

# Final Presentation

Course: Electrical Engineering lab (Automatic Control)

Semester: Fall, 2018

Instructor: Feng-Li Lian

Final Project: **Wheeled Inverted Pendulum**

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# Outline

- ❖ Introduction
- ❖ Design principal
- ❖ System ID
- ❖ Modeling
- ❖ P controller design
- ❖ PID controller design
- ❖ Experiment result
- ❖ Live demo

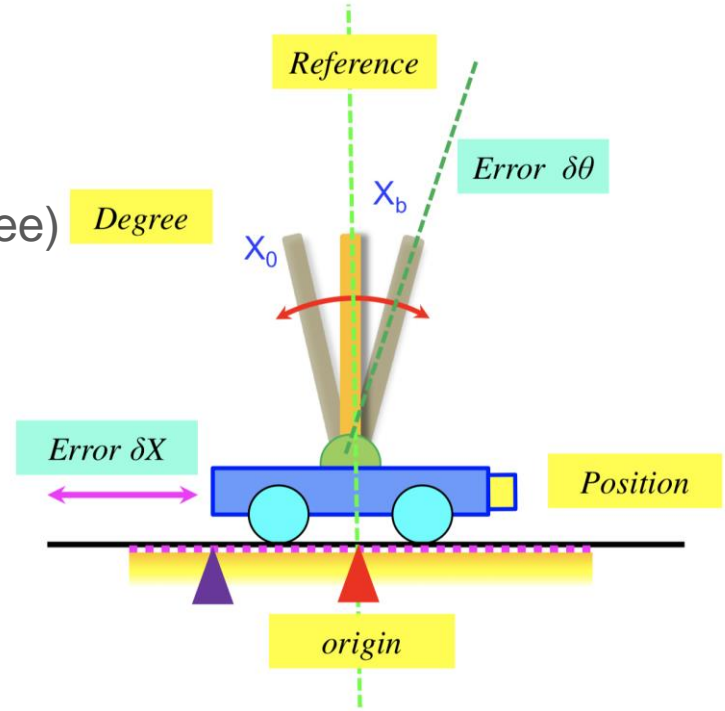
# Introduction

# Wheeled Inverted Pendulum

Actuator: Motor (speed)

Sensor:                  Pendulum Encoder (degree)  
Motor Encoder (degree)

Controller: Arminno



# Wheeled Inverted Pendulum

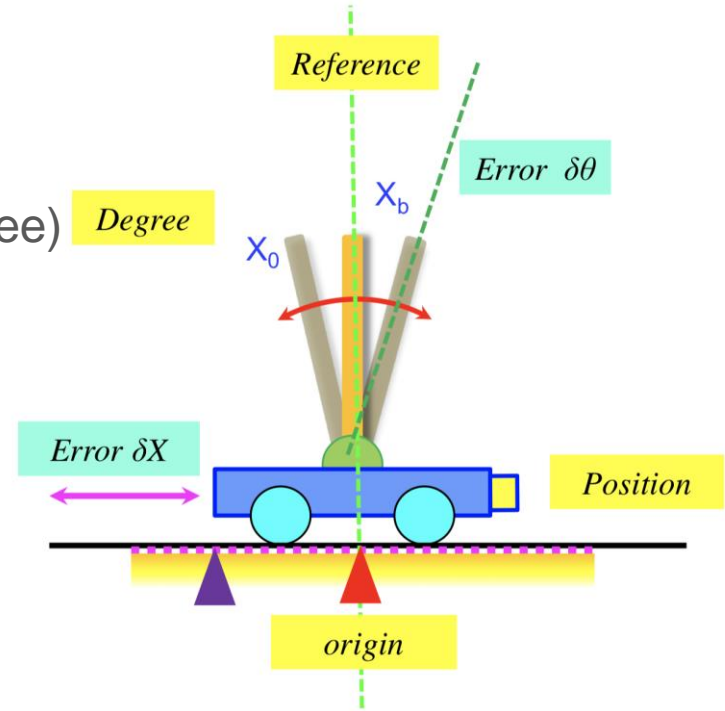
Actuator: Motor (speed)

Sensor: Pendulum Encoder (degree)  
Motor Encoder (degree)

Controller: Arminno

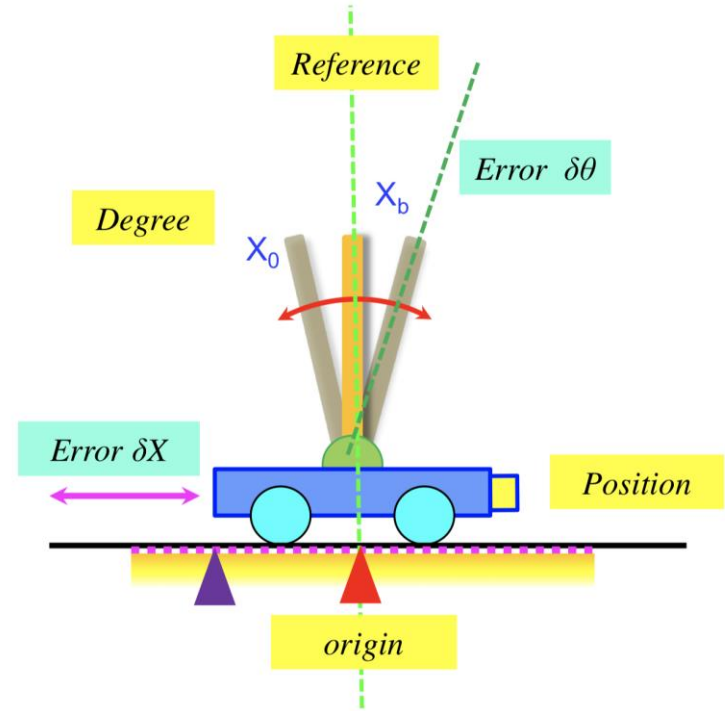
Error:  $\delta\theta, \delta x$

Objective:  $\delta\theta=0, \delta x = 0$



# Challenge

1. Nonlinear System (due to pendulum)
2. Unstable equilibrium point
3. Other nonlinear effects: motor friction etc.



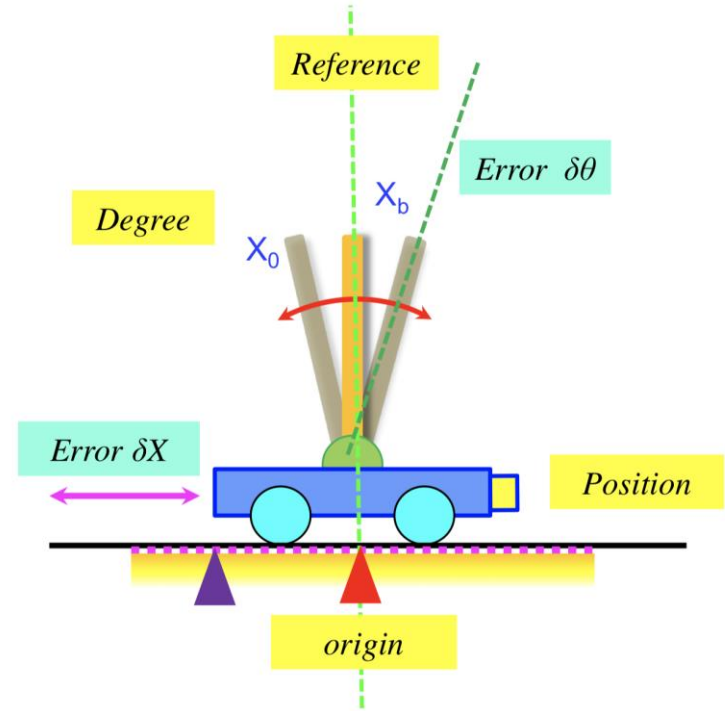
## Time domain spec

## Pendulum Angle $\theta$

- settling time  $< 10$  sec.
- steady state error  $< 5^\circ$

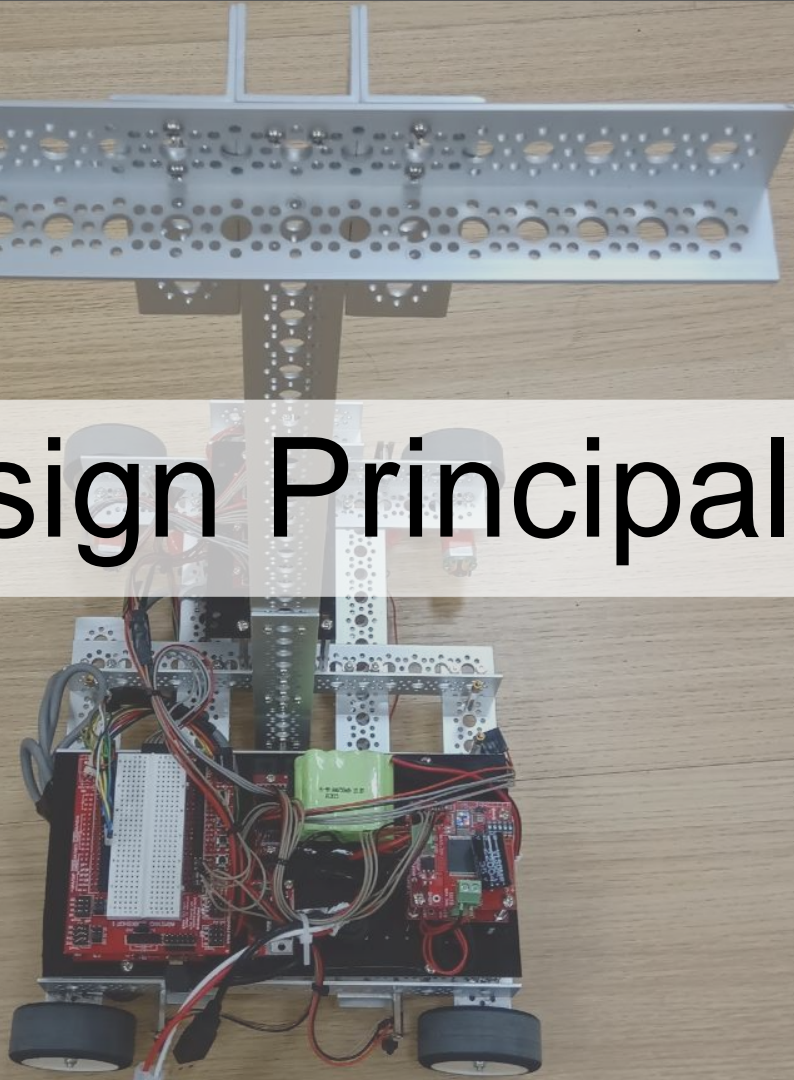
## Vehicle Position $x$

- settling time < 30 sec.





# Design Principal





# Controller Design SOP

System identification from linearization around the equilibrium point

**System ID**

**Modeling**

Modelling from Lagrangian

**P Controller Design**

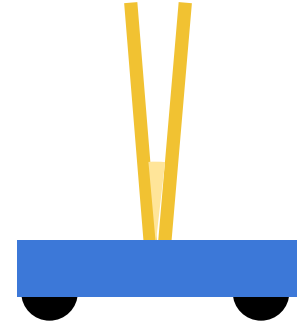
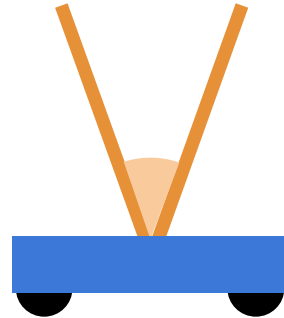
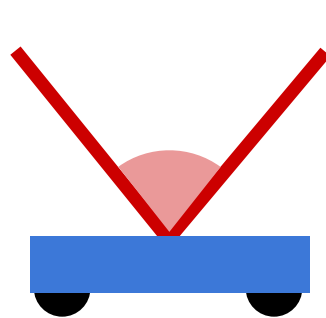
Find the equilibrium point.

**PD controller Design**

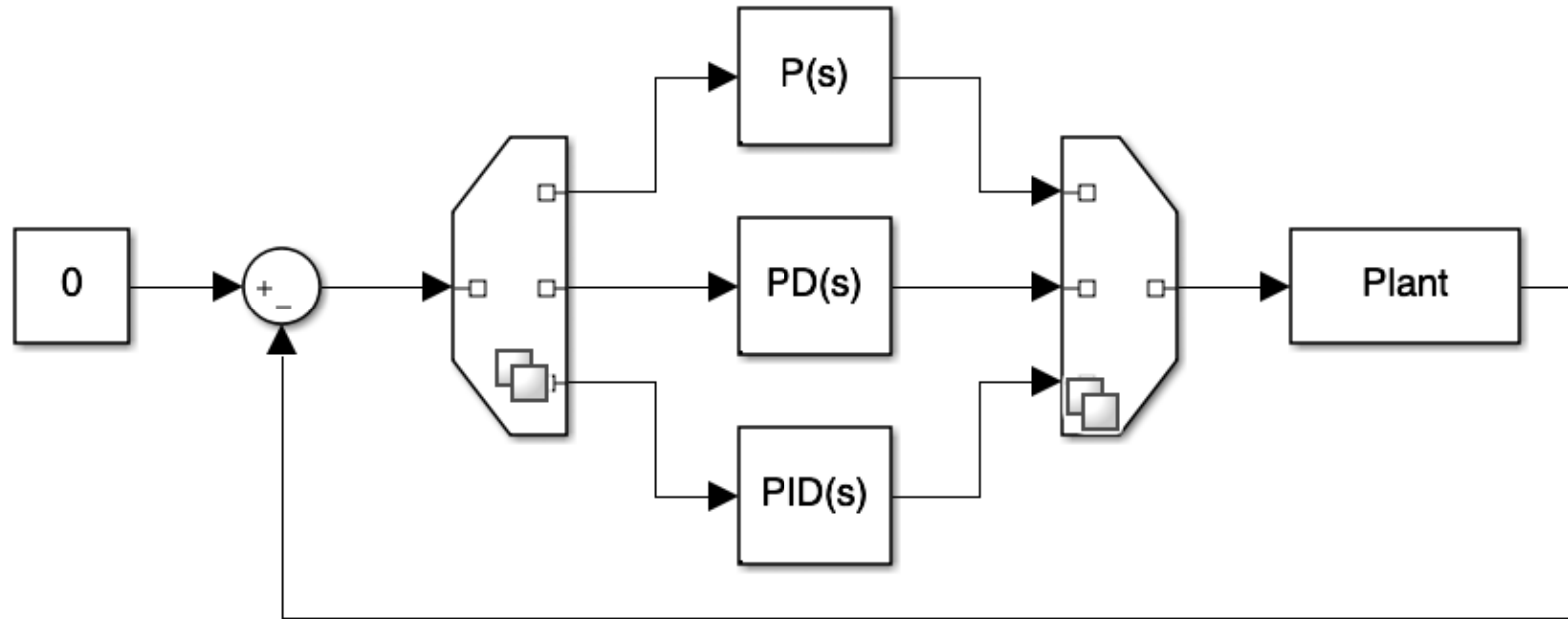
From the linearized model around the equilibrium point, design a PD controller with greater errors.

**PID controller Design**

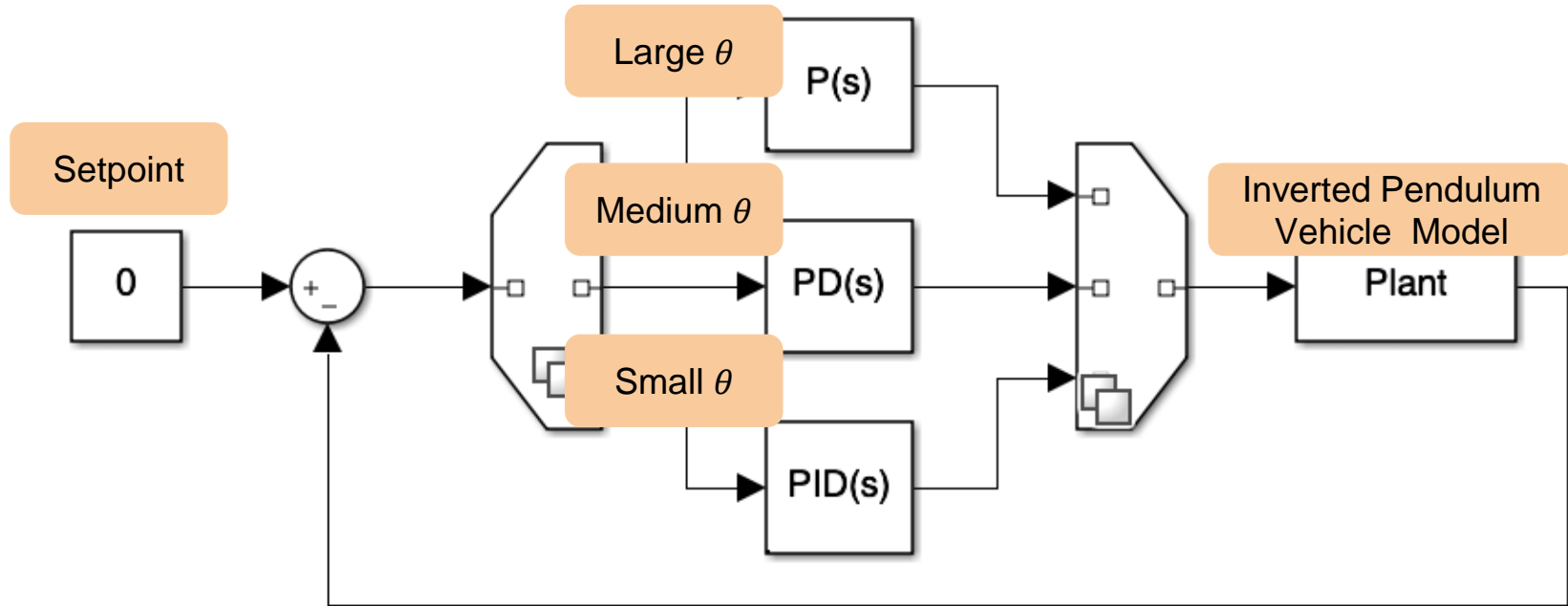
Design a PID controller from the linearized model around the equilibrium point.



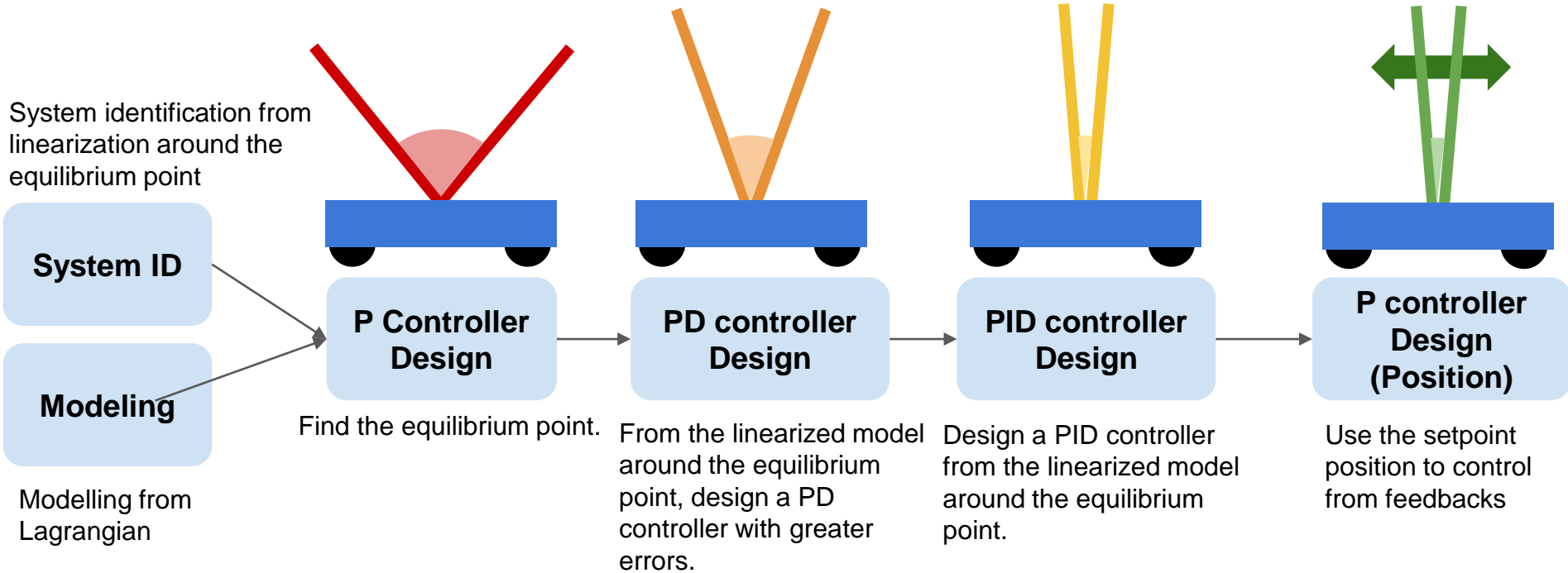
# Controller Block Diagram—tracking $\theta$



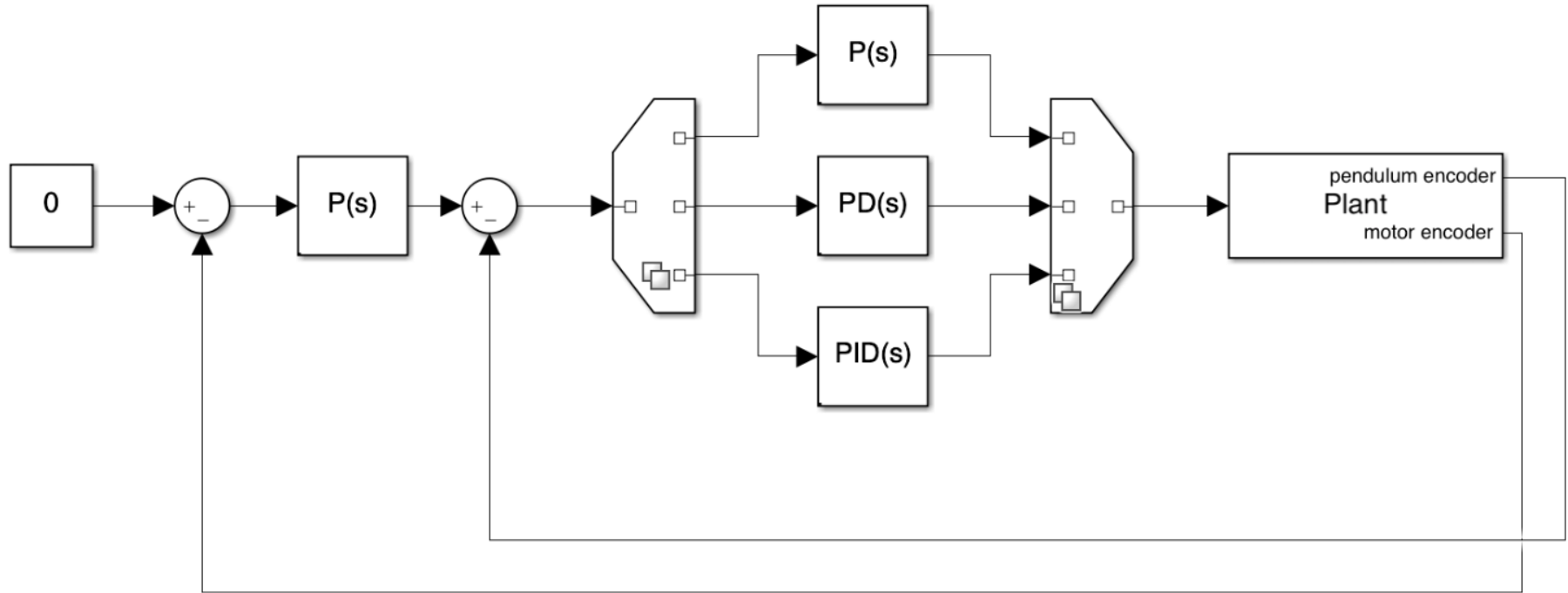
# Controller Block Diagram—tracking $\theta$



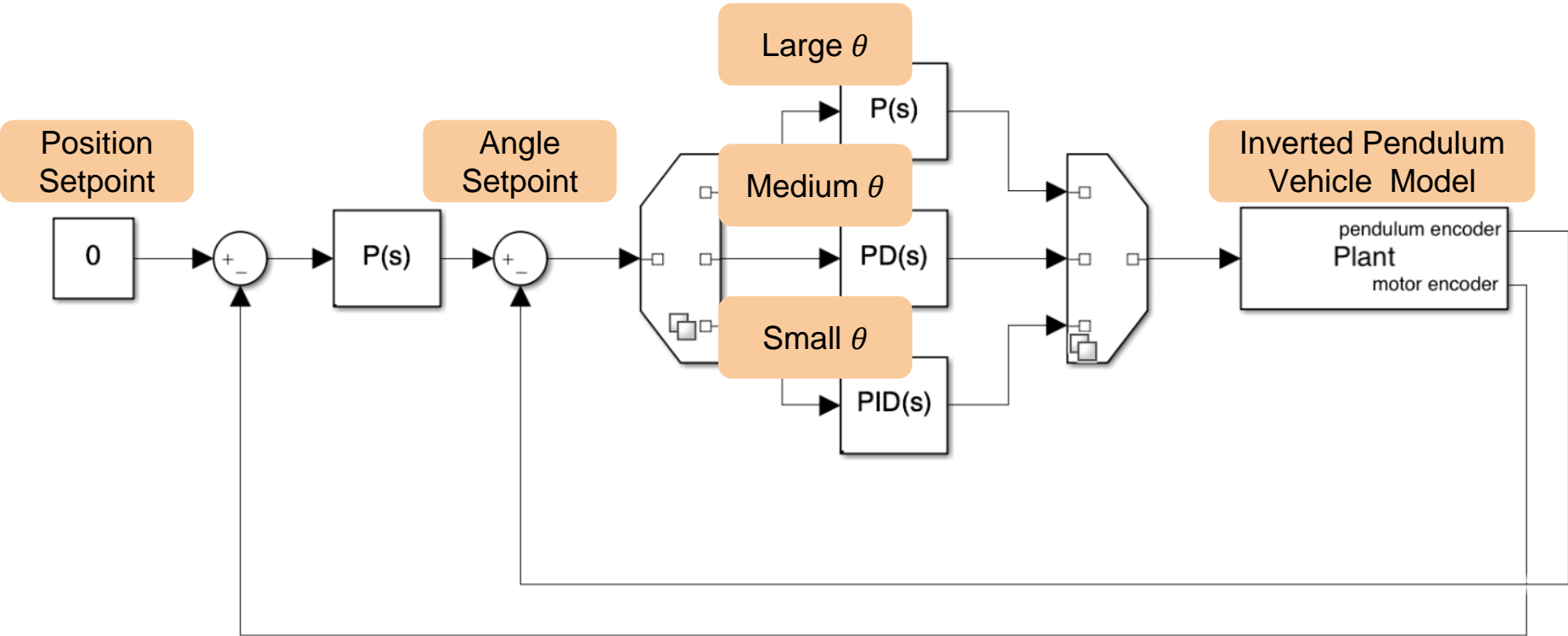
# Controller Design SOP for Position tracking



# Controller Block Diagram—tracking $\theta$ & $x$



# Controller Block Diagram—tracking $\theta$ & $x$





# System ID

# System

Time axis

Left Encoder

Right Encoder

Input Speed

	A	B	C	D	E
1	tm Value	lPdecValue	LdecValue	RdecValue	int(direction)*speed
2	458	0	0	0	0
3	6642181	0	0	0	100
4	12366044	0	22	14	100
5	17373888	0	94	58	0
6	21963747	0	163	117	100
7	26819513	1	228	185	100
8	31515748	2	309	254	0
9	36453164	1	393	335	100
10	41250104	-1	465	411	100
11	45963693	1	547	489	0
12	50720404	1	630	571	0
13	55710127	-1	700	642	100
14	60571922	2	769	712	0
15	65097052	2	839	776	100
16	70104102	0	910	849	100
17	3016683	0	992	925	100
18	7788989	0	1078	1009	0
19	12418820	0	1160	1089	0
20	17248851	1	1229	1160	100
21	22103644	0	1297	1229	100
22	26829926	0	1379	1300	0
23	31648049	2	1460	1377	0
24	36653542	2	1525	1447	100
25	41299016	2	1588	1510	0
26	46067766	-1	1663	1576	100
27	50772267	8	1727	1643	0
28	55690195	-2	1807	1710	0
29	60173741	3	1862	1765	100
30	65413592	2	1927	1832	0
31	69820527	0	1990	1886	0
32	2878127	7	2041	1940	100
33	7666431	0	2097	1994	0
34	12290520	1	2162	2049	100
35	17502802	3	2229	2117	0

speed\_100\_new

# System

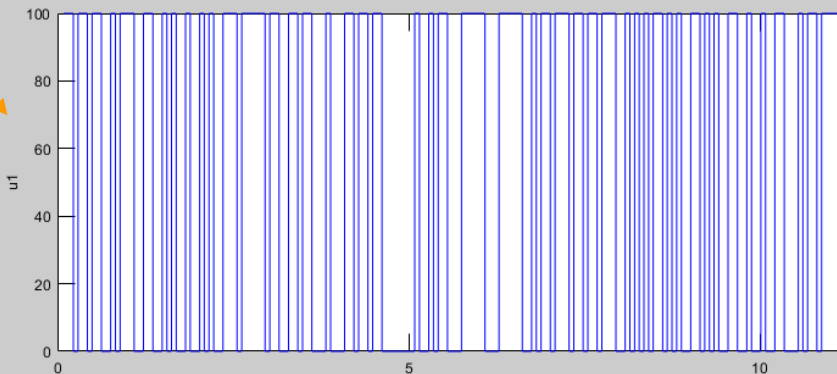
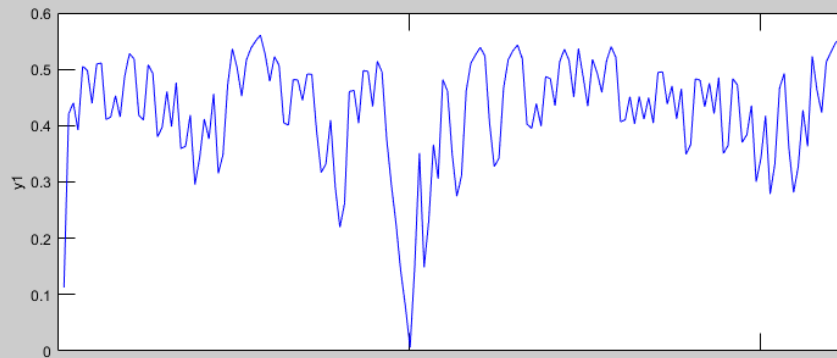
Time axis

Left Speed

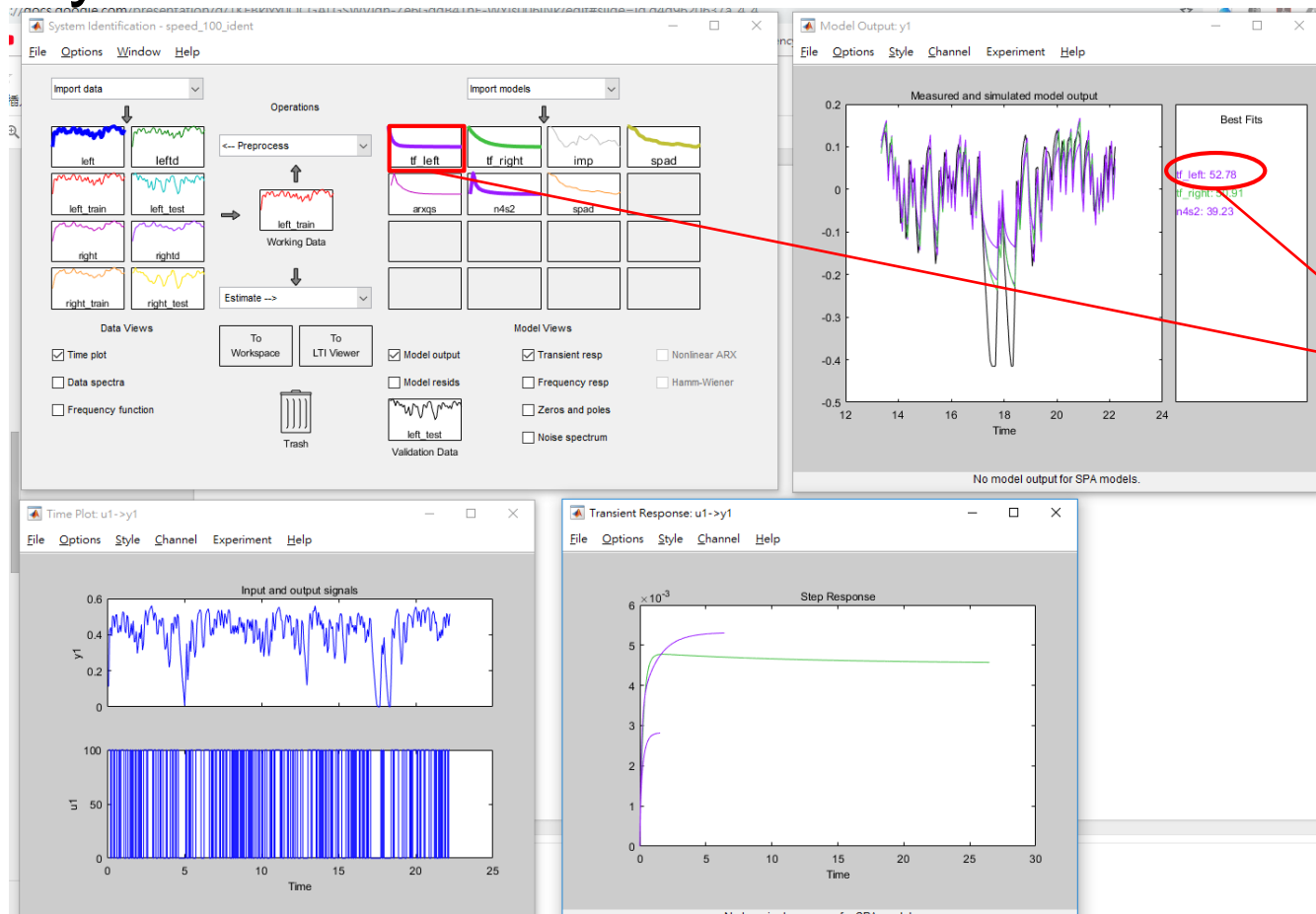
Right Speed

Input Speed

	1	2	3	4
1	0.0923	0.1126	0.0716	100
2	0.1718	0.4211	0.2573	100
3	0.2413	0.4403	0.3765	0
4	0.3051	0.3920	0.4101	100
5	0.3725	0.5051	0.4303	100
6	0.4377	0.4982	0.4804	0
7	0.5063	0.4396	0.4640	100
8	0.5729	0.5095	0.4846	100
9	0.6384	0.5110	0.5049	0
10	0.7045	0.4108	0.4167	0
11	0.7738	0.4156	0.4217	100
12	0.8413	0.4530	0.4142	0
13	0.9041	0.4153	0.4270	100
14	0.9737	0.4888	0.4531	100
15	1.0419	0.5277	0.5155	100
16	1.1082	0.5187	0.5060	0
17	1.1725	0.4184	0.4305	0
18	1.2396	0.4102	0.4162	100
19	1.3070	0.5081	0.4399	100
20	1.3726	0.4923	0.4680	0
21	1.4396	0.3803	0.4096	0
22	1.5091	0.3972	0.3972	100
23	1.5736	0.4606	0.4053	0
24	1.6398	0.3984	0.4171	100
25	1.7052	0.4764	0.3990	0
26	1.7735	0.3593	0.3593	0
27	1.8257	0.2622	0.2715	100



# System Identification



**BEST!!**

# Transfer Function

Input : drive speed

Output : actual speed

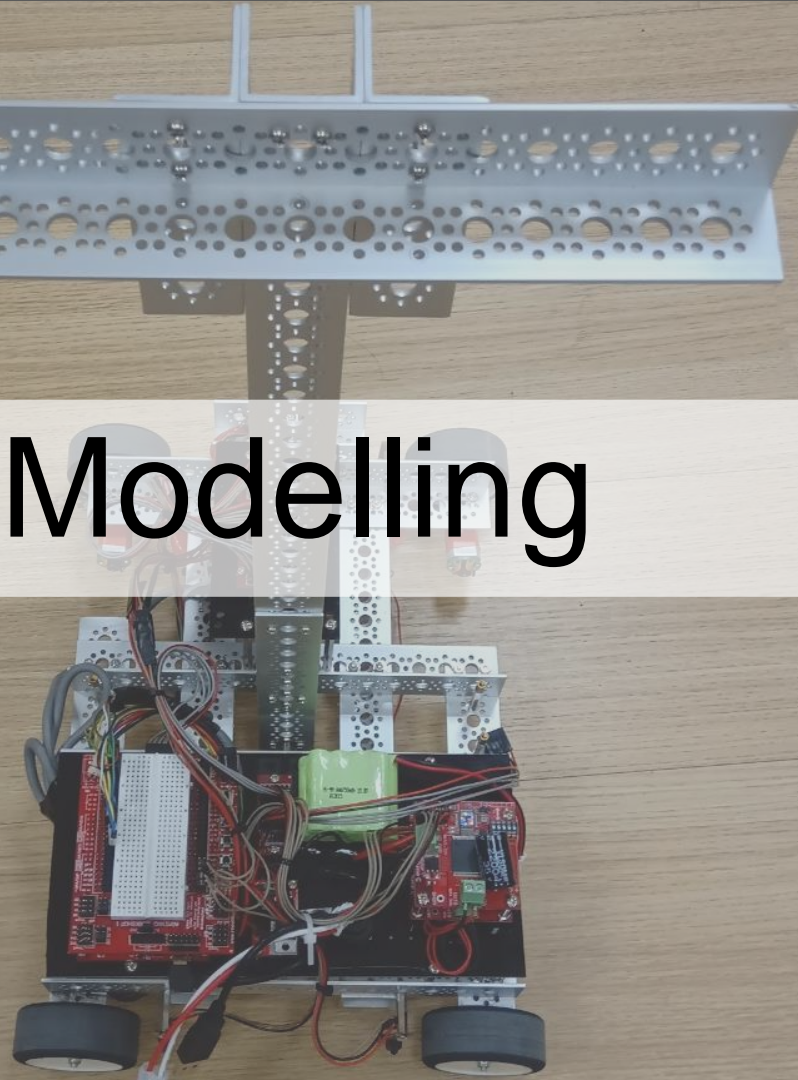
From input to output "y1":

$$\frac{V(s)}{input} = \frac{-7.043 s^3 + 0.5402 s^2 + 236.4 s - 18.13}{3.421 s^3 + 117.4 s^2 + 329 s}$$

$$\frac{A(s)}{input} = \frac{V(s)}{input} \times s$$

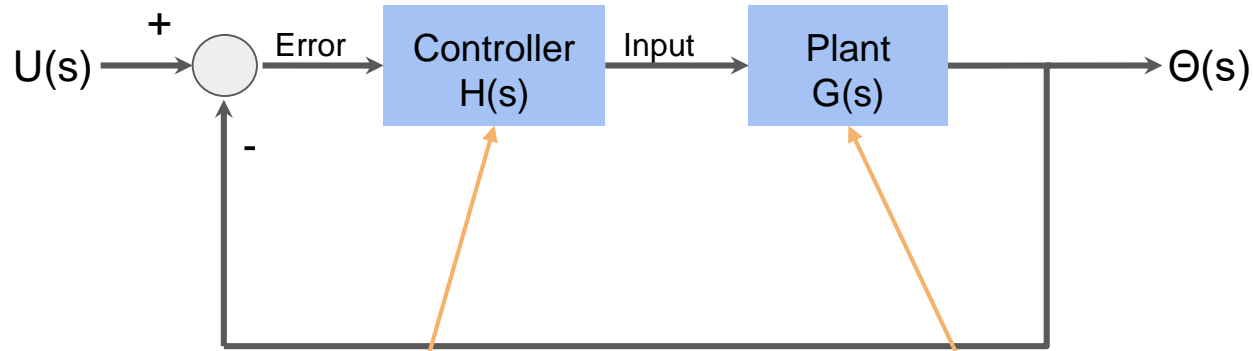


# Modelling





# Overall Model



$\Theta(s)$  = Angle of the Pendulum  
 $A(s)$  = Acceleration of the Vehicle  
Input(s) = Input speed

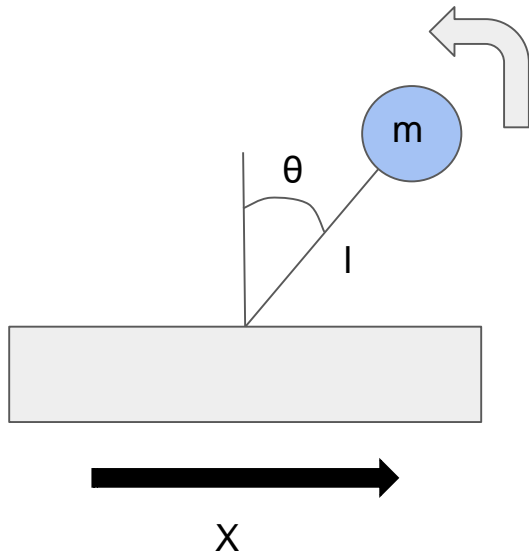
PID Controller

$$G(s) = \frac{\Theta(s)}{\text{Input}(s)} = \frac{\Theta(s)}{A(s)} \frac{A(s)}{\text{Input}(s)}$$

Derived physics model

From System Identification tool

# Physics Model<sub>[2]</sub>



From Newton's Law,

$$\ddot{\theta} = \frac{g \cdot \sin(\theta)}{l} - \frac{\ddot{x} \cdot \cos(\theta)}{l}$$

$\theta \approx 0$

**→**  $-\ddot{x} = l\ddot{\theta} - g\theta$

Laplace

**→**  $\frac{\Theta(s)}{X(s)} = \frac{-s^2}{ls^2 - g}$

or  $\frac{\Theta(s)}{A(s)} = \frac{-1}{ls^2 - g}$

# Physics Model

According to the measurement :

$$l = 0.2923 \text{ m}$$

$$g = 9.81 \text{ m/s}^2$$

$$m = 0.289 \text{ kg}$$

$$\longrightarrow \frac{\Theta(s)}{A(s)} = \frac{-3.421}{s^2 - 33.56}$$

One of the poles  $> 0$   
→ Unstable system

# System Model

Transfer function(speed command to acceleration)

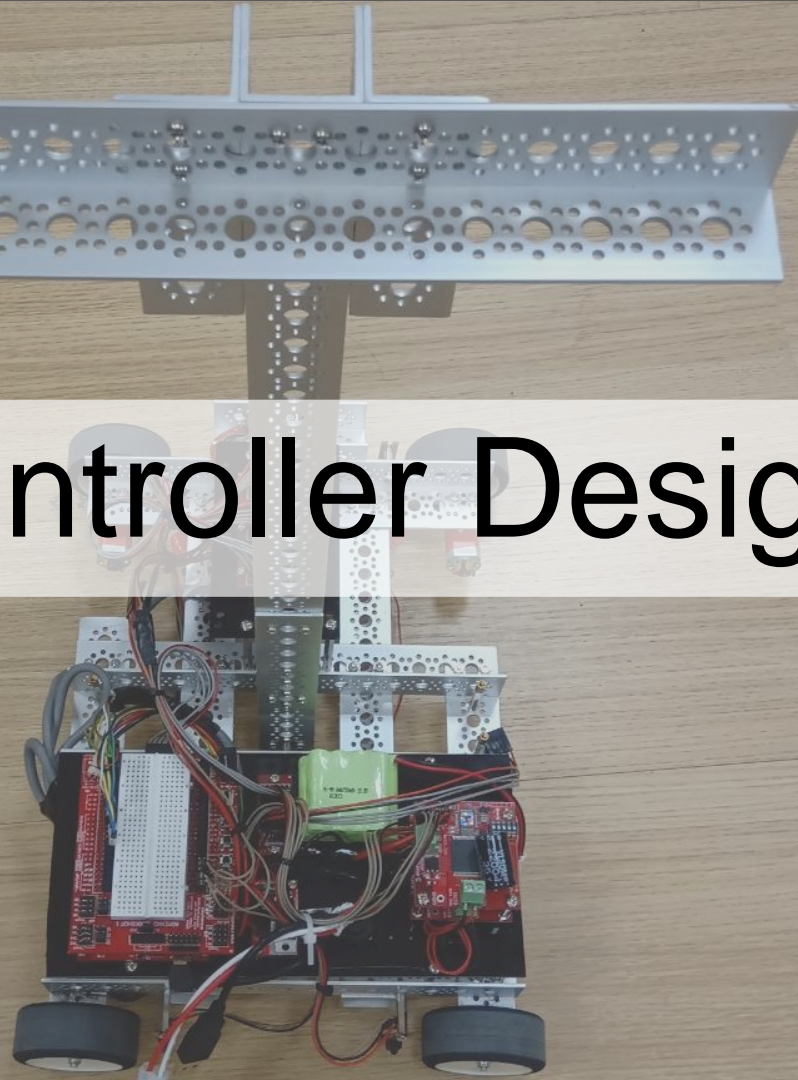
$$\frac{-7.043s^3 + 0.5402s^2 + 236.4s - 18.13}{3.421s^2 + 117.4s + 329}$$

Transfer function(acceleration to pendulum degree)

$$\frac{-3.421}{s^2 - 33.56}$$

Overall transfer function = 
$$\frac{7.043s - 0.5402}{s^2 + 34.32s + 96.16}$$

# P Controller Design

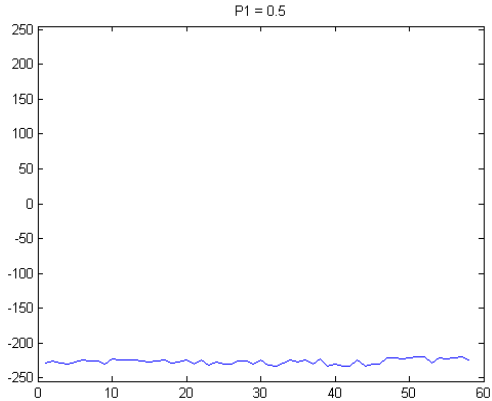
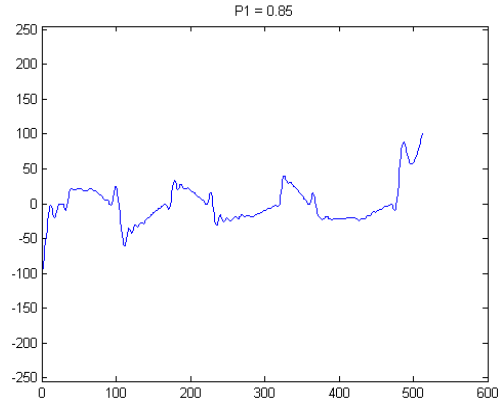
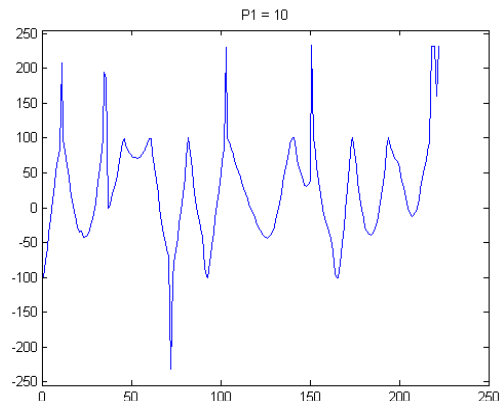


# P Controller ( $\theta > 9^\circ$ )

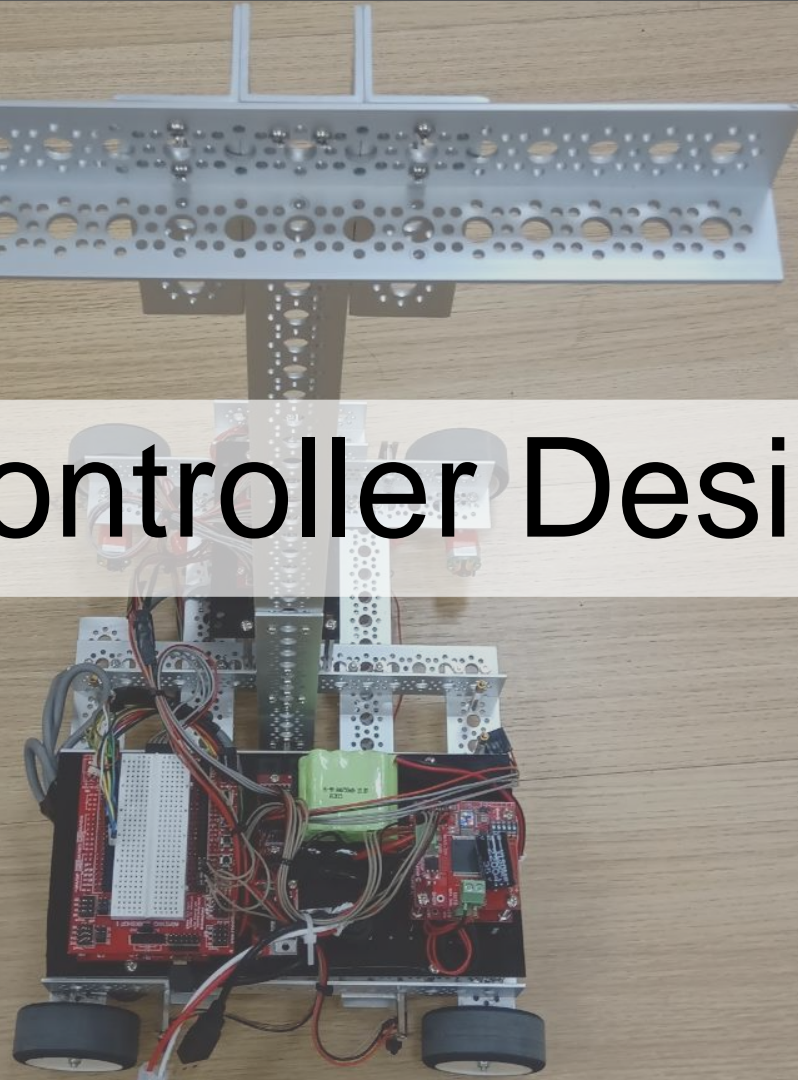
- 最外層的P Controller之目的便是讓倒單擺擺起來，使其到均衡點，進入線性區域。
- P Controller 所使用的區域不適用於線性模型，因此我們也做出了諸多特別的設計來彌補。
- 我們設計讓P Controller在作用之前，先往反方向運動，以增加其速度變化量，如此較容易達成將倒單擺擺起來的目的。



# What happened if the value of $K_p$ is too small or too large?

small $K_p$	moderate $K_p$	large $K_p$
 <p>P1 = 0.5</p>	 <p>P1 = 0.85</p>	 <p>P1 = 10</p>
<p>倒單擺無法進入線性區域，或是進入線性區之速度過小，很快便又跌出來</p>	<p>倒單擺成功地以適當腳速度進入線性區，達成平衡</p>	<p>倒單擺進入線性區之速度過大，很快地衝入另一邊的非線性區，無法達成平衡</p>

# PD Controller Design



# PD controller design ( $5^{\circ} < \theta < 9^{\circ}$ )

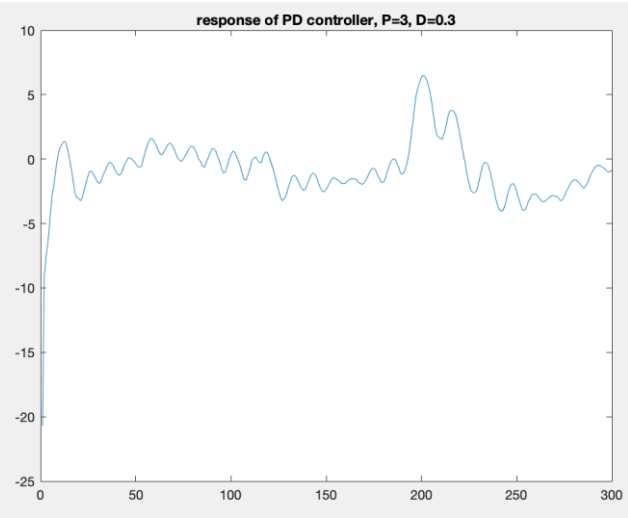
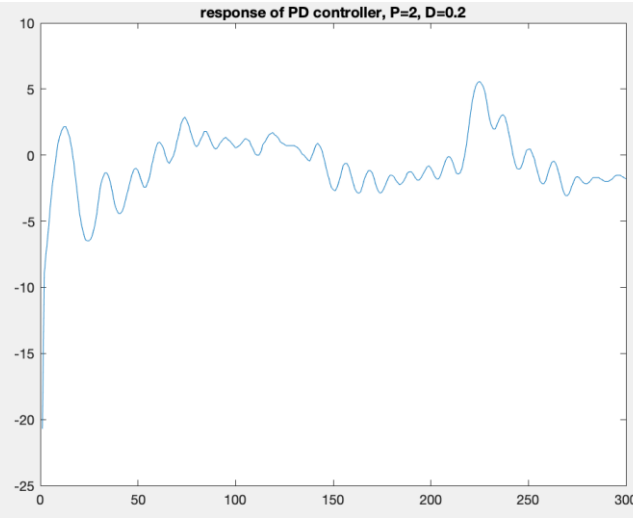
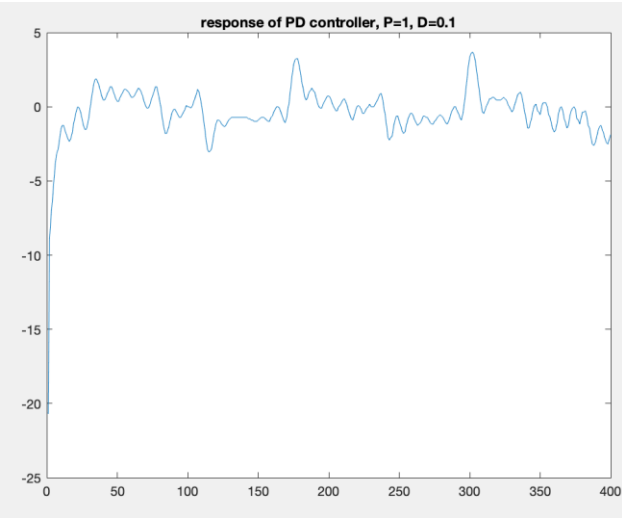


filename	KP	KD	Overshoot(deg)
PD_1_0.1.csv	1	0.1	3.69
PD_1_0.2.csv	1	0.2	4.59
PD_1_0.05.csv	1	0.05	6.84
PD_1_0.15.csv	1	0.15	4.5
PD_2_0.2.csv	2	0.2	5.58
PD_2_0.3.csv	2	0.3	5.56
PD_2_0.4.csv	2	0.4	6.03
PD_3_0.3.csv	3	0.3	6.48
PD_3_0.5.csv	3	0.5	6.75
PD_3_0.7.csv	3	0.7	6.66



# PD controller design

Rising time:  $3 > 2 > 1$   
Overshoot:  $1 < 2 < 3$



$$1 + 0.1s$$

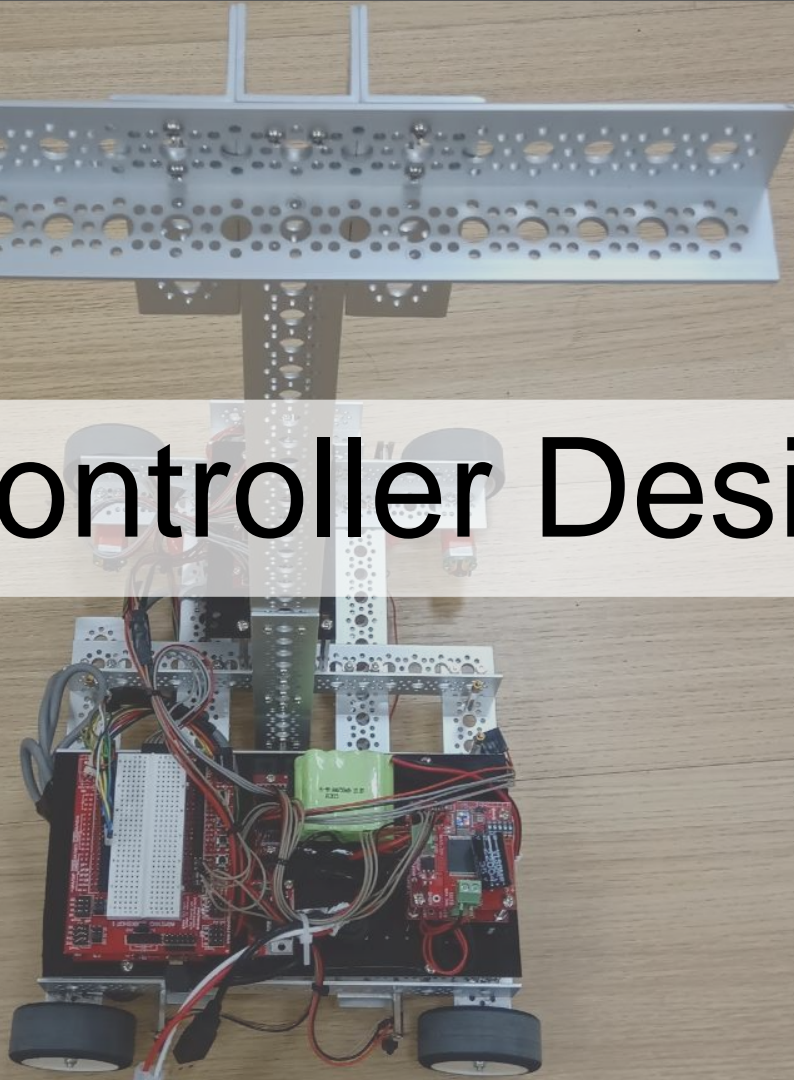


$$2 + 0.2s$$



$$3 + 0.3s$$



# PID Controller Design

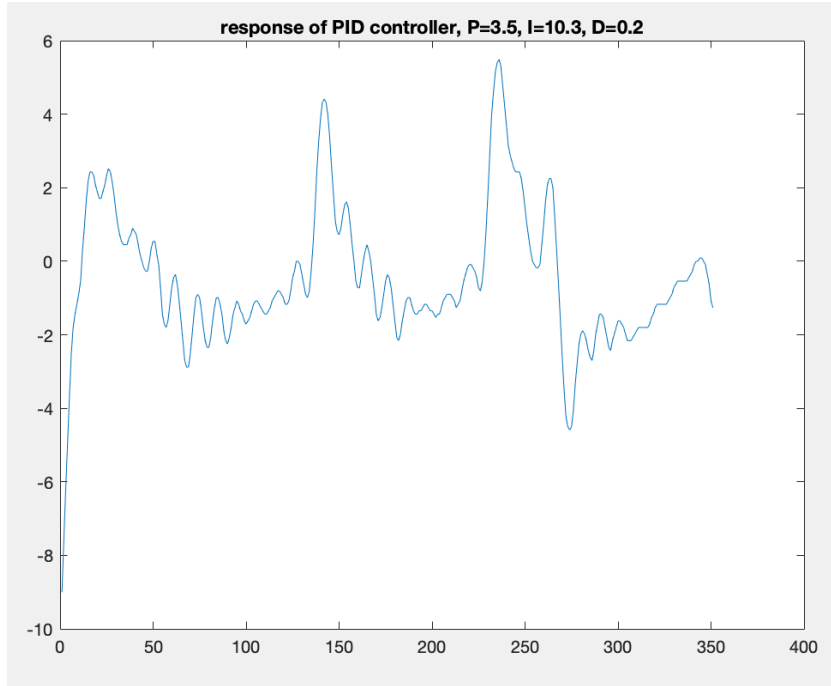


# PID controller design ( $\theta < 5^\circ$ )

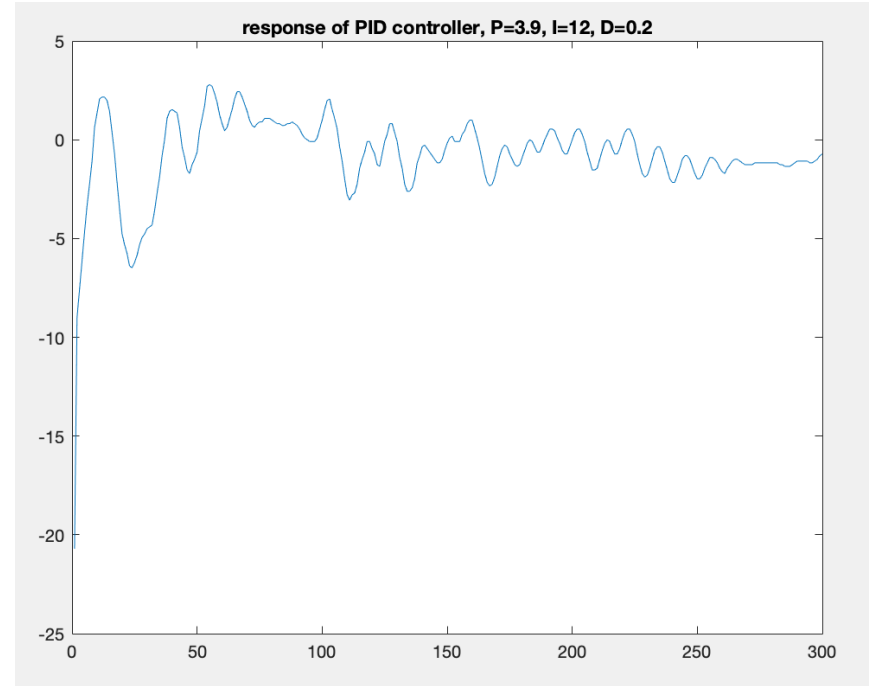


filename	KP	KI	KD	Overshoot(deg )
PID_3.5_9_0.2.csv	3.5	9	0.2	5.49
PID_3.5_10.3_0.2.csv	3.5	10.3	0.2	5.94
PID_3.9_10.3_0.2.csv	3.9	10.3	0.2	6.84
PID_3.9_10.3_0.4.csv	3.9	10.3	0.4	6.48
PID_3.9_10.3_0.05.csv	3.9	10.3	0.05	9.18
PID_3.9_12_0.2.csv	3.9	12	0.2	2.79
PID_4.3_10.3_0.2.csv	4.3	10.3	0.2	3.06

# PID controller design ( $\theta < 5^\circ$ )

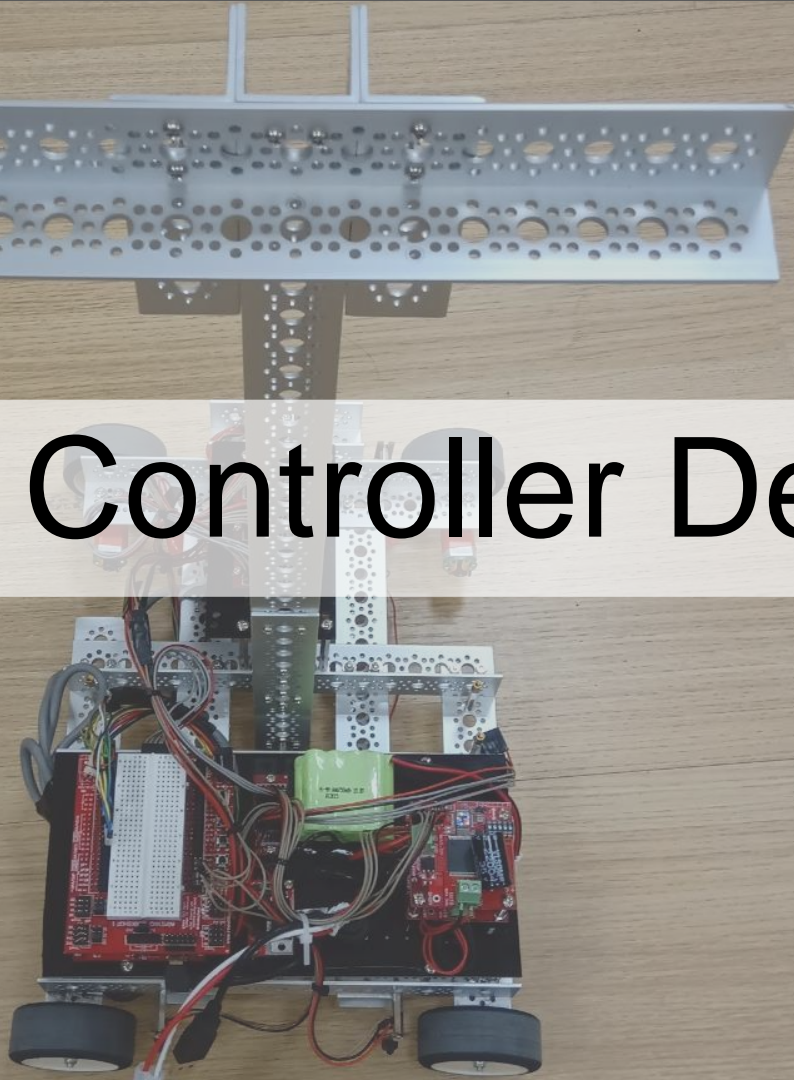


$$3.5 + \frac{10.3}{s} + 0.2s$$



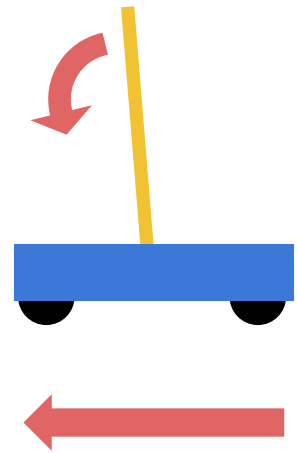
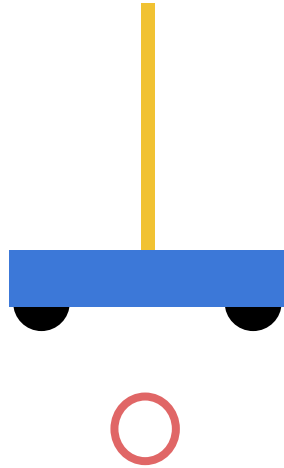
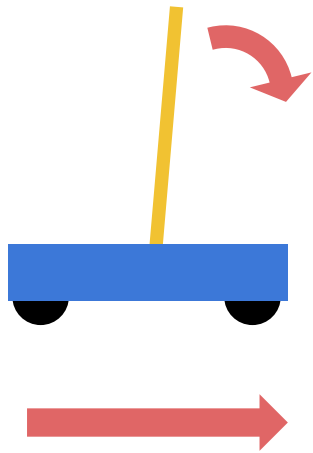
$$3.9 + \frac{12}{s} + 0.2s$$

# Position Controller Design





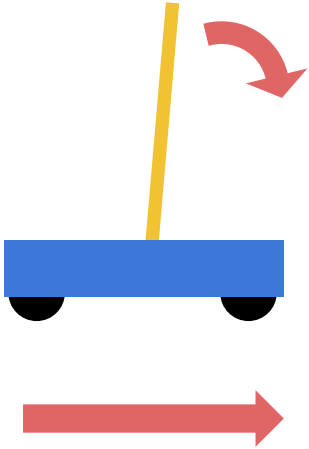
# Inspiration



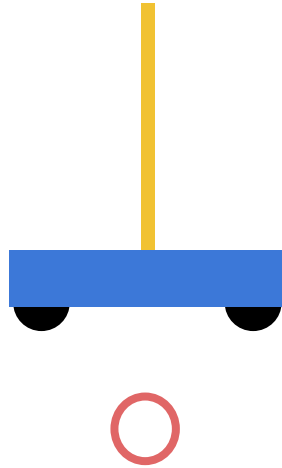
# Inspiration

Using P control to achieve better performance

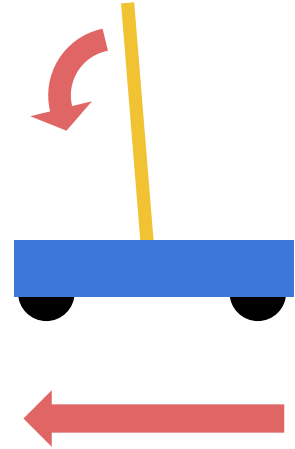
Setpoint  $> 0$



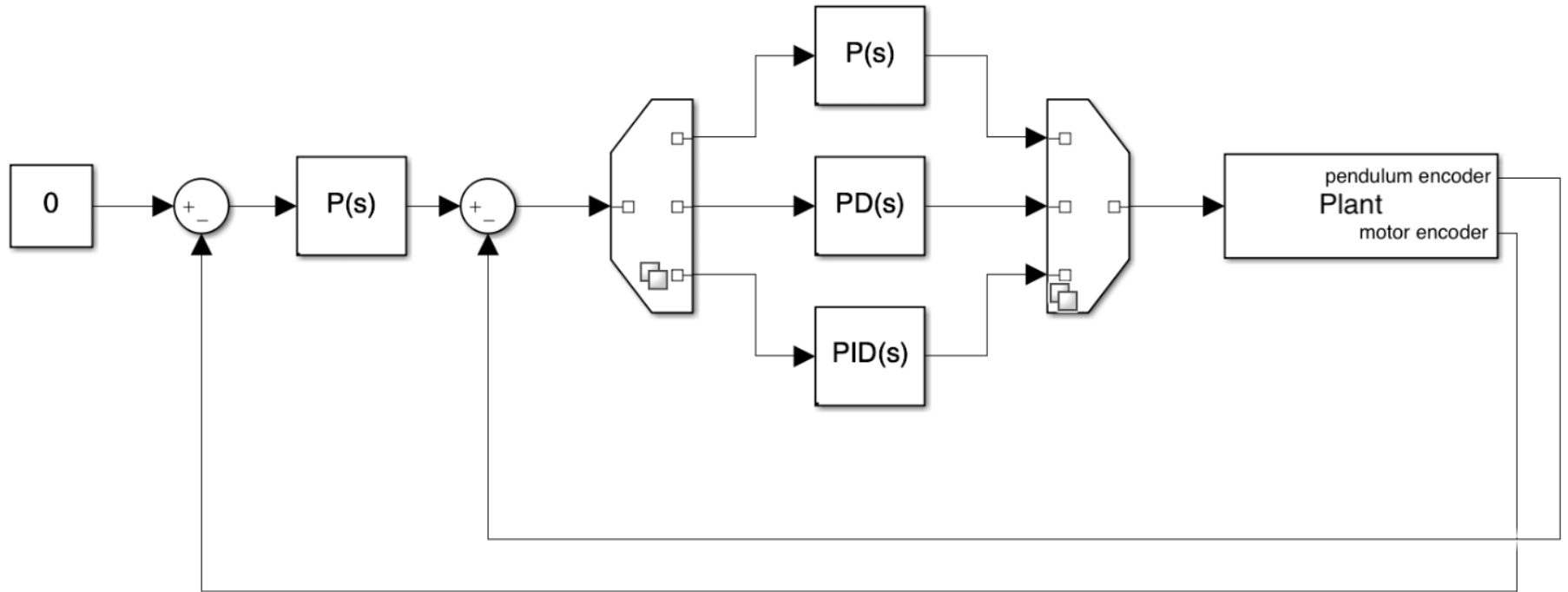
Setpoint  $= 0$



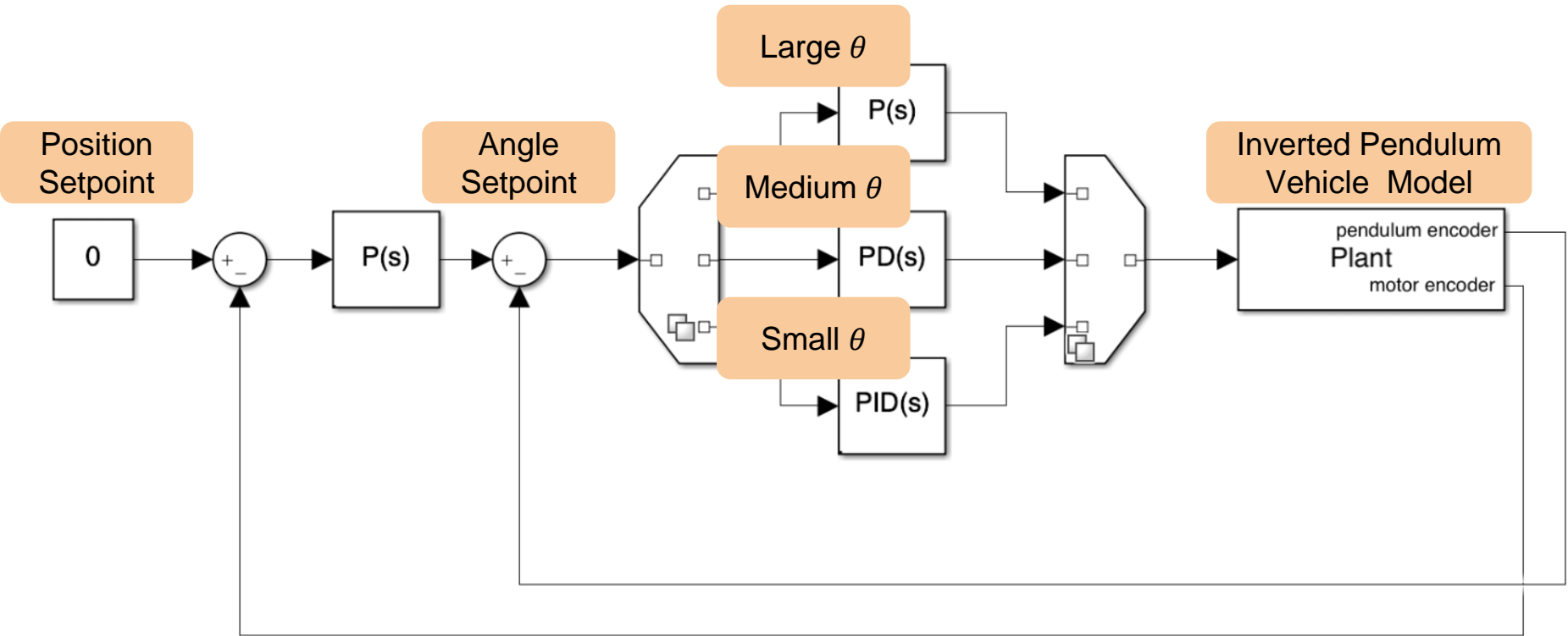
Setpoint  $< 0$



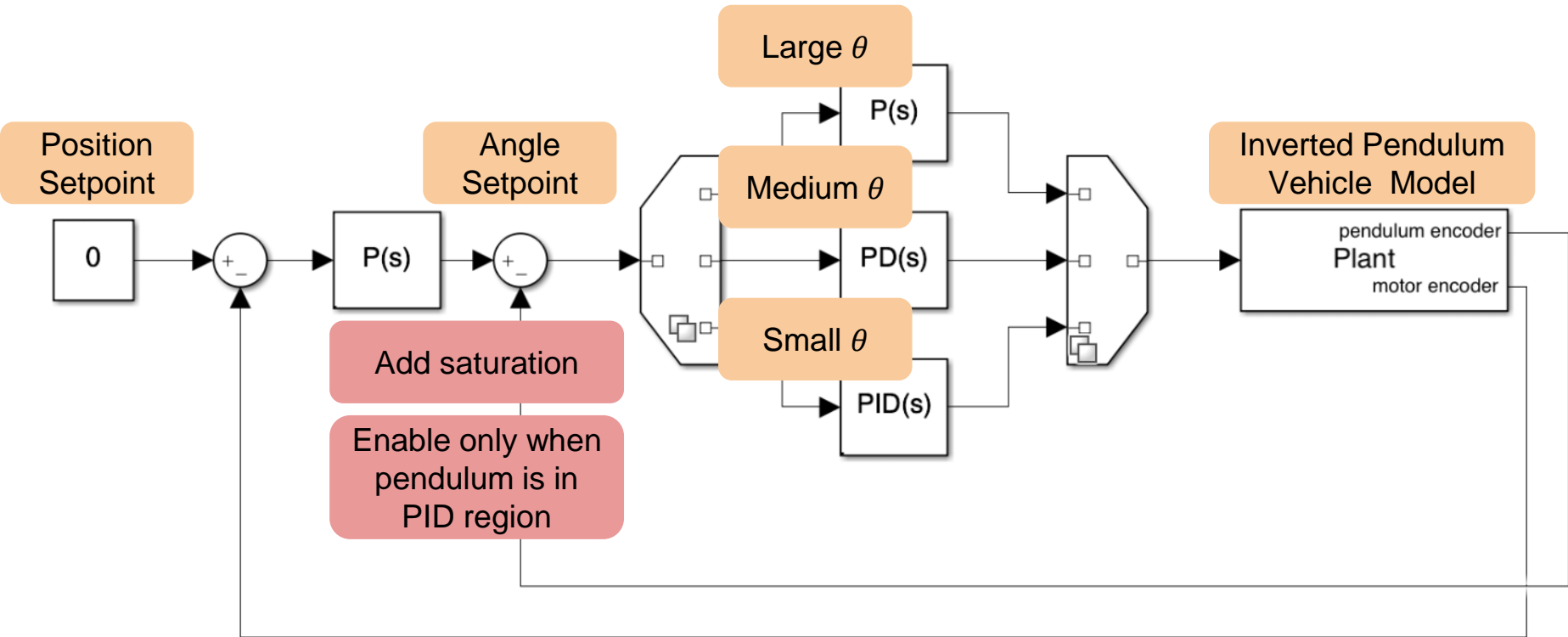
# Position controller design



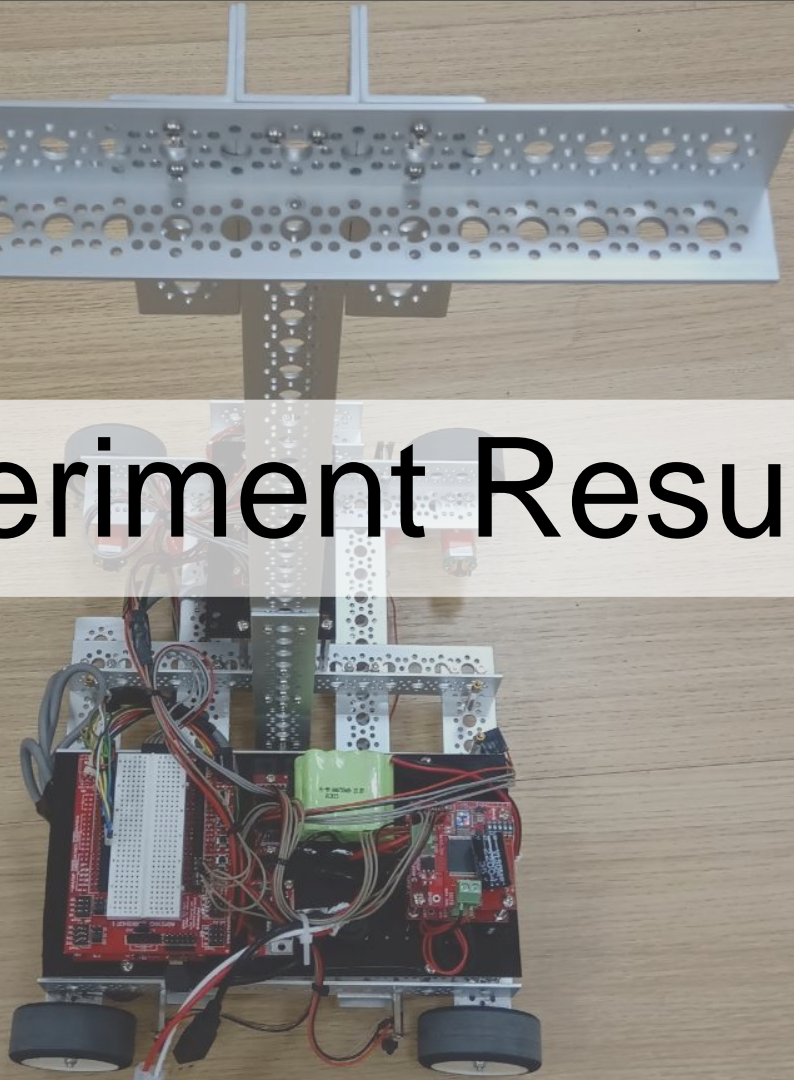
# Position controller design—detail



# Position controller design—further detail

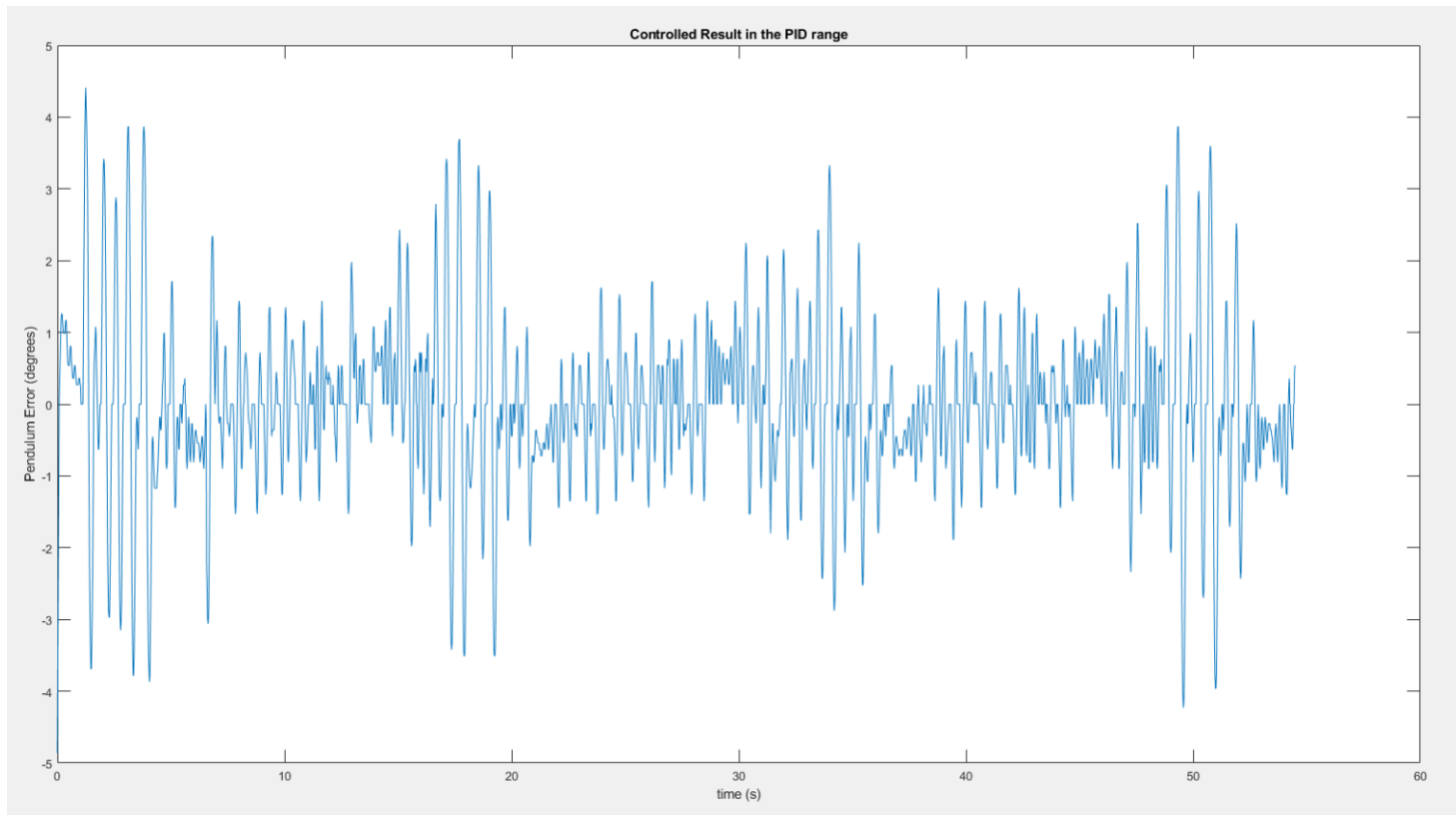


# Experiment Result

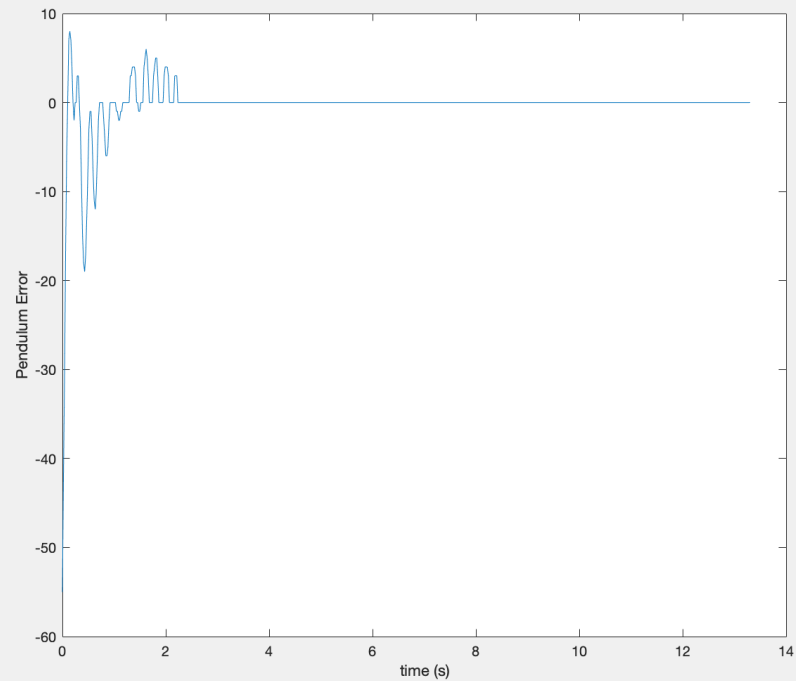
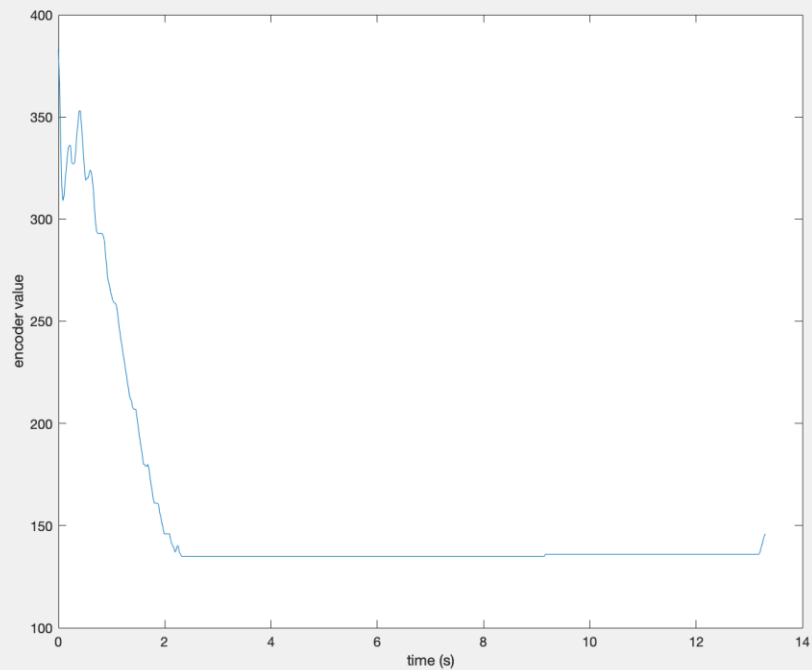


# Pendulum degree

Max	Error : 4.41 degrees
Min	Error : -4.86 degrees
Ave	Error : -0.00074 degree
Std	: 1.1228

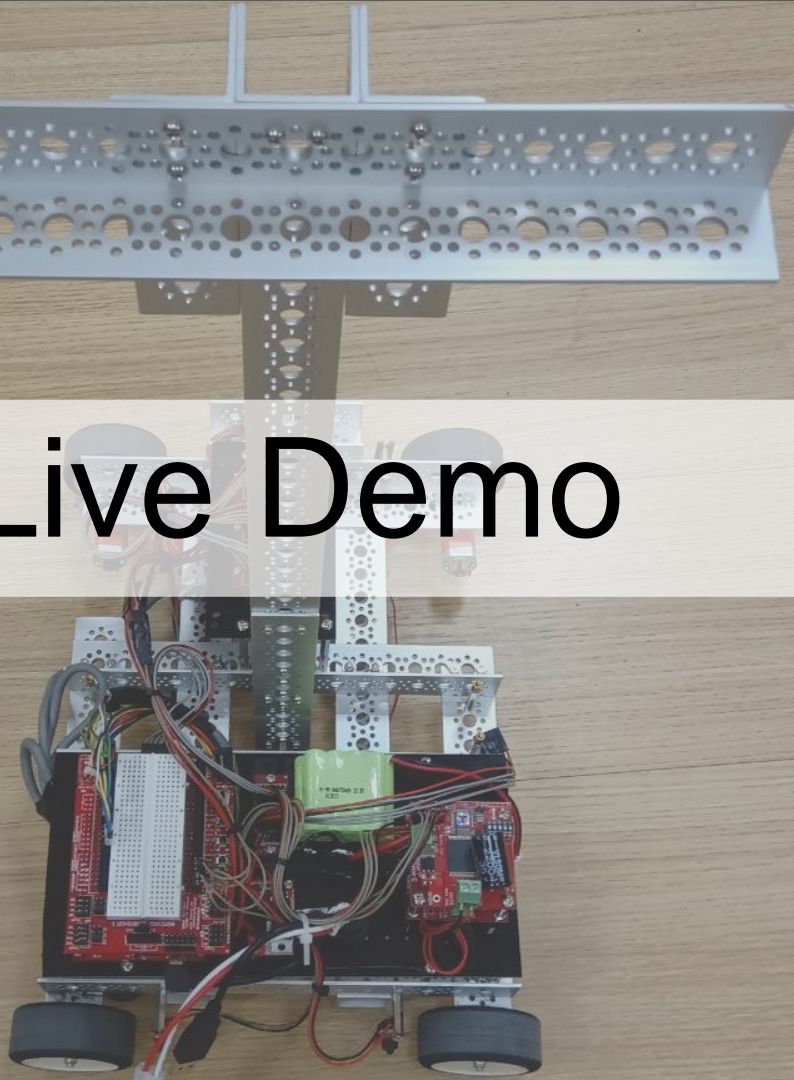


# Vehicle Position



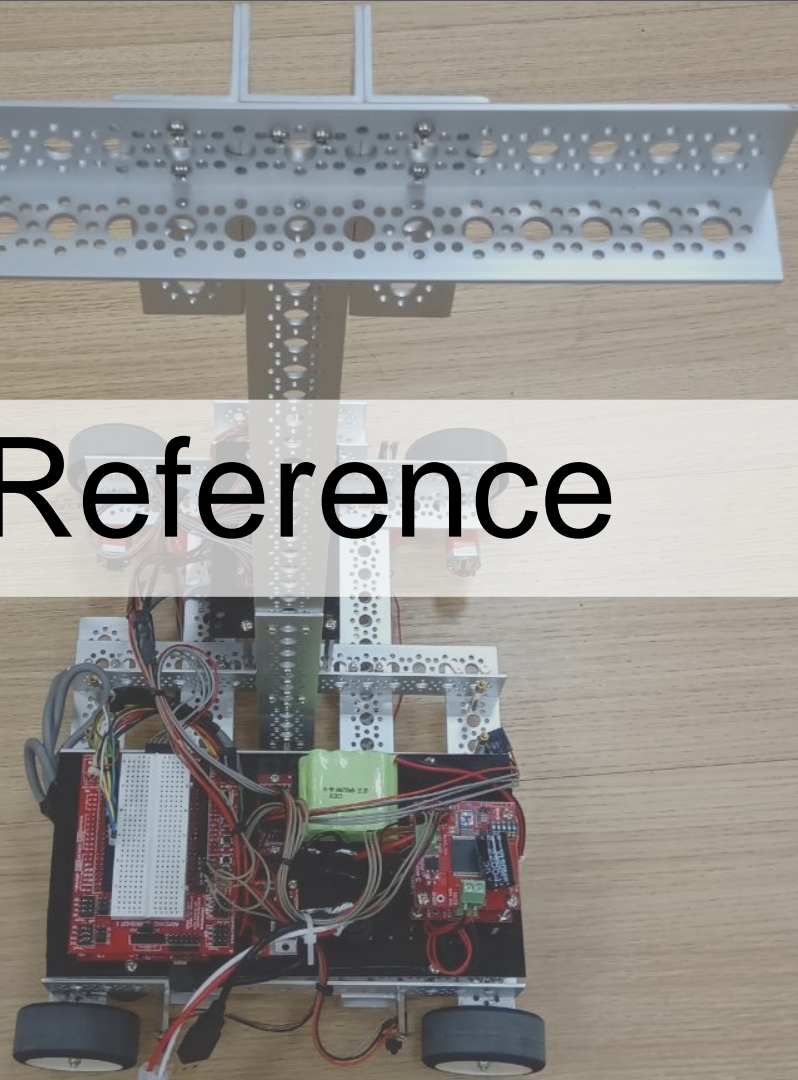


# Live Demo



<https://youtu.be/Kn8Ok2zyNI4>

# Reference



# Reference

[1] 自動控制Final Project實驗講義

[2] Kent Lundberg, “The Inverted Pendulum System” ,  
[http://web.mit.edu/klund/www/papers/UNP\\_pendulum.pdf](http://web.mit.edu/klund/www/papers/UNP_pendulum.pdf)