“Whatever It Takes”: A Good Communication Strategy?*

Federico Innocenti   Tsung-Hsien Li

This version: April 2025

[Check here for the latest version]

Abstract

What is the optimal communication policy for a central bank facing an inattentive public? We develop a model of central bank communication within a Bayesian persuasion framework, incorporating information processing costs and heterogeneous attention budgets across individuals. The central bank seeks to stabilize the economy by shaping inflation expectations in response to fundamental shocks. When shocks are large, it is optimal for the central bank to remain deliberately vague, leveraging inflation surprises to mitigate fluctuations in unemployment. When shocks are small, the central bank communicates more precisely, balancing message accuracy against audience reach. We further examine how belief heterogeneity, shock magnitude and asymmetry, adaptive expectations, the share of inattentive individuals, and skewed attention budget distributions influence the optimal communication strategy. Our findings underscore the strategic role of central bank communication as a flexible and effective instrument for macroeconomic stabilization.

Keywords: Central Bank Communication, Bayesian Persuasion, Rational Inattention, Inflation Expectations, Monetary Policy, Macroeconomic Stabilization

JEL Classifications: D83, E52, E58, E61

*We thank participants at XVIII GRASS Workshop, CEPET Workshop 2024, Academia Sinica, and National Dong Hwa University for helpful comments. **Innocenti**: Postdoctoral Research Fellow at the Department of Economics of the University of Verona (email: f.innocenti93@gmail.com; website: federicoinnocenti.com). **Li**: Assistant Research Fellow at the Institute of Economics, Academia Sinica (email: thli@econ.sinica.edu.tw, tsunghsien1124@gmail.com; website: tsunghsien1124.github.io).

1 Introduction

The primary objective of central banks is to maintain price stability. In practice, most countries adopt an inflation-targeting regime, in which the central bank sets a specific inflation target and adjusts its policy instruments to keep inflation close to that target. The effectiveness of monetary policy in influencing real outcomes hinges critically on how economic agents—or, more broadly, individuals—form their inflation expectations. Central banks may wish to temporarily deviate from their inflation targets or pre-committed monetary measures in response to extraordinary economic disruptions to support broader macroeconomic stabilization. However, such strategic deviations may undermine the credibility of central banks and, in turn, erode the effectiveness of monetary policy.

Communication emerges as an essential complementary policy tool for shaping these expectations (Blinder, Ehrmann, de Haan, and Jansen, 2024; Casiraghi and Perez, 2022). A central bank not only provides information but also strategically designs it to influence public beliefs. This communication can take various forms, ranging from technical reports to policy speeches. While technical reports are typically more precise, they are less effective in reaching the broader public, as individuals often find them too complex or inaccessible. In contrast, speeches, although less detailed, tend to capture greater attention.

Central banks often choose to communicate with intentionally vague terms, even during episodes of heightened economic stress. Such statements can be effective precisely because of their simplicity, especially when individuals face cognitive constraints in processing information. A prominent example is the speech delivered by then-ECB President Mario Draghi in 2012 at the peak of the Eurozone sovereign debt crisis:

> *"Within our mandate, the ECB is ready to do **whatever it takes** to preserve the euro. And believe me, it will be enough."*

Despite its lack of technical detail, the speech is widely credited with calming financial markets and restoring confidence in the euro.

This statement was intentionally vague: while it conveyed a strong commitment to preserving the monetary union and successfully reassured financial markets, it revealed little about the central bank's expectations regarding the evolution of the crisis or the specific instruments it would use. Why would it be optimal for a central bank to withhold such information in the face of a severe crisis? This paper offers a novel, theoretically grounded explanation for that communication strategy.

We study a partial equilibrium model of central bank communication featuring Bayesian persuasion under individual information processing costs and attention budgets. The economy can be in one of two states, *weak* or *strong*, each associated with a state-dependent fundamental shock that drives the economy away from its steady state. These shocks are interpreted as an unemployment shock in the weak state and an employment shock in the strong state. The central bank and individuals may hold different prior beliefs about the likelihood of each state. The central bank seeks to stabilize the economy by minimizing inflation and output gaps across states, captured by a quadratic loss function. The central bank commits to a state-contingent monetary policy to stabilize the economy. According to the classical Phillips curve, the unemployment gap depends on the slope of the curve—that is, the sensitivity of unemployment to inflation surprises—and the inflation surprise itself, defined as the gap between actual inflation and individuals' expected inflation. Through information design, the central bank can influence the magnitude of this inflation surprise.

We consider a Sender–Receiver game in which the central bank (the sender) possesses commitment power—formally modeled as Bayesian persuasion following Kamenica and Gentzkow (2011)—while individuals (the receivers) face limited attention. The central bank can influence individuals' posterior beliefs about the underlying state and, consequently, their inflation expectations by designing a state-contingent communication strategy. However, consuming information is costly. Following Sims (2003), we model these information processing costs using entropy.[1] In essence, the cost increases with the distance between the posterior belief induced by the message and the individual's prior belief. That is, messages conveying more surprising or unfamiliar information are more difficult to process.

Each individual is endowed with an idiosyncratic attention budget, which determines the maximum message complexity they can afford. Given these information processing costs and heterogeneous attention budgets, some individuals may ignore the central bank's message if its complexity exceeds their cognitive capacity. These inattentive individuals form inflation expectations solely based on their prior beliefs. In contrast, attentive individuals—those with sufficiently high attention budgets—process the message and update their expectations based on

[1] Understanding a message requires cognitive effort, such as reading and interpreting content.

the induced posterior beliefs.

Since individuals are rationally inattentive, the central bank faces a fundamental trade-off between message precision and message popularity when using communication to stabilize the economy. We are the first to examine this novel trade-off within a model of central bank communication under commitment.

As our benchmark, we consider a setting in which the central bank and individuals share identical, neutral prior beliefs, and the fundamental shocks are symmetric across states. The attention budget is uniformly distributed, and individuals form rational inflation expectations. Under these assumptions, we theoretically characterize the optimal central bank communication strategy, which exhibits two distinct regimes along the extensive margin: one in which the bank communicates informatively, and another in which it remains uninformative.

When the magnitude of the shocks exceeds the effective power of inflation surprises—defined as the product of the sensitivity parameter and the inflation surprise—the central bank optimally chooses to communicate uninformatively. By deliberately withholding information, it induces full-scale inflation surprises, which help offset the adverse effects of the shocks and thus stabilize the economy.

By contrast, when the effective power of inflation surprises exceeds the magnitude of the shocks, the central bank opts for informative communication to better align inflation expectations with actual inflation across states, thereby mitigating excessive inflation surprises and stabilizing the economy. However, the intensive margin—namely, the extent to which the central bank can communicate precisely—is constrained by the share of inattentive individuals. A higher share of inattentive agents reduces the effectiveness of complex messages, forcing the central bank to trade off precision for popularity. As a result, full revelation is never optimal unless attention constraints are entirely absent or the cost of processing information is negligible.

We then examine the role of belief heterogeneity in central bank communication. In practice, individuals and the central bank often hold different views regarding the likelihood of economic states—that is, they possess distinct prior beliefs. We show that non-neutral priors affect both the extensive and intensive margins of communication, though their impacts differ depending on whether the heterogeneity arises from individuals or the central bank.

When individual prior beliefs shift toward one particular state, that state becomes subjectively more plausible. As a result, inflation surprises diminish in the more plausible state but intensify in the less plausible one. The mismatch between actual and expected inflation in the implausible state grows as individuals assign greater probability to the plausible state. Once this mismatch becomes sufficiently large, the central bank optimally switches from an uninformative to an informative communication strategy, sending a more precise message about the implausible state to reduce its excessive inflation surprise.

This individual belief heterogeneity also influences the intensive margin. The central bank reduces message precision for the state perceived by individuals as more plausible, while increasing precision for the less plausible state to mitigate excessive inflation surprises. However, due to information processing costs, there exists a threshold in individual priors beyond which the central bank overturns this strategy: it becomes optimal to send a more precise message about the most plausible state, in order to reduce communication complexity and indirectly strengthen beliefs about the less plausible state by leveraging message complementarity.[2]

The central bank's belief reflects its perceived priority regarding which state requires more urgent management of inflation surprises. As a result, the optimal communication strategy is to send a more precise message about the state the central bank considers more plausible. This improves the alignment between actual and expected inflation, thereby minimizing the expected loss in that state. However, in order to preserve message popularity among inattentive individuals, the central bank correspondingly reduces the precision of messages about the less plausible state.

The Phillips curve has arguably flattened over time, weakening the sensitivity of unemployment to inflation surprises and reducing the effectiveness of central bank communication. Under this scenario, the central bank finds it increasingly optimal to remain strategically vague, leveraging larger inflation surprises to offset the diminished responsiveness of unemployment to inflation. Informative communication occurs only when the central bank's belief diverges sufficiently from the beliefs held by individuals.

Fundamental shock characteristics, particularly their magnitude and symmetry, critically influence the central bank's optimal communication strategy. Larger shocks prompt increasingly strategic vagueness by the central bank to intentionally trigger larger inflation surprises and stabilize the economy. Only when individuals perceive one of the states as more plausible does the central bank find

[2] Message complementarity results from Bayes' rule. See the discussion in Section 4.

it optimal to communicate informatively, shaping beliefs toward the less plausible state to manage excessive inflation surprises. When shocks are asymmetric, the central bank intervenes promptly with informative communication directed toward the state associated with the smaller shock, aiming to contain inflation surprises amplified by extreme individual priors.

The rational expectations hypothesis has been increasingly questioned, and in practice, many individuals exhibit low attention budgets.[3] To quantify this empirically relevant feature, we introduce adaptive expectations and use the Kumaraswamy distribution to model the skewness of attention budgets toward lower values.[4] This framework enables us to examine how imperfect expectation formation and a population concentrated around low attention capacity affect optimal central bank communication.

On the one hand, if individuals form adaptive inflation expectations that are partially anchored to their priors, this adaptive mechanism weakens the effectiveness of communication, as a message with a given level of precision becomes less effective at guiding individuals toward posterior beliefs that support macroeconomic stabilization. As a result, the central bank optimally responds by communicating more precisely to mitigate information loss during individual belief updating. The greater the degree of adaptiveness, the greater the scope for the central bank to deploy informative communication to offset updating losses and enhance the effectiveness of communication. On the other hand, as the attention budget distribution becomes more right-skewed—that is, as the share of low-budget individuals increases—the central bank optimally reduces message precision to lower complexity and reach a larger portion of the population.

Interestingly, we also uncover that the precision the central bank employs in communication (intensive margin) is a nontrivial function of both the overall share of inattentive individuals and the distribution of attention budgets among them. Typically, the lower the average attention budget across individuals, the less precise the information the central bank is willing to provide to maintain popularity. However, if the population includes a sufficiently large mass of fully attentive individuals, or only a small share of inattentive individuals—most of whom have very limited attention budgets—the central bank may find it optimal to forgo communication with the inattentive group altogether. In such cases, it

[3]For example, many individuals in the United States lack financial literacy, as evidenced by their inability to correctly answer all of the Big Three financial literacy questions.

[4]The uniform distribution is a special case of the Kumaraswamy distribution, allowing for convenient comparisons across different distributional assumptions.

chooses to send fully informative messages targeted at the attentive individuals.

Related Literature Our paper contributes to two strands of literature: Bayesian persuasion and central bank communication.

First, we contribute to the literature on Bayesian persuasion, pioneered by Aumann, Maschler, and Stearns (1995) and Kamenica and Gentzkow (2011). We examine the problem of a sender—a central bank—who designs information for rationally inattentive (Sims, 2003) receivers (individuals) to influence their decision-making. Several studies have incorporated limited attention into Bayesian persuasion frameworks; see, for example, Bloedel and Segal (2020), Lipnowski, Mathevet, and Wei (2020), Lipnowski, Mathevet, and Wei (2022), Wei (2021), Matyskova and Montes (2023), and Innocenti (2024).

In our setting, the central bank and individuals have partially aligned objectives. The central bank can provide perfectly informative messages about the state of the economy, but doing so may exceed the cognitive capacity of inattentive individuals. In certain cases, the bank optimally chooses to remain uninformative when inflation surprises can be leveraged to offset real economic shocks. To our knowledge, we are the first to study information design with commitment in a central bank communication framework that explicitly accounts for rational inattention.

The literature on central bank communication has traditionally relied on cheap talk models, in which the central bank lacks commitment to its messaging strategy (Crawford and Sobel, 1982). Notable examples include Stein (1989), Moscarini (2007), and Bassetto (2019), where communication is inherently imperfect due to strategic interaction between the central bank and the public. By contrast, we assume full commitment to the communication strategy. In this setting, central bank communication emerges as a complementary instrument to inflation-targeting monetary policy, enabling the central bank to mitigate economic fluctuations without deviating from its policy rule. We contribute to this literature by providing the first theoretical characterization of the optimal communication strategy when the central bank faces inattentive individuals and aims to influence inflation expectations through Bayesian persuasion.

There is a considerable body of research studying central bank communication under learning-based frameworks in which agents update expectations based on observed signals. For example, Eusepi and Preston (2010) and Fujiwara and Waki (2022) explore how the disclosure of macroeconomic dynamics or anticipated shocks can guide forward-looking expectations and improve the effectiveness of forward guidance. Cornand and Dos Santos Ferreira (2025) considers the case where the central bank faces firms with motivated beliefs and evaluates whether it should reveal its information or rely on stabilization rules. In contrast, we introduce a novel Bayesian persuasion framework for central bank communication, which allows the central bank to endogenously design the information structure to influence individuals' posterior beliefs and strategically shape inflation expectations in response to economic fundamentals. We characterize the optimal precision of communication and study the trade-offs between message informativeness and macroeconomic stabilization.

More recently, a growing body of research has applied Bayesian persuasion to analyze optimal central bank communication. For example, Herbert (2022) studies communication in a setting characterized by coordination externalities and belief heterogeneity, showing that countercyclical messaging emerges as optimal. The study most closely related to ours is Ko (2023), which corresponds to our benchmark framework when individuals are fully attentive.[5] In contrast to Ko (2023), our paper highlights the importance of audience characteristics in shaping the optimal communication strategy, focusing on rational inattention, belief heterogeneity, and departures from rational expectations. To the best of our knowledge, this is the first study to formally incorporate these dimensions into a unified Bayesian persuasion framework for central bank communication.

The rest of the paper is organized as follows. Section 2 outlines the model framework. In Section 3, we characterize the benchmark theoretical results for the optimal central bank communication strategy. Sections 4 to 6 examine how belief heterogeneity, shock asymmetry, the distribution of attention budgets, and irrational expectations shape the optimal communication design. Section 7 concludes with a discussion of potential extensions. Detailed mathematical derivations are provided in Appendix A.

[5] In Ko (2023), the central bank receives a private, imperfect signal about the state of the economy and chooses the extent of disclosure. Our model yields qualitatively similar insights without assuming private information, thereby simplifying the informational structure while maintaining tractability.

2 Model

We study a partial equilibrium model of central bank communication, framed as a sender-receiver game between a central bank and individuals. Given a predetermined long-run monetary policy, the central bank chooses whether to send informative messages to influence the formation of individual inflation expectations, in order to stabilize the economy promptly in response to fundamental shocks. We examine the extent to which a central bank can leverage communication tools to lean against the wind in the short run—complementing its monetary policy rule without altering the pre-committed policy path and thereby avoiding any compromise of credibility. The central bank commits to a communication strategy prior to the realization of shocks, giving rise to an information design problem known as Bayesian persuasion.

The economy consists of two types of agents: a central bank and a unit-mass continuum of individuals. The central bank faces the following classical Phillips curve:

$$\left(u - u^N\right) = \omega - \gamma\left(\pi - \pi^e\right), \tag{1}$$

where u denotes the unemployment rate, u^N the natural rate of unemployment, ω an exogenous, state-dependent employment shock, $\gamma > 0$ the sensitivity of the unemployment gap to inflation surprises, π actual inflation, and π^e individual expected inflation. Hence, $\pi - \pi^e$ captures the inflation surprise, and $\gamma\left(\pi - \pi^e\right)$ reflects the extent to which unanticipated inflation can offset the adverse shock effect on the unemployment gap.

Equation (1) describes how unemployment responds to inflation surprises. When actual inflation exceeds expected inflation, unemployment falls. Conversely, when inflation is lower than expected, unemployment rises. Ultimately, the central bank cares about the relationship between unemployment and its natural rate. Thus, what matters for the central bank is that the current employment shock and the effect of inflation surprise on unemployment have opposite signs, possibly compensating each other.

We assume that ω reflects a fundamental disturbance independent of inflation expectations and also serves as the state of the economy. It can take on two values: in the weak state, $\omega_1 > 0$, unemployment rises; in the strong state, $\omega_2 < 0$, it falls. We define the state space as $\Omega := \{\omega_1, \omega_2\}$. To stabilize the economy in the face of unemployment fluctuations, the central bank can trade off higher inflation for lower unemployment, or vice versa.

The central bank manages actual inflation according to the following state-contingent monetary policy rule:

$$\pi(\omega, \pi^T, \nu) := \begin{cases} \pi^T + \nu & \text{if } \omega = \omega_1, \\ \pi^T - \nu & \text{if } \omega = \omega_2, \end{cases} \tag{2}$$

where π^T denotes the central bank's inflation target and ν represents its monetary policy flexibility.[6] According to Equation (2), the central bank raises inflation above its target in the weak state to stimulate economic recovery and lowers it below its target in the strong state to preempt overheating or inflationary pressure.

The ultimate goal of the central bank is to stabilize the economy by minimizing quadratic losses across states arising from unemployment and inflation gaps.[7] For a given monetary policy rule and a given state, the central bank's loss, denoted by $L(\omega, \pi^T, \nu)$, is given by:

$$L(\omega, \pi^T, \nu) := \left(u - u^N\right)^2 + \alpha\left(\pi - \pi^T\right)^2 \tag{3}$$

$$= \left[\omega - \gamma\left(\pi(\omega, \pi^T, \nu) - \pi^e\right)\right]^2 + \alpha\left(\pi(\omega, \pi^T, \nu) - \pi^T\right)^2, \tag{4}$$

where α denotes the relative weight the central bank places on the inflation gap relative to the unemployment gap. Equation (4) follows from substituting equations (1) and (2) into equation (3).

Individuals share a common prior belief $\mu_0 \in [0, 1]$ that state ω_1 will occur, while the central bank holds its own prior belief $\mu_0^c \in [0, 1]$. Accordingly, the beliefs assigned to state ω_2 are $1 - \mu_0$ for individuals and $1 - \mu_0^c$ for the central bank. Individuals form their inflation expectations $\pi^e(\mu)$ based on their posterior beliefs $\mu \in [0, 1]$, which can differ from the prior μ_0 following communication by the central bank.

The central bank aims to minimize expected losses under its own prior, taking

[6]For instance, ν can be interpreted as the degree of liquidity expansion or money velocity adjustment used to accommodate inflation.

[7]This parsimonious approach can be micro-founded. See, for example, Woodford (2003).

as given the inflation expectations formed by individuals. Its objective becomes:

$$\begin{aligned}
&\mathbb{E}_{\mu_0^c} L(\mu, \pi^T, \nu) = \\
&\mu_0^c \left\{ \left[\omega_1 - \gamma \left(\pi(\omega_1, \pi^T, \nu) - \pi^e(\mu) \right) \right]^2 + \alpha \left(\pi(\omega_1, \pi^T, \nu) - \pi^T \right)^2 \right\} + \\
&(1 - \mu_0^c) \left\{ \left[\omega_2 - \gamma \left(\pi(\omega_2, \pi^T, \nu) - \pi^e(\mu) \right) \right]^2 + \alpha \left(\pi(\omega_2, \pi^T, \nu) - \pi^T \right)^2 \right\}.
\end{aligned} \tag{5}$$

While the central bank could adjust its inflation target or the flexibility of its monetary policy rule, doing so may come at the cost of reduced credibility—either by violating the predetermined rule or by inviting ex-post policy errors that under- or overshoot the appropriate response to shocks. In the long run, it may be optimal to prudently re-formalize the monetary rule. In the short run, however, the central bank can instead rely on communication to influence individual inflation expectations as a means of stabilizing the economy.

This paper focuses on the latter case, in which the central bank communicates with individuals by providing information σ to influence their posterior beliefs μ and, consequently, their inflation expectations. In what follows, we treat ν as exogenously given and, for notational simplicity, omit it as a function argument. Similarly, since the inflation gap is irrelevant for the information design, it is also omitted from the objective loss function. We therefore focus on how communication shapes the loss function $\mathbb{E}_{\mu_0^c} L(\mu, \sigma)$, given by:

$$\mu_0^c \left[\omega_1 - \gamma \left(\pi(\omega_1) - \pi^e(\mu)\right)\right]^2 + (1 - \mu_0^c) \left[\omega_2 - \gamma \left(\pi(\omega_2) - \pi^e(\mu)\right)\right]^2. \tag{6}$$

The information structure σ consists of a set of messages $S := \{s_1, s_2\}$ and a family of conditional distributions $\{\sigma(\cdot \mid \omega)\}_{\omega \in \Omega}$ over S, that is, $\sigma : \Omega \to \Delta(S)$. Each message $s_j \in S$ sent by the central bank induces the following posterior belief μ_j about state ω_1 among individuals:

$$\mu_j = \frac{\sigma(s_j \mid \omega_1)\mu_0}{\sigma(s_j \mid \omega_1)\mu_0 + \sigma(s_j \mid \omega_2)(1 - \mu_0)}. \tag{7}$$

As a result, σ induces a distribution τ over posteriors μ_j, that is, $\tau \in \Delta(\Delta(\Omega))$. The probability of each message s_j (or, equivalently, each posterior μ_j) as perceived by individuals is:

$$\tau_j = \sigma(s_j \mid \omega_1)\mu_0 + \sigma(s_j \mid \omega_2)(1 - \mu_0). \tag{8}$$

The martingale property—or Bayes plausibility condition—must hold: $\mathbb{E}_\tau[\mu_j] = \mu_0$. Without loss of generality, we assume that the optimal σ satisfies $\mu_1 \geq \mu_0$ and $\mu_2 \leq \mu_0$.

Messages can be interpreted as the arguments the central bank conveys to support its state-contingent monetary policy described in Equation (2), with s_1 associated with ω_1 and s_2 with ω_2.[8] The conditional probability $\sigma(s \mid \omega)$ captures the precision of communication—that is, how strongly a message is conditioned on the underlying state. A value of $\sigma(s_j \mid \omega_j) \to 1$ indicates a fully informative message that strongly supports its corresponding state. Specifically, $\sigma(s_1 \mid \omega_1) = 1$ implies $\mu_1 = 1$, and $\sigma(s_2 \mid \omega_2) = 1$ implies $\mu_2 = 0$. Conversely, $\sigma(s_j \mid \omega_j) = \frac{1}{2}$ corresponds to an uninformative message, as it implies no update from the prior, i.e., $\mu_0 = \mu_1 = \mu_2$.

To contextualize the information structure, consider the weak state ω_1, in which the central bank issues message s_1 to signal its intent to implement expansionary measures—such as liquidity injections—in support of a high-inflation regime. The extent to which the central bank seeks to influence individual inflation expectations depends on the precision of the message. It may choose to convey this intent vaguely—e.g., omitting implementation details—corresponding to $\sigma(s_1 \mid \omega_1) = \frac{1}{2}$, or to communicate it through a fully detailed implementation plan, corresponding to $\sigma(s_1 \mid \omega_1) = 1$. However, understanding detailed policy announcements—or more generally, processing complex information—is costly. As a result, the most informative messages are not necessarily optimal *a priori*.

Individuals can be either attentive or inattentive, with a share $\delta \in [0, 1]$ of the population being inattentive. Attentive individuals can process any information at no cost, whereas inattentive individuals are constrained by limited attention and can process only sufficiently simple information. Each inattentive individual i is endowed with an attention budget κ_i, drawn from an atomless distribution $F(\kappa)$ with support $[0, 1]$. An inattentive individual i processes σ if and only if $c(\sigma) < \kappa_i$, where $c(\sigma)$ denotes the cost of processing the information.

The cost of processing information is defined as:

$$c(\sigma) = \chi \left[H(\mu_0) - \sum_{j \in \{1,2\}} \tau_j \cdot H(\mu_j) \right], \tag{9}$$

[8] In other words, each message is recommended for the state it is intended to substantiate.

where $H(\cdot)$ denotes the Shannon entropy:

$$H(\mu) = -\left[\mu \ln(\mu) + (1-\mu)\ln(1-\mu)\right]. \tag{10}$$

When σ is uninformative—i.e., $\mu_1 = \mu_2 = \mu_0$—the cost is zero: $c(\sigma) = 0$. When σ is fully informative—i.e., $(\mu_1, \mu_2) = (1, 0)$—we have $H(\mu_1) = H(\mu_2) = 0$, and hence $c(\sigma) = \chi H(\mu_0)$. The scaling parameter χ is normalized as $\chi = [H(\mu_0)]^{-1}$, so that the cost of a fully informative σ equals 1. As a consequence, since $F(\chi H(\mu_0)) = 1$, the cost $c(\sigma)$ of processing a generic information structure σ spans the full support of the attention distribution.

It follows that the mass of individuals who pay attention to the central bank is $1 - \delta F(c(\sigma))$. The posterior beliefs of inattentive individuals—those who ignore the central bank's message—remain at the common prior μ_0. Therefore, the central bank's total expected loss under communication is given by:

$$\begin{aligned}
\min_{\sigma} \quad & \delta F(c(\sigma)) \Big\{ \mu_0^c \left[\omega_1 - \gamma\left(\pi(\omega_1) - \pi^e(\mu_0)\right)\right]^2 \\
& + (1-\mu_0^c)\left[\omega_2 - \gamma\left(\pi(\omega_2) - \pi^e(\mu_0)\right)\right]^2 \Big\} \\
& + (1 - \delta F(c(\sigma))) \Big\{ \mu_0^c \sum_{j=1}^{2} \sigma(s_j \mid \omega_1) \left[\omega_1 - \gamma\left(\pi(\omega_1) - \pi^e(\mu_j)\right)\right]^2 \\
& + (1-\mu_0^c) \sum_{j=1}^{2} \sigma(s_j \mid \omega_2) \left[\omega_2 - \gamma\left(\pi(\omega_2) - \pi^e(\mu_j)\right)\right]^2 \Big\}.
\end{aligned} \tag{11}$$

In essence, the central bank determines the precision of its messages—$\sigma(s_1 \mid \omega_1)$ and $\sigma(s_2 \mid \omega_2)$—to minimize expected losses across states. Faced with a trade-off between message precision and popularity (i.e., the need to reach out to inattentive individuals), the central bank strategically influences the inflation expectations of individuals to manage inflation surprises and stabilize the economy.

3 How Should a Central Bank Communicate?

To better understand the trade-off in central bank communication between information precision and popularity, we consider an analytically tractable benchmark case under the following assumptions:

Assumption 1. *Prior beliefs are homogeneous and neutral:* $\mu_0 = \mu_0^c = \frac{1}{2}$.

Assumption 2. *Employment shocks are symmetric:* $\omega_1 = -\omega_2 = \omega$.

Assumption 3. *Individuals form rational inflation expectations:*

$$\pi^e(\mu) = \mu\pi(\omega_1) + (1-\mu)\pi(\omega_2) = \pi^T - (1-2\mu)\nu. \tag{12}$$

Assumption 4. *The attention budget distribution is uniform:* $F(c(\sigma)) = c(\sigma)$.

To facilitate the discussion, we define the inflation surprise under state ω and individual belief μ as $\Pi(\omega,\mu) := \pi(\omega) - \pi^e(\mu)$, which captures the extent to which inflation expectations deviate from actual inflation. The central bank aims to steer the *targeted* inflation surprises $\Pi(\omega_1,\mu_1)$ and $\Pi(\omega_2,\mu_2)$, where each posterior μ_j is induced by the message s_j designed to support state ω_j.[9] Under Assumption 3, we characterize in Lemma 1 the feasible range of these targeted surprises that the central bank can generate through communication.

Lemma 1. *Under Assumption 3 and the state-contingent monetary policy rule in Equation (2), the targeted inflation surprises are characterized by:*

$$\Pi(\omega_1,\mu_1) = 2(1-\mu_1)\nu, \tag{13}$$
$$\Pi(\omega_2,\mu_2) = -2\mu_2\nu. \tag{14}$$

Since central bank communication induces posterior beliefs satisfying $\mu_1 \geq \mu_0$ and $\mu_2 \leq \mu_0$, and the most informative communication yields $(\mu_1,\mu_2) = (1,0)$, the feasible ranges of the targeted inflation surprises are:

$$0 = \Pi(\omega_1,1) \leq \Pi(\omega_1,\mu_1) \leq \Pi(\omega_1,\mu_0) = \nu, \tag{15}$$
$$0 = \Pi(\omega_2,0) \geq \Pi(\omega_2,\mu_2) \geq \Pi(\omega_2,\mu_0) = -\nu. \tag{16}$$

According to Lemma 1, the central bank seeks to generate negative and positive inflation surprises to counteract the state-dependent shocks, given that $\omega_1 \geq 0$ and $\omega_2 \leq 0$. To achieve this, it deliberately calibrates the precision of messages to guide the economy back toward its steady state. Notably, the largest feasible effective inflation surprises arise under uninformative communication, where $\mu_0 = \mu_1 = \mu_2 = \frac{1}{2}$.

[9]Mathematically, Equation (11) indicates that the central bank implicitly places greater weight on targeted inflation surprises as the corresponding message precision increases.

Under Assumptions 1–4, we can analytically characterize the optimal information structure that solves the minimization problem in Equation (11). In particular, the optimal message precision is symmetric across states and satisfies:

$$\sigma(s_1 \mid \omega_1) = \sigma(s_2 \mid \omega_2) \equiv \sigma^*. \tag{17}$$

Note that $\sigma^* = \frac{1}{2}$ corresponds to uninformative communication. This result is formalized in Proposition 1, with the full derivation presented in Appendix A.1.

Proposition 1. *Under Assumptions 1–4, the central bank's optimal information design distinguishes two cases:*

1. *If $2\omega \geq \gamma\nu$, the central bank communicates uninformatively: $\sigma^* = \frac{1}{2}$.*
2. *If $2\omega < \gamma\nu$, the central bank communicates informatively: $\sigma^* \in \left(\frac{1}{2}, 1\right)$, where σ^* solves:*

$$1 + \left(\frac{\mathrm{d}}{\mathrm{d}\sigma} \ln\left(1 - \delta c(\sigma)\right)\bigg|_{\sigma=\sigma^*} \right) \left(\sigma^* - \frac{1}{2} \right) = 0. \tag{18}$$

Proposition 1 provides important insights into central bank communication. First, the extensive margin—namely, whether the central bank chooses to communicate informatively or not—depends on whether there is sufficient scope for communication to stabilize the economy by guiding individual inflation expectations. Recall that γ denotes the sensitivity of the unemployment gap to inflation surprises in Equation (1), while the monetary flexibility parameter ν captures how effectively the central bank can influence inflation expectations through belief formation, as shown in Lemma 1. The effectiveness of central bank communication is thus governed by the joint impact of γ and ν, relative to the magnitude of the fundamental employment shock ω.

When the shock is overwhelming—such that its magnitude exceeds what monetary policy can feasibly offset, i.e., $2\omega \geq \gamma\nu$—the central bank opts for uninformative communication in order to fully exploit inflation surprises to counteract the shock. In contrast, when there is room for communication-based intervention, i.e., $2\omega < \gamma\nu$, the central bank communicates informatively to complement monetary policy by avoiding excessive inflation surprises.

Second, when informative communication is preferred, the intensive margin—that is, the message precision—is constrained by the cost of processing information faced by individuals. Recall that $1 - \delta c(\sigma)$ denotes the mass of individuals who pay attention to central bank communication, given the information

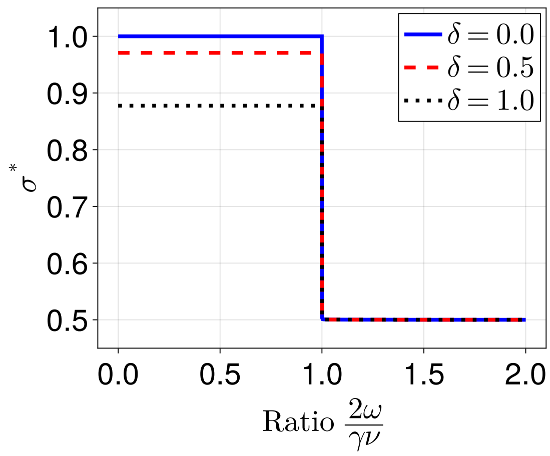

Figure 1: Optimal Central Bank Communication Strategy

structure σ. The derivative $\frac{\mathrm{d}}{\mathrm{d}\sigma}\ln(1-\delta c(\sigma)) \leq 0$ captures the rate at which inattentive individuals disengage as informational complexity increases. The share of inattentive individuals influences this disengagement rate: when δ is larger, disengagement becomes more sensitive to information complexity, leading to a steeper reduction in attention.

To satisfy the condition in Equation (18), the central bank must choose an informative message $\sigma^* > \frac{1}{2}$. However, it faces a fundamental trade-off: while more precise communication improves coordination on inflation expectations, it also imposes higher cognitive costs on inattentive individuals, thereby reducing overall reach. The optimal degree of informativeness thus balances informational precision with popularity.

We illustrate Proposition 1 in Figure 1, which displays the optimal message precision as a function of the ratio $\frac{2\omega}{\gamma\nu}$ for varying shares of inattentive individuals $\delta \in \{0, 0.5, 1\}$. The central bank switches its communication strategy at the threshold $\frac{2\omega}{\gamma\nu} = 1$. When $\frac{2\omega}{\gamma\nu} \geq 1$, the central bank opts for uninformative communication to maximize the effectiveness of inflation surprises in stabilizing the economy. In contrast, for $\frac{2\omega}{\gamma\nu} < 1$, it communicates informatively. However, message precision declines with higher shares of inattentive individuals, as the central bank strategically trades off informativeness for broader audience reach.

Discussion Our benchmark results highlight the critical role of central bank communication in macroeconomic stabilization. During moderate periods, the central bank can strategically deploy informative communication to attract attention and guide inflation expectations, thereby stabilizing the economy without frequently adjusting the underlying monetary policy rule. By contrast, in times of severe distress—when the economy is hit by an unprecedented adverse shock (i.e., $\omega \gg 0$)—it may be optimal for the central bank to remain deliberately vague. In such cases, uninformative communication can amplify inflation surprises, helping to absorb the shock and mitigate unemployment fluctuations.

As shown in Proposition 1, the central bank's decision to communicate informatively or not at the extensive margin hinges on *the fundamental comparison* between the magnitude of the shock and the effective power of inflation surprises. When informative communication is optimal, the central bank strategically designs its messages at the intensive margin by trading off precision against popularity. Although Proposition 1 is derived under Assumptions 1–4, its core insights—the fundamental comparison and the precision-popularity trade-off—remain robust and provide an intuitive framework for interpreting the results in the subsequent extended settings where these assumptions are relaxed.

Moreover, when the Phillips curve flattens—that is, when sensitivity weakens in the form of a reduced γ—the effectiveness of inflation surprises declines. In Section 4.3, we investigate how the flattening of the Phillips curve affects central bank communication. In response to diminished effectiveness, the central bank may need to resort to unconventional monetary tools to enhance its flexibility (i.e., increase ν), thereby improving its ability to influence expectations through communication. A full analysis of unconventional monetary tools is beyond the scope of this paper, and we leave their integration to future research.

4 Communication under Belief Heterogeneity

In practice, the central bank and individuals may hold divergent economic outlooks, which we capture in our framework as belief heterogeneity. To investigate the role of such heterogeneity in shaping central bank communication, we relax Assumption 1, which imposes homogeneous and neutral prior beliefs across individuals and the central bank.

To this end, we numerically solve for the optimal communication strategy defined in Equation (11), allowing for heterogeneous priors. Section 4.1 introduces the benchmark parameterization, while Section 4.2 presents comparative statics that examine how differences in beliefs affect the central bank's optimal communication policy and its broader macroeconomic implications. Finally, Sec-

Table 1: Benchmark Parameterization

Parameter	Value	Description
δ	1	Share of inattentive individuals
ω_1	1	Magnitude of unemployment shock in the weak state
ω_2	-1	Magnitude of employment shock in the strong state
μ_0	0.5	Individual prior belief
μ_0^c	0.5	Central bank prior belief
γ	2.94	Sensitivity of unemployment to inflation surprises
π^T	2	Inflation target
ν	1	Monetary policy flexibility

tion 4.3 explores how a flatter Phillips curve alters the central bank's communication strategy in the presence of belief heterogeneity.

4.1 Benchmark Parameterization

The share of inattentive individuals is set to $\delta = 1$. The shock magnitudes are symmetric, with $\omega_1 = 1$ and $\omega_2 = -1$, representing deviations of one percentage point in the unemployment rate from its natural level. Both individuals and the central bank are assumed to hold homogeneous and neutral prior beliefs, with $\mu_0 = \mu_0^c = 1/2$. The inflation sensitivity parameter is set to $\gamma = 2.94$, calibrated to match the estimate of $\psi = 0.34$ in Hazell, Herreño, Nakamura, and Steinsson (2022).[10] The central bank's inflation target is fixed at $\pi^T = 2$, reflecting the standard 2% target used in practice. The monetary policy parameter is set to $\nu = 1$, indicating that the central bank tolerates a 1 percentage point deviation from the target to stabilize the economy. Table 1 summarizes the benchmark parameter values used in the analysis.

Note that our benchmark parameterization falls into the second case of Proposition 1, where the central bank communicates informatively and trades off message precision for popularity to manage inflation surprises by aligning actual and expected inflation. We also consider a counterfactual scenario with a flattened Phillips curve by setting $\gamma = 1$. This scenario corresponds to the first case in Proposition 1, in which the central bank optimally chooses to remain deliberately vague. Comparing these two cases allows us to investigate how the optimal communication strategy varies along both the intensive and extensive margins.

[10] In our framework, γ corresponds to the reciprocal of the Phillips curve slope parameter ψ in Hazell et al. (2022), i.e., $\gamma = 1/\psi$.

4.2 Heterogeneous Beliefs

Recall that μ_0 and μ_0^c represent the prior beliefs of individuals and the central bank, respectively, about the economy being in state ω_1, which corresponds to a high-unemployment scenario. In our benchmark case, individuals and the central bank share identical priors, each assigning equal probabilities to the two states ω_1 and ω_2, i.e., $\mu_0 = \mu_0^c = 1/2$. Thus, $\mu_0 \to 0$ or $\mu_0^c \to 0$ reflects increasing optimism (a stronger belief in the strong state ω_2), while $\mu_0 \to 1$ or $\mu_0^c \to 1$ indicates rising pessimism (a stronger belief in the weak state ω_1).

Under the benchmark parameterization, inflation surprises are excessive. The central bank thus prefers informative communication as a tool for stabilization. However, due to the cognitive cost of information processing faced by inattentive individuals, the central bank opts for a moderately informative message structure—choosing precisions that are close to but below one. This reflects a deliberate trade-off between precision and popularity.

To examine how belief heterogeneity affects the optimal communication strategy, we consider two scenarios: one in which individuals' prior beliefs vary from 0 to 1 while holding all other parameters fixed, and another in which the central bank's prior beliefs vary while keeping the remaining benchmark parameters constant. We also vary the share of inattentive individuals to explore how inattentiveness interacts with belief heterogeneity in shaping the optimal communication strategy.

Figure 2 presents the optimal message precision as a function of individual prior beliefs ranging from 0 to 1, where $\sigma_1 = \sigma(s_1 \mid \omega_1)$ and $\sigma_2 = \sigma(s_2 \mid \omega_2)$. Figure 2a shows the results when all individuals are inattentive ($\delta = 1$), while Figure 2b displays the results when half individuals are inattentive ($\delta = 0.5$).

As illustrated in Figure 2, individual optimism and pessimism lead to deviations from the benchmark results, exhibiting a mirror-image pattern. For brevity, we focus our discussion on the case of pessimism, with the understanding that the same logic applies symmetrically to the case of optimism.

In Figure 2a, which depicts the case where all individuals are inattentive, the central bank chooses $\sigma_1 \ll 1$ and $\sigma_2 \approx 1$ when individuals are mildly pessimistic (e.g., $\mu_0 = 0.6$). This is because pessimistic beliefs imply that individuals perceive the weak state as more likely *ex ante*. Consequently, their inflation expectations are more closely aligned with actual inflation in the weak state, resulting in smaller inflation surprises. In contrast, the strong state is perceived as less

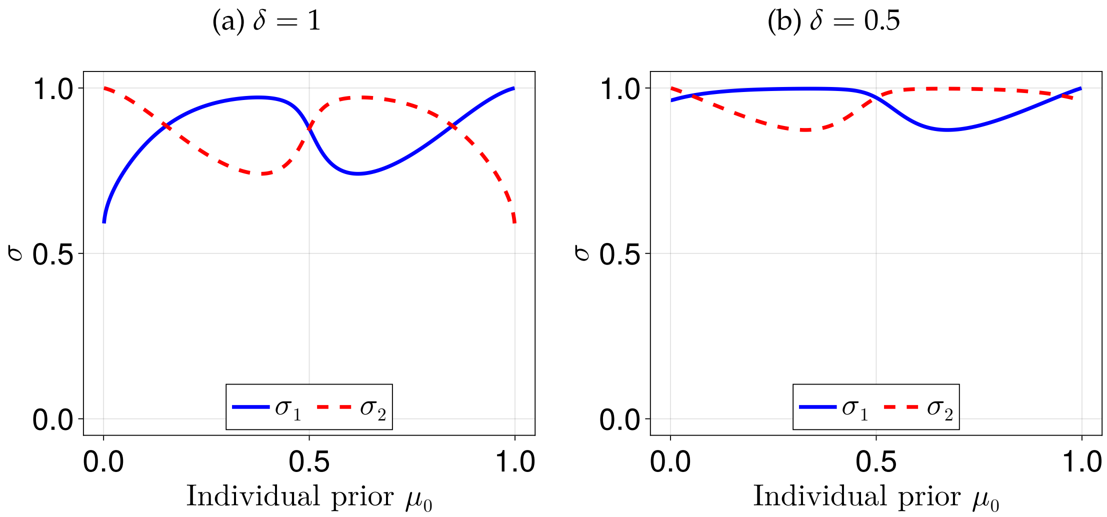

Figure 2: Optimal Message Precision Across Individual Priors

likely, leading to amplified inflation surprises. These dynamics are evident in Equations (13) and (14): as μ_0 increases, $\Pi(\omega_1, \mu_0)$ decreases toward zero, while $\Pi(\omega_2, \mu_0)$ becomes increasingly negative. To manage these larger surprises in the strong state while preserving the popularity of its communication, the central bank optimally reduces message precision in the weak state in order to allocate greater precision to the strong state. This strategy helps mitigate excessive inflation surprises and enhances coordination in the strong state.

However, in Figure 2a, when individuals perceive the weak state as increasingly certain (e.g., $\mu_0 \to 1$), the central bank chooses $\sigma_1 \approx 1$ and $\sigma_2 \ll 1$. As previously discussed, under such pessimistic beliefs, inflation surprises in the weak state become negligible, while those in the strong state grow excessively large. One might therefore expect the central bank to be more informative about the strong state in order to better manage the resulting inflation surprise. Why, then, does the central bank optimally choose to confirm individuals' firmly pessimistic beliefs by setting a high σ_1, while reducing message precision for the strong state through a lower σ_2?

There are two main reasons. First, as shown in Equation (9), the cost of processing information increases with the distance between prior and posterior beliefs. As pessimism intensifies, the cost of shifting individuals toward beliefs favoring the strong state becomes prohibitively high, while confirming beliefs about the weak state becomes virtually costless. Second, according to Equation (7), the central bank can issue a more informative message for the weak state ($\sigma_1 \to 1$) to indirectly shape posterior beliefs about the strong state ($\mu_2 \to 0$) via the law of total probability.[11]

Taken together, these considerations lead the central bank to optimally allocate message precision toward reinforcing the weak state, thereby indirectly guiding beliefs about the strong state at lower information-processing costs. This strategic behavior underscores the role of central bank communication in stabilizing the economy, not only through a trade-off between message precision and popularity, but also through the strategic use of message complementarity in shaping posterior beliefs.

Compared to Figure 2a, the overall message precisions in Figure 2b are higher across the entire range of individual priors. Interestingly, this increase is not uniform but exhibits a non-monotonic pattern, particularly when individuals hold extreme prior beliefs. A reduction in the share of inattentive individuals predictably attenuates the importance of information processing costs. As a result, the central bank can afford to raise message precision, thereby improving its ability to manage inflation expectations. However, persuading individuals with firmly held extreme beliefs remains informationally costly, which limits the degree of precision the central bank can optimally employ in such cases. So why, then, do informative messages become optimal (i.e., $\sigma_1 \to 1$ and $\sigma_2 \to 1$) for extremely pessimistic individuals ($\mu_0 \to 1$) when half of them are inattentive ($\delta = 0.5$)?

The key lies in the composition of individual attentiveness. While communicating with inattentive individuals holding extreme beliefs remains costly, the presence of fully attentive individuals—who can process any messages—enables the central bank to communicate more effectively. Accordingly, the central bank leverages the heterogeneity in attentiveness across individuals to design its communication strategy. Specifically, since some individuals are fully attentive and capable of processing information at any level of complexity,[12] the central bank strategically targets this group with informative messages aimed at correcting extreme priors. Meanwhile, it deliberately refrains from engaging inattentive individuals with limited attention budgets. This targeted approach enhances inflation expectation management and contributes to macroeconomic stabilization at the aggregate level. We will further explore this strategic trade-off in communication, with and without inattentive individuals, in Section 6.

[11] Intuitively, a high $\sigma_1 = \sigma(s_1 \mid \omega_1)$ implies that individuals expect to receive s_1 only if ω_1 occurs. Therefore, upon receiving s_2, they infer it must be conditional on ω_2. Hence, a high σ_1 helps lower μ_2.

[12] Fully attentive individuals can be thought of as having an unlimited attention budget.

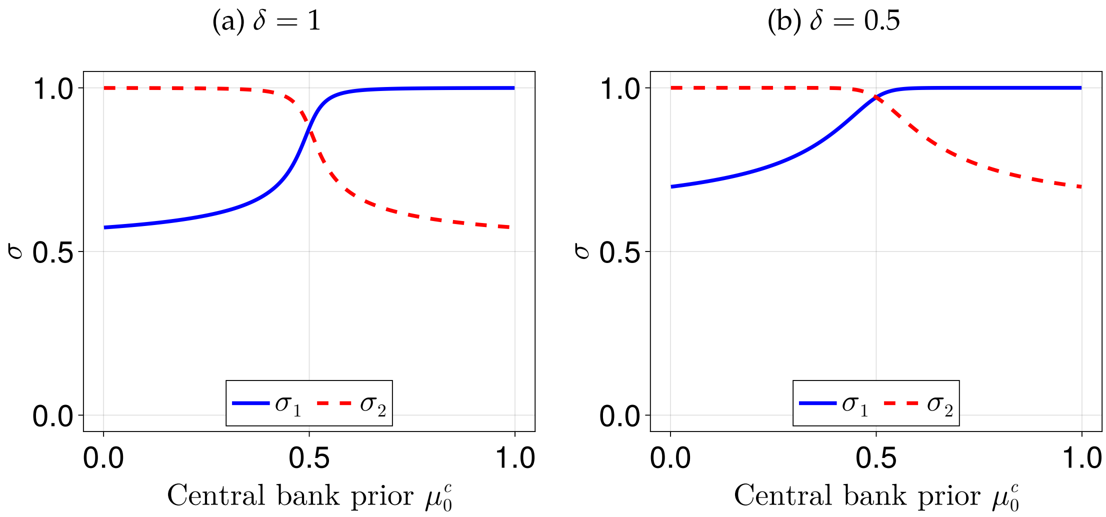

Figure 3: Optimal Message Precision Across Central Bank Priors

Figure 3 presents the optimal message precision as a function of central bank prior beliefs ranging from 0 to 1. Figure 3a shows the case with fully inattentive individuals ($\delta = 1$), while Figure 3b displays the results when half of the individuals are inattentive ($\delta = 0.5$). Again, we focus on the case of pessimism.

Recall that in Equation (11), the central bank minimizes expected losses across states using its own prior beliefs as weights. In other words, the central bank's prior determines the relative importance it assigns to stabilizing each state.[13] Based on this belief, the central bank decides whether and how to construct informative messages to guide individuals—who may hold different priors—in order to stabilize the economy.

In Figure 3a, when the central bank becomes more pessimistic ($\mu_0^c \to 1$), it increases the precision of messages related to the more plausible weak state while reducing precision for the less likely strong state. The opposite holds when the central bank is more optimistic ($\mu_0^c \to 0$). Essentially, the central bank strategically sacrifices message precision for the less likely state in order to enhance precision for the more probable one. This trade-off reflects the central bank's effort to minimize expected losses in the state it perceives as more likely, while still preserving communication popularity among inattentive individuals. Moreover, a reduced share of inattentive individuals leads to an overall increase in message precision, as shown in Figure 3b.

[13]This prior can be interpreted as reflecting the central bank's superior knowledge or informed judgment about the likelihood of different economic conditions.

4.3 Flatter Phillips Curve

The Phillips curve has flattened over time, indicating that unemployment has become less responsive to inflation surprises. In the context of our model, this structural change corresponds to a decline in the sensitivity parameter γ in Equation (1). This trend has raised concerns among policymakers regarding the diminishing effectiveness of monetary policy, particularly in its ability to shape inflation expectations for macroeconomic stabilization.

Under the benchmark parameterization, setting $\gamma = 1$ corresponds to the first case in Proposition 1,[14] in which the central bank optimally adopts uninformative communication to fully exploit inflation surprises for macroeconomic stabilization. Intuitively, as the power of expectation management weakens with a lower γ, and the magnitude of shocks exceeds this diminished power, the central bank resorts to vague messaging to generate the largest feasible inflation surprises.

This benchmark result naturally raises the question of how the optimal communication strategy adapts when the central bank and individuals hold heterogeneous beliefs in the presence of a flattened Phillips curve. As highlighted by the fundamental comparison in Proposition 1, a lower γ narrows the set of conditions under which informative communication is optimal. Meanwhile, belief heterogeneity amplifies inflation surprises in the less likely state, thereby strengthening the central bank's incentive to communicate informatively about that state, as discussed in Section 4.2. We therefore extend our analysis to examine how the interaction between a flatter Phillips curve and belief heterogeneity influences the optimal design of central bank communication.

In Figure 4, we present the optimal communication strategy across individual prior beliefs under a flattened Phillips curve with $\gamma = 1$. Figure 4a shows the case in which all individuals are inattentive ($\delta = 1$), while Figure 4b displays the results when half of the individuals are inattentive ($\delta = 0.5$).

In contrast to the benchmark results shown in Figure 2a, the central bank provides information only when individuals are sufficiently optimistic or pessimistic, as illustrated in Figure 4a. When individual priors are neutral and the Phillips curve is flat, the central bank remains deliberately vague, consistent with Proposition 1. Even when priors deviate slightly from one-half—thereby modestly increasing inflation surprises in the less likely state—the central bank continues to communicate uninformatively. This is because a low value of γ weak-

[14] The condition reads: $2\omega \geq \gamma\nu$. Under $\gamma = 1$, this inequality holds as $2 \cdot 1 \geq 1 \cdot 1$.

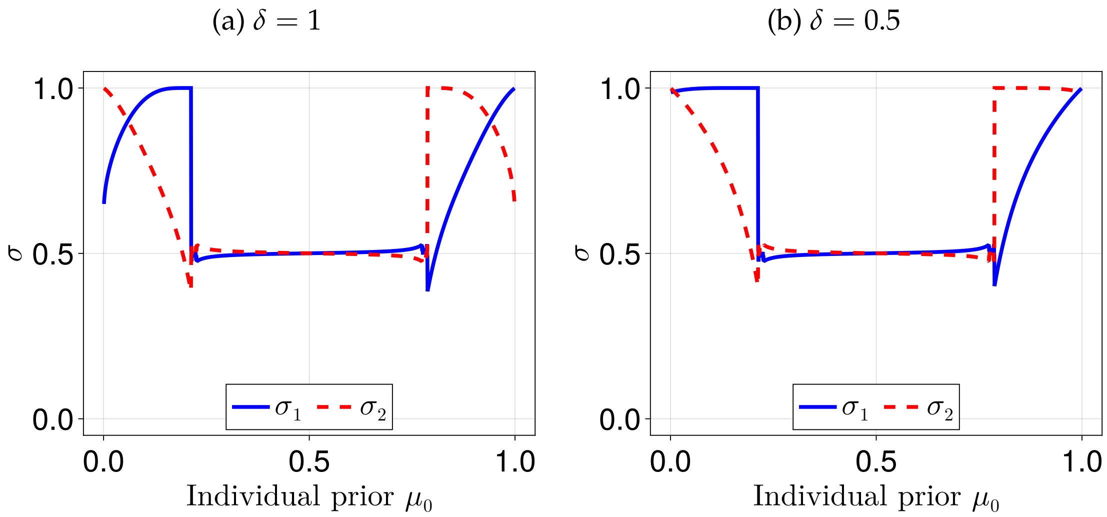

Figure 4: Optimal Message Precision Across Individual Priors when $\gamma = 1$

ens the effective power of inflation surprises, causing the magnitude of shocks to continue dominating. As suggested in Proposition 1, the central bank therefore prefers uninformative communication in such cases.

However, once individuals become sufficiently optimistic or pessimistic, inflation surprises in the less likely state become excessive. At this point, the effective power of inflation surprises outweighs the magnitude of the shock, prompting the central bank to communicate informatively by issuing a more precise message aimed at correcting expectations in the less likely state. As individual beliefs become increasingly extreme, the cost of processing information resurfaces as a binding constraint. In response, the central bank reverts to the previously discussed strategic use of message complementarity: it sends the most precise message for the most plausible state, thereby indirectly guiding beliefs associated with the less likely state.

A comparison between Figure 4a and Figure 4b confirms that the relationship between message precision and inattentiveness persists even under a flat Phillips curve: message precision declines as the share of inattentive individuals increases.

Figure 4 reveals an interesting pattern: as the Phillips curve flattens, a middle region of individual prior beliefs emerges in which uninformative communication becomes optimal for the central bank. We refer to this middle range of beliefs as the *strategic vagueness region*. The flatter the Phillips curve, the broader this region becomes, expanding the set of individual priors over which the central bank

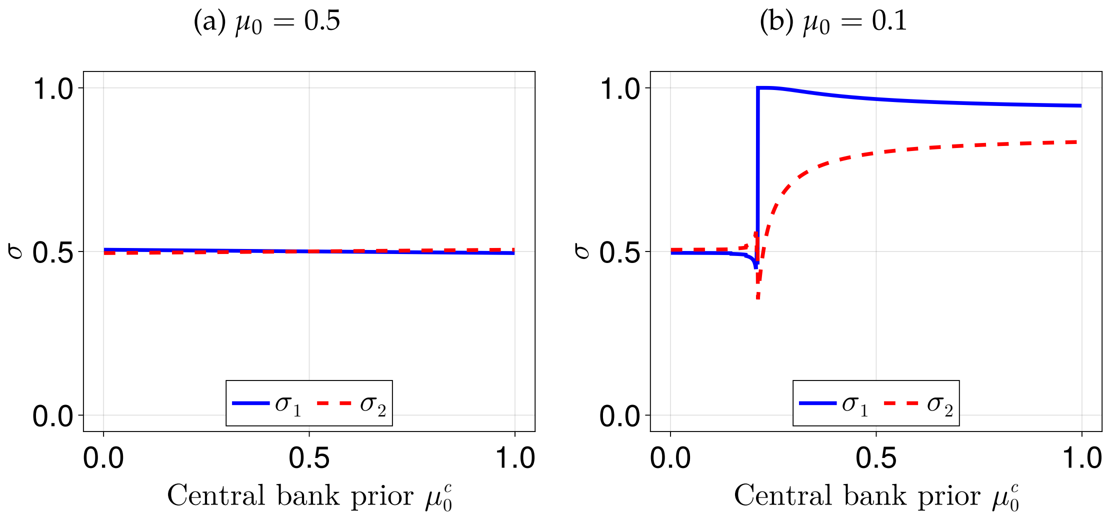

Figure 5: Optimal Message Precision Across Central Bank Priors when $\gamma = 1$

prefers to remain vague in its messaging. In other words, under a flat Phillips curve, the central bank chooses to communicate precisely only when individual beliefs diverge sufficiently from its own.

Following this, we offer a complementary perspective on the observed pattern by examining how the optimal communication strategy varies with central bank prior beliefs under a flattened Phillips curve ($\gamma = 1$). Figure 5a presents the case in which individuals hold neutral priors ($\mu_0 = 0.5$), whereas Figure 5b displays the results for a scenario in which individuals exhibit strong optimism ($\mu_0 = 0.1$).

As discussed, the central bank prefers uninformative communication under $\gamma = 1$ and other benchmark parameters. As shown in Figure 5a, where $\mu_0 = 0.5$, varying the central bank's prior does not affect its communication strategy at the extensive margin. Recall that the central bank's prior serves as a weighting factor on state-contingent expected losses in its objective function, as defined in Equation (11). Since $\mu_0 = 0.5$ implies that the initial inflation surprises are symmetric across states, changing the central bank's prior merely reweights two equivalent terms and therefore does not alter the optimal communication strategy.

By contrast, when individuals are highly optimistic—or more broadly, when they hold prior beliefs that differ significantly from the central bank's—the optimal communication strategy becomes responsive to variations in the central bank's prior, provided it differs substantially from that of the individuals, as depicted in Figure 5b. When $\mu_0 = 0.1$, inflation surprises become excessively large in the weak state, which is perceived as unlikely from the individuals' perspective. If the central bank holds a similar prior belief (i.e., $\mu_0^c \approx 0.1$), it places greater weight on stabilizing losses in the strong state. Accordingly, even though inflation surprises are excessive in the weak state, the central bank intentionally focuses on minimizing losses in the strong state by maintaining uninformative communication.

Nonetheless, when the central bank begins to hold a more pessimistic prior that differs significantly from that of individuals (i.e., $\mu_0^c \gg 0.1$), the large inflation surprises in the weak state begin to factor into its decision-making. In other words, there exists a threshold in the central bank's prior belief above which it switches from uninformative to informative communication in order to correct the excessive inflation surprises associated with the weak state. Once this threshold is exceeded, the central bank issues a more precise message with a high σ_1 to directly manage inflation surprises in the weak state, while strategically reducing message precision in the strong state—with a lower σ_2—to preserve communication popularity.

As the central bank becomes increasingly pessimistic, stabilizing inflation surprises in the weak state emerges as a dominant policy concern. To this end, the central bank seeks to shift individual beliefs from highly optimistic to fully pessimistic. However, persuading *ex ante* highly optimistic individuals to adopt fully pessimistic posterior beliefs is prohibitively costly due to information processing constraints. Consequently, the central bank strategically leverages message complementarity by increasing message precision in the strong state—with a higher σ_2—to indirectly influence beliefs associated with the weak state, thereby managing the corresponding inflation surprises.

5 Central Bank Voice under Turbulence

In reality, shocks are not always moderate or symmetric. Episodes such as financial crises, pandemics, or geopolitical disruptions often involve disturbances that are either unprecedented in scale or asymmetric in nature. During such periods, individuals and the central bank are likely to hold misaligned economic outlooks or divergent prior beliefs about underlying states. These conditions challenge the conclusions derived under Assumptions 1 and 2, which presume neutral beliefs and moderately symmetric disturbances, and underscore the need for a systematic reevaluation of how the central bank should communicate under belief heterogeneity and economic turbulence.

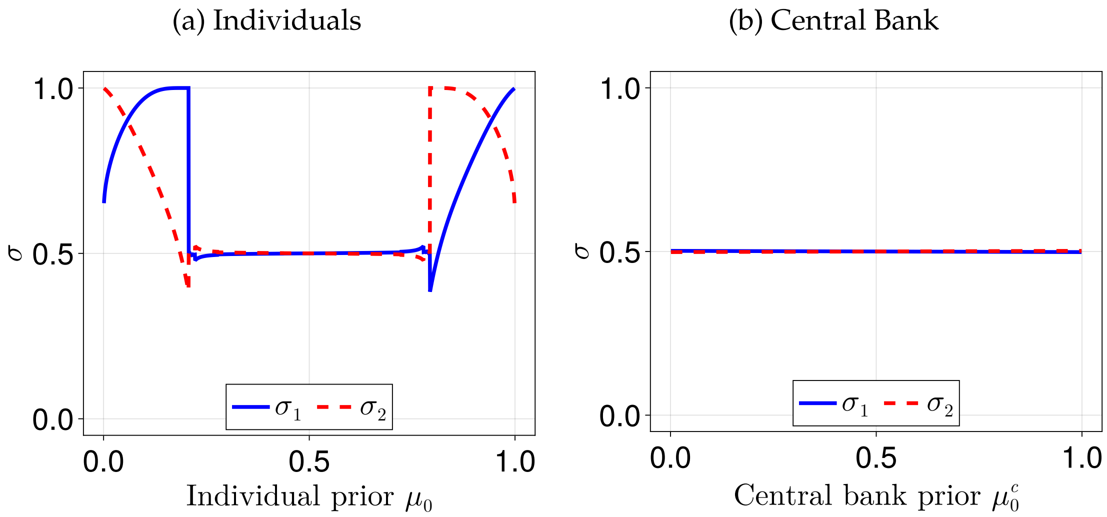

Figure 6: Optimal Message Precision Across Priors when $(\omega_1, \omega_2) = (3, -3)$

To this end, we examine the role of belief heterogeneity and asymmetric unemployment shocks in shaping central bank communication by jointly relaxing Assumptions 1 and 2. Specifically, we explore the optimal communication strategy across varying prior beliefs of both individuals and the central bank, ranging from 0 to 1. Meanwhile, we consider two alternative shock configurations in addition to the benchmark: a large symmetric shock scenario $(\omega_1, \omega_2) = (3, -3)$ and an asymmetric shock scenario $(\omega_1, \omega_2) = (3, -1)$. All other parameters are held at their benchmark values. Recall that the benchmark case corresponds to $(\omega_1, \omega_2) = (1, -1)$. In both counterfactual scenarios, one or both shocks are three times larger in magnitude than in the benchmark, allowing us to assess how the central bank adjusts its communication strategy under more volatile or imbalanced economic conditions.

We present the results under the large symmetric shock scenario $(\omega_1, \omega_2) = (3, -3)$ in Figure 6. Figure 6a displays the optimal communication strategy as individual prior beliefs vary, while Figure 6b illustrates the case in which the central bank's prior belief varies with individual priors held fixed.

Under neutral individual beliefs, the large symmetric shock scenario leads the central bank to optimally choose vague communication, as implied by Proposition 1.[15] When considering heterogeneous individual prior beliefs alongside increased shock magnitude, Figure 6a reveals the emergence of a strategic vagueness region—a middle range of priors over which the central bank finds it op-

[15]The condition reads: $2\omega \geq \gamma\nu$. Under the large symmetric shock scenario, this inequality holds as $2 \cdot 3 \geq 2.94 \cdot 1$.

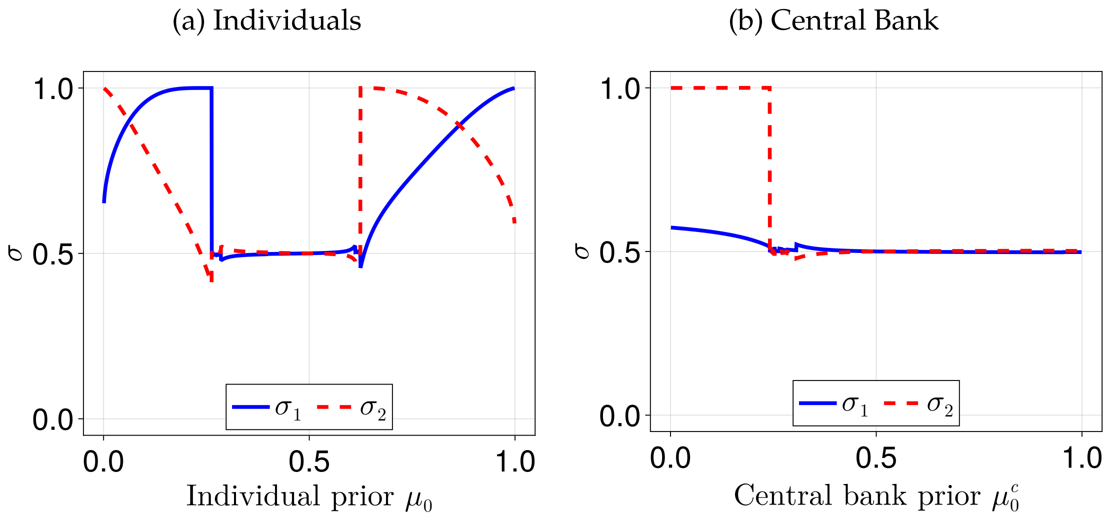

Figure 7: Optimal Message Precision Across Priors when $(\omega_1, \omega_2) = (3, -1)$

timal to remain uninformative, thereby leveraging full-scale inflation surprises to counteract large disturbances. According to the fundamental comparison in Proposition 1, when individual beliefs are relatively neutral, the shock magnitude dominates the effective power of inflation surprises, prompting the central bank to refrain from informative communication. However, as individuals become sufficiently pessimistic or optimistic, this comparison reverses: the effective power of inflation surprises outweighs the shock magnitude, encouraging the central bank to communicate informatively.

This result closely parallels the case of a flat Phillips curve with $\gamma = 1$, as shown in Figure 4a, where a reduced γ weakens the effective power of inflation surprises and generates a comparable strategic vagueness region. See the discussion accompanying Figure 4a for further reference.

Conversely, Figure 6b shows that central bank prior heterogeneity plays no role in shaping communication under the large symmetric shock scenario. As discussed in Figure 5a, varying the central bank's prior simply reweights two equivalent state-contingent losses in the objective function and therefore does not affect the optimal communication strategy. See the discussion accompanying Figure 5a for further reference.

We next present the results under the asymmetric shock scenario $(\omega_1, \omega_2) = (3, -1)$ in Figure 7. Figure 7a displays the optimal communication strategy as individual prior beliefs vary, while Figure 7b illustrates the case in which the central bank's prior belief varies, holding individual priors fixed.

As in the large symmetric shock scenario shown in Figure 6a, the strategic vagueness region also arises under the asymmetric shock scenario, as illustrated in Figure 7a. However, the shape and extent of this region differ between the two settings. Under symmetric shocks, the region is symmetric and centered around the neutral individual prior. In contrast, with asymmetric shocks, the region becomes skewed due to the imbalance in shock magnitudes. Specifically, it shifts toward more optimistic individual priors, given the stronger shock associated with the weak state ($\omega_1 > \omega_2$). This observation naturally raises the question: why does a stronger shock in the weak state cause the strategic vagueness region to extend disproportionately over more optimistic individual priors?

Although the fundamental comparison—between shock magnitude and the effective power of inflation surprises—is originally derived under the assumption of symmetric shocks, it remains informative for analyzing asymmetric scenarios. As previously discussed, inflation surprises intensify in the state that individuals perceive as less likely. In the present asymmetric setting, the relatively small shock magnitude in the strong state implies that, under heightened individual pessimism, inflation surprises in the strong state can quickly exceed the magnitude of the underlying shock, prompting the central bank to adopt informative communication at a relatively low threshold of pessimism. Conversely, the larger shock associated with the weak state allows the central bank to tolerate greater inflation surprises in that state as individual optimism increases, thereby widening the range of optimistic priors over which uninformative communication remains optimal.

Taken together, these dynamics induce an asymmetric response in communication: the central bank begins to communicate informatively earlier under pessimism than under optimism, reflecting the dominance of the weak-state shock magnitude over that of the strong state.

Figure 7b displays a hybrid pattern that combines elements from Figure 6b (large symmetric shocks) and Figure 3a (benchmark small symmetric shocks). Under the large symmetric shock scenario, the central bank optimally chooses to remain vague, leveraging large inflation surprises to counteract substantial shock disturbances. In contrast, under the benchmark scenario with small symmetric shocks, the central bank communicates precisely, tailoring its messages to correct inflation surprises in the state it deems more likely.

In the asymmetric shock scenario, the weak state is subject to a large shock, whereas the strong state remains associated with a smaller shock, as in the benchmark. This asymmetry implies that inflation surprises become relatively excessive in the strong state. In this context, the central bank favors uninformative communication under most non-optimistic prior beliefs. However, as the central bank's prior becomes sufficiently optimistic—or places greater weight on the strong state—it shifts toward informative communication, characterized by a high σ_2 and a moderate σ_1, in order to exclusively correct inflation surprises in the strong state.

6 Talking with Skewed, Behavioral Individuals

The effectiveness of central bank communication depends critically on how individuals form inflation expectations and process information. The former determines the accuracy of individual expectations, while the latter governs the extent to which central bank communication can shape individual beliefs and, in turn, their expectations.

However, the empirical validity of the rational expectations hypothesis has been increasingly questioned, as real-world observations frequently deviate from its core assumptions. Moreover, attention budgets—or more broadly, cognitive capacities—are far from uniformly distributed across individuals. For example, a substantial portion of the population demonstrates limited financial literacy, constraining their ability to interpret complex economic information.

To further generalize our framework, we relax Assumption 3 on rational expectations and Assumption 4 on uniformly distributed attention budgets. Under Assumption 5, we posit that individuals form *adaptive beliefs*, whereby updated beliefs under communication remain partially anchored to their priors. Owing to the linearity of the inflation expectation function, as shown in Lemma 2, these adaptive beliefs give rise to adaptive expectations. Under Assumption 6, we assume that attention budgets among inattentive individuals follow a Kumaraswamy distribution, which offers flexibility in capturing various forms of skewness—including the benchmark uniform distribution as a special case.

Assumption 5. *Individuals form adaptive beliefs:*

$$\hat{\mu} = \mu_0 + \theta(\mu - \mu_0) = (1 - \theta)\mu_0 + \theta\mu, \tag{19}$$

where $\theta \in [0, 1]$ governs the extent to which individuals incorporate the posterior belief under communication, μ, into their updated belief, $\hat{\mu}$, while $1 - \theta$ captures the degree *of anchoring to the prior, μ_0. Notably, $\theta = 1$ corresponds to the rational expectations benchmark.*

Lemma 2. *Given Assumption 5 and the linear inflation expectation function in Equation (12), individuals form adaptive expectations:*

$$\pi^e(\hat{\mu}) = (1-\theta)\pi^e(\mu_0) + \theta\pi^e(\mu). \tag{20}$$

See Appendix A.2 for the proof.

Assumption 6. *The distribution of attention budgets κ among inattentive individuals follows a Kumaraswamy distribution:*

$$f(\kappa; a, b) = a \cdot b \cdot \kappa^{a-1} \cdot (1-\kappa^a)^{b-1}, \tag{21}$$

$$F(\kappa; a, b) = 1 - (1-\kappa^a)^b, \tag{22}$$

where $\kappa \in [0, 1]$, f denotes the probability density function, F the cumulative distribution function, and (a, b) are positive shape parameters. Notably, the case $(a, b) = (1, 1)$ corresponds to the benchmark uniform distribution.

The implication of Assumption 6 is that different combinations of (a, b) flexibly determine the shape of the distribution. In general, a larger value of a shifts more probability mass toward one, producing a left-skewed distribution, whereas a larger value of b concentrates mass near zero, yielding a right-skewed distribution. These effects are illustrated in Figure 8, which depicts the probability density functions of the Kumaraswamy distribution for three parameterizations of (a, b): $(1, 1)$, $(5, 1)$, and $(1, 5)$. In the subsequent analysis, we concentrate on the role of b and, accordingly, focus on the case of $(a, b) = (1, 5)$, which captures the empirically relevant scenario in which a substantial share of individuals possess limited attention budgets.

We begin by examining how Assumptions 5 and 6 modify the benchmark results presented in Section 3. To this end, we extend the theoretical framework established in Proposition 1 and derive new analytical results in Proposition 2, which characterize the optimal communication strategy when individuals form adaptive expectations and exhibit heterogeneous, skewed attention budgets. A detailed derivation of Proposition 2 is provided in Appendix A.1.

Proposition 2. *Under Assumptions 1, 2, 5, and 6, the central bank's optimal information design distinguishes two cases:*

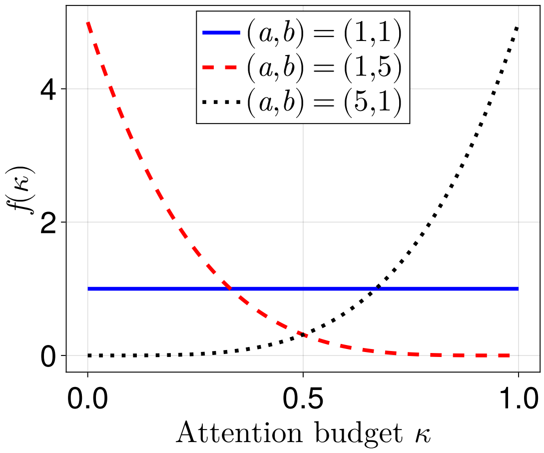

Figure 8: Kumaraswamy Distribution Shapes

1. *If* $2\omega \geq (2-\theta)\gamma\nu$*, the central bank communicates uninformatively:* $\sigma^* = \frac{1}{2}$.
2. *If* $2\omega < (2-\theta)\gamma\nu$*, the central bank communicates informatively:* $\sigma^* \in \left(\frac{1}{2}, 1\right)$*, where* σ^* *solves:*

$$1 + \left(\frac{\mathrm{d}}{\mathrm{d}\sigma} \ln \left(1 - \delta \left[1 - \left(1 - (c(\sigma))^a\right)^b\right]\right)\Bigg|_{\sigma=\sigma^*} \right) \left(\sigma^* - \frac{1}{2}\right) = 0. \tag{23}$$

Compared to Proposition 1, the extensive margin condition—whether the central bank communicates informatively or not—now additionally depends on the degree of adaptive expectations, as captured by the term $2-\theta$. When $\theta = 1$, this additional term drops out, and the condition reduces to the case of rational expectations described in Proposition 1. As θ decreases, the strategic vagueness region contracts, since the condition $2\omega \geq (2-\theta)\gamma\nu$ becomes less likely to hold, *ceteris paribus*. In other words, the presence of adaptive expectations narrows the set of circumstances under which uninformative communication is optimal.

This result arises because, when $\theta < 1$, individuals form inflation expectations that remain partially anchored to their priors. The inertia inherent in this updating process reduces the responsiveness of expectations to new information. Consequently, the central bank becomes more inclined to communicate informatively to compensate for this diminished sensitivity and thus guide inflation expectations more effectively. Figure 9a illustrates the effect of θ on the extensive margin—that is, the threshold determining whether the central bank opts for informative communication. As shown, the optimal region for uninformative communication (the blue-shaded area) shrinks as individuals' expectations become more rational ($\theta \to 1$).

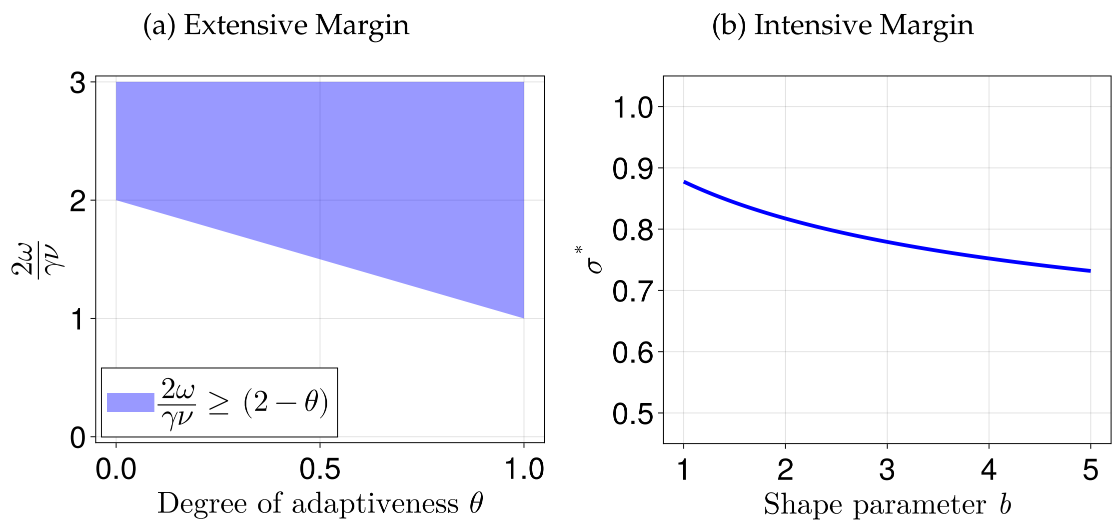

Figure 9: Optimal Communication facing Skewed, Behavioral Individuals

Recall from Proposition 1 that the optimal message precision σ^* is symmetric across states, and that the derivative $\frac{\mathrm{d}}{\mathrm{d}\sigma}\ln(1-\delta F(c(\sigma))) \leq 0$ captures the rate at which inattentive individuals disengage from communication as informational complexity increases. Now, in addition to the share of inattentive individuals δ, as discussed in Section 3, the distribution of attention budgets F also plays a critical role in shaping the trade-off between message precision and message popularity at the intensive margin. According to Equation (22), the cumulative distribution function of the Kumaraswamy distribution with shape parameters (a, b) is given by $F(c(\sigma)) = 1 - \left(1-(c(\sigma))^a\right)^b$.

The parameters a and b shape the curvature of the disengagement rate. A higher value of a shifts probability mass toward higher attention budgets, allowing inattentive individuals to tolerate greater informational complexity before disengaging. Consequently, the marginal disengagement rate flattens, reducing the popularity cost associated with increased message precision. Conversely, a higher value of b concentrates probability mass near lower attention budgets, indicating that most inattentive individuals are more easily overwhelmed by complex messages. In this case, the disengagement rate steepens, intensifying the popularity cost of enhanced message precision. Figure 9b illustrates the influence of b on optimal message precision at the intensive margin.[16]

We next examine how a skewed distribution of attention budgets, the share of

[16]We omit a detailed discussion of parameter a for two reasons. First, the effects of a and b are predictably opposite. Second, the case of an increased b is the empirically relevant counterfactual to consider.

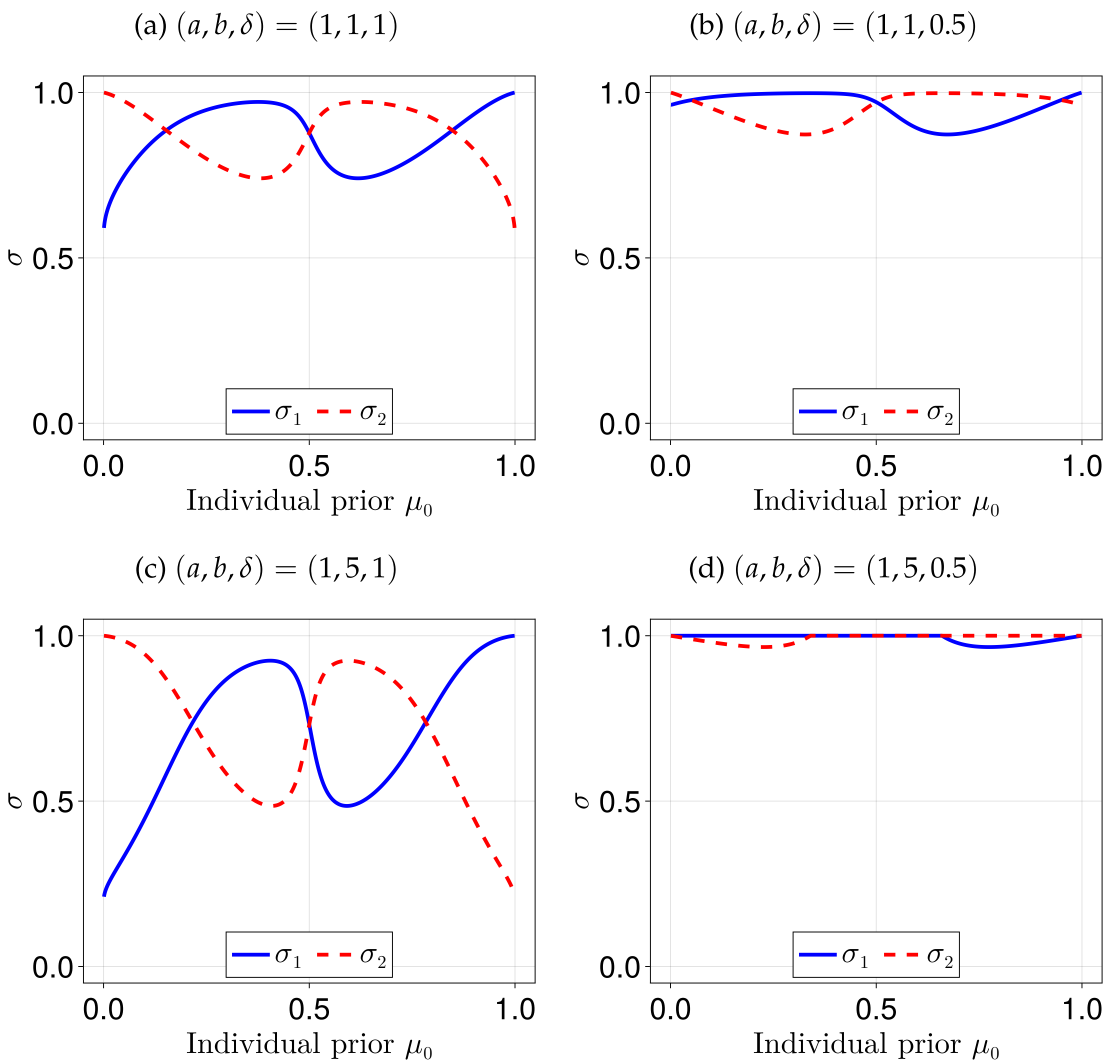

Figure 10: Optimal Message Precision Across Individual Priors by (a, b, δ)

inattentive individuals, and their interaction shape the optimal communication strategy under belief heterogeneity. To this end, we analyze the optimal message precisions as individual prior beliefs vary from 0 to 1, under two parameterizations of the Kumaraswamy distribution, $(a, b) \in \{(1, 1), (1, 5)\}$, and two levels of inattentiveness, $\delta \in \{0.5, 1\}$. The results for each combination of distributional parameters and inattentiveness levels, (a, b, δ), are presented in Figure 10. Note that the cases $(a, b, \delta) = (1, 1, 1)$ and $(a, b, \delta) = (1, 1, 0.5)$ in the upper panel are identical to those in Figure 2, and are repeated here for ease of comparison.

Intuitively, a greater prevalence of individuals with low attention budgets leads the central bank to optimally choose less precise messages in order to maintain broader engagement. This intuition is indeed borne out in the comparison of Figures 10a and 10c for an economy in which all individuals are inattentive ($\delta = 1$): as the distributional parameters shift from $(a, b) = (1, 1)$ to $(a, b) = (1, 5)$—indicating greater mass concentrated on low attention budgets—the central bank sends less precise messages, reflected in overall lower values of σ_1 and σ_2.

However, when only 50% of individuals are inattentive (i.e., $\delta = 0.5$), the pattern reverses. As the attention budget distribution becomes increasingly right-skewed—with more mass concentrated on low attention budgets—the central bank finds it optimal instead to send more precise messages, as reflected in higher values of σ_1 and σ_2, as compared in Figure 10b with Figure 10d.

Why, then, does the logic observed under a uniform attention budget distribution fail to extend to the right-skewed case when $\delta = 0.5$? This is because the central bank considers not only the extent to which inattentive individuals engage with its communication, but also whom it chooses to target—that is, whether to prioritize attentive or inattentive individuals.

When the attention budget distribution among inattentive individuals is heavily right-skewed toward low budgets, and a substantial share of fully attentive individuals exists—those who face no information processing costs—the central bank may instead choose to communicate at the highest level of message precision, targeting only attentive individuals. In doing so, it deliberately forgoes engagement with inattentive individuals altogether. By strategically selecting the target audience for its communication, the central bank can achieve more effective macroeconomic stabilization at the aggregate level.

In Figure 10b, and recalling the discussion of Figure 2b, the central bank sends highly precise messages when individual prior beliefs are extreme in an economy where half of the population is inattentive. This pattern can be rationalized by the strategic trade-off in which the central bank optimally targets only the attentive individuals in its communication, as discussed here.

7 Conclusion

This paper studies the optimal information design of a central bank with commitment. We show that strategically crafted communication can serve as a powerful policy instrument for macroeconomic stabilization. Specifically, the central bank adjusts its information provision depending on whether the inflation surprise is excessive relative to the underlying unemployment shock. When inflation surprises are needed to counteract large shocks, the central bank optimally remains deliberately vague. Otherwise, it communicates informatively, balancing message precision against popularity to manage individual inflation expectations more effectively.

When individuals are either overly optimistic or pessimistic, inflation surprises tend to intensify in the state they consider less likely. In response, the central bank adjusts its communication strategy by providing a more informative message about the less plausible state to better align expectations with actual inflation and mitigate excessive surprises. In contrast, when the central bank holds asymmetric beliefs about the states, it strategically targets precision in the state it deems more probable to minimize expected losses.

We also demonstrate how shock magnitude and asymmetry, adaptive expectation formation, and the distribution of attention budgets shape the optimal communication strategy. In essence, the optimal policy balances the trade-offs among shock magnitude and the effective power of inflation surprises, message precision and popularity, and macroeconomic fundamentals and individual characteristics.

Our results offer a novel interpretation of deliberately vague communication, such as Mario Draghi's "whatever it takes" statement. During a period of severe economic distress, the ECB faced a substantial unemployment shock. In such a context, preserving inflation surprises through vagueness allowed the central bank to stabilize the economy more effectively. In contrast, fully revealing the state of the economy would have neutralized the inflation surprise, reducing its capacity to counteract the shock.

Finally, we leave the study of the coordinated design between monetary and communication policies for future research. As Proposition 1 illustrates, a central bank could, in principle, jointly optimize both instruments to improve stabilization outcomes in the face of economic fluctuations. Moreover, when the effective power of inflation surprises declines, the central bank may turn to unconventional monetary tools to enhance its monetary flexibility (i.e., increase ν), thereby strengthening its capacity to influence expectations through communication.

References

AUMANN, R. J., M. MASCHLER, AND R. E. STEARNS (1995): *Repeated Games with Incomplete Information*, MIT Press.

BASSETTO, M. (2019): "Forward Guidance: Communication, Commitment, or Both?" *Journal of Monetary Economics*, 108, 69–86.

BLINDER, A. S., M. EHRMANN, J. DE HAAN, AND D.-J. JANSEN (2024): "Central Bank Communication with the General Public: Promise or False Hope?" *Journal of Economic Literature*, 62, 5–44.

BLOEDEL, A. W. AND I. SEGAL (2020): "Persuading a Rationally Inattentive Agent," Tech. rep., Working paper, Stanford University.

CASIRAGHI, M. AND L. P. PEREZ (2022): "Central Bank Communications," *MCM Technical Assistance Handbook*.

CORNAND, C. AND R. DOS SANTOS FERREIRA (2025): "Central Bank Communication and Stabilization Policies under Firms' Motivated Beliefs," *Economic Theory*, 79, 687–721.

CRAWFORD, V. P. AND J. SOBEL (1982): "Strategic Information Transmission," *Econometrica*, 50, 1431–1451.

EUSEPI, S. AND B. PRESTON (2010): "Central Bank Communication and Expectations Stabilization," *American Economic Journal: Macroeconomics*, 2, 235–271.

FUJIWARA, I. AND Y. WAKI (2022): "The Delphic Forward Guidance Puzzle in New Keynesian Models," *Review of Economic Dynamics*, 46, 280–301.

HAZELL, J., J. HERREÑO, E. NAKAMURA, AND J. STEINSSON (2022): "The Slope of the Phillips Curve: Evidence from U.S. States," *The Quarterly Journal of Economics*, 137, 1299–1344.

HERBERT, S. (2022): "State-Dependent Central Bank Communication with Heterogeneous Beliefs," *Banque de France Working Paper*, 875.

INNOCENTI, F. (2024): "Can Media Pluralism Be Harmful to News Quality?" *Available at SSRN 4257390*.

KAMENICA, E. AND M. GENTZKOW (2011): "Bayesian Persuasion," *American Economic Review*, 101, 2590–2615.

KO, E. (2023): "Optimal Central Bank Forward Guidance," *International Journal of Central Banking*, 19, 401–447.

LIPNOWSKI, E., L. MATHEVET, AND D. WEI (2020): "Attention Management," *American Economic Review: Insights*, 2, 17–32.

——— (2022): "Optimal Attention Management: A Tractable Framework," *Games and Economic Behavior*, 133, 170–180.

MATYSKOVA, L. AND A. MONTES (2023): "Bayesian Persuasion with Costly Information Acquisition," *Journal of Economic Theory*, 105678.

MOSCARINI, G. (2007): "Competence Implies Credibility," *American Economic Review*, 97, 37–63.

SIMS, C. A. (2003): "Implications of Rational Inattention," *Journal of Monetary Economics*, 50, 665–690.

STEIN, J. C. (1989): "Cheap Talk and the Fed: A Theory of Imprecise Policy Announcements," *American Economic Review*, 79, 32–42.

WEI, D. (2021): "Persuasion under Costly Learning," *Journal of Mathematical Economics*, 94, 102451.

WOODFORD, M. (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton, NJ: Princeton University Press.

A Mathematical Details

A.1 Proof of Proposition 1

We denote with $\sigma_1 = \sigma(s_1|\omega_1)$ and $\sigma_2 = \sigma(s_2|\omega_2)$. It follows that:

$$\mu_1 = \frac{\sigma_1\mu_0}{\sigma_1\mu_0 + (1-\sigma_2)(1-\mu_0)} \tag{24}$$

$$\mu_2 = \frac{(1-\sigma_1)\mu_0}{(1-\sigma_1)\mu_0 + \sigma_2(1-\mu_0)} \tag{25}$$

$$\frac{\partial\mu_1}{\partial\sigma_1} = \frac{\mu_0(1-\mu_0)(1-\sigma_2)}{[\sigma_1\mu_0 + (1-\sigma_2)(1-\mu_0)]^2} \tag{26}$$

$$\frac{\partial\mu_2}{\partial\sigma_1} = \frac{-\mu_0(1-\mu_0)\sigma_2}{[(1-\sigma_1)\mu_0 + \sigma_2(1-\mu_0)]^2} \tag{27}$$

$$\frac{\partial\mu_1}{\partial\sigma_2} = \frac{\mu_0(1-\mu_0)\sigma_1}{[\sigma_1\mu_0 + (1-\sigma_2)(1-\mu_0)]^2} \tag{28}$$

$$\frac{\partial\mu_2}{\partial\sigma_2} = \frac{-\mu_0(1-\mu_0)(1-\sigma_1)}{[(1-\sigma_1)\mu_0 + \sigma_2(1-\mu_0)]^2} \tag{29}$$

$$\tau_1 = \sigma_1\mu_0 + (1-\sigma_2)(1-\mu_0) \tag{30}$$

$$\tau_2 = (1-\sigma_1)\mu_0 + \sigma_2(1-\mu_0) \tag{31}$$

$$\frac{\partial\tau_j}{\partial\sigma_1} = \begin{cases} \mu_0 & \text{if } j = 1 \\ -\mu_0 & \text{otherwise} \end{cases} \tag{32}$$

$$\frac{\partial\tau_j}{\partial\sigma_2} = \begin{cases} 1-\mu_0 & \text{if } j = 2 \\ -(1-\mu_0) & \text{otherwise} \end{cases} \tag{33}$$

$$\frac{\partial H(\mu_j)}{\partial\sigma_k} = -\frac{\partial\mu_j}{\partial\sigma_k}\ln\left(\frac{\mu_j}{1-\mu_j}\right) \tag{34}$$

$$c_k'(\sigma) = -\chi\sum_{j=1,2}\left[\frac{\partial\tau_j}{\partial\sigma_k}H(\mu_j) + \tau_j\frac{\partial H(\mu_j)}{\partial\sigma_k}\right] \tag{35}$$

Under Assumption 3, the problem in (11) becomes:

$$\begin{aligned}
\min_{\sigma}\ \delta F(c(\sigma))&\left\{\mu_0^c\left[\omega_1 - 2\gamma\nu(1-\mu_0)\right]^2 + (1-\mu_0^c)\left[\omega_2 + 2\gamma\nu\mu_0\right]^2\right\} + \\
&+ [1-\delta F(c(\sigma))]\left\{\mu_0^c\left[\sigma_1(\omega_1 - 2\gamma\nu(1-\mu_1))^2 + (1-\sigma_1)(\omega_1 - 2\gamma\nu(1-\mu_2))^2\right] + \right. \\
&\left. + (1-\mu_0^c)\left[\sigma_2(\omega_2 + 2\gamma\nu\mu_2)^2 + (1-\sigma_2)(\omega_2 + 2\gamma\nu\mu_1)^2\right]\right\}
\end{aligned}$$

The F.O.C. are:

$$
\begin{aligned}
&\delta f(c(\sigma))c_1'(\sigma)\left\{\mu_0^c\left[\omega_1-2\gamma\nu(1-\mu_0)\right]^2+(1-\mu_0^c)\left[\omega_2+2\gamma\nu\mu_0\right]^2\right\}+\\
&-\delta f(c(\sigma))c_1'(\sigma)\left\{\mu_0^c\left[\sigma_1(\omega_1-2\gamma\nu(1-\mu_1))^2+(1-\sigma_1)(\omega_1-2\gamma\nu(1-\mu_2))^2\right]+\right.\\
&\left.+(1-\mu_0^c)\left[\sigma_2(\omega_2+2\gamma\nu\mu_2)^2+(1-\sigma_2)(\omega_2+2\gamma\nu\mu_1)^2\right]\right\}+\\
&+[1-\delta F(c(\sigma))]\left\{\mu_0^c\left[(\omega_1-2\gamma\nu(1-\mu_1))^2+4\sigma_1(\omega_1-2\gamma\nu(1-\mu_1))\gamma\nu\frac{\partial\mu_1}{\partial\sigma_1}+\right.\right.\\
&\left.-(\omega_1-2\gamma\nu(1-\mu_2))^2+4(1-\sigma_1)(\omega_1-2\gamma\nu(1-\mu_2))\gamma\nu\frac{\partial\mu_2}{\partial\sigma_1}\right]+\\
&\left.+4\gamma\nu(1-\mu_0^c)\left[\sigma_2(\omega_2+2\gamma\nu\mu_2)\frac{\partial\mu_2}{\partial\sigma_1}+(1-\sigma_2)(\omega_2+2\gamma\nu\mu_1)\frac{\partial\mu_1}{\partial\sigma_1}\right]\right\}=0
\end{aligned}
$$

$$
\begin{aligned}
&\delta f(c(\sigma))c_2'(\sigma)\left\{\mu_0^c\left[\omega_1-2\gamma\nu(1-\mu_0)\right]^2+(1-\mu_0^c)\left[\omega_2+2\gamma\nu\mu_0\right]^2\right\}+\\
&-\delta f(c(\sigma))c_2'(\sigma)\left\{\mu_0^c\left[\sigma_1(\omega_1-2\gamma\nu(1-\mu_1))^2+(1-\sigma_1)(\omega_1-2\gamma\nu(1-\mu_2))^2\right]+\right.\\
&\left.+(1-\mu_0^c)\left[\sigma_2(\omega_2+2\gamma\nu\mu_2)^2+(1-\sigma_2)(\omega_2+2\gamma\nu\mu_1)^2\right]\right\}+\\
&+[1-\delta F(c(\sigma))]\left\{(1-\mu_0^c)\left[(\omega_2+2\gamma\nu\mu_2)^2+4\sigma_2(\omega_2+2\gamma\nu\mu_2)\gamma\nu\frac{\partial\mu_2}{\partial\sigma_2}+\right.\right.\\
&\left.-(\omega_2+2\gamma\nu\mu_1)^2+4(1-\sigma_2)(\omega_2+2\gamma\nu\mu_1)\gamma\nu\frac{\partial\mu_1}{\partial\sigma_2}\right]+\\
&\left.+4\gamma\nu\mu_0^c\left[\sigma_1(\omega_1-2\gamma\nu(1-\mu_1))\frac{\partial\mu_1}{\partial\sigma_2}+(1-\sigma_1)(\omega_1-2\gamma\nu(1-\mu_2))\frac{\partial\mu_2}{\partial\sigma_2}\right]\right\}=0
\end{aligned}
$$

Under Assumptions 1-2, the F.O.C.s becomes symmetric. Thus, we consider symmetric solutions where $\sigma_1=\sigma_2=\sigma^*$ and $\sigma^*\in\left[\frac{1}{2},1\right]$ is the precision of the central bank's information. It follows that the solution to the central bank's information design problem solves the following condition:

$$
2\gamma\nu(4\omega-2\gamma\nu)\left\{[1-\delta F(c(\sigma))](2\sigma^*-1)+2\delta f(c(\sigma))c'(\sigma)\left[\sigma^*(1-\sigma^*)-\frac{1}{4}\right]\right\}=0
$$

Removing irrelevant positive terms yields:

$$(2\omega - \gamma\nu)\left\{[1-\delta F(c(\sigma))]\left(\sigma^* - \frac{1}{2}\right) + \delta f(c(\sigma))c'(\sigma)\left[\sigma^*(1-\sigma^*) - \frac{1}{4}\right]\right\} = 0$$

Observe that:

$$\sigma^*(1-\sigma^*) - \frac{1}{4} = -\left(\sigma^* - \frac{1}{2}\right)^2$$

and that $1 - \delta F(c(\sigma)) > 0$ for any σ and $\delta \in [0,1]$. Substituting the quadratic expression and dividing by $1 - F(c(\sigma))$ yields:

$$(2\omega - \gamma\nu)\left\{\left(\sigma^* - \frac{1}{2}\right) - \frac{\delta f(c(\sigma))}{1-\delta F(c(\sigma))}c'(\sigma)\left(\sigma^* - \frac{1}{2}\right)^2\right\} = 0$$

Observe that:

$$\frac{\mathrm{d}}{\mathrm{d}\sigma}\ln(1-\delta F(c(\sigma))) = -\frac{\delta f(c(\sigma))}{1-\delta F(c(\sigma))}c'(\sigma) \leq 0$$

This implies that:

$$(2\omega - \gamma\nu)\left(\sigma^* - \frac{1}{2}\right)\left[1 + \left(\frac{\mathrm{d}}{\mathrm{d}\sigma}\ln(1-\delta F(c(\sigma)))\right)\left(\sigma^* - \frac{1}{2}\right)\right] = 0$$

Under Assumption 4, F denotes the CDF for a uniform distribution on $[0,1]$:

$$F(c(\sigma)) = c(\sigma)$$

This implies:

$$\ln(1-\delta F(c(\sigma))) = \ln(1-\delta c(\sigma))$$

Accordingly, two possible candidates satisfy the necessary condition: $\underline{\sigma} = \frac{1}{2}$ and $\overline{\sigma} \in (\frac{1}{2}, 1)$ such that:

$$1 + \left(\frac{\mathrm{d}}{\mathrm{d}\sigma}\ln(1-\delta c(\sigma))\bigg|_{\sigma=\overline{\sigma}}\right)\left(\overline{\sigma} - \frac{1}{2}\right) = 0$$

Therefore $\sigma^* \in \{\underline{\sigma}, \overline{\sigma}\}$. To show which candidate is sufficient, we compare utilities across them. Central bank's utility with a generic $\widehat{\sigma}$ is:

$$\begin{aligned} u_{\widehat{\sigma}} = &- \delta F(c(\sigma))(\omega - \gamma\nu)^2 \\ &- (1-\delta F(c(\sigma)))\left[\widehat{\sigma}(\omega - 2\gamma\nu(1-\widehat{\sigma}))^2 + (1-\widehat{\sigma})(\omega - 2\gamma\nu\widehat{\sigma})^2\right] \end{aligned}$$

Its utility with uninformative communication $\underline{\sigma}$ is:

$$u_{\underline{\sigma}} = -(\omega - \gamma\nu)^2$$

For $\sigma^* = \underline{\sigma}$ to be optimal, $u_{\underline{\sigma}} - u_{\widehat{\sigma}} \geq 0$. Under which condition does this relationship hold?

In the first step, $u_{\widehat{\sigma}}$ can be rewritten into:

$$\begin{aligned}
u_{\widehat{\sigma}} = & -\delta F(c(\sigma))(\omega - \gamma\nu)^2 \\
& - (1 - \delta F(c(\sigma)))(\omega - \gamma\nu)^2 + (1 - \delta F(c(\sigma)))(\omega - \gamma\nu)^2 \\
& - (1 - \delta F(c(\sigma))) \left[\widehat{\sigma}(\omega - 2\gamma\nu(1 - \widehat{\sigma}))^2 + (1 - \widehat{\sigma})(\omega - 2\gamma\nu\widehat{\sigma})^2\right]
\end{aligned}$$

It follows that:

$$u_{\widehat{\sigma}} = u_{\underline{\sigma}} - (1 - \delta F(c(\sigma))) \left[\widehat{\sigma}(\omega - 2\gamma\nu(1 - \widehat{\sigma}))^2 + (1 - \widehat{\sigma})(\omega - 2\gamma\nu\widehat{\sigma})^2 - (\omega - \gamma\nu)^2\right]$$

Thus, showing $u_{\underline{\sigma}} - u_{\widehat{\sigma}} \geq 0$ is equivalent to show:

$$\widehat{\sigma}(\omega - 2\gamma\nu(1 - \widehat{\sigma}))^2 + (1 - \widehat{\sigma})(\omega - 2\gamma\nu\widehat{\sigma})^2 - (\omega - \gamma\nu)^2 \geq 0$$

In the second step,

$$\begin{aligned}
\widehat{\sigma}(\omega - 2\gamma\nu(1 - \widehat{\sigma}))^2 &= \widehat{\sigma}\omega^2 - 4\omega\gamma\nu(1 - \widehat{\sigma})\widehat{\sigma} + 4\gamma^2\nu^2(1 - \widehat{\sigma})^2\widehat{\sigma} \\
(1 - \widehat{\sigma})(\omega - 2\gamma\nu\widehat{\sigma})^2 &= (1 - \widehat{\sigma})\omega^2 - 4\omega\gamma\nu(1 - \widehat{\sigma})\widehat{\sigma} + 4\gamma^2\nu^2(1 - \widehat{\sigma})\widehat{\sigma}^2 \\
(\omega - \gamma\nu)^2 &= \omega^2 - 2\omega\gamma\nu + \gamma^2\nu^2
\end{aligned}$$

The above inequality can be rearranged as:

$$-8\omega\gamma\nu(1 - \widehat{\sigma})\widehat{\sigma} + 4\gamma^2\nu^2(1 - \widehat{\sigma})\widehat{\sigma} + 2\omega\gamma\nu - \gamma^2\nu^2 \geq 0$$

It can be further expressed nicely as:

$$\begin{aligned}
& -4\gamma\nu(1 - \widehat{\sigma})\widehat{\sigma}(2\omega - \gamma\nu) + \gamma\nu(2\omega - \gamma\nu) \geq 0 \\
& \gamma\nu(2\omega - \gamma\nu)\left[1 - 4(1 - \widehat{\sigma})\widehat{\sigma}\right] \geq 0
\end{aligned}$$

Given that $\gamma\nu > 0$ by construction and $1 - 4(1 - \widehat{\sigma})\widehat{\sigma} \geq 0$ for any $\widehat{\sigma} \in [\frac{1}{2}, 1)$, we find that, consistent with the necessary condition, $\sigma^* = \underline{\sigma} = \frac{1}{2}$ if $2\omega - \gamma\nu \geq 0$ while $\sigma^* = \overline{\sigma} \in (\frac{1}{2}, 1)$ if $2\omega - \gamma\nu < 0$.

A.2 Proof of Lemma 2

Proof.

$$
\begin{aligned}
\pi^e(\hat{\mu}) &= \pi^T - \nu(1-2\hat{\mu}) \\
&= \pi^T - \nu\left\{1 - 2[(1-\theta)\mu_0 + \theta\mu]\right\} \\
&= \pi^T + 2\nu(1-\theta)\mu_0 + 2\nu\theta\mu - \nu \\
&= (1-\theta)\pi^T + \theta\pi^T + 2\nu(1-\theta)\mu_0 + 2\nu\theta\mu - (1-\theta)\nu - \theta\nu \\
&= (1-\theta)\left[\pi^T - \nu(1-2\mu_0)\right] + \theta\left[\pi^T - \nu(1-2\mu)\right] \\
&= (1-\theta)\pi^e(\mu_0) + \theta\pi^e(\mu)
\end{aligned}
$$

□

A.3 Proof of Proposition 2

Under adaptive belief, the problem in (11) becomes:

$$
\begin{aligned}
\min_{\sigma}\ & \delta F(c(\sigma))\left\{\mu_0^c\left[\omega_1 - 2\gamma\nu(1-\mu_0)\right]^2 + (1-\mu_0^c)\left[\omega_2 + 2\gamma\nu\mu_0\right]^2\right\} + \\
& + \left[1-\delta F(c(\sigma))\right]\left\{\mu_0^c\left[\sigma_1(\omega_1 - \gamma\nu\left(1 + (1-\theta)(1-2\mu_0) + \theta(1-2\mu_1)\right))^2 + \right.\right. \\
& \left. + (1-\sigma_1)(\omega_1 - \gamma\nu\left(1 + (1-\theta)(1-2\mu_0) + \theta(1-2\mu_2)\right))^2\right] + \\
& + (1-\mu_0^c)\left[\sigma_2(\omega_2 + \gamma\nu\left(1 - (1-\theta)(1-2\mu_0) - \theta(1-2\mu_2)\right))^2 + \right. \\
& \left.\left. + (1-\sigma_2)(\omega_2 + \gamma\nu\left(1 - (1-\theta)(1-2\mu_0) - \theta(1-2\mu_1)\right))^2\right]\right\}
\end{aligned}
$$

The F.O.C. are:

$$
\begin{aligned}
&\delta f(c(\sigma))c_1'(\sigma)\left\{\mu_0^c\left[\omega_1 - 2\gamma\nu(1-\mu_0)\right]^2 + (1-\mu_0^c)\left[\omega_2 + 2\gamma\nu\mu_0\right]^2\right\} + \\
&-\delta f(c(\sigma))c_1'(\sigma)\left\{\mu_0^c\left[\sigma_1(\omega_1 - \gamma\nu\left(1 + (1-\theta)(1-2\mu_0) + \theta(1-2\mu_1)\right))^2 + \right.\right. \\
&\qquad \left. + (1-\sigma_1)(\omega_1 - \gamma\nu\left(1 + (1-\theta)(1-2\mu_0) + \theta(1-2\mu_2)\right))^2\right] + \\
&\qquad + (1-\mu_0^c)\left[\sigma_2(\omega_2 + \gamma\nu\left(1 - (1-\theta)(1-2\mu_0) - \theta(1-2\mu_2)\right))^2 + \right. \\
&\qquad \left.\left. + (1-\sigma_2)(\omega_2 + \gamma\nu\left(1 - (1-\theta)(1-2\mu_0) - \theta(1-2\mu_1)\right))^2\right]\right\} + \\
&+[1-\delta F(c(\sigma))]\left\{\mu_0^c\left[(\omega_1 - \gamma\nu\left(1 + (1-\theta)(1-2\mu_0) + \theta(1-2\mu_1)\right))^2 + \right.\right. \\
&\qquad + 4\sigma_1(\omega_1 - \gamma\nu\left(1 + (1-\theta)(1-2\mu_0) + \theta(1-2\mu_1)\right))\gamma\theta\nu\frac{\partial\mu_1}{\partial\sigma_1} + \\
&\qquad - (\omega_1 - \gamma\nu\left(1 + (1-\theta)(1-2\mu_0) + \theta(1-2\mu_2)\right))^2 + \\
&\qquad \left. + 4(1-\sigma_1)(\omega_1 - \gamma\nu\left(1 + (1-\theta)(1-2\mu_0) + \theta(1-2\mu_2)\right))\gamma\theta\nu\frac{\partial\mu_2}{\partial\sigma_1}\right] + \\
&\qquad + 4\gamma\theta\nu(1-\mu_0^c)\left[\sigma_2(\omega_2 + \gamma\nu\left(1 - (1-\theta)(1-2\mu_0) - \theta(1-2\mu_2)\right))\frac{\partial\mu_2}{\partial\sigma_1} + \right. \\
&\qquad \left.\left. + (1-\sigma_2)(\omega_2 + \gamma\nu\left(1 - (1-\theta)(1-2\mu_0) - \theta(1-2\mu_1)\right))\frac{\partial\mu_1}{\partial\sigma_1}\right]\right\} = 0
\end{aligned}
$$

$$\delta f(c(\sigma))c_2'(\sigma)\left\{\mu_0^c\left[\omega_1-2\gamma\nu(1-\mu_0)\right]^2+(1-\mu_0^c)\left[\omega_2+2\gamma\nu\mu_0\right]^2\right\}+$$

$$-\delta f(c(\sigma))c_2'(\sigma)\left\{\mu_0^c\left[\sigma_1(\omega_1-\gamma\nu\left(1+(1-\theta)(1-2\mu_0)+\theta(1-2\mu_1)\right))^2+\right.\right.$$

$$\left.+(1-\sigma_1)(\omega_1-\gamma\nu\left(1+(1-\theta)(1-2\mu_0)+\theta(1-2\mu_2)\right))^2\right]+$$

$$+(1-\mu_0^c)\left[\sigma_2(\omega_2+\gamma\nu\left(1-(1-\theta)(1-2\mu_0)-\theta(1-2\mu_2)\right))^2+\right.$$

$$\left.\left.+(1-\sigma_2)(\omega_2+\gamma\nu\left(1-(1-\theta)(1-2\mu_0)-\theta(1-2\mu_1)\right))^2\right]\right\}+$$

$$+[1-\delta F(c(\sigma))]\left\{(1-\mu_0^c)\left[(\omega_2+\gamma\nu\left(1-(1-\theta)(1-2\mu_0)-\theta(1-2\mu_2)\right))^2+\right.\right.$$

$$4\sigma_2(\omega_2+\gamma\nu\left(1-(1-\theta)(1-2\mu_0)-\theta(1-2\mu_2)\right))\gamma\theta\nu\frac{\partial\mu_2}{\partial\sigma_2}+$$

$$-(\omega_2+\gamma\nu\left(1-(1-\theta)(1-2\mu_0)-\theta(1-2\mu_1)\right))^2+$$

$$\left.4(1-\sigma_2)(\omega_2+\gamma\nu\left(1-(1-\theta)(1-2\mu_0)-\theta(1-2\mu_1)\right))\gamma\theta\nu\frac{\partial\mu_1}{\partial\sigma_2}\right]+$$

$$+4\gamma\theta\nu\mu_0^c\left[\sigma_1(\omega_1-\gamma\nu\left(1+(1-\theta)(1-2\mu_0)+\theta(1-2\mu_1)\right))\frac{\partial\mu_1}{\partial\sigma_2}+\right.$$

$$\left.\left.(1-\sigma_1)(\omega_1-\gamma\nu\left(1+(1-\theta)(1-2\mu_0)+\theta(1-2\mu_2)\right))\frac{\partial\mu_2}{\partial\sigma_2}\right]\right\}=0$$

Applying assumptions 1-2, the F.O.C.s becomes symmetric. Thus, we consider symmetric solutions where $\sigma_1=\sigma_2=\sigma^*$. It follows that the solution to the central bank's information design problem solves the following condition:

$$2\theta\gamma\nu(4\omega-2(2-\theta)\gamma\nu)\left\{[1-\delta F(c(\sigma))](2\sigma^*-1)+2\delta f(c(\sigma))c'(\sigma)\left[\sigma^*(1-\sigma^*)-\frac{1}{4}\right]\right\}=0$$

Removing irrelevant positive terms yields:

$$(2\omega-(2-\theta)\gamma\nu)\left\{[1-\delta F(c(\sigma))]\left(\sigma^*-\frac{1}{2}\right)+\delta f(c(\sigma))c'(\sigma)\left[\sigma^*(1-\sigma^*)-\frac{1}{4}\right]\right\}=0$$

Observe that:

$$\sigma^*(1-\sigma^*)-\frac{1}{4}=-\left(\sigma^*-\frac{1}{2}\right)^2$$

and that $1-\delta F(c(\sigma))>0$ for any σ and $\delta\in[0,1]$. Substituting the quadratic expression and dividing by $1 - F(c(\sigma))$ yields:

$$(2\omega - (2-\theta)\gamma\nu)\left\{\left(\sigma^* - \frac{1}{2}\right) - \frac{\delta f(c(\sigma))}{1-\delta F(c(\sigma))}c'(\sigma)\left(\sigma^* - \frac{1}{2}\right)^2\right\} = 0$$

Observe that:

$$\frac{\mathrm{d}}{\mathrm{d}\sigma}\ln(1-\delta F(c(\sigma))) = -\frac{\delta f(c(\sigma))}{1-\delta F(c(\sigma))}c'(\sigma) \leq 0$$

This implies that:

$$(2\omega - (2-\theta)\gamma\nu)\left(\sigma^* - \frac{1}{2}\right)\left[1 + \left(\frac{\mathrm{d}}{\mathrm{d}\sigma}\ln(1-\delta F(c(\sigma)))\right)\left(\sigma^* - \frac{1}{2}\right)\right] = 0$$

Accordingly, two possible candidates satisfy the necessary condition: $\underline{\sigma} = \frac{1}{2}$ and $\overline{\sigma} \in (\frac{1}{2}, 1)$ such that:

$$1 + \left(\frac{\mathrm{d}}{\mathrm{d}\sigma}\ln(1-\delta F(c(\sigma)))\right)\left(\sigma^* - \frac{1}{2}\right) = 0.$$

Therefore, $\sigma^* \in \{\underline{\sigma}, \overline{\sigma}\}$. To show which candidate is sufficient, we compare utilities across them. Central bank's utility with a generic $\widehat{\sigma}$ is:

$$\begin{aligned} u_{\widehat{\sigma}} = &- \delta F(c(\sigma))(\omega - \gamma\nu)^2 \\ &- (1-\delta F(c(\sigma)))\left[\widehat{\sigma}(\omega - \gamma\nu(1+\theta(1-2\widehat{\sigma})))^2 + (1-\widehat{\sigma})(\omega - \gamma\nu(1-\theta(1-2\widehat{\sigma})))^2\right] \end{aligned}$$

Its utility with uninformative communication $\underline{\sigma}$ is:

$$u_{\underline{\sigma}} = -(\omega - \gamma\nu)^2$$

For $\sigma^* = \underline{\sigma}$ to be optimal, $u_{\underline{\sigma}} - u_{\widehat{\sigma}} \geq 0$. Under which condition does this relationship hold?

In the first step, $u_{\widehat{\sigma}}$ can be rewritten into:

$$\begin{aligned} u_{\widehat{\sigma}} = &- \delta F(c(\sigma))(\omega - \gamma\nu)^2 \\ &- (1-\delta F(c(\sigma)))(\omega - \gamma\nu)^2 + (1-\delta F(c(\sigma)))(\omega - \gamma\nu)^2 \\ &- (1-\delta F(c(\sigma)))\left[\widehat{\sigma}(\omega - \gamma\nu(1+\theta(1-2\widehat{\sigma})))^2 + (1-\widehat{\sigma})(\omega - \gamma\nu(1-\theta(1-2\widehat{\sigma})))^2\right] \end{aligned}$$

It follows that:

$$u_{\widehat{\sigma}} = u_{\underline{\sigma}} - (1 - \delta F(c(\sigma))) \Big[\widehat{\sigma}(\omega - \gamma\nu(1 + \theta(1 - 2\widehat{\sigma})))^2$$
$$+ (1 - \widehat{\sigma})(\omega - \gamma\nu(1 - \theta(1 - 2\widehat{\sigma})))^2 - (\omega - \gamma\nu)^2 \Big]$$

Thus, showing $u_{\underline{\sigma}} - u_{\widehat{\sigma}} \geq 0$ is equivalent to show:

$$\widehat{\sigma}(\omega - \gamma\nu(1 + \theta(1 - 2\widehat{\sigma})))^2 + (1 - \widehat{\sigma})(\omega - \gamma\nu(1 - \theta(1 - 2\widehat{\sigma})))^2 - (\omega - \gamma\nu)^2 \geq 0$$

In the second step,

$$\begin{aligned}
\widehat{\sigma}(\omega - \gamma\nu(1 + \theta(1 - 2\widehat{\sigma})))^2 &= \widehat{\sigma}[\omega^2 - 2\omega\gamma\nu(1 + \theta(1 - 2\widehat{\sigma})) + \gamma^2\nu^2(1 + \theta(1 - 2\widehat{\sigma}))^2] \\
(1 - \widehat{\sigma})(\omega - \gamma\nu(1 - \theta(1 - 2\widehat{\sigma})))^2 &= (1 - \widehat{\sigma})[\omega^2 - 2\omega\gamma\nu(1 - \theta(1 - 2\widehat{\sigma})) + \gamma^2\nu^2(1 - \theta(1 - 2\widehat{\sigma}))^2] \\
(\omega - \gamma\nu)^2 &= \omega^2 - 2\omega\gamma\nu + \gamma^2\nu^2
\end{aligned}$$

The above inequality can be rearranged as:

$$-2\omega\gamma\nu[1 - \theta(1 - 2\widehat{\sigma})^2] + \gamma^2\nu^2[1 + \theta^2(1 - 2\widehat{\sigma})^2 - 2\theta(1 - 2\widehat{\sigma})^2] + 2\omega\gamma\nu - \gamma^2\nu^2 \geq 0$$

It can be further expressed nicely as:

$$\begin{aligned}
&2\omega\gamma\nu\theta(1 - 2\widehat{\sigma})^2 + \gamma^2\nu^2\theta(\theta - 2)(1 - 2\widehat{\sigma})^2 \geq 0 \\
&\gamma\nu\theta(1 - 2\widehat{\sigma})^2[2\omega - \gamma\nu(2 - \theta)] \geq 0
\end{aligned}$$

Given that $\gamma\nu\theta \geq 0$ by construction and $(1 - 2\widehat{\sigma})^2 \geq 0$ for any $\widehat{\sigma} \in [\frac{1}{2}, 1)$, we find that, consistent with the necessary condition, $\sigma^* = \underline{\sigma} = \frac{1}{2}$ if $2\omega - \gamma\nu(2 - \theta) \geq 0$ while $\sigma^* = \overline{\sigma} \in (\frac{1}{2}, 1)$ if $2\omega - \gamma\nu(2 - \theta) < 0$.