear terms are retained in equation (2.18) for f. The fluctuations modify the average distribution, f, but interactions among the fluctuations, such as mode coupling, are neglected.

Finally, in third order we retain the right hand side of (2.13), so that the third order equations are formally identical with the exact equations, (2.12) through (2.15). As we shall see later, the approximation consists in solving (2.12) and (2.13) by a perturbation expansion, keeping only terms of third order in the fluctuations. (Similarly, we could go on to fourth or higher orders, but these remain largely unexplored at the present time.) Only in this order do we have mode coupling of fluctuations, nonlinear wave-particle interactions, self interaction of large amplitude waves and similar exotic phenomena, so we may describe it as the <u>nonlinear wave approximation</u>.

For a plasma in equilibrium, one can prove that this "expansion in fluctuations" is tantamount to an expansion in the plasma parameter, ϵ_p, and hence well justified if $\epsilon_p << 1$. However, the most interesting problems in plasma physics involve non-equilibrium phenomena, where this expansion procedure can really be justified only on an a posteriori basis, for each problem.

To avoid confusion with other treatments, we should emphasize that even within the Vlasov approximation one may make an expansion in the fields $\underline{E}$ and $\underline{B}$, (or in their deviation from the values characterizing some elementary solution) and hence encounter equations of "second order" or "third order" in the fields. These equations will be formally similar to those describing what we have called the quasilinear and nonlinear wave approximations, simply because of the obvious <u>formal</u> similarity between the Vlasov and Klimontovich equations. However, the physical interpretations are quite different, since within the Vlasov approximation we deal <u>only</u> with ensemble-averaged quantities, whereas the quasilinear and nonlinear wave approximations involve, in an essential way, stochastic variables. A crude designation of the difference is to describe the nonlinear Vlasov theory as one involving "coherent" waves. The confusion is compounded by the circumstance that in many problems the formal analysis (expansion in diagrams, etc.) may be quite similar, but the distinction between the physical significance is an important one, as we shall see in later chapters.)

E. An Overview

In subsequent chapters, we shall study systematically the consequences of these various orders of approximation. Before doing so, we have a few comments on their general properties.

1. The Vlasov equations show clearly the self-consistent aspect of plasma physics, with f determined by $\tilde{\underline{E}}$, and $\tilde{\underline{E}}$ having sources given partly by f.

2. Most of our understanding of the properties of the Vlasov system is based on a linearization of f about some time and space independent "equilibrium" or "unperturbed" function, $f_0(\underline{v})$, with only terms of first order in $(f-f_0)$ and $\tilde{\underline{E}}$ retained. Since <u>any</u> $f_0(\underline{v})$ satisfies (2.16) when $\tilde{\underline{E}} = 0$, we must look elsewhere for guidance in making a sensible choice of f_0. For this, we need to consider the second order effects. The neglect of fluctuations at the Vlasov level means that of the total force on a given particle we are including only the average part, $\tilde{\underline{E}}$, and ignoring the rapidly fluctuating portions which arise from the discrete, particulate character of the plasma. However, it is just the latter which determine the equilibrium f_0. In fact, the $\langle\delta \mathcal{F} \delta \tilde{\underline{\mathcal{E}}}\rangle$ terms in (2.13) correspond to two physical effects:

a) "Close" collisions (meaning those with impact parameter less than the Debye length, as we shall see later); and

b) "Quasilinear" modifications of the average distribution function by the fluctuations.

As we shall see, for a plasma in equilibrium, where the fluctuations are and remain, small, of order $\mathcal{E}_p$, only the first of these two effects is important and the right hand side of (2.13), which we write as

$$\delta f/\delta t \equiv -(q/m)\nabla_v \cdot \langle \delta \mathcal{F} \delta \tilde{\underline{\mathcal{E}}} \rangle \tag{2.20}$$

satisfies an H-theorem, i.e. tends to drive f_0 towards a Maxwellian distribution,

$$f_M(\underline{v}) = \exp(-v^2a^2)/a^3\pi^{3/2} \tag{2.21}$$

so we shall often make this choice for f_0.

3. For many purposes, even the Vlasov description is too difficult to solve and we deal instead with the moments of f, i.e. density, n, mean velocity, $\underline{v}$ and pressure tensor, p, as functions of $\underline{x}$ and t. It is easy to derive equations for these quantities from (2.12), albeit the set does not close without further approximations. This leads to "two fluid magnetohydrodynamics", so-called because there are equations and dependent variables for each of the two (or more) species in the plasma. A further approximation, valid at low frequencies and large wavelengths, reduces this to "one fluid mhd". These fluid approximations are clearly justified in the case of high collision frequencies, when the short mean free path tends to preserve the initial grouping of particles. However, they can often give a good account of many phenomena even when their use is not clearly justified, probably because they represent the basic conservations laws--mass, momentum and energy.

4. Within the Vlasov equations, it is often useful to make the "electrostatic approximation," neglecting the Lorentz force, $\underline{v} \times \underline{B}$, part of $\underline{\tilde{E}}$, and similarly for $\delta\underline{\tilde{\mathcal{E}}}$, when we go to second or third order. This represents an enormous simplification for the analysis, but it must be justified in each particular context.

5. So far as an external magnetic field, $\underline{B}_0$, is concerned, the simplest case is, of course, $\underline{B}_0 = 0$, and we shall consider that first in discussing the Vlasov equation. Next simplest is the case of very strong $\underline{B}_0$ (cyclotron frequency >> all other significant frequencies, cyclotron radius << all other significant lengths) when the Alfven guiding center approximation and related techniques which we shall discuss later, are applicable.

6. The relation amongst the various approximations or "models" of the plasma can be summarized in a block diagram:

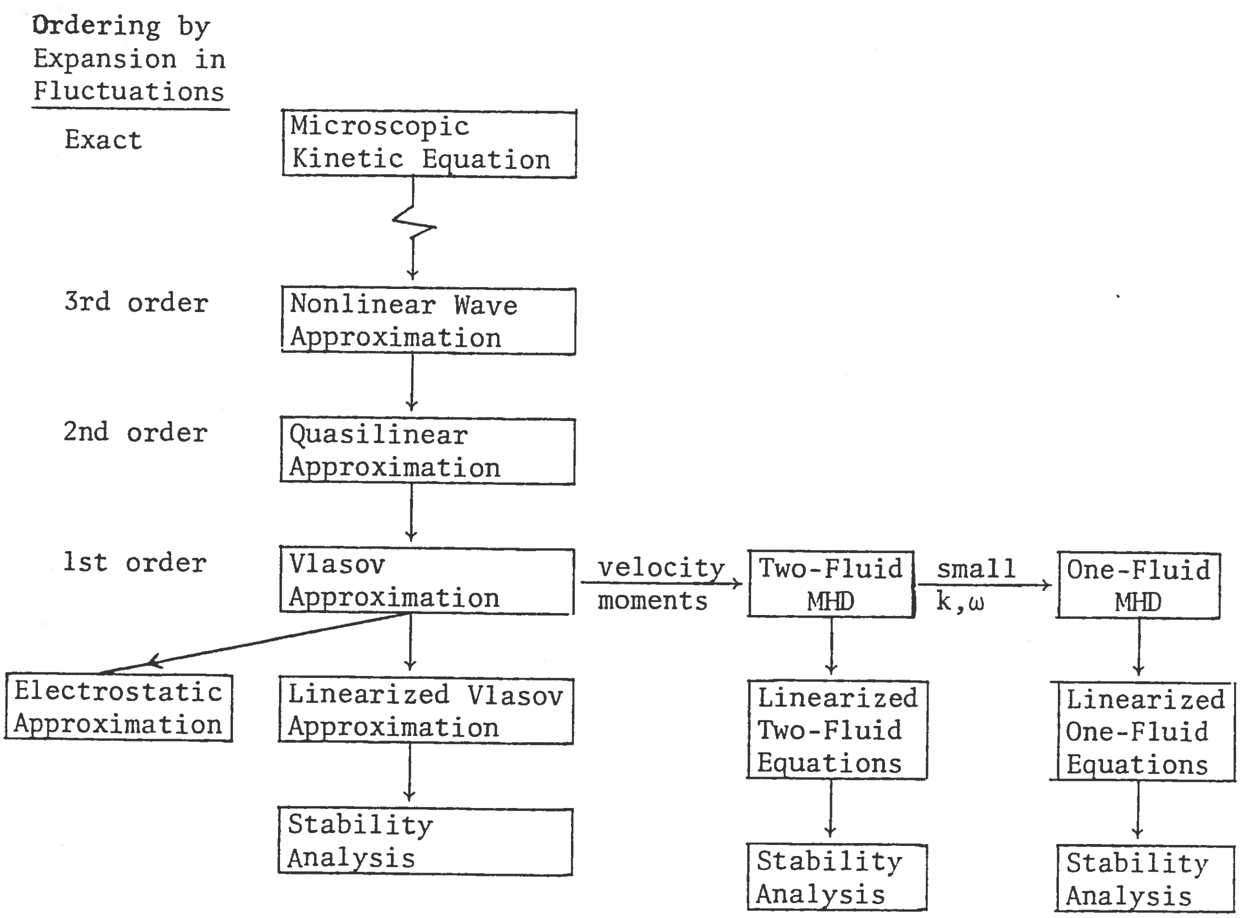

Fig. 1.1

7. Many of the approximations depicted here can be derived directly, on phenomenological grounds. For example, if we simply introduce a density function, $f(\underline{x},\underline{v},t)$ in the six dimensional phase space, then in absence of "collisions" between particles, conservation of particles gives a six dimensional continuity equation,

$$\partial f/\partial t + \nabla\cdot(\underline{v}f) + \nabla_v\cdot(\dot{\underline{v}}f) = 0$$

With external fields $\underline{E}_e$, $\underline{B}_e$,

$$\dot{\underline{v}} = (q/m)(\underline{E}_e + \underline{v} \times \underline{B}_e/c)$$

so

$$\partial f/\partial t + \underline{v}\cdot\nabla f + (q/m)(\underline{E}_e + \underline{v} \times \underline{B}_e/c)\cdot \nabla_v f = 0$$

If we allow the "external" fields to have as sources also the plasma charge and current densities described by f, i.e. replace $\underline{E}_e$ and $\underline{B}_e$ with $\underline{E}$ and $\underline{B}$ satisfying

$$\nabla \cdot \underline{E} = 4\pi(\rho_e + \int d\underline{v}\bar{n}qf)$$

$$\nabla \times \underline{B} = 4\pi c^{-1}(\underline{j}_e + \int d\underline{v}\bar{n}q\underline{v}f) + \dot{E}/c$$

$$\nabla \cdot \underline{B} = \nabla \times \underline{E} + \dot{\underline{B}}/c = 0$$

then we have just the Vlasov formulations. As we will see later, this includes, in the "self-consistent" fields, $\underline{E}$-$\underline{E}_e$, $\underline{B}$-$\underline{B}_e$, the particle interactions associated with impact parameter, b, greater than L_D and neglects the "close" collisions, $b < L_D$. It is the latter which are described by the $<\delta \not{f} \delta \tilde{\underline{\mathcal{E}}}>$ terms neglected in the Vlasov approximation.

In Chapter III we shall derive the two-fluid equations and consider the linear theory in absence of a magnetic field in order to make contact with some elementary plasma phenomena of physical interest and also illustrate certain techniques which we will frequency employ. Following that, we derive in Chapter IV the one-fluid mhd equations and examine the linearized waves which they predict in the presence of an external magnetic field, thus getting our first taste of magnetic effects. We then return to the full Vlasov equations in Chapter V and reexamine some of the phenomena studied in Chapters III and IV in order to understand the differences between the Vlasov and fluid treatments.