Consumer Bankruptcy: the Role of Financial Frictions*

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Abstract

Bankruptcy leniency leads to higher default premia in equilibrium. The higher relative borrowing-to-saving price also renders financial intermediaries more externally funded via deposits. However, more leveraged intermediaries face an extra premium because of the agency problem between banks and depositors (financial frictions). The premium results in increased borrowing prices (leverage channel). In addition, the premium reduces banks' investment in physical capital due to the no-arbitrage condition. Lower capital leads to lower earnings for all households (divestment channel). Both channels jointly influence the welfare implication of bankruptcy laws. To understand to what extent financial frictions shape the welfare implication, I develop an Aiyagari-type model with consumer default and financial frictions. I find that households instead prefer a stricter bankruptcy regime to offset the adverse effects of financial frictions. The preliminary result also suggests that a lenient bankruptcy code might not be optimal even when households face significant idiosyncratic risks.

Keywords: Consumer Credit, Bankruptcy, Default, Financial Frictions

JEL Classifications: D14, E44, K35

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1 Introduction

Consumer bankruptcy serves as essential insurance for households. In the face of financially adverse shocks, households can be released from unaffordable debt burdens by filing for bankruptcy. However, bankruptcy leniency results in increased borrowing costs due to higher default premia. Such higher borrowing costs worsen households' ability to smooth consumption intertemporally. The welfare implication of bankruptcy laws thus hinges upon the insurance-efficiency trade-off between *smoothing across states* and *smoothing over time* (Zame, 1993). This trade-off has been studied in the literature with the assumption of "frictionless" financial markets.[1]

However, financial frictions exist prevalently and affect the practice of financial intermediaries. One type of financial friction is the agency problem between banks and depositors. The incentive for banks to deviate from continuation increases as banks obtain more external funds via deposits. To prevent such an action taken by banks, banks are required to have skin in the game—an extra premium attached to the future returns on assets makes today's deviation costly. As a result, banks' loan issuance is subjective to not only the refinancing and operating costs, but also the extra premium.[2]

Financial frictions may affect the welfare implication of bankruptcy regimes. For example, under a lenient bankruptcy regime, aggregate savings increase as the relative price between saving and borrowing drops due to a higher default premium. Banks, in turn, become more externally financed. With the presence of financial frictions, an extra premium accordingly emerges to disincentivize banks to deviate. However, the literature has not explored such a channel when evaluating the welfare implication of bankruptcy laws. In this paper, I attempt to fill this void by addressing two questions:

1. How do financial frictions affect the welfare implication of bankruptcy laws?

2. To what extent does the welfare implication of bankruptcy laws vary with the pres-

[1] The "frictionless" means that financial intermediaries can efficiently distribute available funds to those in need, in addition to incomplete markets. For example, Athreya (2002) study the welfare implication of abolishing bankruptcy regulation and mean-testing. Livshits, MacGee, and Tertilt (2007) stress the role of the life-cycle component and idiosyncratic uncertainty in evaluating the welfare implication of bankruptcy regimes. See Exler and Tertilt (2020) for a complete survey.

[2] There are some related evidence to consumer finance. For example, Nakajima and Ríos-Rull (2014) and Fieldhouse, Livshits, and MacGee (2016) both suggest that a countercyclical shock to intermediation cost can help match business cycles of unsecured credit and the number of bankruptcy. Dempsey and Ionescu (2020) find only the default premium cannot explain the borrowing premium on the revolving credit card debts identified in the data.

ence of financial frictions?

To this end, I extend the standard consumer default model (Chatterjee, Corbae, Nakajima, and Ríos-Rull, 2007; Livshits et al., 2007) by introducing financial frictions following Gertler and Karadi (2011). Households can file for bankruptcy to insure themselves against the idiosyncratic risk of labor productivity. Filers can discharge all debts at the costs of wage garnishment and temporary exclusion from credit markets. Banks use net worth and deposits to invest in physical capital and issue one-period defaultable unsecured loans. Crucially, an agency problem exists between banks and depositors: banks are tempted to divert funds if deposits outweigh the accumulated net worth. In other words, banks might default, provided that the major funding is obtained from outside of banks. An incentive constraint is imposed to restrict banks from expanding their balance sheets willingly, thus avoiding banks' deviation.

Under this framework, borrowing term depends on loan size, idiosyncratic household characteristics, and the induced premium due to the agency problem. First of all, if a household takes out a larger borrowing amount or has a worse future income prospect, then the likelihood that the household will repay the debt decreases. As a result, banks charge a higher borrowing price today to compensate for this potential loss in the future. In addition, when deposits dominate banking capitalization, banks face a higher leverage ratio and are tempted to deviate. To incentivize banks to continue, an extra leverage premium is attached to the borrowing interest rates. The higher returns on unsecured loans in the future thus make it unprofitable for banks to deviate today.

The current parameterization is suggestive. Most parameters are exogenously determined by direct empirical evidence or taken from the related literature. The model period is a year. Nonetheless, the uncalibrated model can already account for some untargeted moments fairly well, such as the fraction of households in debt and the average borrowing interest rate.

I find that a stricter bankruptcy regime is favored with financial frictions compared to without financial frictions. This distinction can be explained by the leverage and divestment channels. Under lenient bankruptcy law, banks face a higher leverage ratio in equilibrium. Due to the agency problem, an extra leverage premium must be attached to the return on assets to make it profitable for banks to continue their business. As a result, a wedge arises between the return on unsecured loans and their refinancing costs, thus leading to increased borrowing prices. Higher borrowing costs worsen households' ability to smooth consumption intertemporally (leverage channel). In addition, the higher return on assets crowds out physical capital due to no-arbitrage. Lower capital gives rise to lower production and wages owing to factor complementarity. As a result, all households receive fewer earnings (divestment channel). Both channels emerge when financial frictions exist and thus affect the welfare outcome of bankruptcy laws. Therefore, households rather prefer a stricter bankruptcy regime to minimize the negative effects of bankruptcy leniency, coming into effect through the two channels.

The rest of the paper is organized as follows. Section 2 presents the model. Parameterization is discussed in Section 3. In section 4, I first show how a lenient bankruptcy regulation results in a higher banking leverage ratio. I then explain to what extent the welfare implication of bankruptcy laws varies with or without financial frictions. Section 5 concludes and points out future extensions.

2 The Model

Time is discrete and infinite.[3] The market is incomplete. There is a measure-one continuum of households. There also exist firms and banks. Both operate in perfectly competitive markets. Firms produce homogeneous goods using a decreasing returns to scale technology. Banks offer saving and lending services in the form of one-period assets and unsecured defaultable loans.

In each period, households receive persistent labor productivity e following an AR(1) process $Q^e(e'|e)$ and transitory labor productivity z determined by an i.i.d. process $Q^z(z)$. All income realizations are independent across individuals. An i.i.d. preference shock ν following $Q^\nu(\nu)$ arrives at the beginning of each period which changes households' degrees of patience measured by their discount factors β_H.

Households are risk-averse and derive utility from consumption c. Households supply their unit labor endowment inelastically and receive efficiency wages $w \cdot e \cdot z$ dependent on their labor productivity. Households can either borrow or save an amount b' at the discount price q with banks. At the beginning of each period, if a household has any bank

[3] I follow the convention of dynamic programming that the time subscript is removed, and the next-period variable is expressed with prime $'$.

debt $b < 0$, she can choose to repay $d = 0$ or file for bankruptcy $d = 1$. Defaulters can discharges all their bank debts $b = 0$ but are subject to wage garnishment at the rate η and temporary exclusion from the bank asset market in the filing period $b' = 0$. Household states are summarized as (b, e, z, ν). The cross-sectional distribution of households is denoted by $\mu(b, e, z, \nu)$.

Banks are risk-neutral. symmetric,[4] and owned by foreign investors that are not modeled in the economy. Banks can either borrow or save at the risk-free rate r_f in the international financial markets. Banks use net worth N and deposit D' to invest in physical capital K' and issue one-period unsecured defaultable loans L'. Compared to investment, loan issuance is subjective to a transaction cost τ. Banks' profits π are determined by the realized returns on physical capital and loans, net of the costs of deposits. Banks then follow an exogenous retention policy ψ to accumulate net worth $N = \psi\pi$ and pay out the remaining as dividends $(1 - \psi)\pi$. The objective of banks is to maximize the sum of discounted future dividends at the rate β_B.

Financial frictions arise endogenously because an agency problem exists between banks and depositors. Banks can divert θ fraction of assets $K' + L'$, sell them at a secondary frictionless market, and leave the market without being noticed. To prevent banks from deviation, an incentive constrain is imposed to restrict banks' balance sheets. When this constraint binds, banks charge an extra leverage premium ι to make it more (less) profitable to continue (deviate). This premium introduces a wedge between the return on assets and the external financing cost. Banks can observe all characteristics of households that influence the repayment probability. Given the perfect competition, banks use the zero-profit condition to pin down the risk-based discount borrowing price schedule of borrowing amounts and household persistent types $q(b', e)$.

2.1 Timing

The timing in every period is summarized as follows:

1. Households begin each period with state (b, e, z, ν).

2. Given the discount borrowing price schedule $q(b', e)$, households choose to either

[4] Symmetry means that every bank chooses the same portfolio. This is a standard assumption in the literature. With this assumption imposed, it is equivalent to studying a representative bank.

repay debt $d = 0$ or file for bankruptcy $d = 1$.

- If $d = 0$, they also choose b' and consume $c = w \cdot e \cdot z + b - q(b', e) \cdot b'$.
- If $d = 1$, they consume the leftover earnings $c = (1 - \eta) \cdot w \cdot e \cdot z$

3. e', z', and ν' are drawn from $Q^e(e'|e)$, $Q^z(z')$, and $Q^\nu(\nu')$.

2.2 Households

Households take as given the bank discount pricing function $q(b', e)$. As the beginning of each period, households can choose between full repayment and filing for bankruptcy.

The value function of repayment $d = 0$ is given by:

$$\begin{aligned} V^r(b, e, z, \nu) = \max_{b'} \Big[& u\left(w \cdot e \cdot z + b - q(b', e) \cdot b'\right) \\ & + \nu \beta_H \cdot \sum_{(e', z', \nu')} Q^e(e'|e) \cdot Q^z(z') \cdot Q^\nu(\nu') \cdot V(b', e', z', \nu') \Big], \end{aligned} \tag{1}$$

where the utility function defined on consumption $u(c)$ is additively separable over time, continuous, increasing, and concave. The value function of defaulting $d = 1$ is given by:

$$\begin{aligned} V^d(b, e, z, \nu) = & u\left((1 - \eta) \cdot w \cdot e \cdot z\right) \\ & + \nu \beta_H \cdot \sum_{(e', z', \nu')} Q^e(e'|e) \cdot Q^z(z') \cdot Q^\nu(\nu') \cdot V(0, e', z', \nu'), \end{aligned} \tag{2}$$

where recall that η denotes the garnishment rate. η will be used as a proxy for bankruptcy strictness: higher η means a stricter bankruptcy regime. Filers can neither save nor borrow with banks only in the current period $b' = 0$.[5]

The unconditional value function is then determined by the maximum of these two conditional value functions on the default decision:

$$V(b, e, z, \nu) = \max_{d \in [0,1]} \left[(1 - d) \cdot V^r(b, e, z, \nu) + d \cdot V^d(b, e, z, \nu) \right]. \tag{3}$$

Solving household maximization problem yields the policy functions for consumption $g_c(b, e, z, \nu)$, bank asset choice $g_b(b, e, z, \nu)$, and default decision $g_d(b, e, z, \nu)$.

[5] This simplification follows Livshits et al. (2007).

2.3 Firms

Firms produce homogeneous goods Y using physical capital K and aggregate labor in efficiency units E with the standard Cobb-Douglas technology:

$$Y = F(K, E) = K^{\alpha} E^{1-\alpha}, \tag{4}$$

where α denotes the capital share and aggregate labor in efficiency units is defined as:

$$E = \sum_{(b,e,z,\nu)} e \cdot z \cdot \mu(b, e, z, \nu). \tag{5}$$

The rate of return on physical capital and wage can be derived as:

$$r_k = F_K(K, E), \tag{6}$$

$$w = F_E(K, E). \tag{7}$$

2.4 Banks

Banks issue one-period contracts of a amount b' conditional on persistent labor productivity e. The total number of a contract with term (b', e) is denoted by $\mathcal{B}'(b', e)$. Negative amount $b' < 0$ refers to a unsecured borrowing contract. Ex-post breach of contract is allowed for households but at the costs of wage garnishment and temporary exclusion from the bank asset market. Positive amount $b' > 0$ refers to a saving contract with the risk-free rate r_f. The aggregate defaultable unsecured loans L' and deposits D' can be defined as:

$$L' = \sum_{(b'<0,\, e)} \mathcal{B}'(b', e) \cdot q(b', e) \cdot -b', \tag{8}$$

$$D' = \sum_{(b'>0,\, e)} \mathcal{B}'(b', e) \cdot q(b', e) \cdot b', \tag{9}$$

where $q(b' > 0, e) \equiv (1 + r_f)^{-1}$ by construction.

Recall that banks use net worth N and deposits D' to invest in physical capital K' and issue loans L'. Loan issuance is subjective to transaction costs τ. The balance sheet of banks can be accordingly characterized as:

$$K' + (1+\tau)L' = D' + N. \tag{10}$$

The next-period profits π' are determined by the gains from investment and lending net of the gross interest payments to depositors. In particular,

$$\pi' = (1 + r_k' - \delta)K' + (1 + r_l')L' - (1 + r_f)D', \tag{11}$$

where δ is the depreciation rate and $1 + r_l'$ denotes the gross expected rate of return on the unsecured loans. The rate is defined as the ratio of expected total amounts repaid tomorrow to the total amounts lent out today. To be specific,

$$1 + r_l' \equiv \frac{-\sum_{(b'<0,e,e')} \mathcal{B}'(b',e) \cdot Q^e(e'|e) \cdot R(b',e')}{L'} \tag{12}$$

where $R(b', e')$ denotes the amount that banks receive tomorrow for a contract amount of b' conditional on persistent labor productivity being e' tomorrow. This term consists of full repayment if $d' = 0$ and wage garnishment if $d' = 1$, defined as:

$$R(b', e') = (1 - d') \cdot b' + d' \cdot \eta \cdot w' \cdot e' \cdot z'. \tag{13}$$

Banks can divert θ fraction of assets $K' + L'$ and sell them at an international frictionless secondary market without being noticed (Gertler and Karadi, 2011). For depositors to stay with banks, the banking value function $W(N)$ must be greater than or equal to the diverting gain $\theta(K' + L')$. The incentive constraint is imposed as:

$$W(N) \geq \theta(K' + L'). \tag{14}$$

As widely shown in the literature, the banking value function can be rewritten as $W(N) \equiv \xi \cdot N$ where ξ denotes the marginal benefit of holding one additional unit of net worth.[6] Plugging it into (14) yields:

$$\frac{\xi}{\theta} \geq \left(\frac{K' + L'}{N}\right) \equiv LR', \tag{15}$$

[6] This linear form results from the assumption of risk neutrality and all constraints are linear in net worth.

where LR' denotes the banking leverage ratio. Higher LR' means that banks are more leverage or more externally financed through deposits. As can be seen in Equation (15), the incentive constraint translates into an endogenous leverage ratio constraint.

The objective of banks is to maximize the sum of discounted dividends. The banking constrained optimization problem in the recursive form can be summarized below:

$$
\begin{aligned}
W(N) = & \max_{K', \mathcal{B}'} \left[\beta_B(1-\psi)\pi' + \beta_B W(N')\right] \\
\text{s.t.} \quad & K' + (1+\tau)L' = D' + N, \\
& \pi' = (1 + r'_k - \delta)K' + (1 + r'_l)L' - (1 + r_f)D', \\
& N' = \psi\pi', \\
& W(N) \geq \theta(K' + L').
\end{aligned}
\tag{16}
$$

Solving this problem gives rise to the no-arbitrage conditions:

$$
r'_k - (\delta + r_f) = r'_l - (\tau + r_f) = \iota \equiv \frac{\lambda\theta}{\Lambda'} \geq 0, \tag{17}
$$

where ι denotes the leverage premium; λ stands for the multiplier on the incentive constraint; and $\Lambda' = \beta_B(1 - \psi + \psi\xi')$ denotes the adjusted discount factor for banks.

These conditions are intuitive. If there are no financial frictions $\theta = 0$, the leverage premium is trivially fixed at zero $\iota = 0$. Under this case, the excess returns on either capital investment or lending must be both zeros, otherwise banks will change their portfolio until equalizing the two. When there are financial frictions $\theta > 0$, the premium may emerge if the banking leverage ratio is too high under which banks are tempted to divert assets (i.e., the incentive constraint is binding). This extra premium in turn introduces a wedge into the excess returns. Positive returns result in increased profits in the future, thus rendering deviation undesirable for banks.

By plugging Equation (12) and (13) to (17), the bank discount pricing schedule can be derived as:

$$
q(b', e) = \frac{\sum_{e'}\left[(1-d') \cdot b' + d' \cdot \eta \cdot w' \cdot e' \cdot z'\right] Q^e(e'|e)}{1 + r_f + \tau + \iota}. \tag{18}
$$

As in the existing literature,[7] the numerator of Equation (18) captures the expected amount received by banks in the next period across all realizations of persistent labor productivity, weighted by its transition probability. In other words, this term measures the individual-level default premium. However, my setting additionally introduces a leverage premium into the denominator of Equation (18) due to the presence of leverage frictions. A shortage in banking net wroth or increased banking leverage ratio leads to higher leverage premium, thus lowering all discount borrowing prices (i.e., higher borrowing costs).

2.5 Evolution of the Household Distribution

The probability for an individual to move from state (b, e, z, ν) to (b', e', z', ν') is governed by the following mapping:

$$T^*(b', e', z', \nu' | b, e, z, \nu) = \mathbb{I}_{[b' = g_b(b,e,z,\nu)]} \cdot Q^e(e'|e) \cdot Q^z(z') \cdot Q^\nu(\nu'). \tag{19}$$

Therefore, the cross-sectional distribution of households μ evolves according to:

$$\mu'(b', e', z', \nu') = \sum_{(b,e,z,\nu)} T^*(b', e', z', \nu' | b, e, z, \nu) \cdot \mu(b, e, z, \nu). \tag{20}$$

2.6 Stationary Recursive Competitive Equilibrium

A stationary Recursive Competitive Equilibrium (RCE) is a set of value functions V^* and W^*, household policy functions g_c^*, g_b^*, and g_d^*, factor prices r_k^* and w^*, bank loan pricing function q^*, leverage premium ι^*, aggregate variables L^*, D^*, and K^*, and a distribution $\bar{\mu}^*$ such that:

1. Household Optimality: $V^*(b, e, z, \nu)$, $g_c^*(b, e, z, \nu)$, $g_b^*(b, e, z, \nu)$, and $g_d^*(b, e, z, \nu)$ satisfy Equation (1), (2), and (3) for all (b, e, z, ν).

2. Factor Prices: r_k^* and w^* satisfy Equation (6) and (7).

3. Bank Optimality: $W^*(N)$, ι^*, L^*, D^*, K^* solves Equation (16) and $q^*(b', e)$ satisfies Equation (18) for all (b', e).

[7] For example, Nakajima and Ríos-Rull (2014) in consumer finance or Arellano (2008) in sovereign debts. In both papers, the bank loan pricing function depends mainly on default premia.

4. Stationary Distribution: $\bar{\mu}^*(b, e, z, \nu)$ solves Equation (20).

To numerically solve the model, I implement the one-loop algorithm where the value functions, loan pricing functions, and the household distribution are updated simultaneously in each iteration until convergence.[8] Solving the equilibrium leverage premium is not trivial since the premium is determined by the loan pricing function and the household distribution. However, these three objects are mutually dependent, leading to slow convergence. In particular, a better approximation for the loan pricing function is essential; otherwise, a non-smooth pricing scheme often results in discrete jumps in aggregate unsecured loans and thus the leverage premium. Refer to Appendix A and B for details.

3 Parameterization

The current parameterization is suggestive. Most parameters are taken from the related literature or exogenously determined by direct empirical evidence. The model period is a year. Following Livshits et al. (2007), I set the household discount factor equal to 0.94. The benchmark wage garnishment rate and the borrowing transaction cost are set to 0.355 and 0.04, respectively. The bank discount factor is chosen to be 0.96. The implied risk-free rate is 4.17%, aligned with the common value used in the literature. The coefficient of relative risk aversion (CRRA) is set equal to 2, the standard value in the macro literature. The depreciation rate is fixed at 8%. The banking diverting fraction and retention ratio are taken from Gertler and Karadi (2011). They are $\theta = 0.381$ and $\psi = 0.972$, meaning that banks can divert 38.1% of assets secretly and pay out 2.8% of profits as dividends.

The labor productivity processes are taken from Nakajima and Ríos-Rull (2014). The AR(1) coefficient of persistent labor productivity is 0.963. The standard deviations of persistent and transitory labor productivity are 0.13 and 0.35, respectively. For the scale of patience/preference shock, I consider two states for convenience: $\nu_L = 0$ and $\nu_H = 1$. A household hit by $\nu_L = 0$ means that she becomes extremely myopic with zero weight on the future value. The probability that a household is hit by ν_L is 1%. The specification implies that in each period there is 1% of households who become extremely myopic with zero weight on the future value. The other 99% of households possess the benchmark discount factor. All are summarized in Table 1.

[8] See, for example, Hatchondo, Martinez, and Sapriza (2010) in details.

Parameter		Value	Source
β_H	Household discount factor	0.94	Livshits et al. (2007)
η	Wage garnishment rate	0.355	Livshits et al. (2007)
τ	Transaction cost	0.04	Livshits et al. (2007)
β_B	Bank discount factor	0.96	risk-free rate = 4.17%
σ	CRRA	2	standard value
α	Capital share	0.33	standard value
δ	Depreciation rate	0.08	standard value
θ	Diverting fraction	0.381	Gertler and Karadi (2011)
ψ	Retention ratio	0.972	Gertler and Karadi (2011)
ρ_p	AR(1) of persistent labor productivity	0.963	Nakajima and Ríos-Rull (2014)
σ_p	S.D. of persistent labor productivity	0.13	Nakajima and Ríos-Rull (2014)
σ_t	S.D. of transitory labor productivity	0.35	Nakajima and Ríos-Rull (2014)
$\nu_{impatient}$	Impatience shock	0	extreme myopia
$\mathbb{P}_\nu$	Probability of being impatient	0.01	1% impatient households

Table 1: Suggestive Parameterization

Variables (in %)	Data	Model
Default rate (%)	0.99	0.32
Debt-to-income ratio (cond.)	0.35	0.23
Average borrowing interest rate	12.87	14.51
Fraction of households in debt	10.43	15.89

Table 2: Data v.s. Model Moments

Table 2 illustrates a few important moments in the literature of consumer finance, including the default rate, the debt-to-income ratio (condition on having any loans), the average borrowing interest rate, and the fraction of households in debt. The second column reports the value generated by the model with the given parameterization. The third column denotes the corresponding data moments taken from Chatterjee, Corbae, Dempsey, and Ríos-Rull (2020).[9] One can see that some model moments are already quite close to the data counterparts, even though the model is uncalibrated.

4 Results

In this section, I will first compare the equilibria across the different degrees of bankruptcy strictness in Section 4.1. This comparison demonstrates how bankruptcy law affects the

[9] The default rate in the data is computed as the total number of non-business Chapter 7 filings normalized by the total number of U.S. households. The debt-to-income ratio is calculated conditional on the households that possess negative net worth. The fraction of households in debt follows the same definition.

leverage ratio of banks. If the banking leverage ratio is sufficiently high, financial frictions come into play. Second, the welfare implication of bankruptcy laws concerning different levels of strictness is computed with and without financial frictions in Section 4.2. Finally, I will also explain that the above welfare conclusion is affected by financial frictions through the leverage and divestment channels.

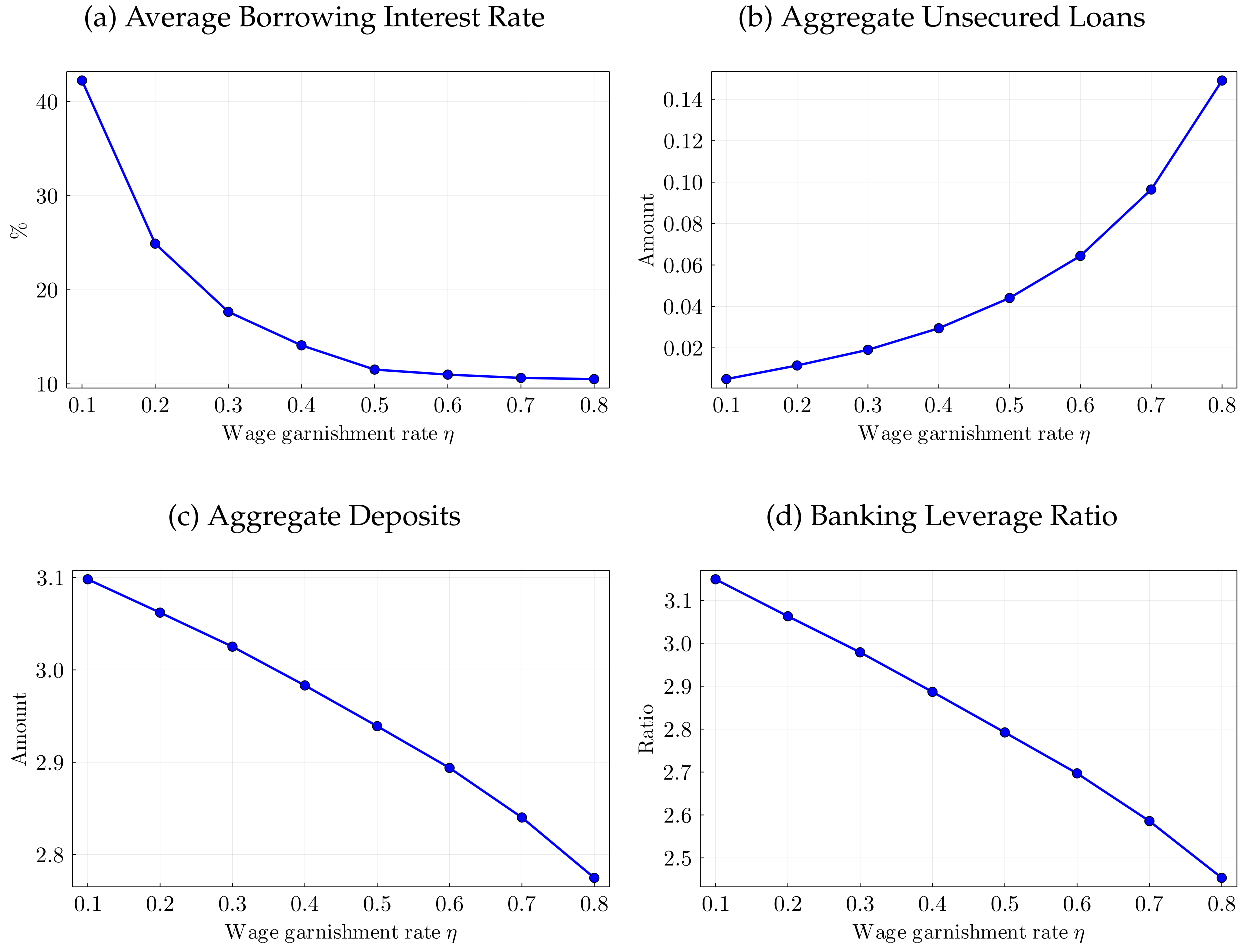
4.1 Equilibrium Comparison across Bankruptcy Strictness

I solve equilibria across the degree of bankruptcy strictness first without financial frictions to demonstrate in which direction the equilibrium banking leverage varies with the degree of bankruptcy strictness. In particular, I use the wage garnishment rate as a proxy for such strictness and consider the rates ranging from 10% to 80%, increased by 10% each. A lower garnishment rate refers to a more lenient bankruptcy regime.

Figure 1 presents the average borrowing interest rate (Figure 1a), the aggregate unsecured loans (Figure 1b), the aggregate deposits (Figure 1c), and the banking leverage ratio (Figure 1d), in equilibrium across wage garnishment rates. One can see that the average borrowing interest rate decreases in bankruptcy strictness as a stricter bankruptcy code results in a lower default premium. Lower borrowing prices lead to a lower relative borrowing-to-saving price since the saving rate is constant at the risk-free rate. The decreased borrowing-to-saving price gives rise to more aggregate unsecured loans and fewer aggregate deposits than a more lenient bankruptcy regime. With fewer deposits, banks become less externally financed and thus have a lower leverage ratio.

Then, I redo the exercises with financial frictions. Figure 2 plots the leverage premium across wage garnishment rates without financial frictions (solid line) and with financial frictions (dashed line). Leverage premia are mechanically all zeros given the absence of financial frictions. With financial frictions, the leverage premium instead starts to increase for lower wage garnishment rates. The reason is straightforward. Recall that, in the financially frictional world, banks will deviate from continuation if the external financing dominates. Since the leniency of bankruptcy regulations induces the rise in the banking leverage ratio (as shown in Figure 1d), an extra leverage premium must be attached to the returns on assets to make continuation more profitable or, equivalently, to prevent banks from deviation. As a result, this additional leverage premium leads to increased interest rates for all borrowing contracts. See Equation (18) for details.

Figure 1: Equilibria across Bankruptcy Strictness



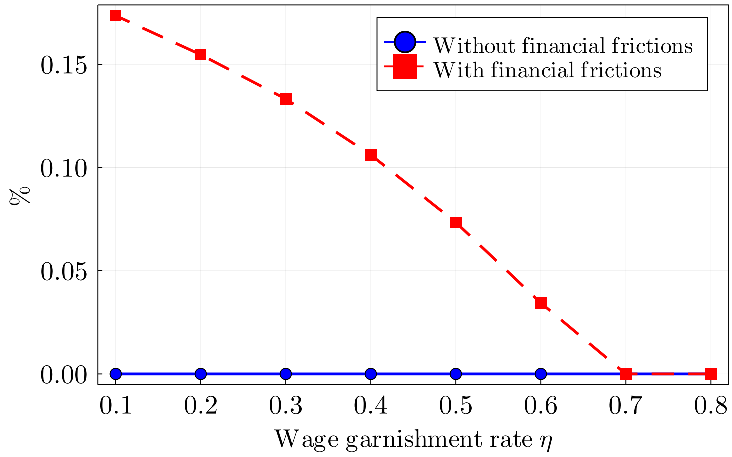
Notes: Each equilibrium is computed at the given wage garnishment rate "without" financial frictions. Banking leverage ratio is defined as the sum of physical capital and unsecured loans divided by the banking net worth.

4.2 Welfare Implication of Bankruptcy Laws

To understand how financial frictions affect the welfare implication of bankruptcy laws, I first compute the equilibria at different levels of wage garnishments rates, without and with financial frictions. Then, I measure the welfare outcome at each wage garnishments rate in consumption equivalent variation (CEV) units relative to the most lenient case $\eta = 0.1$ in percentage points in each scenario.

Figure 3 summarizes the results, where the solid (dashed) line depicts the results without (with) financial frictions and the vertical dotted line marks the wage garnishment rate that delivers the highest welfare gain in each scenario. The welfare outcomes without and with financial frictions are distinct. Without financial frictions, a medium wage garnishment rate $\eta = 0.5$ results in the largest welfare gain. Yet, with financial frictions, people

Figure 2: Leverage Premium across Bankruptcy Strictness



Notes: Without financial frictions, leverage premium is zero by construction. With financial frictions, the premium depends on whether and to what extent the incentive constraint binds. Higher banking leverage ratio results in higher leverage premium. See Equation (17) for the detailed discussion.

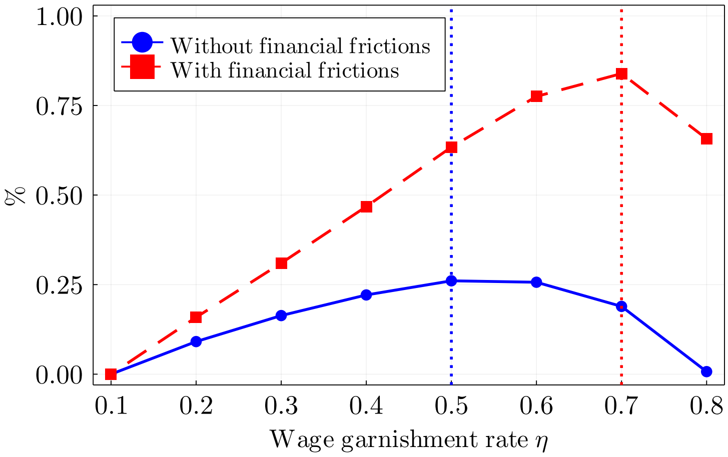
prefer a higher garnishment rate $\eta = 0.7$, meaning that a stricter bankruptcy code is preferred when financial frictions exist.

The leverage and divestment channels explain the difference in welfare conclusions with and without financial frictions. These two channels are defined as:

1. **Leverage channel**: when there exist financial frictions, bankruptcy leniency results in a higher banking leverage ratio. Higher leverage, however, magnifies the principal-agent tension between banks and depositors. As a result, an extra leverage premium must be charged and attached to the returns on assets in the future in order to prevent banks from diverting assets in the current period. This extra premium, in turn, pushes up the interest rates for all borrowing contracts.

2. **Divestment channel**: the returns on all assets must be identical in equilibrium; otherwise, banks could switch funds to the more profitable one until all returns are equivalent. Recall that, on the asset side, banks can invest in physical capital and issue unsecured loans. Due to the extra leverage premium, banks, in turn, require a higher return on physical capital. As a result, the capital investment is "crowded out" because of the assumption of diminishing returns on production factors. Given the factor complementarity, fewer capital results in a lower wage for households. This effect is labeled as the divestment channel and visualized in Figure 4. Without financial frictions (solid lines), the capital rental rate is fixed at the risk-free rate, and physical capital is constant across bankruptcy strictness. When there exist finan-

Figure 3: Welfare Implication of Bankruptcy Laws

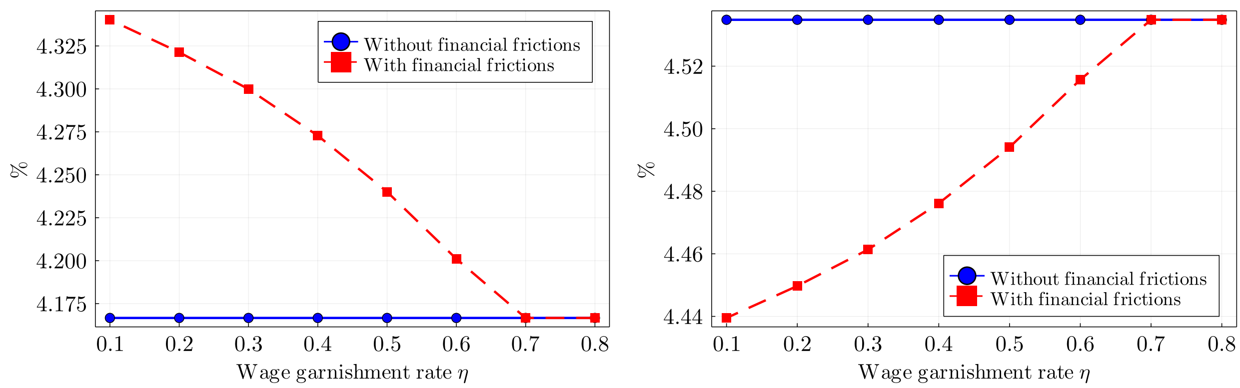


Notes: Welfare is measured in CEV units relative to the most lenient case $\eta = 0.1$, expressed in percentage points. The results with and without financial frictions are calculated independently. The vertical dotted line marks the wage garnishment rate associated with the highest welfare variation.

Figure 4: Divestment Channel

(a) Capital Rental Rate

(b) Physical Capital



cial frictions (dashed lines), capital rental rate (physical capital) decreases (increases) with the wage garnishment rate, i.e., the strictness of bankruptcy regulations.

So why do households prefer a stricter bankruptcy law when there exist financial frictions compared to the case without financial frictions? First, bankruptcy leniency results in an extra leverage premium through the leverage channel. Banks internalize this additional premium by increasing borrowing interest rates for all households. As a consequence, it becomes harder for households to smooth consumption over time by borrowing. Second, bankruptcy leniency leads to a lower equilibrium wage via the divestment channel. Ceteris paribus, households thus receive fewer earnings given the inelastic labor supply assumption. Lower earnings make households worse off across all states. Therefore, when there are financial frictions, households instead prefer a stricter bankruptcy regime to avoid the adverse effects of the leverage and divestment channels.

5 Conclusion

To understand how the presence of financial frictions affects the welfare implication of bankruptcy laws, I build an Aiyagari-type mode of consumer default and financial frictions. In particular, I introduce a principal-agent problem between banks and depositors to trigger financial frictions. Under this framework, banks cannot expand their balance sheets willingly for any given bankruptcy law. The strong incentive for highly-leveraged banks to deviate from continuation is confronted by a premium attached to the return on assets. Such a higher future return thus motivates banks not to divert their assets today.

Using a quantitative macroeconomic model, I find that households prefer a stricter bankruptcy regime with financial frictions than without financial frictions. This distinction can be explained by the leverage and divestment channels. Under lenient bankruptcy law, banks face a higher leverage ratio. A leverage premium is thus charged and attached to the required returns on assets to prevent banks from deviation (leverage channel). However, higher borrowing prices accordingly worsen the household's ability to smooth consumption intertemporally. Moreover, this extra premium crowds out banks' position on physical capital (divestment channel). Decreased capital gives rise to a lower equilibrium wage and fewer earnings for households.

In the future, calibration is primary. For example, the parameters related to financial frictions could be internally calibrated by matching relevant moments. A better fit to data can, in turn, make the policy conclusions more robust. Moreover, the current welfare conclusion suggests that, when financial frictions exist, a lenient bankruptcy code is not necessarily optimal even when households face significant idiosyncratic risks. This observation necessitates two further investigations: (1) how idiosyncratic risks affect the equilibrium outcomes with and without financial frictions; (2) to what extent the magnitude of idiosyncratic risks quantitatively changes the welfare implication of bankruptcy laws.

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A Grid Specifications

Variable	Symbol	# Points	Value/Range
Persistent productivity	e	9	$\{0.23, 0.33, 0.48, 0.69, 1.0, 1.43, 2.06, 2.96, 4.25\}$
Transitory productivity	z	3	$\{0.78, 1.0, 1.26\}$
Preference/patience	ν	2	$\{0.0, 1.0\}$
Bank loan	$b < 0$	201	$[-8.0, 0.0]$
Bank asset	$b > 0$	51	$[0.0, 300.0]$

Table 3: Grids Used for Model Computation

I discretize the persistent and transitory labor productivity processes with 9 and 3 points using Tauchen's method, respectively. I choose the lower bounds for bank loans to ensure that the endogenous borrowing limits across all levels of labor productivity are included. The upper bound for bank assets is chosen in the way that the optimal choice for bank assets is included. I then consider an equally-spaced grid of 201 points for bank loans from -8.0 to 0.0 and an exponentially-spaced grid of 51 points for bank assets from 0.0 to 300.0. Refer to Figure 5 for the pricing schedules and discounted borrowing amounts in equilibrium.

B Semi-Continuous Approximation for Default Risk

Assessing default probability plays an essential role in solving endogenous default models. However, as shown in Hatchondo et al. (2010), an inaccurate computational approximation could, unfortunately, result in "wrong" results, not just quantitatively but also sometimes qualitatively. Compared to their suggestion on using quadrature methods, I propose a tractable and accurate way to evaluate the default risk.

The basic idea is to use the value functions to identify where the kinks happen, at which to default and to repay are indifferent to households with given states. Using the monotone relationship of these kinks and the associated states, the threshold in labor productivity below which household defaults on a given loan size can be identified. The exact default risk can be analytically computed by evaluating the likelihood that tomorrow's labor probability is lower then the threshold based on the distributional assumption on labor productivity. Algorithm details: TBC.

Figure 5: Pricing Schedule and Discounted Borrowing Amount

(a) Discount Loan Pricing Schedule

(b) Discounted Borrowing Amount

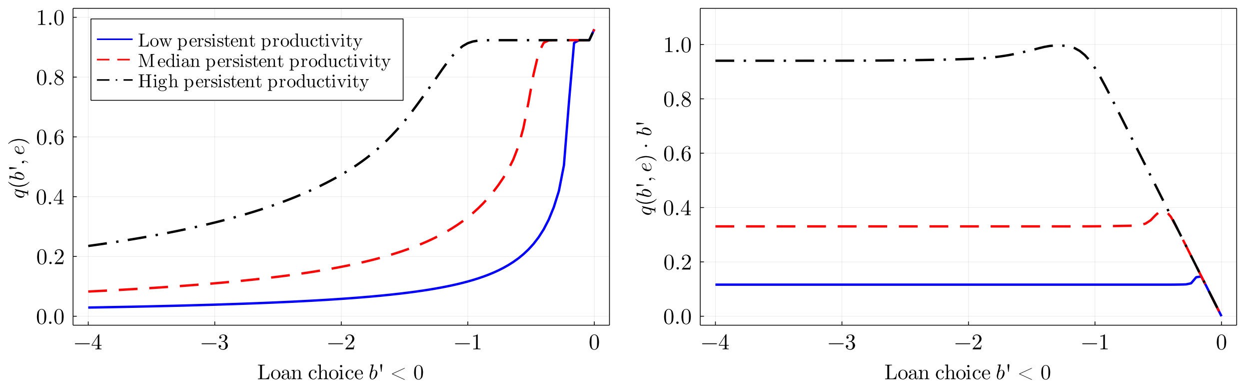

Figure 5a and 5b plot the resulting loan discount pricing functions and the associated discounted borrowing amounts across persistent labor productivity. One can see that all functions are smooth and have shapes as what the theorem would predict.