Rationalizing the Payday Loan Puzzle: A Credit Scoring Explanation*

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Abstract

Many credit cardholders in the U.S. turn to expensive payday loans, even though they have not yet exhausted their credit lines. This results in significant monetary losses and has been coined the "Payday Loan Puzzle". One proposed but not yet examined explanation is that households use payday loans to protect their credit scores (reputation) since payday lenders do not report to credit bureaus. To verify this hypothesis, we build a two-asset Huggett-type model with two default options as well as both hidden information and actions. Our calibrated model is the first able to endogenously generate the puzzle. In particular, we can quantitatively account for 40% of the puzzle occurrence and match the magnitude of monetary losses due to this seemingly pecuniary mistake. To inform the policy debate over payday lending, we assess the welfare implications of several policy counterfactuals such as banning payday loans or increasing the default costs. The welfare conclusion hinges upon the trade-off between the pooling of unobservable payday loan usage and the insurance role of payday loans.

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1 Introduction

Why do households take out expensive payday loans while they have far cheaper credit options (e.g., on their credit cards) available?[1] It has been documented that many credit card holders turn to payday loans, even though they have not yet exhausted their card limits. For example, Agarwal, Skiba, and Tobacman (2009) observe in their data that two-thirds of individuals, who use both credit cards and payday loans, have more than twice the amount of a typical payday loan left in available credit on their cards. The authors calculate that this seeming "pecuniary mistake" is very costly: These people could have saved on average \$200 by borrowing up to their credit card limits first. In a similar vein, Carter, Skiba, and Tobacman (2011) find monetary losses of a comparable magnitude for individuals using payday loans when cheaper options are available. This phenomenon has been termed the "Payday Loan Puzzle" and a variety of explanations have been put forward, such as irrationality and financial illiteracy.

The objective of this project is to examine to what extent this at-first-glance puzzling behavior can be explained by the optimal decision of rational and well-informed agents. In particular, the following "reputation protection" mechanism has been proposed:[2] People do not exhaust their credit card limits as they want to protect their credit score, since payday lenders in the U.S. in general do not report to the credit bureaus whereas borrowing or defaulting on credit cards will appear on your credit history.[3] People care about their credit score as it influences credit access, credit cost, mortgage terms, and even job application prospects.

To better understand the reasons behind the occurrence of the payday loan puzzle and to examine the above hypothesis, we build on the credit scoring framework by Chatterjee, Corbae, Dempsey, and Ríos-Rull (2020) which is a Huggett-type model with consumer default and asymmetric information.[4] We extend their framework by adding a second asset,

[1] A payday loan is a short-term loan (usually with a duration of a few weeks) for a typically small amount of around \$300. Payday loans are usually charged with very high interests corresponding to annualized percentage rates up to several hundred percent.

[2] See for example Servon (2017).

[3] A credit score is a statistic that tells lenders how likely a person is to pay back his debts. The most well-known credit score in the U.S. is the FICO score, 35% of which is determined by the payment history and 30% by the debt burden.

[4] The workhorse consumer default frameworks were developed by Chatterjee, Corbae, Nakajima, and Ríos-

payday loans, an additional financial sector, payday lenders, and a further default option to the model. Under such a framework, households can be one of two types, impatient or patient, and can take up either bank loans, (expensive) payday loans, or both to achieve consumption smoothing. They also have the option to strategically default on both kinds of loans to insure themselves against economic hardship driven by earnings shocks. Both types of financial intermediaries are unable to observe a household's type but resort to type scores as a proxy for an individual's credit worthiness, thereby providing a tailored price schedule dependent on credit score and borrowing amount. In addition, banks are also unable to observe the payday loan choices of households. Essentially, this introduces a trade-off for households when borrowing between the marginal benefit of maintaining type (credit) scores versus the marginal cost of using payday loans instead of cheaper bank loans.

To quantify the payday loan puzzle identified with the payday loan data in 2004 and to avoid the effects of the 2005 bankruptcy reform, we calibrate our model to data in 2004 by matching the formal and payday default rates. Our calibrated model can account for various untargeted moments. For instance, the average interest rates for bank and payday loans.

Our calibrated model can can quantitatively account for 40% of the puzzle households identified in the data as well as match the magnitude of the monetary costs. In addition, our model also endogenously gives rise to a reputation protection channel through which households are willing to pay higher interest costs in order to maintain their type scores.

It is important to gain a better understanding of how and why households use payday loans because these loans have been a popular target of legislation in the United States in recent years. Without a thorough understanding, legislation designed to protect households can easily end up harming them. However, the current literature does not offer much guidance for policy makers.[5]

We contribute to this policy debate by using our framework to conduct a series of counterfactual policy experiments. First, we investigate the effects of limiting the max-

Rull (2007) and Livshits, MacGee, and Tertilt (2007). See also Section 2 for related literature.

[5] The empirical papers studying the effects of payday loans on consumer welfare are largely divided between finding positive and negative effects. See Section 2 for a more detailed discussion.

imum payday loan size (a quantity cap) or even outright banning payday loans. In the former case, we find that this policy change is overall welfare decreasing. However, there is heterogeneity across households: Impatient households lose while patient ones gain. The underlying economic reasons for this outcome are intuitive: In our economy, impatient households are more likely to borrow larger payday loans and are thus more directly affected by the quantity cap. In addition, the quantity cap also reduces the amount of information asymmetry in the bank market[6] and thus the amount of pooling. In turn, this leads to a decline in cross-subsidization of impatient by patient households which additionally hurts impatient households. At the same time, this reduction in pooling and cross-subsidization is also the reason why welfare increases for patient households. In contrast to the quantity cap, the policy implementing a full ban on payday loans is in fact welfare decreasing for both types of households. The reason for the welfare loss of patient households is the reduction in available risk-sharing/insurance. Payday loans are used by both impatient and patient households to smooth idiosyncratic shocks without harming their type scores.[7] In the full ban counterfactual this loss outweighs the gain for patient households from the reduction in cross-subsidization. These results imply that current regulatory efforts in certain U.S. states to ban payday loans may be misguided in the sense that they hurt all households.

Second, we examine the implications of increasing either the formal or payday default cost. The increase in default costs is calibrated to reflect the increase in Chapter 7 filing costs after the 2005 Bankruptcy Abuse Prevention and Consumer Protection Act in the U.S.[8] We find that increasing formal default costs leads to a welfare gain whereas increasing payday default costs leads to a welfare loss for households. Theoretically, higher default costs make it harder to smooth consumption across states by defaulting but easier to smooth over time by borrowing through lower default premia. In equilibrium households prefer to default (smooth across states) using payday loans while smoothing over

[6] Recall that banks cannot observe payday loan choices of households.

[7] See Section 7.1 for a more detailed explanation.

[8] Chapter 7 is one of the bankruptcy alternatives available to households in the U.S. It entails the liquididation of non-exempt assets in return for debt dischargement. The 2005 Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) was the most significant reform of bankruptcy law in recent years. Among other changes it significantly increased the total out-of-pocket filing costs for households. See also Albanesi and Nosal (2020).

time using bank loans for three reasons: 1) Defaulting on payday loans does not directly affect a household's type score whereas formally defaulting on a bank loan does, 2) interest rates are much lower for bank loans, and 3) payday default costs are lower than formal default costs. Higher formal (payday) default cost exactly help (hamper) households in doing so.

The contributions of our paper are fourfold. To our knowledge, we are the first to explicitly model payday loans using a two-asset structure and study its interaction with bank loans and strategic defaulting behavior with two default options.[9] Second, our calibrated model is the first that can generate and quantitatively match the empirically identified payday loan puzzle. Third, we are the first to examine the reputation protection explanation rigorously in a model. Fourth, we are the first to inform the payday loan policy debate under a richer and structural framework where there are hidden information and actions as well as the substitution between bank and payday loans.

The rest of the paper is organized as follows: Section 2 gives an overview of the related literature. Section 3 details the model framework. Section 4 illustrates the current parameterization of the model. Section 5 presents the fundamental mechanism of pooling and cross-subsidization in our framework. In Section 6 we discuss in detail the payday loan puzzle and the reputation protection channel in our model. Section 7 presents the policy experiments and Section 8 concludes with some potential extensions.

2 Related Literature

In this section we discuss the literature related to our paper. The consumer finance literature (both empirical and theoretical) is extensive, thus we will only focus here on the papers most directly related to our own. We start with the papers that we build on methodologically and then give a brief summary of the literature on payday loans.

Methodologically, our theoretical framework is based on the credit scoring framework developed by Chatterjee et al. (2020). In their paper they build on the consumer default workhorse models developed by Chatterjee et al. (2007) and Livshits et al. (2007) in which

[9] Until now, there are very few theoretical studies on payday loans. See Section 2 for an overview of related papers.

households save and borrow in order to smooth consumption over time. Additionally, households are allowed to default on their loans as insurance against idiosyncratic risk. Crucially, both Chatterjee et al. (2007) and Livshits et al. (2007) assume that lenders are fully informed about all household's characteristics that affect repayment in the next period. Chatterjee et al. (2020) depart from this assumption and introduce heterogeneity across households in the form of different discount factors, which is not observable by the banks. As the patience of households affects their loan repayment probability, banks try to infer households' types by computing an individual-specific type or credit score. This score captures the individual type assessment of the banks based on observable household behavior and Bayesian updating. Our paper extends their model by introducing a second asset and an additional default option.

Our paper is also closely related to the literature on payday loans. Using matched credit card and payday loan data Agarwal et al. (2009) document that many borrowers use payday loans when they still have sufficient credit left on their credit cards, even though payday loans carry much higher interest rates. They compute that this behavior is very costly and leads to monetary losses of several hundred US dollar over one year and they coin this finding the "Payday Loan Puzzle". Furthermore, Carter et al. (2011) look at a dataset of credit union members and their payday loan borrowing behavior. They find a pecuniary loss due to usage of payday loans instead of cheaper alternatives similar to the previous paper. We contribute to this literature by generating the payday loan puzzle in a theoretical model for the first time and by offering a rational explanation behind its occurrence.[10]

Payday loans and their effects on consumers are also a hotly debated regulatory topic in the United States. The literature on the effects of payday loans on consumer welfare is in disagreement about its sign. On the one hand, using household panel survey data Zinman (2010) finds that restricting access to payday loans leads consumers to shift to bank overdrafts and late payments. The result is a decline in the financial health of affected households and an overall harmful effect of restricting payday loans. Similarly, Morse (2011) uses natural disasters to estimate that access to payday lenders leads to welfare increases.

[10] To the best of our knowledge.

Morgan, Strain, and Seblani (2012) find that the banning of payday lending leads to an increase in bounced checks and overdraft fees. These papers stress that payday loans are instrumental for households to mitigate the negative effects of transitory income or expenditure shocks, especially when access to the mainstream financial system is impaired. On the other hand, many papers point out that employing such high-cost alternatives can potentially even further worsen the financial situation of these households afterwards. Skiba and Tobacman (2019) estimate that using payday loans significantly increases bankruptcy rates by depressing the cash flow of households. Melzer (2011) finds that access to payday loans worsens the ability of households to pay mortgage, rent and utility bills. Carrell and Zinman (2014) use exogenous variation in payday loan access for military personnel to estimate that usage of payday loans decreases job performance, retention and readiness. Campbell, Martínez-Jerez, and Tufano (2012) find that access to payday lending increases rates of involuntary bank account closures. Our paper contributes to this literature by offering a theoretic framework which jointly models mainstream financial and payday loans as well as their interaction with credit scores. We then use our framework to conduct counterfactual policy exercises, such as banning payday loans, and to investigate the resulting welfare implications for households.

Our paper is also related to Exler (2020). He examines the welfare impact of different alternatives to regulate small dollar loans, such as payday loans, by building and calibrating a quantitative model of unsecured lending where individuals can declare bankruptcy and become delinquent. His findings suggest scope for possible welfare improving regulation compared to current and proposed legislation by the Consumer Financial Protection Bureau. In contrast to our approach, he considers only one asset and does not model credit scores. Saldain (2021) considers a model of only payday loans with irrational households and studies the policy regulations on payday lending.

3 The Model

Time is discrete and infinite.[11] There is a continuum of measure one households populating the economy. Every period households can survive at rate ρ and receive persistent earnings $e \in \mathcal{E}$ following a stationary finite state Markov process $Q^e(e'|e)$ and transitory earnings $z \in \mathcal{Z}$ determined by an i.i.d. process $Q^z(z)$. All income realizations are independent across individuals. There is no aggregate uncertainty.

Households discount future utility with rate $\beta \in \mathcal{B}$ which follows a finite state Markov process $Q^\beta(\beta'|\beta)$. The process evolves independently across individuals. We will call an individual's discount factor her type. Since all financial intermediaries cannot see individual's type, they would like to infer it using type score s, which denotes the probability of being certain types.[12] The individual's utility function defined on consumption $u(c)$ is additively separable over time, continuous, increasing, and concave.

In a given period, households can either borrow $b' < 0$ at the discount price q_b or save $b' > 0$ at the risk-free interest rate r_f with banking institutions which operate in a competitive market. Furthermore, there exists a second type of financial sector, payday lenders. If a household has not saved at banks, she may also borrow from payday lenders p' at the discount price q_p.

At the beginning of each period, if a household has any kind of debt, i.e., either $b < 0$ or $p < 0$, then she can choose to repay both ($d = R$). Additionally, households also have two default choices available: If a household has a bank loan ($b < 0$), it can also choose to formally default ($d = FD$). In this case all debt (including potential payday loans) is discharged. In this case, we assume that households are not allowed to borrow or save in the current period $b' = p' = 0$ and have to pay out-of-pocket bankruptcy costs κ_{FD}.[13] If a household has a payday loan ($p < 0$), she may alternatively choose to default on payday loans only ($d = PD$) at the lump-sum bankruptcy costs κ_{PD}. In this case, potential bank loans $b < 0$ still need to be repaid but households retain access to the bank asset market. An individual's consumption $c^{(d,b',p')}$ is thus pinned down by her actions (d, b', p'), income

[11] We follow the convention of dynamic programming that the time subscript is removed and the next-period variable is expressed with prime $'$.

[12] See more explanations below.

[13] Bankruptcy is thus modeled as Chapter 7 bankruptcy.

(z, e), previous asset/debt holdings (b, p), prices (q_b, q_p), type (discount factor) β, and type score s.[14]

Following the discrete choice literature, for each action (d, b', p') and individual, an unobervable additive utilitarian shock $\epsilon^{(d,b',p')}$ is drawn from a extreme value distribution $EV(\epsilon)$ with scale parameter α. These shocks are i.i.d across both time and individuals. More importantly, they can capture other unobservable heterogeneity that are not explicitly modeled in a reduced but tractable way. With these shocks, choices become probabilistic with $\sigma^{(d,b',p')}(\beta, z, e, b, s, p)$.

Bank intermediaries can observe individuals' persistent earnings e, bank loan holdings and choices (b, b'), and type scores s. We denote (e, b, s) as ω_b, the bank-observable state. Banks cannot observe payday loan positions or choices (p, p'), and preferences (ϵ, β). Since banks only observe bank loan but not payday loan choices, we also assume that they can observe only if formal default $d = FD$ has occurred (i.e., bank loans are not repaid), but that they cannot distinguish between full repayment $d = R$ or payday default $d = PD$ (i.e., whether or not payday loans are repaid). As a result, banks can differentiate formal default $\tilde{d} = FD$ from non-formal default $\tilde{d} = NFD$ consisting of R and PD.

As these unobservable variables are relevant for the repayment probability of loans in the next period, banks would like to infer them. While extreme value shock ϵ and transitory earnings z cannot be inferred by banks as they are i.i.d. across time and households, β, p, p', and d can be inferred.

In particular, past actions are informative for inferring β' as it follows a persistent process. In equilibrium, the fraction of individuals of each type is given by the invariant distribution induced by the transition $Q^{\beta}(\beta'|\beta)$. The banks' prior assessment of an individual's type at the beginning of a given period before any actions (or an individual's bank credit score) is denoted as $s = \left(s(\beta_1), ..., s(\beta_{n_\beta})\right)$ with $\sum_{i=1}^{n_\beta} s(\beta_i) = 1$. Assume $s \in \mathcal{S} \subset [0, 1]^{n_\beta}$. Given the bank-observable state variables ω_b and current choices $(\tilde{d}, b')$, the bank will update the individual's credit score s' using Bayes' rule. The posterior type score is denoted as $\psi_{\beta'}^{(\tilde{d},b')}(\omega_b)$. As it may not lie on the score grid, it will be randomly assigned to one of the two nearest points. This assignment is captured by the function

[14] More on the type score in Section 3.2.1.

$Q^s(s'|\psi)$. Thus, the bank loan pricing function $q_b^{(NFD,b')}(\omega_b)$ will be affected by an individual's choices.[15]

In terms of bank inference regarding a household's payday loan assets p, we assume that banks have available aggregate information on the distribution of payday loans in the population and are not able to track payday loan holdings at an individual level. As a result, banks exploit the cross-sectional distribution to form their expectation about a household's payday loan assets.[16] As for payday loan and default choices (p', PD), they can be filtered out with choice probabilities.

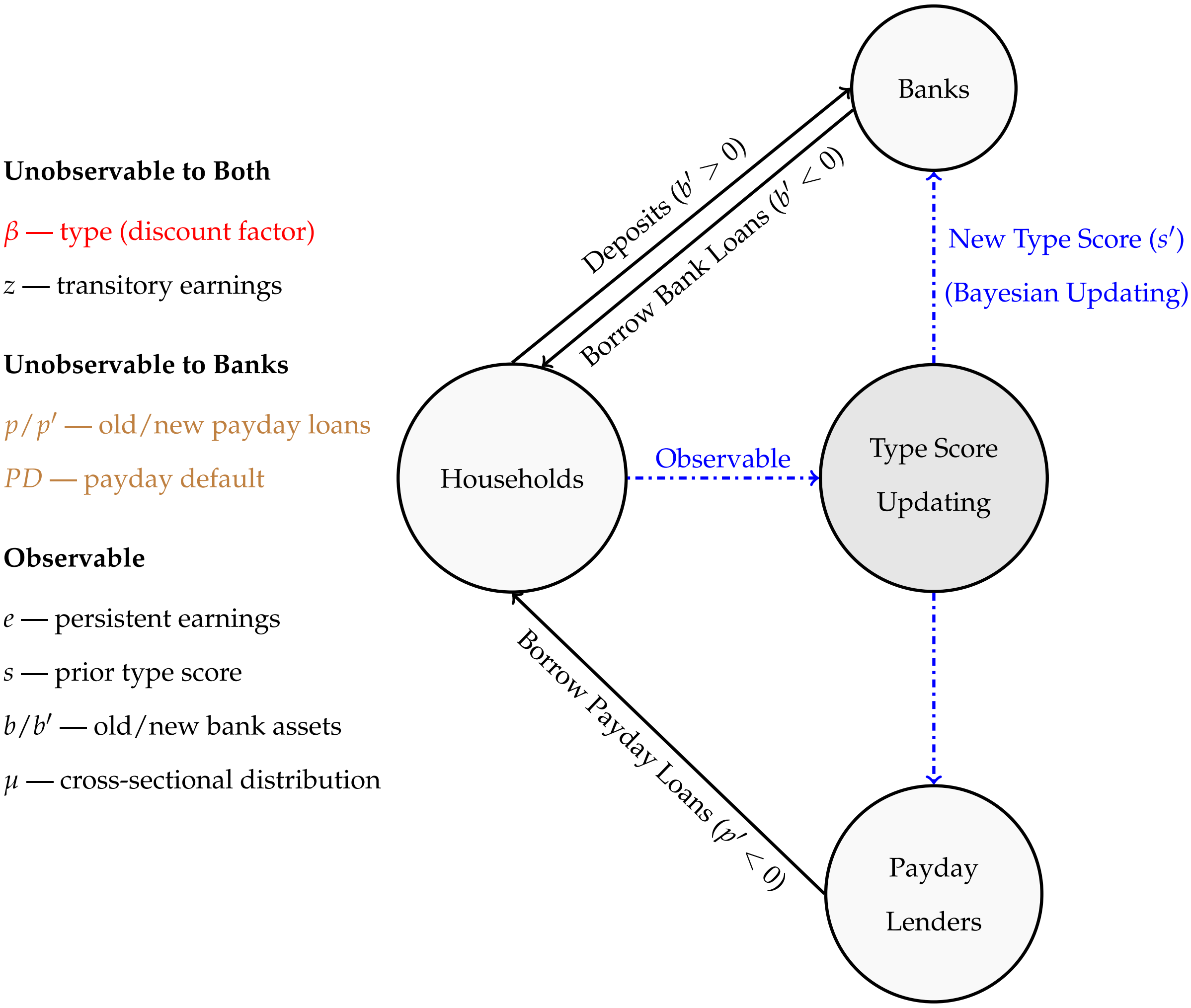
The payday lending sector is modeled as more informed than banks—in addition to what banks can observe, payday lenders can of course tell payday loan decisions. We assume payday lenders use the the same type score as banks for simplicity.[17] The payday loan pricing function $q_p^{(R,b',p')}(\omega_b)$ is thus determined in the way of achieving zero profit. The layout of the economy is summarized in Figure 1.

[15] Note that, compared to most papers in the consumer finance literature, there is no exogenous exclusion imposed in this model.

[16] See Section 3.2.1 for the mathematical details. In principle it is also possible to assume that banks form a joint score over type and payday loan choices $s(\beta, p)$ for each household.

[17] In principle payday lenders are able to form another type score using their richer information set compared to banks. In reality, they exists a payday loan specific credit score, called Teletrack. This simplifying assumption is meant to keep computation numerically tractable. In our model economy, payday lenders still have a better predictive power in defaulting behavior as they are more informed.

Figure 1: Layout of the Economy



The timing in every period is as follows:

1. Individuals begin each period with state (β, z, e, b, s, p) and an action-specific extreme value shock ϵ.

2. Given bank prices $q_b^{(NFD,b')}(\omega_b)$ and payday prices $q_p^{(R,b',p')}(\omega_b)$, individuals choose to either repay all debts $d = R$, default on payday loans only $d = PD$, or formally default $d = FD$.

   - If $d = R$, they also choose b' and p' and consume $c^{(R,b',p')}$.
   - If $d = PD$, they also choose b' and $p' = 0$ by construction and consume $c^{(PD,b',0)}$.
   - If $d = FD$, they consume the leftover earnings $c^{(FD,0,0)}$.

3. Based on individual bank-observable choices $(\tilde{d}, b')$ and states ω_b, banks update their type scores from s to $\psi_{\beta'}^{(\tilde{d},b')}(\omega_b)$.

4. β', z', e', and s' are drawn from $Q^{\beta}(\beta'|\beta)$, $Q^{z}(z')$, $Q^{e}(e'|e)$, and $Q^{s}(s'|\psi)$. Newborn households begin with discount factor β' drawn from the initial distribution G_{β}, transitory earnings z' from G_z, persistent earnings e' from G_e, no bank or payday loan assets $(b', p') = (0, 0)$, and a type score s' consistent with G_{β}.

3.1 Households

Individuals take as given the bank and payday loan pricing functions $q_b^{(NFD,b')}(\omega_b)$ and $q_p^{(R,b',p')}(\omega_b)$ as well as the type scoring function $\psi_{\beta'}^{(\tilde{d},b')}(\omega_b)$. Recall the bank-observable state $\omega_b \equiv (e, b, s)$ includes persistent earnings e, bank assets b, and prior type score s. Households can choose between repayment ($d = R$), defaulting on payday loans only ($d = PD$), or formally defaulting on both bank and payday loans ($d = FD$). The value function is given by:

$$V(\epsilon, \beta, z, \omega_b, p) = \max_{(d,b',p')} v^{(d,b',p')}(\beta, z, \omega_b, p) + \epsilon^{(d,b',p')}, \tag{1}$$

where $\epsilon^{(d,b',p')}$ denotes the action-specific preference shocks which follow an extreme value distribution with the following cumulative distribution function:

$$EV(\epsilon) = \exp\left\{ -\exp\left(-\frac{\epsilon - \mu_{\epsilon}}{\alpha} \right) \right\}, \tag{2}$$

where $\alpha > 0$ determines the variance of the shocks and $\mu_{\epsilon} = -\alpha\gamma_E$ makes the shocks mean zero where γ_E is the Euler's constant.

The conditional value function is given by:

$$\begin{aligned} v^{(d,b',p')}(\beta, z, \omega_b, p) &= u\left(c^{(d,b',p')}(z, \omega_b, p) \right) \\ &+ \beta\rho \cdot \sum_{(\beta',z',e',s')} Q^{\beta}(\beta'|\beta) \cdot Q^{z}(z') \cdot Q^{e}(e'|e) \cdot Q^{s}(s'|\psi) \cdot W(\beta', z', \omega_b', p'), \end{aligned} \tag{3}$$

where consumption $c^{(d,b',p')}(z,\omega_b,p)$ is defined as:

$$\begin{cases} e \cdot z + b + p - q_b^{(NFD,b')}(\omega_b) \cdot b' - q_p^{(R,b',p')}(\omega_b) \cdot p', & \text{if } (d,b',p') = (R,b',p') \\ e \cdot z - \kappa_{PD} + b - q_b^{(NFD,b')}(\omega_b) \cdot b', & \text{if } (d,b',p') = (PD,b',0) \\ e \cdot z - \kappa_{FD}, & \text{if } (d,b',p') = (FD,0,0) \end{cases}, \tag{4}$$

where κ_{PD} and κ_{FD} denotes the out-of-pocket bankruptcy costs for payday and formal defaults, respectively. There are two technical notions worth being mentioned here. First, to avoid the situation where households might hold bank assets while borrowing payday loans for the insurance purpose (i.e., $(d = R, b' > 0, p' < 0)$), we assume households can only take up payday loans if they do not save at the banking sector. Second, we assume households do not make any pecuniary mistakes in z,defaulting. Defaulting is restricted to those who have debts larger than the respective monetary bankruptcy costs. For example, formal default FD is feasible only if $d + p < -\kappa_{FD}$.

Let the set of feasible actions then be defined as:

$$\mathcal{F}(z,\omega_b,p) = \left\{ (d,b',p') | c^{(d,b',p')}(z,\omega_b,p) > 0 \right\}. \tag{5}$$

The standard Logit assumption implies the following choice probabilities:

$$\sigma^{(d,b',p')}(\beta,z,\omega_b,p) = \begin{cases} \frac{\exp\left\{v^{(d,b',p')}(\beta,z,\omega_b,p)/\alpha\right\}}{\sum_{(\hat{d},\hat{b}',\hat{p}')\in\mathcal{F}} \exp\left\{v^{(\hat{d},\hat{b}',\hat{p}')}(\beta,z,\omega_b,p)/\alpha\right\}} & \text{if } (d,b',p') \in \mathcal{F}(z,\omega_b,p) \\ 0 & \text{otherwise} \end{cases}. \tag{6}$$

In addition, the unconditional value function is given by:

$$\begin{aligned} W(\beta,z,\omega_b,p) &= \mathbb{E}_\epsilon V(\epsilon,\beta,z,\omega_b,p) \\ &= \alpha \cdot \ln \left(\sum_{(d,b',p')\in\mathcal{F}(\beta,z,\omega_b,p)} \exp\left\{ \frac{v^{(d,b',p')}(\beta,z,\omega_b,p)}{\alpha} \right\} \right). \end{aligned} \tag{7}$$

3.2 Financial Intermediaries

There are two types of financial intermediaries, banks and payday lenders. Both operate in perfectly competitive markets. They differ in their information set and lending costs. In particular, banks cannot observe payday-related variables and have lower lending costs compared to payday lenders.

3.2.1 Banks

Banks have deep pockets so can borrow from the international credit market at risk-free interest rate r_f. Due to perfect competitiveness, every bank takes pricing schedule $q_b^{(NFD,b')}(\omega_b)$ and type score updating function $\psi_{\beta'}^{(\tilde{d},b')}(\omega_b)$ as given. The bank's profit $\pi_b^{(NFD,b')}(\omega_b)$ for a contract (NFD, b') is then given by:

$$\pi_b^{(NFD,b')}(\omega_b) = \begin{cases} \rho \cdot \frac{\mathbb{P}_b^{(NFD,b')}(\omega_b)\cdot(-b')}{1+r_f} - q_b^{(NFD,b')}(\omega_b) \cdot (-b') & \text{if } b' < 0 \\ q_b^{(NFD,b')}(\omega_b) \cdot b' - \rho \cdot \frac{b'}{1+r_f} & \text{if } b' \geq 0 \end{cases},$$

where ρ is the survival probability and $\mathbb{P}_b^{(NFD,b')}(\omega_b)$ denotes the probability of repayment of a contract (NFD, b') for an individual with bank-observable state ω_b. Given perfect competition and constant returns to scale in lending, optimization by banks implies for each contract that:

$$q_b^{(NFD,b')}(\omega_b) = \begin{cases} \rho \cdot \frac{\mathbb{P}_b^{(NFD,b')}(\omega_b)}{1+r_f} & \text{if } b' < 0 \\ \frac{\rho}{1+r_f} & \text{if } b' \geq 0 \end{cases}. \tag{8}$$

In order to determine the probability of repayment tomorrow $\mathbb{P}_b^{(NFD,b')}(\omega_b)$, the banking intermediary solves an inference problem since type β, transitory earnings z, utility shock ϵ, payday loan holdings and choices (p, p'), as well as the exact choice of repayment or payday default ($d = PD \vee R$) are unobserved by the bank. Banks' tasks can be summarized into three steps.

1. Filter out a number of unobservable states and actions (ϵ, p, p', R, PD) to obtain the bank-observable choice probabilities $\tilde{\sigma}_b^{(\tilde{d},b')}(\beta, z, \omega_b)$.

2. Assess the probability that an individual is of type β' tomorrow given observable state ω_b and choices $(\tilde{d}, b')$, i.e., $\psi_{\beta'}^{\tilde{d},b'}(\omega_b)$. Then, assign resulting posterior to type score grid using $Q^s(s'|\psi)$.

3. Compute for each possible β' the individual's repayment probability given transition over ω_b and then use the weighted sum over β' to compute $\mathbb{P}_b^{(NFD,b')}(\omega_b)$.

First of all, use the standard Logit formula to filter out utility shock ϵ and obtain the general choice probabilities $\sigma^{(d,b',p')}(\beta, z, \omega_b, p)$ in Equation (6). Then, payday loan holdings p can be weighted with the cross-sectional distribution μ and sum out payday loan choices p' as follows.

$$\sigma_b^{(d,b')}(\beta, z, \omega_b) = \sum_{p'} \underbrace{\sum_{p} \sigma^{(d,b',p')}(\beta, z, \omega_b, p) \cdot \frac{\mu(\beta, z, \omega_b, p)}{\sum_{\hat{p}} \mu(\beta, z, \omega_b, \hat{p})}}_{=\hat{\sigma}_b^{(d,b',p')}(\beta,z,\omega_b)}, \tag{9}$$

where the last fraction denotes the marginal distribution of p conditional on (β, z, ω_b). Finally, form the probability of formal default $\tilde{d} = FD$ versus non-formal default $\tilde{d} = NFD$ to obtain the bank-observable choice probabilities.

$$\tilde{\sigma}_b^{(\tilde{d},b')}(\beta, z, \omega_b) = \begin{cases} \sigma_b^{(d,b')}(\beta, z, \omega_b) & \text{if } \tilde{d} = FD \\ \sum_{d \in \{R,PD\}} \sigma_b^{(d,b')}(\beta, z, \omega_b) & \text{if } \tilde{d} = NFD \end{cases}. \tag{10}$$

Accordingly, the feasible set from the bank's perspective can be defined as:

$$\tilde{\mathcal{F}}_b(\beta, z, \omega_b) = \left\{ (\tilde{d}, b') \,\middle|\, \tilde{\sigma}_b^{(\tilde{d},b')}(\beta, z, \omega_b) > 0 \right\}. \tag{11}$$

The intuition behind the different σ variables is as follows: As banks are not able to observe a household's payday loan holdings p and they have rational expectation, they impute the distribution of unobservables conditional on the observables, namely, using the marginal distribution of p conditional on (β, z, ω_b) to compute the weighted choice probabilities. This yields $\hat{\sigma}_b$ in Equation (9) based on the "true" choice probabilities σ in Equation (6). Furthermore, the payday loan choices p' are also unobservable by the banks.

As σ denotes probabilities, banks can simply compute σ_b as in equation (9) to eliminate this unobservable. Finally, banks can not distinguish between payday default ($d = PD$) and full repayment ($d = R$). In order to form an expectation of this unobservable choice as $\tilde{d} = NFD$, banks compute $\tilde{\sigma}_b$ in Equation (10).

Second, an individual's type probability distribution in the next period is computed using Bayes rule and transitory earnings z is weighted with its i.i.d. distribution Q^z. The type scoring function $\psi_{\beta'}^{(\tilde{d},b')}(\omega_b) = \left(\psi_{\beta'_1}^{(\tilde{d},b')}(\omega_b), ..., \psi_{\beta'_{\#\beta}}^{(\tilde{d},b')}(\omega_b)\right)$ is computed as:[18]

$$\psi_{\beta'}^{(\tilde{d},b')}(\omega_b) = \begin{cases} \sum_z Q^z(z) \cdot \sum_\beta Q^\beta(\beta'|\beta) \cdot \frac{\tilde{\sigma}_b^{(\tilde{d},b')}(\beta,z,\omega_b)\cdot s(\beta)}{\sum_{\hat{\beta}} \tilde{\sigma}_b^{(\tilde{d},b')}(\hat{\beta},z,\omega_b)\cdot s(\hat{\beta})} & \text{for } (\tilde{d},b') \in \tilde{\mathcal{F}}_b \\ \sum_\beta Q^\beta(\beta'|\beta) \cdot s(\beta) & \text{for } (\tilde{d},b') \notin \tilde{\mathcal{F}}_b \end{cases}, \tag{12}$$

where an infeasible action is updated with prior and exogenous type transition in an uninformative way.

As the updated type score ψ may not lie on the original type score grid, it is randomly assigned to one of the two nearest grid points $s'_i(\beta')$ and $s'_j(\beta')$ for all β' with $s'_i(\beta') \leq \psi_{\beta'}^{(\tilde{d},b')} \leq s'_j(\beta')$, and assign probability $\chi(\beta'|\psi)$ to $s'_i(\beta')$ and $1 - \chi(\beta'|\psi)$ to $s'_j(\beta')$, where

$$\chi(\beta'|\psi) = \frac{s'_j(\beta') - \psi_{\beta'}^{(\tilde{d},b')}(\omega_b)}{s'_j(\beta') - s'_i(\beta')}, \quad \forall \beta'. \tag{13}$$

For all s' such that $s'(\beta') \in \{s'_i(\beta'), s'_j(\beta')\}$ for all β', the probability of receiving score s' in the next period is thus equal to

$$Q^s(s'|\psi) = \prod_{s'(\beta')=s'_i(\beta')} \chi(\beta'|\psi) \cdot \prod_{s'(\beta')=s'_j(\beta')} (1 - \chi(\beta'|\psi)). \tag{14}$$

For all other s', $Q^s(s'|\psi) = 0$.

In the final step, the probability of the next period's repayment of a contract (NFD, b')

[18] Note that $\sum_{\beta'} \psi_{\beta'}^{(d,b')}(\omega_b) = 1$ for all ω_b and (d, b') in the feasible set.

for the bank is computed as:

$$\mathbb{P}_b^{(NFD,b')}(\omega_b) = \sum_{(\beta',z',e',s')} s'(\beta') \cdot Q^z(z') \cdot Q^e(e'|e) \cdot Q^s \left(s'(\beta') \left| \psi_{\beta'}^{(NFD,b')}(\omega_b) \right. \right)$$
$$\left[\mathcal{W}_{PD}^{b'}(\omega_b) \cdot \left(1 - \sigma^{(FD,0,0)}(\beta', z', \omega_b', p' = 0) \right) + \right.$$
$$\left. \left(1 - \mathcal{W}_{PD}^{b'}(\omega_b) \right) \cdot \sum_{p'} \mathcal{W}_{p'}^{(R,b')}(\omega_b) \cdot \left(1 - \sigma^{(FD,0,0)}(\beta', z', \omega_b', p') \right) \right], \quad (15)$$

where the weighting factor $\mathcal{W}_{PD}^{b'}(\omega_b)$ denotes the probability that a household in a bank-observable state ω_b and bank loan choice b' chooses to partially default $d = PD$ in the current period. It is given by:

$$\mathcal{W}_{PD}^{b'}(\omega_b) = \sum_z Q^z(z) \cdot \frac{\sum_\beta s(\beta) \cdot \sigma_b^{(PD,b')}(\beta, z, \omega_b)}{\sum_{\hat{d} \in \{PD,R\}} \sum_\beta s(\beta) \cdot \sigma_b^{(\hat{d},b')}(\beta, z, \omega_b)}. \quad (16)$$

In this case, provided that an individual has chosen to default on his payday loans in the current period, the bank realizes the only possible payday loan choice in the next period is zero $p' = 0$.

Analogously, $1 - \mathcal{W}_{PD}^{b'}(\omega_b)$ gives the probability of choosing full repayment $d = R$ in the current period. As banks do not observe p', they must additionally form an expectation over the individual's payday loan choice. Conditional on full repayment, $\mathcal{W}_{p'}^{(R,b')}(\omega_b)$ denotes the probability of a household choosing a certain payday loan p' and is given by:

$$\mathcal{W}_{p'}^{(R,b')}(\omega_b) = \sum_z Q^z(z) \cdot \frac{\sum_\beta s(\beta) \cdot \hat{\sigma}_b^{(R,b',p')}(\beta, z, \omega_b)}{\sum_{\hat{p}'} \sum_\beta s(\beta) \cdot \hat{\sigma}_b^{(R,b',\hat{p}')}(\beta, z, \omega_b)}. \quad (17)$$

3.2.2 Payday Lenders

The payday loan pricing schedule is also endogenously determined under a zero profit assumption. We assume that the payday lending sector cannot observe type β and transitory earnings z but all the others. For simplicity, we assume payday lenders use the same type score as banks to infer individual's hidden type. The repayment probability for a contract (R, b', p') for an bank-observable state ω_b is thus given by:[19]

$$\mathbb{P}_p^{(R,b',p')}(\omega_b) = \sum_{(\beta',z',e',s')} s(\beta') \cdot Q^z(z') \cdot Q^e(e'|e) \cdot Q^s\left(s'(\beta')|\psi_{\beta'}^{(NFD,b')}(\omega_b)\right) \left(1 - \sum_{d' \in \{FD,PD\}} \sum_{b''<0} \sigma^{(d',b'',0)}(\beta', z', \omega_b', p')\right), \tag{18}$$

where note that payday lenders have to take into account both formal default FD and payday default PD because payday loans can be discharged in both cases. Moreover, a payday loan can be taken only if she does not save at banks $b'' < 0$. The payday loan pricing function is thus given by:

$$q_p^{(R,b',p')}(\omega_b) = \rho \cdot \frac{\mathbb{P}_p^{(R,b',p')}(\omega_b)}{1 + r_b}, \tag{19}$$

where r_b denotes the lending costs in the payday lending industry.

3.3 Evolution of Distribution

The probability for an individual to move from state (β, z, e, b, s, p) to $(\beta', z', e', b', s', p')$ is governed by the following mapping:

$$\begin{aligned}
&T^*(\beta', z', \omega_b', p'|\beta, z, \omega_b, p) \\
&= \rho \cdot Q^\beta(\beta'|\beta) \cdot Q^z(z') \cdot Q^e(e'|e) \cdot \sigma^{(d,b',p')}(\beta, z, \omega_b, p) \cdot Q^s\left(s'(\beta')|\psi_{\beta'}^{(\tilde{d},b')}(\omega_b)\right) \\
&+ (1-\rho) \cdot G_\beta(\beta') \cdot G_z(z') \cdot G_e(e') \cdot \mathbb{1}_{b'=0} \cdot \mathbb{1}_{s'=G_\beta} \cdot \mathbb{1}_{p'=0}.
\end{aligned} \tag{20}$$

Then, the cross-sectional distribution μ evolves according to:

$$\mu'(\beta', z', \omega_b', p') = \sum_{(\beta,z,\omega_b,p)} T^*(\beta', z', \omega_b', p'|\beta, z, \omega_b, p) \cdot \mu(\beta, z, \omega_b, p). \tag{21}$$

[19] Note that the current payday loan holdings are not predictive of payday default in the next period.

3.4 Equilibrium

A stationary Recursive Competitive Equilibrium (RCE) is a set of (un)conditional value functions v^* and W^*, bank loan pricing functions q_b^* and repayment probability $\mathbb{P}_b^*$, payday loan pricing functions q_p^* and repayment probability $\mathbb{P}_p^*$, a type scoring function ψ^*, choice probability functions σ^* and $\tilde{\sigma}_b^*$, and a steady state distribution $\overline{\mu}^*$ such that:

1. Optimality: $v^{*(d,b',p')}(\beta, z, \omega_b, p)$, $W^*(\beta, z, \omega_b, p)$, and $\sigma^{*(d,b',p')}(\beta, z, \omega_b, p)$ satisfy Equation (3), (7), and (6) for all (β, z, ω_b, p), respectively.

2. Zero Profits for Bank Lenders: $q_b^{*(NFD,b')}(\omega_b)$ and $\mathbb{P}_b^{*(NFD,b')}(\omega_b)$ satisfy Equation (8) and (15) for all ω_b, respectively.

3. Zero Profits for Payday Lenders: $q_p^{*(R,b',p')}(\omega_b)$ and $\mathbb{P}_p^{*(R,b',p')}(\omega_b)$ satisfy Equation (19) and (18) for all ω_b, respectively.

4. Type Score Updating: $\psi_{\beta'}^{*(\tilde{d},b')}(\omega_b)$ and $\tilde{\sigma}_b^{*(\tilde{d},b')}(\beta, z, \omega_b)$ satisfy (12) and (10) for all (β, z, ω_b), respectively.

5. Stationary Distribution: $\overline{\mu}^*(\beta, z, \omega_b, p)$ solves (21).

4 Calibration

The goal of the paper is to rationalize the payday loan puzzle documented in Agarwal et al. (2009). Given they used a payday loan dataset collected from 2000 to 2004 and to circumvent the effects of the 2005 BAPCPA reform, we set the baseline calibration year to 2004. The model period is one year. The median earnings is thus set to \$33,176 in 2004 from the Current Population Survey (CPS).[20] Our strategy for the parameterization is threefold: Standard parameters are taken from the literature, parameters with a direct empirical counterpart are exogenously calibrated, and the rest are internally calibrated to match targeted data moments.

The persistent and transitory earnings processes e and z are directly taken from Floden and Lindé (2001) because they estimated the processes using wage earnings in the U.S.

[20] $\$638 \times 52 = \$33,176$.

of the same frequency, within the same periods considered in our paper, and without life-cycle components. We assume the newborn is endowed with the lowest persistent earnings G_e while with the transitory earnings subject randomly to the estimated process G_z. These assumptions are meant to capture the fact of low earnings at the early life cycle. Following Chatterjee et al. (2020), we set discount factors to 0.886 and 0.915, respectively. The turn-over rates are $Q^{\beta}(\beta_L|\beta_H) = 0.013$ and $Q^{\beta}(\beta_H|\beta_L) = 0.011$, implying households are more likely to become patient by 20%. The share of newborn's types G_{β} is set to 72% of impatient households. This is consistent with the fact that households are more impatient at the early stage of their life cycle.[21] All are summarized in Table 1.

We choose the CRRA parameter to be 2, the standard value in the macro literature. The working life span of people is targeted at 40 years, implying the survival probability of 0.975. The risk-free rate r_f is 1.4% and implies the effective interest rate of 4% consistent with the commonly used value in the literature. Albanesi and Nosal (2020) estimated the out-of-pocket filing costs for Chapter 7 before the 2005 bankruptcy reform is approximately \$697, implying $\kappa_{FD} = 0.02$. As Montezemolo and Wolff (2015) pointed out payday defaults in practice involves two bounced checked fees (one by banks and the other by payday lenders, \$35 each), we set the out-of-pocket filing costs for payday defaults κ_{PD} to 0.002. According to Flannery and Samolyk (2005), the average operating costs (without default losses) per two-week payday loan of size \$230 is around \$19, thus implying the annualized lending cost for payday lenders r_p is 1.925. The dispersion parameter of the extreme value distribution is set to 0.005.[22] Table 1 provides a summary.

We internally calibrate stigma costs for formal default κ_{FD} and for payday default κ_{PD} jointly in order to match formal default rate and conditional payday default rate. Results are summarized in Table 2. The formal default rate in the data is computed with the total number of non-business Chapter 7 filings from American Bankruptcy Institute (ABI) normalized by the total number of U.S. households in 2004. The conditional payday default rate is taken from Skiba and Tobacman (2018) where the same payday loan data is used. The formal and payday stigma costs are accordingly set to 0.02235 and 0.00702,

[21] It implies there are 41% of patient and 59% of impatient households in our economy.

[22] In order to rule out the contribution of extreme value shocks to the payday loan puzzle, we check whether households are making such a seemingly pecuniary mistake with higher values. See Section 6.1. In addition, our value is comparable the those used in Chatterjee et al. (2020).

Parameter		Value	Target/Source
Low discount factor	β_L	0.886	Chatterjee et al. (2020)
High discount factor	β_H	0.915	Chatterjee et al. (2020)
Transition from low to high	$Q^{\beta}(\beta_L\|\beta_H)$	0.013	Chatterjee et al. (2020)
Transition from high to low	$Q^{\beta}(\beta_H\|\beta_L)$	0.011	Chatterjee et al. (2020)
Discount factor at birth	G_{β}	(0.72,0.28)	Chatterjee et al. (2020)
Persistence of persistent earnings	ρ_e	0.9136	Floden and Lindé (2001)
Innovation to persistent earnings	σ_e^2	0.0426	Floden and Lindé (2001)
Innovation to transitory earnings	σ_z^2	0.0421	Floden and Lindé (2001)
Persistent earnings at birth	G_e	(1,0,0)	Upward earnings profile
Transitory Earnings at birth	G_z	(1/3,1/3,1/3)	Upward earnings profile
CRRA	γ	2	Standard
Survival probability	ρ	0.975	40 years
Risk-free rate	r_f	0.014	Effective interest rate = 4%
Formal default cost	κ_{FD}	0.02	Albanesi and Nosal (2020)
Payday default cost	κ_{PD}	0.002	Montezemolo and Wolff (2015)
Lending cost for payday lenders	r_p	1.925	Flannery and Samolyk (2005)
Dispersion of extreme value shocks	α	0.005	

Table 1: Exogenously Chosen Parameters

respectively.

Parameter		Value	Target	Data	Model
Formal stigma cost	ξ_{FD}	0.02235	Formal default rate	0.99%	0.99%
Payday stigma cost	ξ_{PD}	0.00702	Payday default rate (cond.)	29.7%	29.7%

Table 2: Internally Calibrated Parameters

We also evaluate our model fit on a set of untargeted moments standard in the consumer finance literature. The data and model moments are summarized in Table 3.[23] For the fraction of bank loan users in the data, we use the 2004 Survey of Consumer Finances (SCF) and construct a measure of liquid net worth.[24] We then compute the fraction of households with negative net worth. The fraction of payday loan users is computed with the 2010 SCF since information on payday loans was firstly collected in the 2010 wave. We also compute the bank debt-to-earnings ratio using the 2004 SCF.[25] Debt is measured

[23]Note that for all SCF related data moments, we restrict the sample to households with household heads aged between 20 and 60. We do this since our model does not account for retirement or childhood.

[24]We follow Herkenhoff (2019) in constructing this measure of liquid net worth. It is calculated as the difference between a household's liquid assets, such as checking and savings accounts, and credit card debt. We prefer this measure of net worth as we do not explicitly model illiquid assets such as housing in our framework.

[25]More specifically, we compute the ratio of average debt relative to average earnings conditional on having debt.

using the same liquid net worth definition as above. Income is computed as wage income as measured in the SCF.

The average interest rate for bank loans is computed as the average credit card interest rate in the 2004 SCF (among those having a positive credit card balance, net of the one-year ahead CPI inflation). As for the average interest rate for payday loans, we use the payday loan statistics reported in Skiba and Tobacman (2018) to calculate the average payday interest rate, deflated with the one-year ahead CPI inflation.[26]

Moment (in %)	**Data**	**Model**
Households in Debt		
Fraction of bank loan users	20.9	24.26
Fraction of payday loan users	5.61	9.46
Bank debt-to-earnings (cond. on borr.)	11.75	6.48
Interest Rate		
Ave. interest rate for bank loans	9.26	8.56
Ave. interest rate for payday loans	447.88	410.85

Table 3: Untargeted Moments: Data v.s. Model

5 Pooling and Cross-Subsidization

In our economy there is hidden information (about a household's type) in addition to hidden actions (a household's payday loan choice is unobservable to banks). Because banks cannot observe households' types and payday loan choices, they cannot design contracts that condition on these variables directly.[27] This then leads to two dimensional pooling across household type and payday loans when banks price their loans.[28]

Because our model generates significant heterogeneity in behavior across types and payday loan users, this pooling will in turn lead to cross-subsidization of bank loans between these groups. Figure 2 illustrates these differences in borrowing and defaulting

[26] In particular, the average bi-weekly payday loan size is \$317.55 with average interest payment of \$56.4. It follows that $\frac{56.4}{317.55} \times \frac{365}{14} \times \frac{1}{1.03388} \times 100 = 447.88\%$.

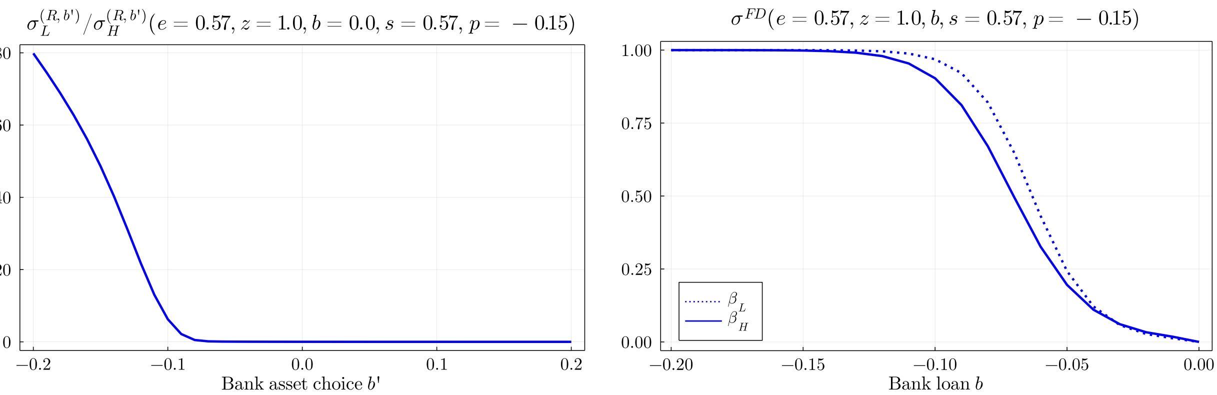
[27] As we discussed in Section 3.2.1 banks will instead use type *scores* and the conditional distribution of payday loans given observed variables.

[28] For payday lenders, there is only pooling across types since they can of course observe a household's payday loan choice and assets. In this section we will focus on pooling and cross-subsidization in the bank lending market.

behavior compared across low and high types. In Figure 2a, we can see how low-type (impatient) households are much more likely to borrow and to borrow more relative to the high type (patient). This is intuitive as households with a lower discount factor value consumption today more and will therefore tend to borrow more. Figure 2b illustrates how the formal default probability varies across levels of debt and household types. It can be seen that the low type is more likely to formally default than the high type. As a result, conditional on the same state (and in particular, the same bank loan size), low types are riskier borrowers for banks.

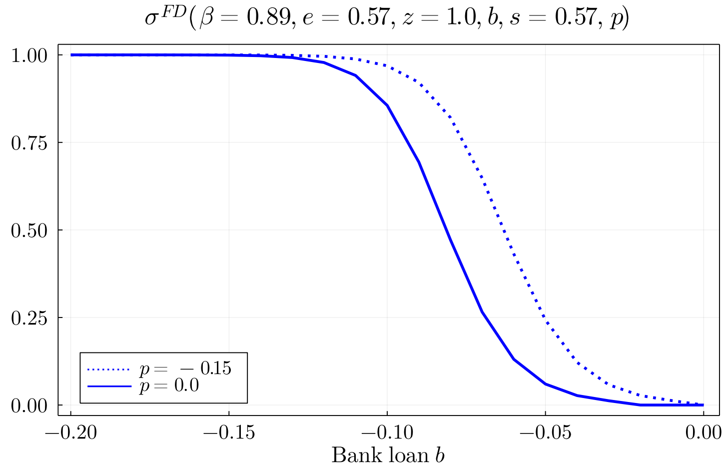
Figure 2: Borrowing and Defaulting Behavior Across Types

(a) Choice Likelihood Ratio (b) Formal Default Probability



Moreover, there are also differences in defaulting behavior across payday loan users. In Figure 3 we can see that, conditional on the same bank loan size, households indebted with extra payday loans are more likely to formally default on both loans. As a result, borrowers who use payday loans are riskier for banks.

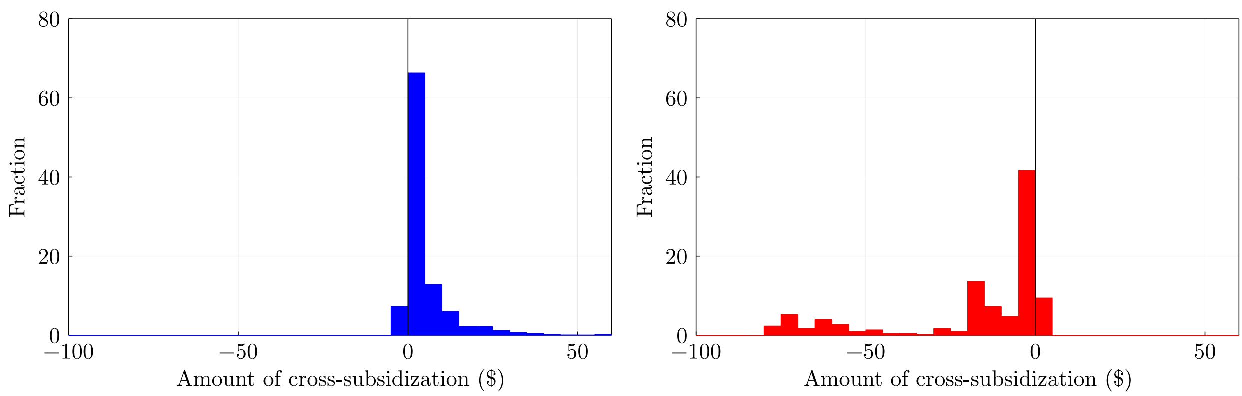
Figure 3: Formal Default Probability Across Payday Loans



Putting the pooling across types and payday loan users together with differences in default and borrowing behavior between these groups leads to cross-subsidization in the bank loan market. Conditional on the level of bank borrowing, low types face lower bank loan rates than actuarially fair rates (i.e. the rates borrowers would face if banks were able to observe households' types). This is shown in Figure 4, where we can see that it is mostly low types who are cross-subsidized by high types. This is due to the fact that low types tend to be riskier borrowers as they are more likely to default.

Figure 4: Cross-Subsidization of Bank Loans across Types

(a) Low Type (b) High Type



In addition, there also exists cross-subsidization of bank loans between borrowers who use payday loans and those who do not. Because banks cannot observe payday loan usage by households, borrowers who use payday loans face the same bank loan pricing schedule as borrowers who do not use payday loans. Conditional on the same level of bank loan payday loan users tend to have a higher default probability as they have more total debt. As a result payday (non-payday) loan users pay rates on bank loans that are lower (higher) than actuarially fair rates (i.e. the rates borrowers would face if banks were able to observe payday loan usage). This is illustrated in Figure 5 where distribution of the difference between the actual and actuarially fair rates across payday loan and non-payday loan users is plotted.

Figure 5: Cross-Subsidization of Bank Loans across (Non-)Payday Loan Users

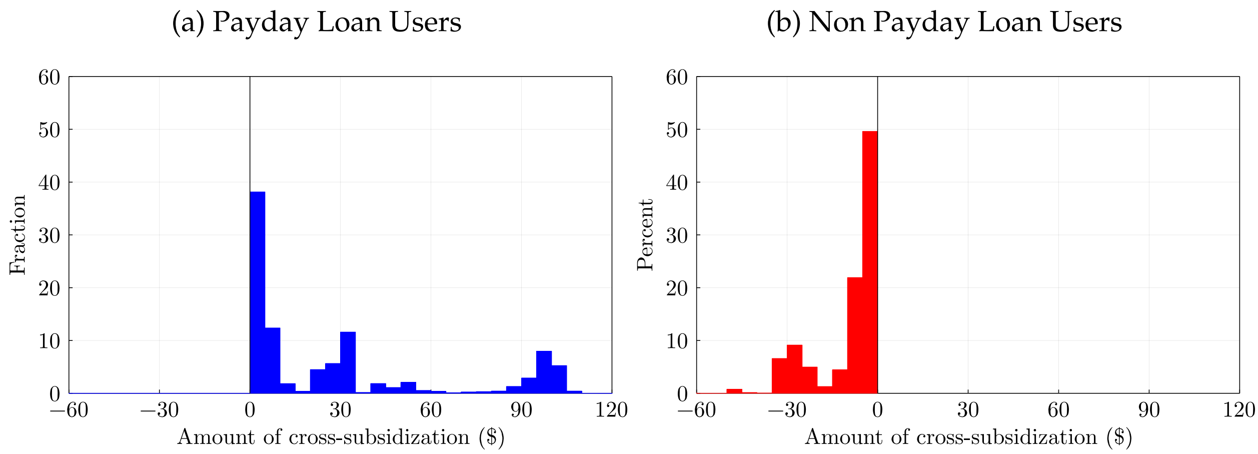


Table 4 summarizes the main equilibrium outcomes across types. Compared to patient households, impatient households are more likely to default and borrow, and hold larger debts for both bank and payday loans. This leads to overall higher borrowing costs for the impatient even though they are partially cross-subsidized by patient households as we saw above.

6 The Payday Loan Puzzle

The central result of our paper is that our model can offer a rational explanation for the payday loan puzzle observed in the data. In this section we first illustrate how we identify the payday loan puzzle in our model. Then, we show that our model can account for around a third of the fraction of puzzle households observed in the data as well as match the magnitude of monetary losses caused by this behavior. In addition, we quantify the type score gains and interest costs from using payday loans and investigate when households engage in this kind of puzzle behavior in our model.

Moment	Aggregate	Impatient	Patient
Default			
Formal default rate	0.99	1.27	0.57
Payday default rate	2.81	3.27	2.13
Households in debt			
Fraction of bank loan users	24.26	27.5	19.55
Fraction of payday loan users	9.46	10.7	7.65
Bank debt-to-earnings (cond.)	6.48	6.54	6.36
Interest rate			
Ave. interest rate for bank loans	8.56	8.79	8.06
Ave. interest rate for payday loans	410.85	433.89	362.74

Table 4: Equilibrium Across Types

6.1 Identification of Payday Loan Puzzle

Our calibrated model can generate the payday loan puzzle found in the empirical literature. In the model, we identify the households who make this seemingly pecuniary mistakes that are consistent with the payday loan puzzle in the following way: For each possible state (β, z, ω_b, p), we identify those feasible borrowing choices with repayment $(R, b' < 0, p' < 0) \in \mathcal{F}(z, \omega_b, p)$ that involve a payday loan where the same total amount of borrowing $\hat{b}' = b' + p'$ could have been achieved at lower borrowing costs using bank loans only. That is:

$$\left| q_b^{(NFD,b')}(\omega_b) \cdot b' + q_p^{(R,b',p')}(\omega_b) \cdot p' \right| < \left| q_b^{(NFD,\hat{b}')}(\omega_b) \cdot \hat{b}' \right| . \tag{22}$$

The borrowing choices that fulfil the above condition are choices that we classify as the payday loan puzzle. Let the set of these choices be called $\mathcal{P}(\beta, z, \omega_b, p)$.[29]

To illustrate where the region with payday loan puzzle can happen, Condition (22) is

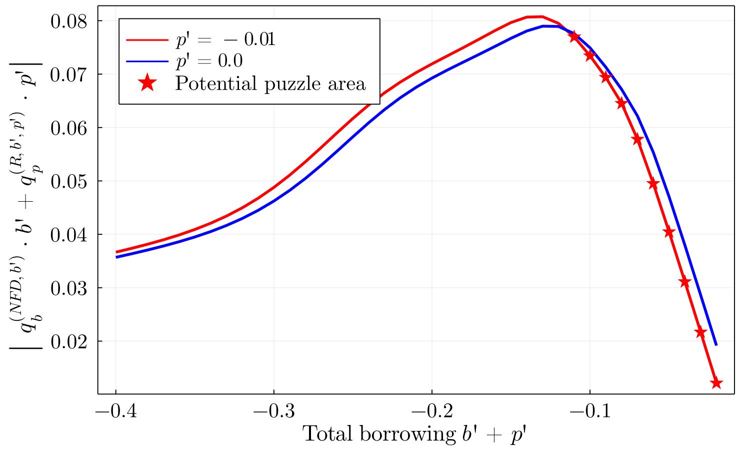
[29] Recall that in a model with taste shocks, any feasible action will be chosen with positive probability (not just the choice with the highest value). As a result, households might take up payday loans because mainly of such shocks. To control for this nuisance, we additionally check whether households are conscious of making this decision with higher values. Specifically, for each state (β, z, ω_b, p), the feasible borrowing choices with repayment $(R, b' < 0, p' < 0) \in \mathcal{F}(z, \omega_b, p)$ where the value of borrowing a certain amount is higher when using payday loans compared to only using bank loans. That is:

$$v^{(R,b',p')}(\beta, z, \omega_b, p) > v^{(R,\hat{b}',p=0)}(\beta, z, \omega_b, p). \tag{23}$$

Hence, there exists the general dependency of $\mathcal{P}(\cdot)$ on β.

visualized in Figure 6, where red line denotes the term on the left-hand side for a given payday loan size p' and blue line represents the term on the right-hand side in Condition (22), respectively. The region of choices satisfying the condition is marked by asterisks and labeled as "Potential Puzzle Area".[30]

Figure 6: Identification of Payday Loan Puzzle



The central result of the model is the average rate of puzzle occurrence which is computed as the fraction of households that make a choice that would be classified as the payday loan puzzle relative to all households that borrow using both bank and payday loans. More specifically, this rate $\mathcal{R}$ is calculated as follows in the model:

$$\mathcal{R} \equiv \frac{\sum_{\beta,z,\omega_b,p} \mu(\beta,z,\omega_b,p) \cdot \sum_{(d,b',p') \in \mathcal{P}(\beta,z,\omega_b,p)} \sigma^{(d,b',p')}(\beta,z,\omega_b,p)}{\sum_{\beta,z,\omega_b,p} \mu(\beta,z,\omega_b,p) \cdot \sum_{(d,b',p') \in \mathcal{F}_{both}(z,\omega_b,p)} \sigma^{(d,b',p')}(\beta,z,\omega_b,p)}, \tag{24}$$

where $\mathcal{P}$ is defined above and the feasible set of borrowings choices using both loans $\mathcal{F}_{both}(z,\omega_b,p)$ is defined as:

$$\mathcal{F}_{both}(z,\omega_b,p) \equiv \left\{(d,b',p') | (d = R, b' < 0, p' < 0) \in \mathcal{F}(z,\omega_b,p)\right\}. \tag{25}$$

Table 5 shows the average rate of puzzle occurrence across types. Our model can account for a significant fraction of puzzle households identified in the data. In the model the average rate of puzzle occurrence is around 26.44%. Using matched credit card and payday loan data, Agarwal et al. (2009) have found that in their data that the correspond-

[30] In fact, condition (23) is weak as almost all borrowing choices using both loans are fulfilled. Please refer to Figure in Appendix.

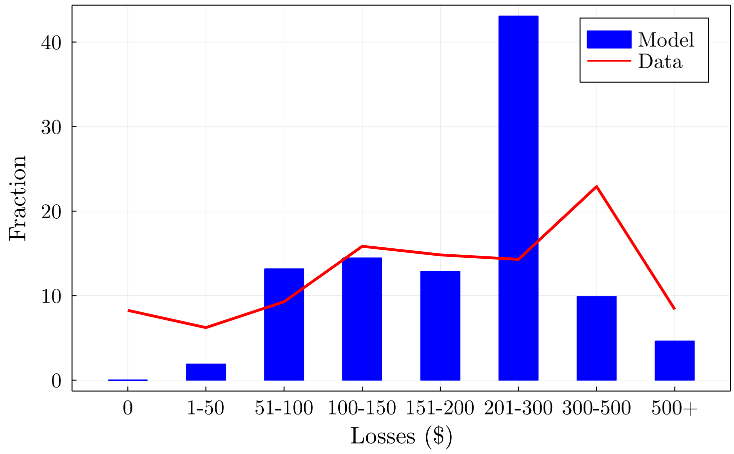
ing moment amounts to around two-thirds. That is, our model can account for about 40% of the payday loan puzzle found in the data.

Moment	Aggregate	Impatient	Patient
Average rate of puzzle occurrence	26.44	25.55	28.31

Table 5: Payday Loan Puzzle Across Types

Our model can also match the magnitude of monetary losses from the payday loan puzzle (the price of type score/reputation protection). Figure 7 shows the distribution of the corresponding monetary losses in both data and our model. We can see that the magnitude of the losses amount to a level very similar to the one observed in the data.

Figure 7: Histogram of Monetary Costs of Payday Loan Puzzle



Data source: Agarwal et al. (2009).

6.2 The Reputation Protection Channel

Does the proposed reputation protection hypothesis work theoretically in our model? Yes, because: (1) Borrowing more bank loans leads to a lower type score; (2) Households with lower type scores face lower discounted bank loan prices. Hence, households have an incentive to borrow using payday loans instead of bank loans in order to avoid a negative impact on their type scores, thus giving them access to cheaper bank credit in the future.

Figure 2 and 8 illustrate how this mechanism works. Figure 2a and 8a explains the first point. In Figure 2a, we could see how low types are more likely to borrow and to borrow more relative to the high type. Figure 8a illustrates how this difference in borrowing behavior translates into different type scores. As banks realize that the low type is more likely to borrow, they will also assign a lower type score to households who borrow more (ceteris paribus). The second point is shown in Figure 2b and 8b. Figure 2b illustrated how low types are more likely to formally default across different levels of debt. This in turn leads to heterogeneity in bank loan discounted price schedules across types as shown in Figure 8b. Banks will charge households with lower type scores, who are more likely to be of low type, lower discounted prices (higher interest rates) in order to be compensated for the additional default risk.

Figure 8: Reputation Protection Incentive

(a) Type Score Update

(b) Bank Loan Discounted Price Schedule

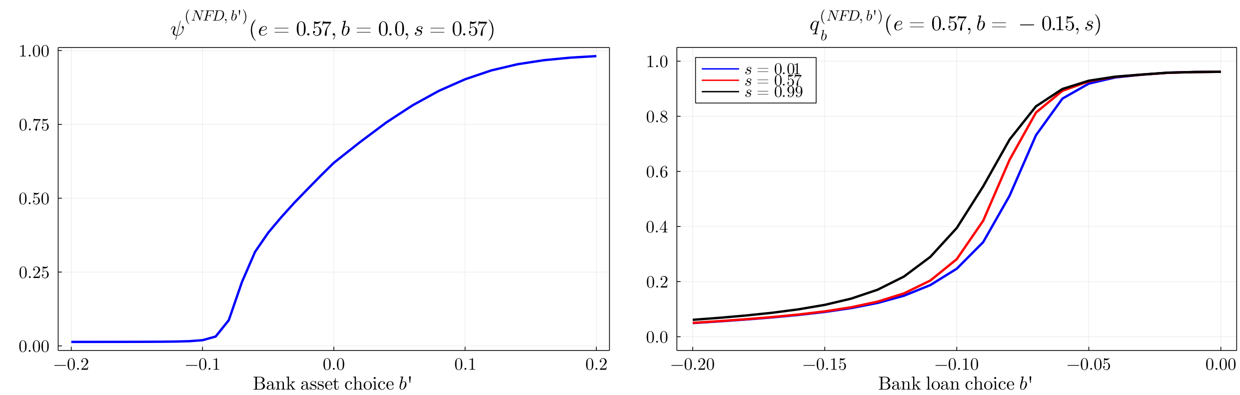
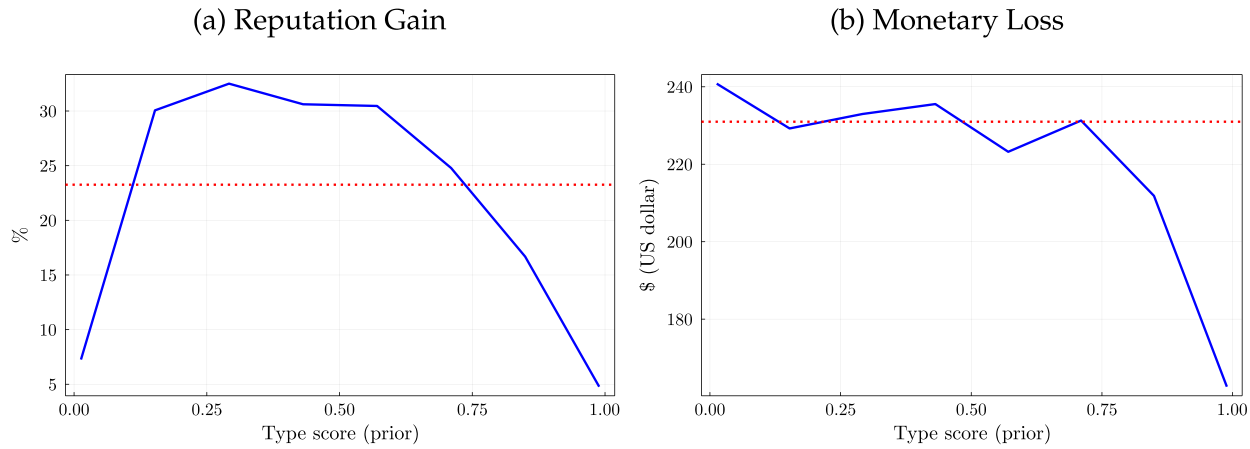


Figure 9 looks at the trade-off between type score protection and monetary costs (extra interest payments) for puzzle users. First of all, Figure 9a illustrates the reputation gain of puzzle users compared to the counterfactual where they were to use only bank loans conditional on prior type scores. There exists significant heterogeneity dependent on priors. Particularly, the gain is even over 30% for those who have lower medium type scores. On the other hand, Figure 9b calculates the interest losses relative to the case if they were to firstly borrow using bank loans in percentage points. Such pecuniary losses are significant. Those with lowest type scores are willing to pay extra \$240 for achieving higher posterior type scores than the ones if there were to be fully indebted in bank loans. On average, puzzle users are willing to pay extra \$230 interest payments in exchange for a increase in type scores by 23%. This is aligned with the average magnitude of the monetary losses reported in Agarwal et al. (2009) where the average losses are around $200.

Figure 9: Cost-Benefit Analysis among Puzzle Users



6.3 Profile of Puzzle Users

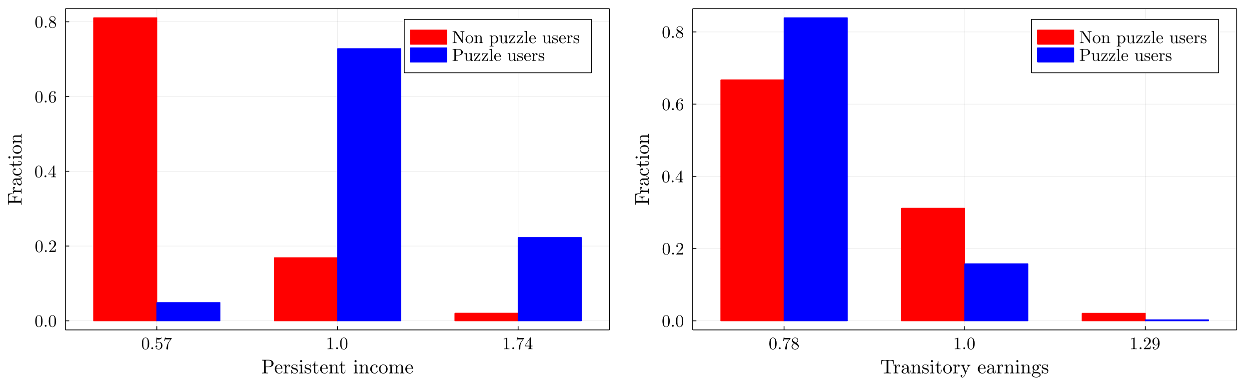
In the previous subsection we illustrated how using payday instead of bank loans can lead to significant reputation gains at the cost of substantially increased interest costs. In this subsection we further investigate when households engage in this puzzle behavior.

The first result is that households engage in puzzle behavior when they have medium to high persistent earnings but low transitory earnings in our model. This is illustrated in Figure 10, where Figure 10a depicts the distribution of persistent earnings conditional on puzzle versus non-puzzle users (the same for transitory earnings in Figure 10b). Non-puzzle users are those households which borrow using both loans but do not fulfill the two conditions in equations 22 and 23, whereas puzzle users are those which borrow using both loans and fulfil the two conditions. Compared to the non-puzzle users, who possess mainly low persistent earnings, puzzle users have higher persistent income. On the other hand, puzzle users tend to have lower transitory earnings than non-puzzle users.

These results indicate that puzzle users borrow using payday loans to smooth out the shortfall in transitory earnings without damaging theirs type scores too much (such a trade-off will be explained in Figure 9). Recall that persistent earnings are observable to banks, whereas transitory earnings are not. As a result, borrowing using bank loans to smooth out a negative transitory earnings shocks while having high persistent earnings will lead to a type score downgrade. This is illustrated in Figure 11 which shows the type score updating for different bank asset choices across persistent income (similar to Figure 2b). The intuition is simple: Borrowing a lot is more indicative of impatience (low type) when you have high compared to low persistent earnings. Instead, by complementing bank loans with payday loans, which are unobservable to banks, households can reduce the negative impact on their type score while still being able to smooth out the transitory earnings shock.

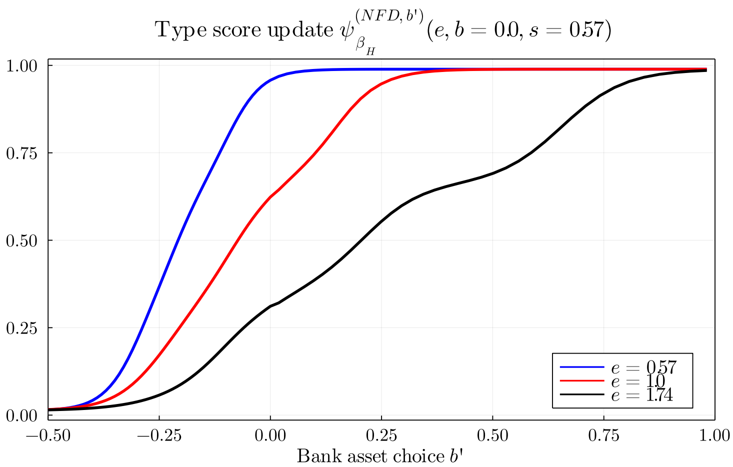
Figure 10: Earnings Distribution among Puzzle Users

(a) Persistent Earnings (b) Transitory Earnings



Note: Puzzle users: Households which borrow using both loans and fulfil condition 22 and 23. Non-puzzle users: Households which borrow using both loans but do not fulfil both conditions.

Figure 11: Type Score Updating Across Persistent Earnings



7 Policy Experiments

Our rich framework allows us to simulate policy counterfactuals and investigate the (potentially heterogeneous) welfare implications. In this section we consider two different policy experiments that are highly relevant in the consumer credit market: policies curtailing (or outright banning) payday loans and bankruptcy law regulation.

7.1 Payday Loan Regulation

Payday loans have been a subject of intensive public debate. Opponents of payday loans have long argued that payday lenders prey on poor households and should be banned. Advocates emphasized the role of payday loans in smoothing consumption and claimed that regulating payday loans would lead to alternatives carrying even higher interest rates.

We contribute to this debate by investigating the welfare implications of limiting access to payday loans through quantity caps or an outright ban on payday loans in our model. Table 6 summarizes the key results of these policy counterfactuals. The column "Quantity Cap" denotes the counterfactual where the possible payday loan choices are limited to the smallest possible payday loan of size \$300 in the benchmark economy. The column "Full Ban" describes the counterfactual where payday loans become unavailable in the economy.

Compared to the benchmark, a quantity cap leads to a lower fraction of payday loan users as there are less payday loan choices available. It also leads to a decrease in the (unconditional) payday default rate to 2.2% since it becomes less advantageous to default on smaller payday loans.[31] The unconditional payday default rate also drops mechanically as there are less payday loan borrowers. However, it can be easily seen that the conditional effective default rate on payday loans also decreases from around 34.68% in the benchmark to 31.24%. Accordingly, the average payday interest rate decreases slightly. The formal default also decreases slightly and as such there is no substitution from payday default to formal default as a consequence of the payday loan cap. Surprisingly, the

[31] The monetary filing cost stays the same as in the benchmark economy.

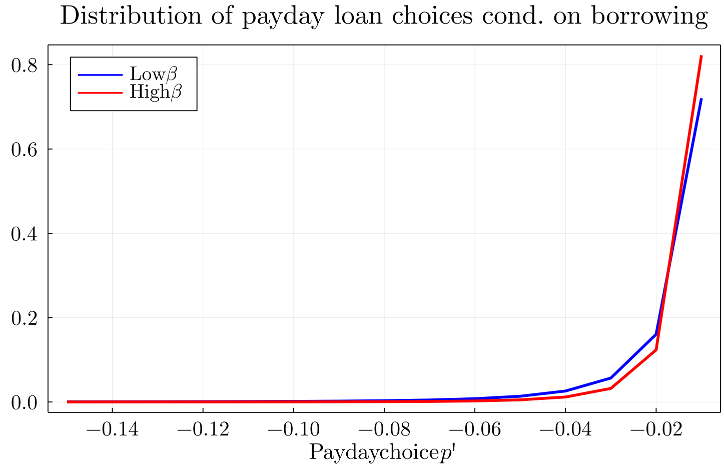
Variables	Benchmark	Quantity Cap	Full Ban
Formal Default Rate	0.99	0.96	0.89
Payday Default Rate	2.81	2.19	–
Eff. Cond. Payday Default Rate	34.68	31.24	–
Fraction of Bank Loan Users	24.26	24.06	23.15
Fraction of Payday loan Users	9.46	8.22	–
Bank Debt-to-Earnings	6.48	6.61	6.84
Ave. Bank Interest Rate	8.56	8.53	8.46
Ave. Payday Interest Rate	410.85	341.88	–
Welfare (CEV)	–	-0.0012	-0.0291
Welfare (CEV) – Impatient	–	-0.0029	-0.0331
Welfare (CEV) – Patient	–	0.0013	-0.0233

Table 6: Policy Counterfactual: Quantity Caps

extensive margin of bank loan borrowing also decreases: The fraction of bank loan users drops slightly. The lack of an increase in the extensive margin of bank loan users is explained by the fact that most payday loan users were already borrowing bank loans in the benchmark economy. Instead, limiting the size of payday loans leads to an increase in the *intensive* margin of bank loan usage: Conditional on borrowing bank debt-to-earnings rises. This is because borrowers now partially substitute bank loans for payday loans. This in turn gives rise to a mild decrease in average bank interest rate. In the full ban counterfactual all of these changes are magnified.

The overall welfare effects of both policy counterfactuals are negative. More interestingly, the welfare implications of experiments are heterogenous across household types. Impatient households lose in terms of welfare whenever the payday loan market becomes more constrained. In contrast, patient households have higher welfare in the quantity cap counterfactual but lower welfare in the full ban counterfactual compared to the benchmark economy. The reasons for the declines in welfare for impatient households are intuitive: Firstly, they are more likely to borrow larger payday loans in the benchmark economy and are thus more affected by the quantity cap/ban, as shown in Figure 12 of the distribution of payday loan size in the benchmark. In addition, imposing a payday loan quantity cap (or banning them) also reduces the informational asymmetry in the bank market and thus reduces pooling. As we could see in Figure 4 this pooling led to cross-subsidization of impatient households by patient ones in the benchmark economy. The end result is that constraining the payday loan market leads to less subsidization of impatient households which lowers their welfare. In turn, this also explains the increase in welfare for patient households in the quantity cap counterfactual.

Figure 12: Distribution of Payday Loan Size



So what explains the decrease in welfare for patient households when payday loans are fully banned? The answer is that there is a second factor at play apart from cross-subsidization: Risk-sharing/insurance. Constraining payday loan choices makes it harder for everyone in the economy, including patient households, to insure against idiosyncratic shocks. When payday loans are quantity capped but still available in the economy the reduction in cross-subsidization outweighs this loss in insurance for patient households. But patient households do depend on payday loans to smooth shocks, for example in order to reduce the negative effect on type scores of a transitory earnings shock as discussed in Section 6. In the full ban economy this loss of insurance outweighs the gain from reduced cross-subsidization for patient households. This result implies that in our model fully banning payday loans makes *everyone* worse off.

7.2 Bankruptcy Regulation

Another approach to regulation in the consumer finance market taken by policy makers is through bankruptcy laws. The most notable overhaul of bankruptcy regulation in recent years is the 2005 Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA). Among other changes, this legislation increased the total out-of-pocket filing cost for Chapter 7 filings by around 35% (see Albanesi and Nosal (2020)). To examine the effect of such an increase in monetary filing cost in our model, we simulate a counterfactual where the formal default cost is increased by 35% ($1.35 \times \kappa_{FD}$). Additionally, we also consider the policy counterfactual where the payday default cost rises by the same magnitude ($1.35 \times \kappa_{PD}$) to assess the implication of a stricter regulation on payday lending. This allows to fairly compare the roles of bank and payday loans in terms of the efficiency-insurance trade-off. The essential results of these policy counterfactuals are summarized in Table 7.

Variables	**Benchmark**	$\mathbf{1.35 \times \kappa_{FD}}$	$\mathbf{1.35 \times \kappa_{PD}}$
Formal Default Rate	0.99	0.84	0.99
Payday Default Rate	2.81	3.03	2.60
Eff. Cond. Payday Default Rate	34.68	33.59	33.78
Fraction of Bank Loan Users	24.26	26.35	24.21
Fraction of Payday loan Users	9.46	10.11	9.07
Bank Debt-to-Earnings	6.48	7.56	6.48
Ave. Bank Interest Rate	8.56	7.51	8.56
Ave. Payday Interest Rate	410.85	395.01	398.23
Welfare (CEV)	–	0.1236	-0.0032
Welfare (CEV) – Impatient	–	0.1404	-0.0036
Welfare (CEV) – Patient	–	0.0991	-0.0026

Table 7: Policy Counterfactual: Higher Default Costs

Focusing first on the increased formal default cost, we can observe that this change leads to a significant decrease in the formal default rate. This is caused by a substitution from formal default to payday default as the payday default rate rises. The drop in the formal default rate leads to a decrease in the average bank interest rate as banks require a lower default premium on their loans. This in turns makes borrowing using bank loans cheaper and increases bank loan borrowing both in terms of the extensive as well as the intensive margins. Interestingly, the increase in bank loan borrowing is not accompanied by a decrease in payday loan borrowing. Rather payday loan usage also increases leading to an overall higher level of debt in the economy. This is because the conditional effective default rate on payday loans actually drops from 34.68% in the benchmark to 33.59%, thus implying cheaper borrowing costs for payday loans.

The explanation for the heterogeneous welfare implications of the two policy experiments is somewhat subtle. In consumer finance models, the key insurance-efficiency trade-off between different bankruptcy regimes/strictness is that between smoothing *over time* and *across states* (Zame, 1993). Smoothing over time means households borrow to achieve consumption smoothing intertemporally while smoothing across states means households could file for bankruptcy to discharge debt burdens under worse earnings realization ex-post. Apparently, more lenient regime makes smoothing across states easier while smoothing over time more costly as higher default premium results in higher borrowing costs.

In the benchmark economy, we can observe that it is much cheaper to achieve smoothing over time using bank loans rather payday loans since the average bank interest rates $\bar{r}_b = 8.56$ is much smaller than the one of payday loans $\bar{r}_p = 410.85$. In contrast, it is less costly in terms of filing costs and reputation to smooth across states using payday loans rather bank loans since payday default is not perfectly captured in type score updating and the fixed filing cost for payday default is ten times less than the one for formal default in the absolute sense ($\kappa_{PD} = 0.002 < \kappa_{FD} = 0.02$). As a result, households tend to borrow bank loans for smoothing over time and payday loans for smoothing across states.[32]

A higher formal filing cost regulates the financial market toward the direction that all would prefer. Essentially, higher κ_{FD} not only renders a stricter regime on bank loans—easier to smooth over time but harder to smooth across states using bank loans—but also indirectly makes it cheaper to borrow payday loans as they can be discharged with formal default as well. Plus, the partial insurance role of payday loans across states are untouched. All lead to the welfare-improving outcomes for all types.

As for the rising filing cost for payday default, the payday default rate drops mechanically associated with a lower average payday loan interest rate. All bank-related variables remain roughly the same except the fraction of bank loan users decreases slightly. Sur-

[32] This argument is also valid across types. As shown in Table 4, the average payday interest rates are far higher than the ones for bank loans for both types.

prisingly, the fraction of payday loan users drops. This in fact echos the way of household using both loans in the benchmark economy—bank loans for smoothing over time and payday loans for smoothing across states. A higher payday default cost makes it harder to smooth across states with payday loans, thus diminishing the major benefit of households from taking up payday loans. Increasing κ_{PD} thus gives rise to a overall welfare-reducing outcomes.

8 Conclusion

One puzzle in the consumer finance literature is the so-called ”payday loan puzzle”: Households use expensive payday loans even when they still have cheaper alternatives, like their credit cards, available. To investigate this hypothesis, we build a two-asset Huggett-type model with two types of consumer default as well as asymmetric information and hidden actions. Households have access to bank as well as payday loans and can be of one of two types: patient or impatient. This household type is unobservable to lenders. Lenders compute a household-specific type score in order to form an expectation of a household's type.

Our model can successfully replicate the payday loan puzzle observed in the empirical literature and match both the fraction of households that show behavior consistent with the payday loan puzzle as well as the magnitude of the monetary costs. We also illustrate how the reputation protection channel can lead to the emergence of the payday loan puzzle in this model. Furthermore, we conduct a series of policy experiments: We show that a full ban on payday loans is welfare decreasing for both impatient and patient households. In addition, we also show that increasing the costs of defaulting on payday loans is welfare decreasing whereas increasing the costs of formal default is beneficial in terms of welfare. This implies that current regulatory efforts in the United States to curtail or even ban the payday loan sectors may potentially be harmful to U.S. households.

In the future, estimating the model with systematic procedures (e.g., SMM) could be desirable. As a result, the policy conclusion could be more robust. However, such an estimation is often constrained by the availability of payday loan data at the individual level. In addition, we are planning to consider a case where banks can observe payday loan usage (as if payday lenders become required to report). This would allow us to more cleanly separate the effect on policy outcomes of pooling across types versus pooling across payday loan users, thus guiding the regulation of payday lending industry.

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A Theoretical Appendix

A.1 Additional Model Details

B Computation

B.1 Grid Specifications

Variable	Symbol	# Grid points	Range
Persistent earnings	e	3	$\{0.57, 1.00, 1.74\}$
Transitory earnings	z	3	$\{0.78, 1.00, 1.29\}$
Bank assets	b	201	$[-0.40, 15.00]$
Payday loans	p	16	$[-0.15, 0.00]$
Type score	s	8	$[0.013, 0.989]$

Table 8: Grids used in model computation

B.2 One-Loop Algorithm

1. Set parameters and tolerances for convergence.

2. Create grids for (β, z, e, b, s, p) with lengths $(n_\beta, n_z, n_e, n_b, n_s, n_p)$.

3. Initialize algorithm with starting guesses:

   (a) $W(:,:,:,s,p) = W^{FI}$ for all s and p, where W^{FI} denotes the unconditional value function under full information.

   (b) $\psi_{\beta'}^{(\tilde{d},b')}(\omega_b) = sQ^{\beta}(\beta_H|\beta_H) + (1-s)Q^{\beta}(\beta_H|\beta_L)$ for all ω_b and $(\tilde{d}, b')$.

      i. $s_i' = \max\left\{s \in \mathcal{S} | s \leq \psi_{\beta'}^{(\tilde{d},b')}(\omega_b)\right\}$ and $s_j' = \min\left\{s \in \mathcal{S} | s \geq \psi_{\beta'}^{(\tilde{d},b')}(\omega_b)\right\}$.

      ii. $Q^s\left(s_i'(\beta_H) | \psi_{\beta'}^{(\tilde{d},b')}(\omega_b)\right) = \frac{s_j' - \psi_{\beta'}^{(\tilde{d},b')}(\omega_b)}{s_j' - s_i'}$ and $Q^s\left(s_j'(\beta_H) | \psi_{\beta'}^{(\tilde{d},b')}(\omega_b)\right) = \frac{\psi_{\beta'}^{(\tilde{d},b')}(\omega_b) - s_i'}{s_j' - s_i'}$.

   (c) $q_b^{(NFD,b')}(:,:,s) = q_b^{FI}$ for all b' and ω_b, where q_b^{FI} denotes the bank loan price function under full information.

   (d) $q_p^{(R,p')}(\beta, \omega_b) = \frac{\rho}{1+r_f+\iota}$ for all b', p' and β, ω_b.

   (e) μ is uniformly distributed.

4. Begin the one-loop algorithm:

   (a) Solve for new W_1 taking as given W_0.

      i. Find set of feasible actions (d, b', p') using (**??**).

      ii. For each (β, z, ω_b, p), compute the value $v^{(d,b',p')}(\beta, z, \omega_b, p)$ for each feasible action (d, b', p') according to (3).

      iii. Compute new W_1 using (7).

   (b) Compute $\sigma^{(d,b',p')}(\beta, z, \omega_b, p)$ according to (**??**), (**??**), and (**??**).

   (c) Compute new equilibrium functions.

      i. On bank side:

         A. Compute $\sigma_b^{(\tilde{d},b')}(\beta, \omega_b)$ using (**??**).

         B. Then $\psi_{\beta'}^{(\tilde{d},b')}(\omega_b)$ using (**??**).

         C. Then $\chi(\beta'|\psi)$ using (**??**) for all ψ from previous step.

         D. Then $Q^s(s'|\psi)$ using (**??**) for all ψ from previously.

         E. Then $\mathbb{P}_b^{(NFD,b')}(\omega_b)$ using (**??**).

         F. Finally $q_b^{(NFD,b')}(\omega_b)$ using (**??**).

      ii. On payday lender side:

         A. Compute $\mathbb{P}_p^{(R,b',p')}(\beta, \omega_b)$ using (**??**).

         B. Then $q_p^{(R,b',p')}(\beta, \omega_b)$ using (**??**).

   (d) Compute stationary distribution μ' using (**??**).

   (e) Assess convergence of W, ψ, q_b, q_p, and μ.

      i. If achieved, continue to the next step.

      ii. Otherwise, update the initialization of the targeted objects with relaxation and return to step (a).

5. Compute moments.

C Model Moment Definitions

The fraction of households choosing to default on payday loans $\mu(d = PD)$ is defined as:

$$\mu(d = PD) \equiv \sum_{\beta,z,e,b,s,p} \sum_{b'} \sigma^{(PD,b',0)}(\beta, z, e, b, s, p) \cdot \mu(\beta, z, e, b, s, p). \tag{26}$$

The fraction of households choosing to formally default on both bank and payday loans $\mu(d = FD)$ is defined as:

$$\mu(d = FD) \equiv \sum_{\beta,z,e,b,s,p} \sigma^{(FD,0,0)}(\beta, z, e, b, s, p) \cdot \mu(\beta, z, e, b, s, p). \tag{27}$$

The fraction of bank loan users among all households $\mu(b < 0)$ is defined as:

$$\mu(b < 0) \equiv \sum_{\beta,z,e,b<0,s,p} \mu(\beta, z, e, b, s, p). \tag{28}$$

The fraction of payday loan users among all households $\mu(p < 0)$ is defined as:

$$\mu(p < 0) \equiv \sum_{\beta,z,e,b,s,p<0} \mu(\beta, z, e, b, s, p). \tag{29}$$

The fraction of bank or payday loan users $\mu(b < 0 \vee p < 0)$ is defined as:

$$\mu(b < 0 \vee p < 0) \equiv \sum_{\beta,z,e,b<0,s,p<0} \mu(\beta, z, e, b, s, p). \tag{30}$$

The fraction of both loan users $\mu(b < 0 \wedge p < 0)$ is defined as:

$$\mu(b < 0 \wedge p < 0) \equiv \sum_{\beta,z,e,b<0,s,p<0} \mu(\beta, z, e, b, s, p). \tag{31}$$

The debt-to-income ratio for banks loans DI_b is defined as:

$$DI_b \equiv \sum_{\beta,e,b<0,s,p} \left(\frac{|b|}{e} \right) \cdot \frac{\mu(\beta, e, b, s, p)}{\sum_{\hat{\beta},\hat{e},\hat{b}<0,\hat{s},\hat{p}} \mu(\hat{\beta}, \hat{e}, \hat{b}, \hat{s}, \hat{p})}. \tag{32}$$

The debt-to-income ratio for payday loans DI_p is defined as:

$$DI_p \equiv \sum_{\beta,e,b,s,p<0} \left(\frac{|p|}{e}\right) \cdot \frac{\mu(\beta,e,b,s,p)}{\sum_{\hat{\beta},\hat{e},\hat{b},\hat{s},\hat{p}<0} \mu(\hat{\beta},\hat{e},\hat{b},\hat{s},\hat{p})}. \tag{33}$$

The total-debt-to-income ratio DI is defined as:

$$DI \equiv \sum_{\beta,e,b<0,s,p<0} \left(\frac{|b+p|}{e}\right) \cdot \frac{\mu(\beta,e,b,s,p)}{\sum_{\hat{\beta},\hat{e},\hat{b}<0,\hat{s},\hat{p}<0} \mu(\hat{\beta},\hat{e},\hat{b},\hat{s},\hat{p})}. \tag{34}$$

The average interest rate for bank loans i_b is defined as:

$$i_b \equiv \sum_{\beta,\omega_b,p} \sum_{d\in\{R,PD\},b'<0,p'} \left(\frac{1}{q_b^{(NFD,b')}(\omega_b)} - 1\right) \cdot \widehat{\sigma\mu}^{(d,b',p')}(\beta,\omega_b,p), \tag{35}$$

where recall that $\omega_b = (e,b,s)$ and the relative share is defined as:

$$\widehat{\sigma\mu}^{(d,b',p')}(\beta,\omega_b,p) \equiv \frac{\sigma^{(d,b',p')}(\beta,\omega_b,p) \cdot \mu(\beta,\omega_b,p)}{\sum_{\hat{\beta},\hat{\omega}_b,\hat{p}} \sum_{\hat{d}\in\{R,PD\},\hat{b}'<0,\hat{p}'} \sigma^{(\hat{d},\hat{b}',\hat{p}')}(\hat{\beta},\hat{\omega}_b,\hat{p}) \cdot \mu(\hat{\beta},\hat{\omega}_b,\hat{p})}.$$

The debt-weighted interest rate for bank loans is similarly defined as:

$$i_{b,weighted} \equiv \sum_{\beta,\omega_b,p} \sum_{d\in\{R,PD\},b'<0,p'} \left(\frac{1}{q_b^{(NFD,b')}(\omega_b)} - 1\right) \cdot \widehat{\sigma\mu}^{(d,b',p')}_{weighted}(\beta,\omega_b,p), \tag{36}$$

where the relative share is defined as:

$$\widehat{\sigma\mu}^{(d,b',p')}_{weighted}(\beta,\omega_b,p) \equiv \frac{\sigma^{(d,b',p')}(\beta,\omega_b,p) \cdot \mu(\beta,\omega_b,p) \cdot |b'|}{\sum_{\hat{\beta},\hat{\omega}_b,\hat{p}} \sum_{\hat{d}\in\{R,PD\},\hat{b}'<0,\hat{p}'} \sigma^{(\hat{d},\hat{b}',\hat{p}')}(\hat{\beta},\hat{\omega}_b,\hat{p}) \cdot \mu(\hat{\beta},\hat{\omega}_b,\hat{p}) \cdot |b'|}.$$

The average interest rate for payday loans i_p is defined as:

$$i_p \equiv \sum_{\beta,\omega_b,p} \sum_{b',p'<0} \left(\frac{1}{q_p^{(R,p')}(\beta,\omega_b)} - 1\right) \cdot \widehat{\sigma\mu}^{(R,b',p')}(\beta,\omega_b,p), \tag{37}$$

where the relative share is defined as:

$$\widehat{\sigma\mu}^{(R,b',p')}(\beta,\omega_b,p) \equiv \frac{\sigma^{(R,b',p')}(\beta,\omega_b,p) \cdot \mu(\beta,\omega_b,p)}{\sum_{\hat{\beta},\hat{\omega}_b,\hat{p}} \sum_{\hat{b}',\hat{p}'<0} \sigma^{(R,\hat{b}',\hat{p}')}(\hat{\beta},\hat{\omega}_b,\hat{p}) \cdot \mu(\hat{\beta},\hat{\omega}_b,\hat{p})}.$$

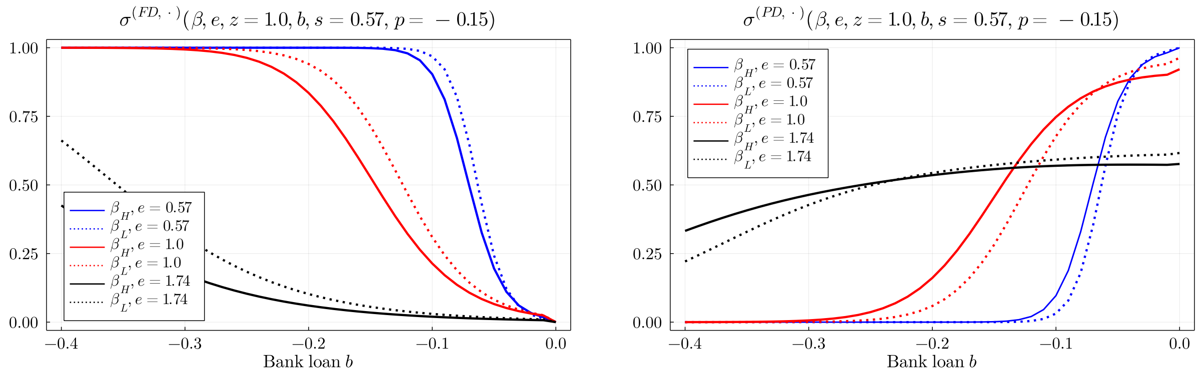
D Data Moment Details

E General Results

Figure 13 depicts how default probabilities vary across (persistent) earnings e and types β. The left-hand side shows how the probability of a household choosing formal default increases as its debt burden grows (b becomes more negative). Households with lower earnings start to formally default at lower debt burdens compared to households with higher earnings. Furthermore, more impatient households (β_L) also start to formally default at smaller debt levels. In contrast, as can be seen on the right-hand side the probability of payday default decreases as the debt burden grows. This is due to the switching from payday to formal default: As bank loans increase households switch from payday defaulting on their payday loans only to formally defaulting on all debt in order to discharge their larger bank loans. We can see in Figure 13b that this switching starts earlier at lower debt levels for households with less income (black line starts dropping at lower b) and for households that are more impatient (dashed lines drop more quickly than solid lines). This happens because low types are less concerned about the long-term reputational damage from formal default.

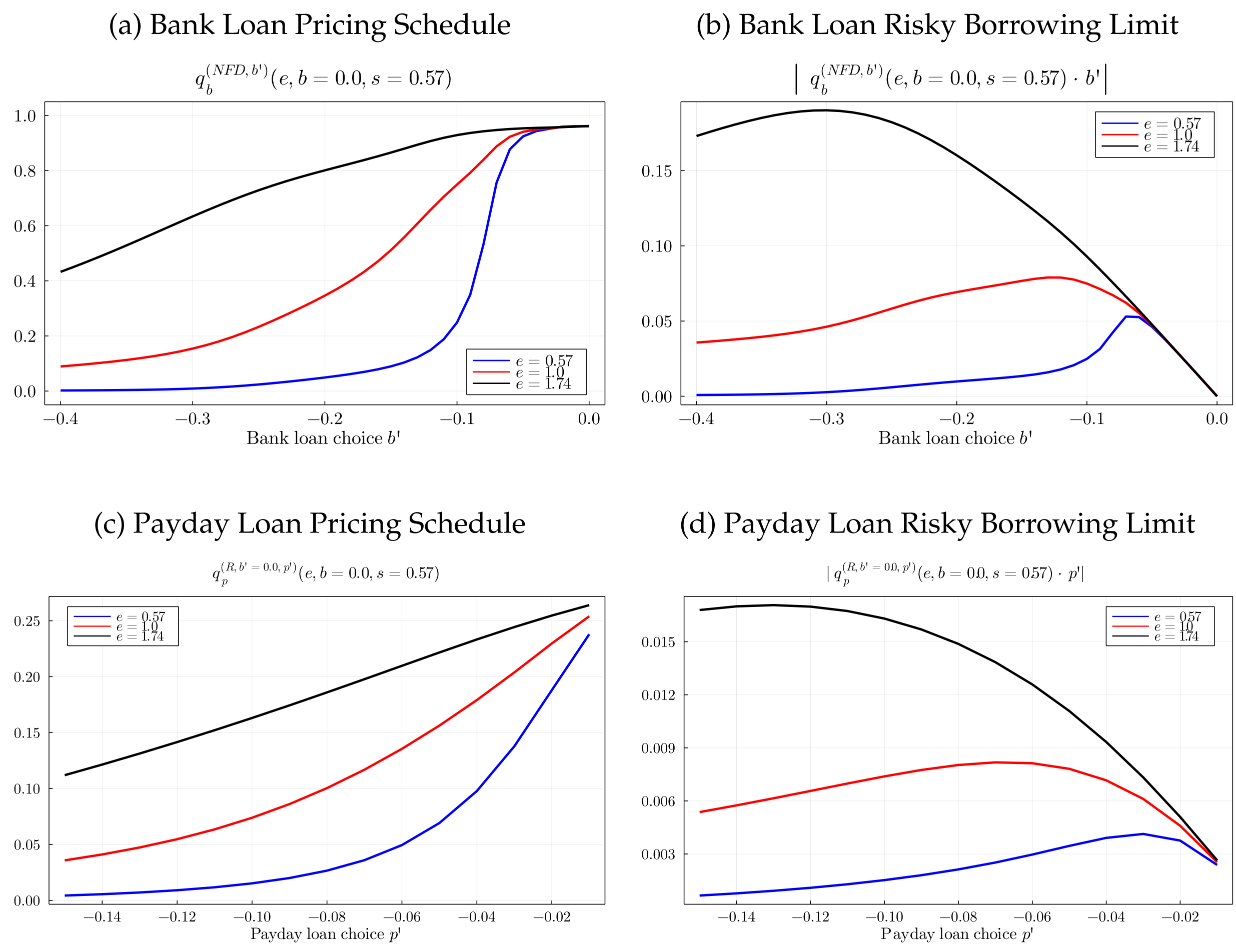
Figure 13: Default Probabilities

(a) Formal Default (b) Payday Default



The pricing schedules and the risky borrowing limits of bank and payday loans across earnings in the model are depicted in Figure 14. These results are quite standard in consumer default models. The intuition is clear: On the one hand, borrowing more this period will lead to a higher default probability next period c.p. as the gain from defaulting is larger. As a result we can see in Figure 14a that borrowing more (more negative b') leads to lower prices/higher interest rates. Furthermore, an individual with lower persistent earnings e will face lower prices compared to one with higher e c.p. due to the difference in default probability in the following period. Similarly, the payday loan pricing schedules and the risky borrowing limits across earnings in the model are in the bottom panel. These results are similar to those of bank loans. The significant disparity in levels across bank and payday loans results from the fact that payday lenders have higher operating costs than banks (i,e, higher lending costs).

Figure 14: Pricing Schedule and Risky Borrowing Limit Across Persistent Earnings



(a) Bank Loan Pricing Schedule

(b) Bank Loan Risky Borrowing Limit

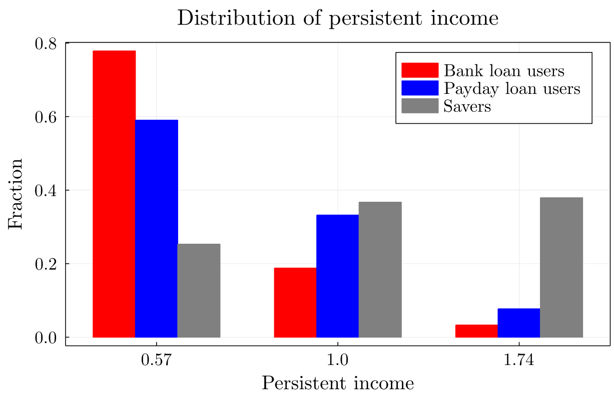
(c) Payday Loan Pricing Schedule

(d) Payday Loan Risky Borrowing Limit

Figure 15 illustrates what kind of household in our economy saves or borrows. On the left, Figure 15a shows the distribution of savers and borrowers across *persistent* income. Unsurprisingly, savers in our economy tend to have higher (persistent) income compared to borrowers. We can also see that households who use bank loans (either only bank loans or together with payday loans) are overwhelmingly poor (the red bars). Perhaps more interestingly, payday loan users, while still being poor compared to savers, tend to have higher persistent income than bank loan users. On the right, Figure 15b shows the distribution of households across *transitory* income. Compared to Figure 15a it can be seen that payday loan users tend to have lower transitory income than bank loan users. These two figures suggest that the two types of loans are used to smooth different types of income shocks in our model: households use bank loans to smooth persistent income shocks whereas payday loans are used to smooth transitory shocks. This makes sense: Payday loans are more expensive than bank loans and are much more costly to smooth a persistent negative income shock. On the other hand, using payday loans does not (directly) affect your type score. As a result, it can make sense to smooth transitory income shocks using payday loans in order avoid long-term reputational damage to a household.

Figure 15: Earnings Distribution among Borrowers/Savers

(a) Persistent Earnings



(b) Transitory Earnings

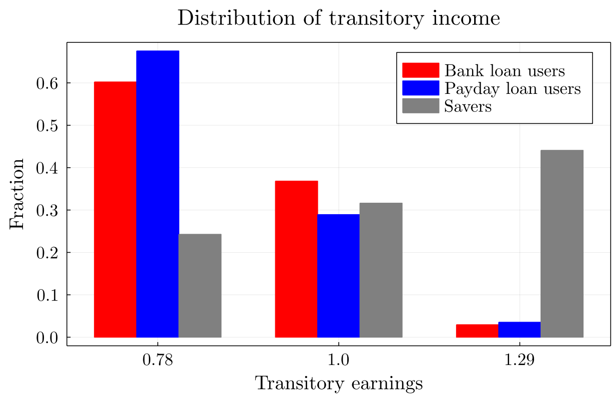
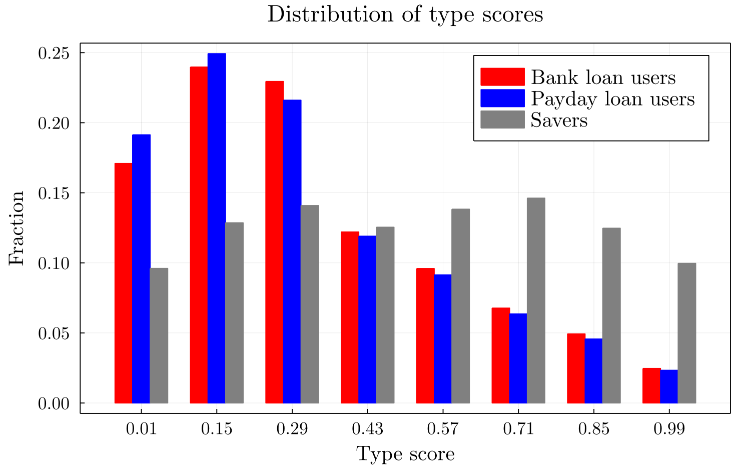


Figure 16 shows how savers in our economy tend to have higher type scores (more likely to be the patient type from the viewpoint of the bank) compared to borrowers. Interestingly, payday loan users tend to have slightly worse type scores compared to bank loan users.

Figure 16: Type Score Distribution among Borrowers/Savers



In addition, puzzle users tend to have lower prior type scores in contrast to non-puzzle users. This is shown in Figure 17 which depicts the type score distribution of puzzle vs. non-puzzle users. We can see that the distribution of prior type scores among different kinds of borrowers is all skewed to the right. This is because the reputation gain/interest costs for puzzle users are higher/lower for low type score households (see Figure 9).

(ADD TYPE DISTRIBUTION HERE?)

Figure 17: Prior Type Score Distribution

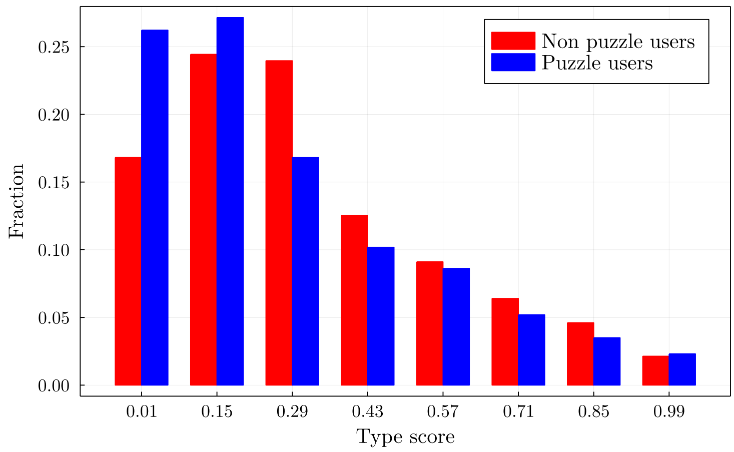


Figure 18 plots the percentage variation in updated type scores relative to priors conditionally for puzzle users (solid line) and if they were to borrow the same amount using instead only bank loans (dotted line). The borrowing portfolio of only banks loans results in overall lower posterior type scores across all priors, compared to using some payday loans.

Figure 18: Prior Type Score Distribution