15

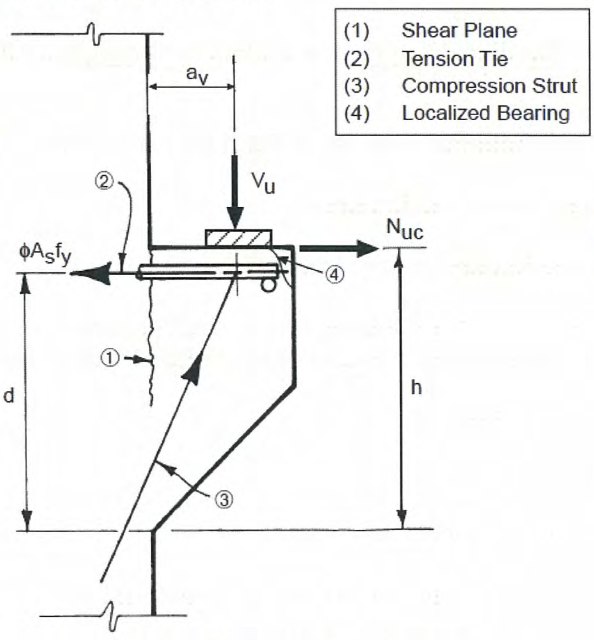
Brackets, Corbels and Beam Ledges

GENERAL CONSIDERATIONS

Provisions for the design of brackets and corbels were introduced in ACI 318-71. These provisions were derived based on extensive test results. The 1977 edition of the code permitted design of brackets and corbels based on shear friction, but maintained the original design equations. The provisions were completely revised in ACI 318-83, eliminating the empirical equations of the 1971 and 1977 codes, and simplifying design by using the shear-friction method exclusively for nominal shear-transfer strength V_n. From 1971 through 1999 code, the provisions were strictly limited to shear span-to-depth ratio a_v/d less than or equal to 1.0. Since 2002, the code allows the use of the provisions of Appendix A, Strut-and-tie models, to design brackets and corbels with a_v/d ratios less than 2.0, while the provisions of 11.9 continue to apply only for a_v/d ratios less than or equal to 1.0.

11.9 LIMITATIONS OF BRACKET AND CORBEL PROVISIONS

The design procedure for brackets and corbels recognizes the deep beam or simple truss action of these short-shear-span members, as illustrated in Fig. 15-1. Four potential failure modes shown in Fig. 15-1 shall be prevented: (1) Direct shear failure at the interface between bracket or corbel and supporting member; (2) Yielding of the tension tie due to moment and direct tension; (3) Crushing of the internal compression "strut;" and (4) Localized bearing or shear failure under the loaded area.



$N_{uc} \geq 0.2 V_u$

Figure 15-1 Structural Action of Corbel

$a_v/d < 2$ For brackets and corbels with a shear span-to-depth ratio a_v/d less than 2, the provision of Appendix A may be used for design. The provisions of 11.9.3 and 11.9.4 are permitted with $a_v/d \le 1$ and the horizontal force $N_{uc} \le V_n$.

Regardless which design method is used, the provisions of 11.9.2, 11.9.3.2.1, 11.9.3.2.2, 11.9.5, 11.9.6, and 11.9.7 must be satisfied.

$a_v/d > 2$ When a_v/d is greater than 2.0, brackets and corbels shall be designed as cantilevers subjected to the applicable provisions of flexure and shear.

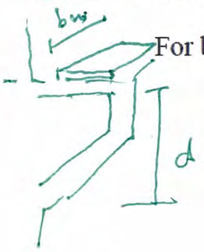
11.9.1 - 11.9.5 Design Provisions

The critical section for design of brackets and corbels is taken at the face of the support. This section should be designed to resist simultaneously a shear V_u, a moment $M_u = V_u a_v + N_{uc}(h - d)$, and a horizontal tensile force N_{uc} (11.9.3). The value of N_{uc} must be not less than $0.2V_u$, unless special provisions are made to avoid tensile forces (11.9.3.4). This minimum value of N_{uc} is established to account for the uncertain behavior of a slip joint and/or flexible bearings. Also, the tension force N_{uc} typically is due to indeterminate causes such as restrained shrinkage or temperature stresses. In any case it shall be treated as a live load with load factor of 1.6 (11.9.3.4). Since corbel and bracket design is predominantly controlled by shear, 11.9.3.1 specifies that the strength reduction factor ϕ shall be taken equal to 0.75 for all design conditions.

$N_{uc} \ge 0.2V_u$

$V_n < 0.2 f'_c b_w d$ and $800 b_w d$

For normal weight concrete, shear strength V_n is limited to the smaller of $0.2 f'_c b_w d$ and $800 b_w d$ (11.9.3.2). For lightweight concrete, V_n is limited by the provisions of 11.9.3.2.2, which are somewhat more restrictive than those for normal weight concrete. Tests show that for lightweight concrete, V_n is a function of f'_c and a_v/d.



For brackets and corbels, the required reinforcement is:

A_{vf} = area of shear-friction reinforcement to resist direct shear V_u, computed in accordance with 11.7 (11.9.3.2).

A_f = area of flexural reinforcement to resist moment $M_u = V_u a_v + N_{uc}(h - d)$, computed in accordance with 10.2 and 10.3 (11.9.3.3).

A_n = area of tensile reinforcement to resist direct tensile force N_{uc}, computed in accordance with 11.9.3.4.

Actual reinforcement is to be provided as shown in Fig. 15-2 and includes:

A_{sc} = primary tension reinforcement

A_h = shear reinforcement (closed stirrups or ties)

$A_{sc}+A_h$ が満たす条件

This reinforcement is provided such that total amount of reinforcement $A_{sc} + A_h$ crossing the face of support is the greater of (a) $A_{vf} + A_n$, and (b) $3A_f/2 + A_n$ to satisfy criteria based on test results.[15.1]

If case (a) controls (i.e., $A_{vf} > 3A_f/2$):

$$A_{sc} = A_{vf} + A_n - A_h$$

$$= A_{vf} + A_n - 0.5\,(A_{sc} - A_n) \qquad \textit{11.9.4}$$

or $\quad A_{sc} = 2A_{vf}/3 + A_n$ (primary tension reinforcement)

and $\quad A_h = (0.5)(A_{sc} - A_n) = A_{vf}/3$ (closed stirrups or ties) $\qquad$ *11.9.4*

If case (b) controls (i.e., $3A_f/2 > A_{vf}$):

$$A_{sc} = 3A_f/2 + A_n - A_h$$

$$= 3A_f/2 + A_n - 0.5\,(A_{sc} - A_n)$$

or $\quad A_{sc} = A_f + A_n$ (primary tension reinforcement)

and $\quad A_h = (0.5)\,(A_{sc} - A_n) = A_f/2$ (closed stirrups or ties)

In both cases (a) and (b), $A_h = (0.5)\,(A_{sc} - A_n)$ determines the amount of shear reinforcement to be provided as closed stirrups parallel to A_{sc} and uniformly distributed within (2/3)d adjacent to A_{sc} per 11.9.4.

A minimum ratio of primary tension reinforcement $\rho_{min} = 0.04\, f_c'/f_y$ is required to ensure ductile behavior after cracking under moment and direct tensile force (11.9.5).

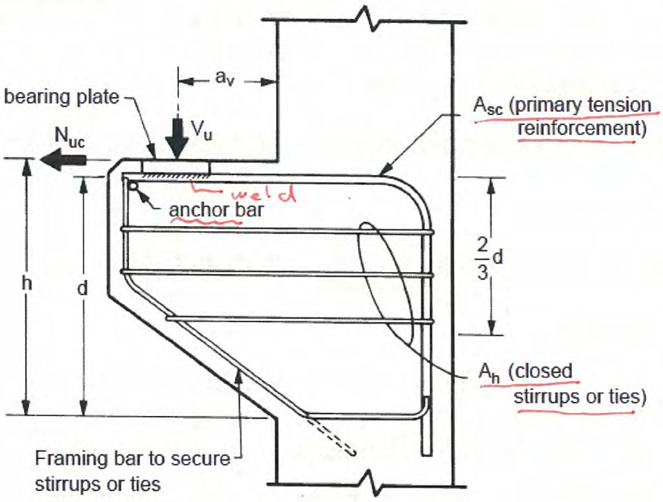


Figure 15-2 Corbel Reinforcement

BEAM LEDGES

Beam with ledges shall be designed for the overall member effects of flexure, shear, axial forces, and torsion, as well as for local effects in the vicinity of the ledge (Refs. 15.2-15.6). The design of beam ledges is not specifically addressed by the code. This section addresses only local failure modes and reinforcement requirements to prevent such failure.

Design of beam ledges is somewhat similar to that of a bracket or corbel with respect to loading conditions. Additional design considerations and reinforcement details need to be considered in beam ledges. Accordingly, even though not specifically addressed by the code, special design of beam ledges is included in this Part. Some failure modes discussed above for brackets and corbels are also shown for beam ledges in Fig. 15-3. However, with beam ledges, two additional failure modes shall be considered (see Fig. 15-3): (5) separation between ledge and beam web near the top of the ledge in the vicinity of the ledge load and (6) punching shear. The vertical load applied to the ledge is resisted by a compression strut. In turn, the vertical component of the inclined compression strut must be picked up by the web stirrups (stirrup legs A_v adjacent to the side face of the web) acting as "hanger" reinforcement to carry the ledge load to the top of beam. At the reentrant corner of the ledge to web intersection, a diagonal crack would extend to the stirrup and run downward next to the stirrup. Accordingly, a slightly larger shear span, a_f, is used to compute the moment due to V_u. Therefore, the critical section for moment is taken at center of beam stirrups, not at face of beam. Also, for beam ledges, the internal

$d < 0.8h$ moment arm should not be taken greater than 0.8h for flexural strength.

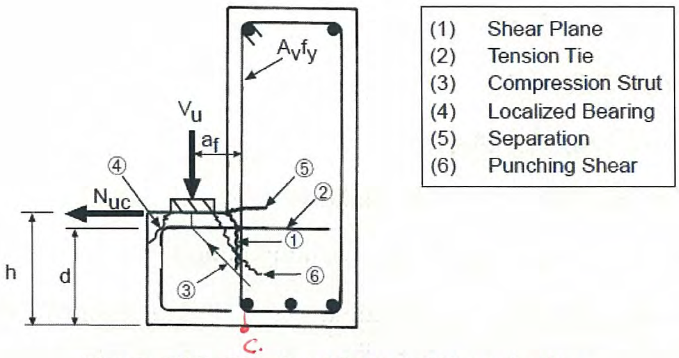


Figure 15-3 Structural Action of Beam Ledge

The design procedure described in this section is based on investigations performed by Mirza and Furlong (Refs. 15.3 to 15.5). The key information needed by the designer is establishing the effective width of ledge for each of the potential failure modes. These effective widths were determined by Mirza and Furlong through analytical investigations, with results verified by large scale testing. Design of beam ledges can also be performed by the strut-and-tie procedure (refer to Part 17 for discussion).

Design to prevent local failure modes requires consideration of the following actions:

1. Shear V_u
2. Horizontal tensile force N_{uc} greater or equal to $0.2V_u$, but not greater than V_u. $V_u > N_{uc} \geq 0.2V_u$
   N_{uc}は最低でも$0.2V_u$を仮定
3. Moment $M_u = V_u a_f + N_{uc}$ (h-d)

Reinforcement for the different failure modes is determined based on the effective widths or critical sections discussed below. In all cases, the required strengths (V_u, M_u, or N_u) should be less than or equal to the design strengths (ϕV_n, ϕM_n, or ϕN_n). The strength reduction factor ϕ is taken equal to 0.75 for all actions, as for brackets and corbels. The strength requirements for different failure modes are shown below for normal weight concrete. When lightweight aggregate concrete is used, modifications should be made per 11.2.

許容低減
$\phi = 0.75$

a. Shear Friction

Parameters affecting determination of the shear friction reinforcement are illustrated in Figure 15-4.

$$V_u \leq 0.2\phi f_c' (W + 4a_v)d \tag{1}$$

$$\leq \phi \mu A_{vf} f_y$$

where

d = effective depth of ledge from centroid of top layer of ledge transverse reinforcement to the bottom of the ledge (see Fig. 15-4)

μ = coefficient of friction per 11.7.4.3. $\mu = 1.4$ Concrete placed monolithically

Note that per 11.7.5, $0.2 f_c' \leq 800$ psi.
56 kg/cm² 5.5 N/mm²

If $(W + 4a_v) > S$, then $V_u \leq 0.2\phi f_c' Sd$, where S is the distance between center of adjacent bearings on the same ledge.
S：隣接しある場合の中心距離

At ledge ends, $V_u \leq 0.2\phi f_c' (2c)d$, where c is the distance from center of end bearing to the end of the ledge. However, 2c should be less than or equal to the smaller of $(W + 4a_v)$ and S.
c：端部からの距離

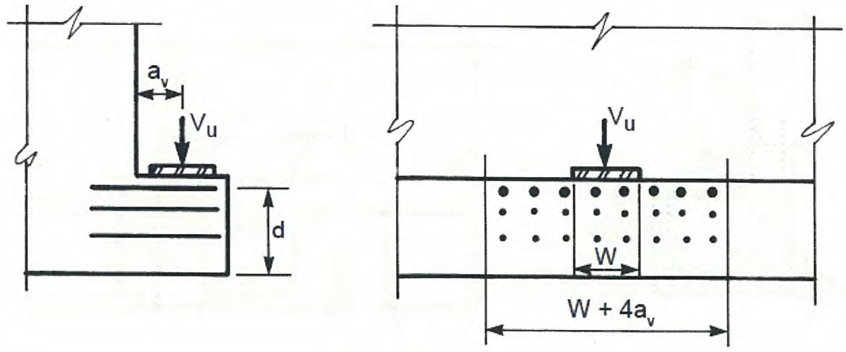


Figure 15-4 Shear Friction

b. Flexure

Conditions for flexure and direct tension are shown in Figure 15-5.

$$V_u a_f + N_{uc} (h\text{-}d) \le \phi A_f f_y \,(jd) \qquad (2)$$

$$N_{uc} \le \phi A_n f_y$$

The primary tension reinforcement A_{sc} should equal the greater of $(A_f + A_n)$ or $(2A_{vf}/3 + A_n)$. If $(W + 5a_f) > S$, reinforcement should be placed over distance S. At ledge ends, reinforcement should be placed over distance (2c), where c is the distance from the center of the end bearing to the end of the ledge, but not more than 1/2 $(W + 5a_f)$. Reference 15.5 recommends taking jd = 0.8d.

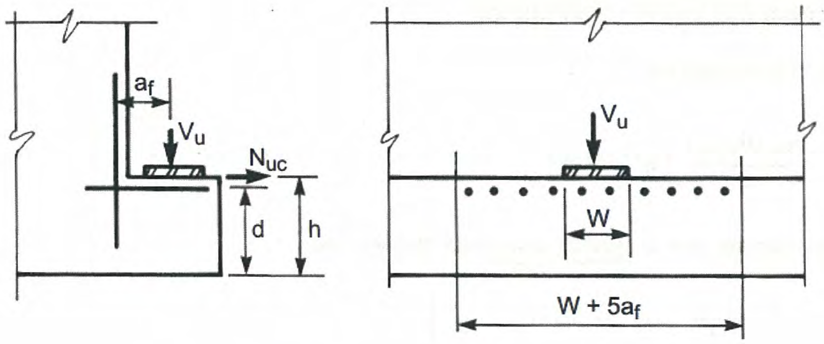


Figure 15-5 Flexure and Direct Tension

c. Punching Shear

Critical perimeter for punching shear design is illustrated in Fig. 15-6.

$$V_u \le 4\phi\sqrt{f'_c}\,(W + 2L + 2d_f)\,d_f \qquad (3)$$

where d_f = effective depth of ledge from top of ledge to center of bottom transverse reinforcement (see Fig. 15-6)

Truncated pyramids from adjacent bearings should not overlap. At ledge ends,

$$V_u \le 4\phi\sqrt{f'_c}\,(W + L + d_f)\,d_f$$

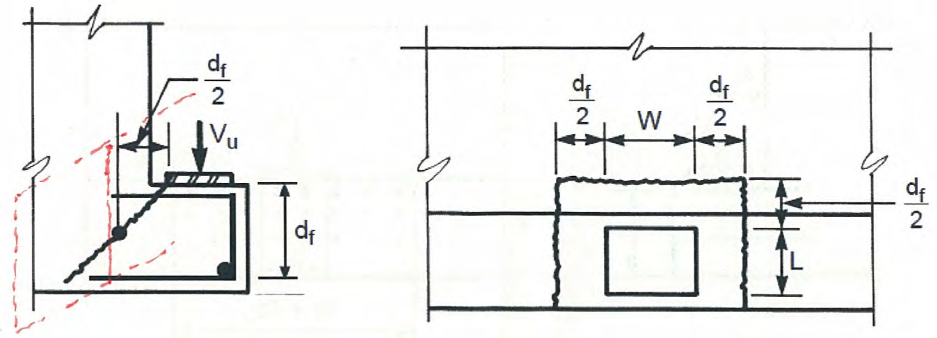


Figure 15-6 Punching Shear

d. Hanger Reinforcement

Hanger reinforcement should be proportioned to satisfy strength. Furthermore, serviceability criteria should be considered when the ledge is subjected to a large number of live load repetitions, as in parking garages and bridges. As shown in Figure 15-7, strength is governed by

$$V_u \leq \phi \frac{A_v f_y}{s} S \tag{4}$$

where A_v = area of one leg of hanger reinforcement

S = distance between ledge loads

s = spacing of hanger reinforcement

Serviceability is governed by

$$V \leq \frac{A_v\,(0.5f_y)}{s}\,(W + 3av) \tag{5}$$

where V is the reaction due to service dead load and live load.

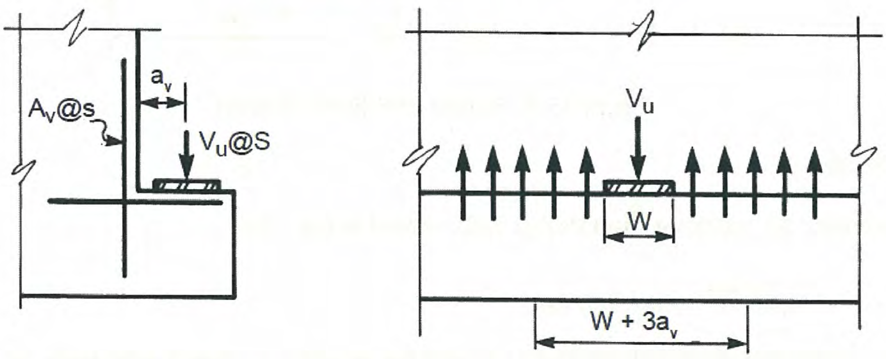


Figure 15-7 Hanger Reinforcement to Prevent Separation of Ledge from Stem

In addition, hanger reinforcement in inverted tees is governed by consideration of the shear failure mode depicted in Figure 15-8:

$$2V_u \le 2\left[2\phi\sqrt{f'_c}b_f d'_f\right] + \phi\frac{A_v f_y}{s}(W + 2d'_f) \qquad (6)$$

where d'_f = flange depth from top of ledge to center of bottom longitudinal reinforcement (see Fig. 15-8)

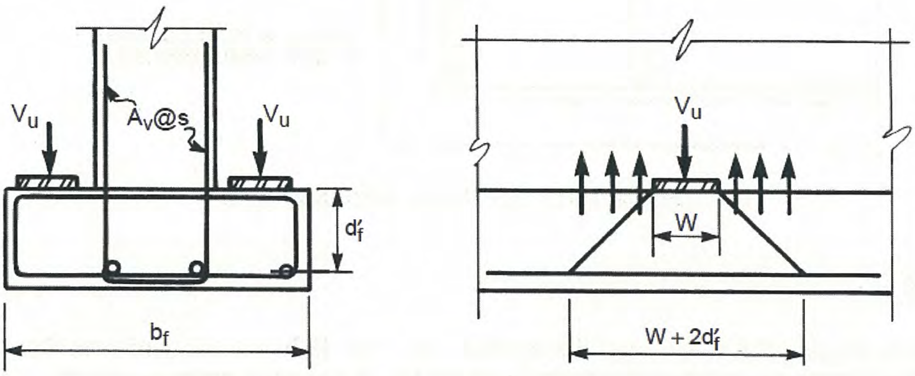


Figure 15-8 Hanger Reinforcement to Prevent Partial Separation of Ledge from Stem and Shear of the Ledge

11.9.6 Development and Anchorage of Reinforcement

All reinforcement must be fully developed on both sides of the critical section. Anchorage within the support is usually accomplished by embedment or hooks. Within the bracket or corbel, the distance between load and support face is usually too short, so that special anchorage must be provided at the outer ends of both primary reinforcement A_{sc} and shear reinforcement A_h. Anchorage of A_{sc} is normally provided by welding an anchor bar of equal size across the ends of A_{sc} (Fig. 15-9(a)) or welding to an armor angle. In the former case, the anchor bar must be located beyond the edge of the loaded area. Where anchorage is provided by a hook or a loop in A_{sc}, the load must not project beyond the straight portion of the hook or loop (Fig. 15-9(b)). In beam ledges, anchorage may be provided by a hook or loop, with the same limitation on the load location (Fig. 15-10). Where a corbel or beam ledge is designed to resist specific horizontal forces, the bearing plate should be welded to A_{sc}.

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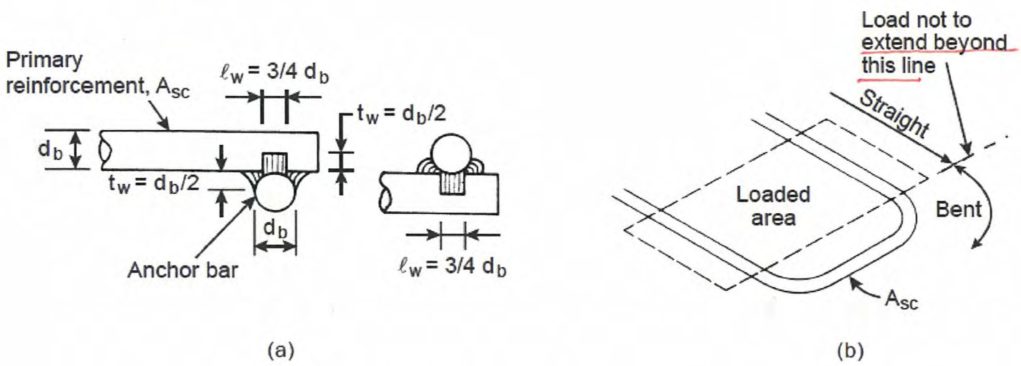


Figure 15-9 Anchorage Details Using (a) Cross-Bar Weld and (b) Loop Bar Detail

The closed stirrups or ties used for A_h must be similarly anchored, usually by engaging a "framing bar" of the same diameter as the closed stirrups or ties (see Fig. 15-2).

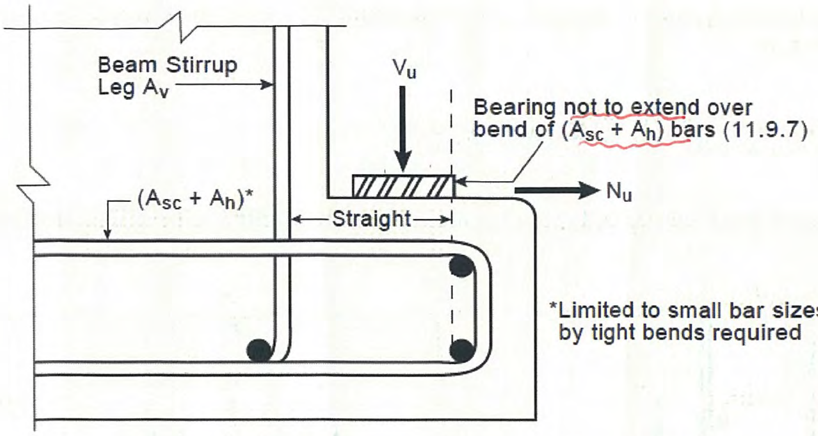


Figure 15-10 Bar Details for Beam Ledge

REFERENCES

15.1 Mattock, Alan H., Chen, K.C., and Soongswang, K., "The Behavior of Reinforced Concrete Corbels," *PCI Journal*, Prestressed Concrete Institute, V. 21, No. 2, Mar.-Apr. 1976, pp. 52-77.

15.2 Klein, G. J., "Design of Spandrel Beams," *PCI Journal*, Vol. 31, No. 5, Sept.-Oct. 1986, pp. 76-124.

15.3 Mirza, Sher Ali, and Furlong, Richard W., "Strength Criteria for Concrete Inverted T-Girder," *Journal of Structural Engineering*, V. 109, No. 8, Aug. 1983, pp. 1836-1853.

15.4 Mirza, Sher Ali, and Furlong, Richard W., "Serviceability Behavior and Failure Mechanisms of Concrete Inverted T-Beam Bridge Bent Caps," *ACI Journal, Proceedings*, V. 80, No. 4, July-Aug. 1983, pp. 294-304.

15.5 Mirza, S. A., and Furlong, R. W., "Design of Reinforced and Prestressed Concrete Inverted T-Beams for Bridge Structures," *PCI Journal*, Vol. 30, No. 4, July-Aug. 1985, pp. 112-136.

15.6 "Design of Concrete Beams for Torsion," Portland Cement Association, Skokie, Illinois, 1999.

Example 15.1—Corbel Design

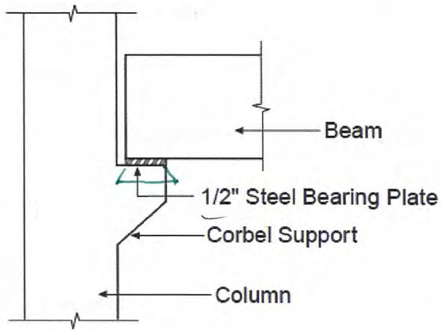
Design a corbel with minimum dimensions to support a beam as shown below. The corbel is to project from a 14-in. square column. Restrained creep and shrinkage create a horizontal force of 20 kips at the welded bearing.

f'_c = 5000 psi (normal weight)

f_y = 60,000 psi

Beam reactions:

DL = 24 kips
LL = 37.5 kips
T = 20 kips



Calculations and Discussion	Code Reference

1. Size bearing plate based on bearing strength on concrete according to 10.17. Width of bearing plate = 14 in.

設計荷重 U = 1.2 (D+F+T) + 1.6 (L+H) + 0.5 (Lr or S or R)

V_u = 1.2 (24) + 1.6 (37.5) = 88.8 kips — *Eq. (9-2)*

D: Dead L
F. weight and Pressure of fluid
H. 〃 of soil, water in soil
T: temperature, creep, shrinkage
L: Live L
Lr: roof live L.
S: snow
R: rain

$$V_u \le \phi P_{nb} = \phi(\underbrace{0.85 f'_c}_{\text{許容應力}} A_1)$$ — *10.17.1* bearing support 0.85f'c

ϕ = 0.65 — *9.3.2.4*

$$88.8 = 0.65\,(0.85 \times 5 \times A_1) = 2.763 A_1$$

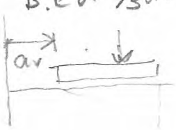
$$A_1 = \frac{88.8}{2.763} = 32.14 \text{ in.}^2$$

$$\text{Bearing length} = \frac{32.14}{14} = 2.30 \text{ in.}$$

Use 2.5 in. × 14 in. bearing plate.

2. Determine shear span 'a_v' with 1 in. max. clearance at beam end. Beam reaction is assumed at third point of bearing plate to simulate rotation of supported girder and triangular distribution of stress under bearing pad.

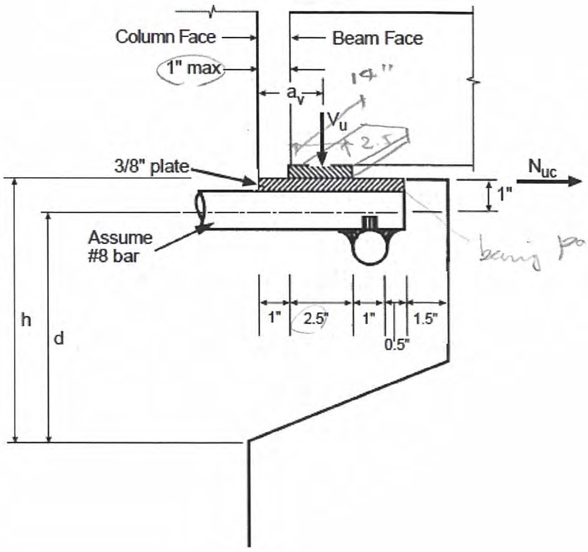
B.R の 1/3の位置



$$a_v = \frac{2}{3}(2.5) + 1.0 = 2.67 \text{ in.}$$

Use a_v = 3 in. maximum.

Detail cross bar just outside outer bearing edge.



Example 15.1 (cont'd)	Calculations and Discussion	Code Reference

3. Determine total depth of corbel based on limiting shear-transfer strength V_n.

V_n is the least of $V_n = 800b_wd$ (governs) — 11.9.3.2.1

or $V_n = 0.2f'_cb_wd = (0.2 \times 5000)b_wd = 1000b_wd$

Thus, $V_u \le \phi V_n = \phi(800b_wd)$

$$\text{Required } d = \frac{88{,}800}{0.75\,(800 \times 14)} = 10.57 \text{ in.}$$

Assuming No. 8 bar, 3/8 in. steel plate, plus tolerance,

$h = 10.57 + 1.0 = 11.57$ in. Use $h = 12$ in.

For design, $d = 12.0 - 1.0 = 11.0$ in.

$$\frac{a_v}{d} = 0.27 < 1 \quad \text{O.K.}$$ — 11.9.1

Also, $N_{uc} = 1.6 \times 20 = 32.0$ kips (treat as live load)

1.6×T

$N_{uc} < V_u = 88.8$ kips O.K.

4. Determine shear-friction reinforcement A_{vf}. — 11.9.3.2

$$A_{vf} = \frac{V_u}{\phi f_y \mu} = \frac{88.8}{0.75\,(60)\,(1.4 \times 1)} = 1.41 \text{ in.}^2$$ — 11.7.4.1

5. Determine direct tension reinforcement A_n.

$$A_n = \frac{N_{uc}}{\phi f_y} = \frac{32.0}{0.75 \times 60} = 0.71 \text{ in.}^2$$ — 11.9.3.1

6. Determine flexural reinforcement A_f. — 11.7.4.3

$M_u = V_ua_v + N_{uc}(h - d) = 88.8\,(3) + 32\,(12 - 11) = 298.4$ in.-kips — 11.9.3.3

Find A_f using conventional flexural design methods or conservatively use $j_ud = 0.9d$.

$$A_f = \frac{298.4}{0.75\,(60)\,(0.9 \times 11)} = 0.67 \text{ in.}^2$$

$\phi\, A_f f_y\, 0.9d = M_u$

Note that for all design calculations, $\phi = 0.75$ — 11.9.3.1

7. Determine primary tension reinforcement A_s. *11.9.3.5*

$\frac{2}{3}A_{vf} = \frac{2}{3}(1.41) = 0.94 \text{ in.}^2 > A_f = 0.67 \text{ in.}^2$; Therefore, $\frac{2}{3}A_{vf}$ controls design   case (a)

$$A_{sc} = \frac{2}{3}A_{vf} + A_n = 0.94 + 0.71 = 1.65 \text{ in.}^2$$

Use 2-No. 9 bars, $A_{sc} = 2.0 \text{ in.}^2$

×3

Check minimum reinforcement: *11.9.5*

$$\rho_{min} = 0.04\left(\frac{f'_c}{f_y}\right) = 0.04\left(\frac{5}{60}\right) = 0.0033$$

$A_{sc(min)} = 0.0033\,(14)\,(11) = 0.51 \text{ in.}^2 < A_{sc} = 2.0 \text{ in.}^2$ O. K.

8. Determine shear reinforcement A_h *11.9.4*

$$A_h = 0.5\,(A_{sc} - A_n) = 0.5\,(2.0 - 0.71) = 0.65 \text{ in.}^2$$

Use 3-No. 3 stirrups, $A_h = 0.66 \text{ in.}^2$

Distribute stirrups in two-thirds of effective corbel depth adjacent to A_{sc}.

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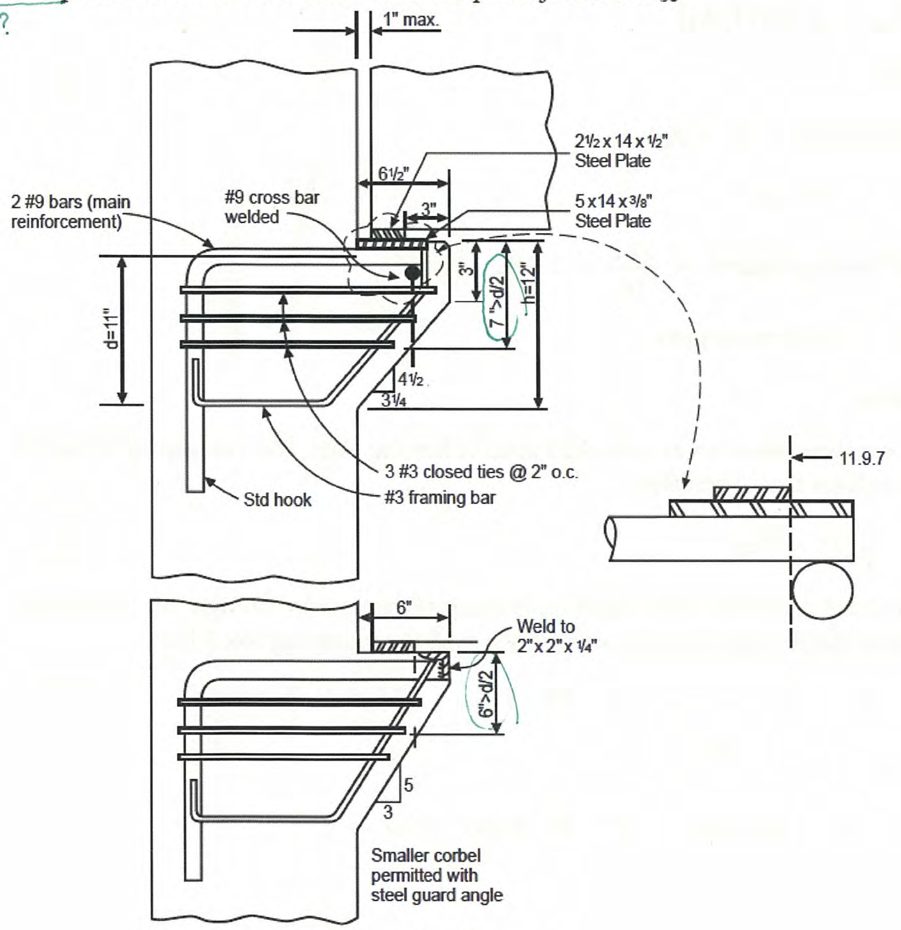


Example 15.2—Corbel Design . . . Using Lightweight Concrete and Modified Shear-Friction Method

Design a corbel to project from a 14-in.-square column to support the following beam reactions:

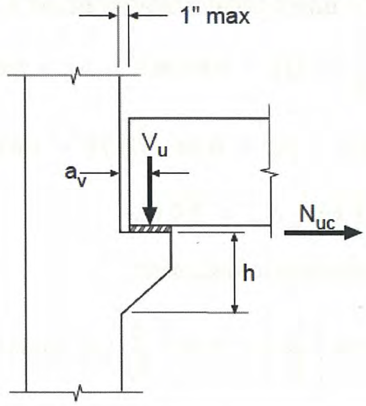
Dead load = 32 kips

Live load = 30 kips

Horizontal force = 24 kips

f'_c = 4000 psi (all lightweight)

f_y = 60,000 psi



Calculations and Discussion	Code Reference

1. Size bearing plate

$V_u = 1.2\,(32) + 1.6\,(30) = 86.4$ kips — *Eq. (9-2)*

$V_u \le \phi P_{nb} = \phi\,(0.85 f'_c A_1)$ — *10.17.1*

$\phi = 0.65$ — *9.3.2.4*

$86.4 = 0.65\,(0.85 \times 4 \times A_1)$

Solving, $A_1 = 39.1$ in.^{2}

Length of bearing required $= \dfrac{39.1}{14} = 2.8$ in.

Use 14 in. × 3 in. bearing plate.

2. Determine a_v.

Assume beam reaction to act at outer third point of bearing plate, and 1 in. gap between back edge of bearing plate and column face. Therefore:

$a_v = 1 + \dfrac{2}{3}(3) = 3$ in.

3. Determine total depth of corbel based on limiting shear-transfer strength V_n. For easier placement of reinforcement and concrete, try h = 15 in. Assuming No. 8 bar:

$d = 15 - 0.5 - 0.375 = 14.13$ in., say 14 in.

$\dfrac{a_v}{d} = \dfrac{3}{14} = 0.21 < 1.0$ — *11.9.1*

$N_{uc} = 1.6 \times 24 = 38.4$ kips $< V_u = 86.4$ kips O.K.

For lightweight concrete and $f'_c = 4000$ psi, V_n is the least of: 11.9.3.2.2

$$V_n = \left(800 - 280\frac{a_v}{d}\right)b_w d = [800 - (280 \times 0.21)]\ 14 \times \frac{14}{1000} = 145.3 \text{ kips}$$

$$V_n = \left(0.2 - 0.07\frac{a_v}{d}\right)f'_c b_w d = [0.2 - 0.07\ (0.21)]\ (4{,}000)\ (14)\ \frac{14}{1000} = 145.3 \text{ kips}$$

$\phi V_n = 0.75\ (145.3) = 109.0$ kips $> V_u = 86.4$ kips O.K.

4. Determine shear-friction reinforcement A_{vf}. 11.9.3.2

Using a Modified Shear-Friction Method as permitted by 11.7.3 (see R11.7.3):

$$V_n = 0.8A_{vf}f_y + K_1 b_w d, \text{ with } \frac{A_{vf}f_y}{b_w d} \text{ not less than 200 psi}$$

For all lightweight concrete, $K_1 = 200$ psi R11.7.3

$V_u \le \phi V_n = \phi\ (0.8A_{vf}f_y + 0.2b_w d)$

Solving for A_{vf}:

$$A_{vf} = \frac{V_u - \phi(0.2b_w d)}{\phi(0.8f_y)}, \text{ but not less than } 0.2 \times \frac{b_w d}{f_y}$$

$$= \frac{86.4 - (0.75 \times 0.2 \times 14 \times 14)}{0.75\ (0.8 \times 60)} = 1.58 \text{ in.}^2 \text{ (governs)}$$

but not less than $0.2 \times \dfrac{b_w d}{f_y} = 0.2 \times \dfrac{14 \times 14}{60} = 0.65 \text{ in.}^2$

For comparison, compute A_{vf} by Eq. (11-25): 11.7.4.3

For lightweight concrete,

$\mu = 1.4\lambda = 1.4\ (0.75) = 1.05$

$$A_{vf} = \frac{V_u}{\phi f_y \mu} = \frac{86.4}{0.75 \times 60 \times 1.05} = 1.83 \text{ in.}^2 > 1.58 \text{ in.}^2$$

Note: Modified shear-friction method presented in R11.7.3 would give a closer estimate of shear-transfer strength than the conservative shear-friction method in 11.7.4.1.

5. Determine flexural reinforcement A_f. 11.9.3.3

$M_u = V_u a_v + N_{uc}\ (h - d) = 86.4\ (3) + 38.4\ (15 - 14.0) = 297.6$ in.-kips

Find A_f using conventional flexural design methods, or conservatively use $j_u d = 0.9d$

$$A_f = \frac{M_u}{\phi f_y j_u d} = \frac{297.6}{0.75 \times 60 \times 0.9 \times 14} = 0.53 \text{ in.}^2$$

Note that for all design calculations, $\phi = 0.75$ *11.9.3.1*

6. Determine direct tension reinforcement A_n. *11.9.3.4*

$$A_n = \frac{N_{uc}}{\phi f_y} = \frac{38.4}{0.75 \times 60} = 0.85 \text{ in.}^2$$

7. Determine primary tension reinforcement A_{sc}. *11.9.3.5*

$\left(\frac{2}{3}\right)A_{vf} = \left(\frac{2}{3}\right) 1.83 = 1.22 \text{ in.}^2 > A_f = 0.53 \text{ in.}^2$; Therefore, $\left(\frac{2}{3}\right)A_{vf}$ controls design.

$$A_{sc} = \left(\frac{2}{3}\right)A_{vf} + A_n = 1.22 + 0.85 = 2.07 \text{ in.}^2$$

Use 3-No. 8 bars, $A_{sc} = 2.37 \text{ in.}^2$

Check $A_{sc(min)} = 0.04 \left(\frac{4}{60}\right) 14 \times 14 = 0.52 \text{ in.}^2 < A_{sc} = 2.37 \text{ in.}^2$ O. K. *11.9.5*

8. Determine shear reinforcement A_h. *11.9.4*

$A_h = 0.5\,(A_{sc} - A_n) = 0.5\,(2.37 - 0.85) = 0.76 \text{ in.}^2$

Use 4-No. 3 stirrups, $A_h = 0.88 \text{ in.}^2$

The shear reinforcement is to be placed within two-thirds of the effective corbel depth adjacent to A_{sc}.

$s_{max} = \left(\frac{2}{3}\right)\frac{14}{4} = 2.33$ in. Use 2¼ in. o.c. stirrup spacing.

9. Corbel details

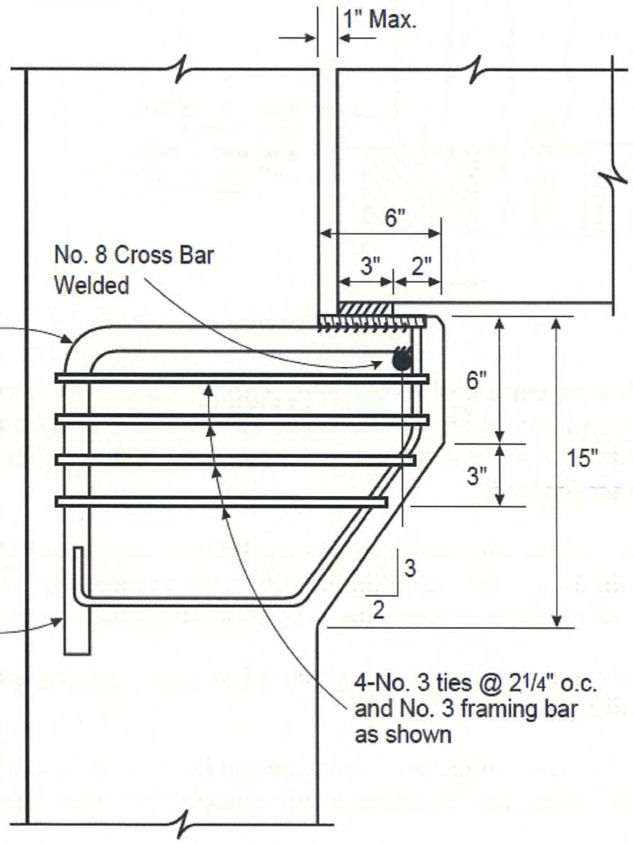
Corbel will project $(1 + 3 + 2) = 6$ in. from column face.

Use 6-in. depth at outer face of corbel, then depth at outer edge of bearing plate will be

$6 + 3 = 9 \text{ in.} > \frac{14}{2} = 7.0$ in. O.K. *11.9.2*

A_{sc} to be anchored at front face of corbel by welding a No. 8 bar transversely across ends of A_{sc} bars. *11.9.6*

A_{sc} must be anchored within column by standard hook.

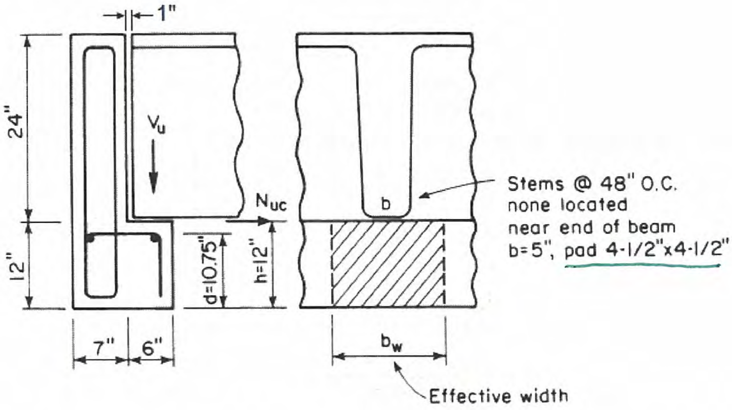


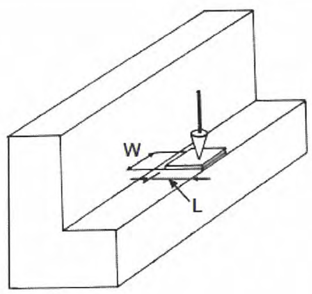
1" Max.
6"
3"
2"
No. 8 Cross Bar
Welded
3-No. 7 Bars
Welded to
Bearing Plate
6"
3"
15"
3
2
Standard
Hook
4-No. 3 ties @ 21/4" o.c.
and No. 3 framing bar
as shown

Example 15.3—Beam Ledge Design

f'_c = 5000 psi (normal weight)

f_y = 60,000 psi





The L-beam shown is to support a double-tee parking deck spanning 64 ft. Maximum service loads per stem are: DL = 11.1 kips; LL = 6.4 kips; total load = 17.5 kips. The loads may occur at any location on the L-beam ledge except near beam ends. The stems of the double-tees rest on 4.5 in. × 4.5 in. × 1/4 in. neoprene bearing pads (1000 psi maximum service load).

Design in accordance with the code provisions for brackets and corbels may require a wider ledge than the 6 in. shown. To maintain the 6-in. width, one of the following may be necessary: (1) Use of a higher strength bearing pad (up to 2000 psi); or (2) Anchoring primary ledge reinforcement A_{sc} to an armor angle.

This example will be based on the 6-in. ledge with 4.5-in.-square bearing pad. At the end of the example an alternative design will be shown.

Note: This example illustrates design to prevent potential local failure modes. In addition, ledge beams should be designed for global effects, not considered in this example. For more details see References 15.2 to 15.6.

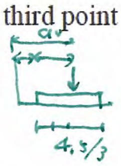
Calculations and Discussion	Code Reference

1. Check 4.5 × 4.5 in. bearing pad size (1000 psi maximum service load).

   Capacity = 4.5 × 4.5 × 1.0 = 20.3 kips > 17.5 kips O.K.

2. Determine shear spans and effective widths for both shear and flexure [Ref. 15.3 to 15.5]. The reaction is considered at outer third point of the bearing pad.

   a. For shear friction

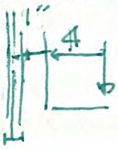


$$a_v = 4.5\left[\frac{2}{3}\right] + 1.0 = 4 \text{ in.}$$

Effective width = W + $4a_v$ = 4.5 + 4 (4) = 20.5 in.

b. For flexure, critical section is at center of the hanger reinforcement (A_v)

Assume 1 in. cover and No. 4 bar stirrups



12

$a_f = 4 + 1 + 0.25 = 5.25$ in.

½D'

Effective width $= W + 5a_f = 4.5 + 5\,(5.25) = 30.75$ in.

3. Check concrete bearing strength.

$V_u = 1.2\,(11.1) + 1.6\,(6.4) = 23.6$ kips — *Eq. (9-2)*

D.L L.L

$\phi P_{nb} = \phi(0.85 f'_c A_1)$ — *10.17.1*

$\phi = 0.65$ — *9.3.2.4*

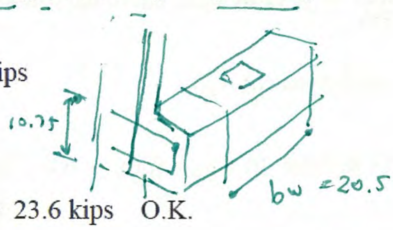
V_u

$\phi P_{nb} = 0.65\,(0.85 \times 5 \times 4.5 \times 4.5) = 55.9$ kips > 23.6 kips O.K.

4. Check effective ledge section for maximum nominal shear-transfer strength V_n. — *11.9.3.2.1*

For $f'_c = 5000$ psi: $V_n(\max) = 800 b_w d$, where $b_w = (W + 4a_v) = 20.5$ in.

$$V_n = \frac{800\,(20.5)\,(10.75)}{1000} = 176.3 \text{ kips}$$



$\phi = 0.75$ — *11.9.3.1*

$\phi V_n = 0.75\,(176.3) = 132.2$ kips > 23.6 kips O.K.

5. Determine shear-friction reinforcement A_{vf}. — *11.9.3.2*

$$A_{vf} = \frac{V_u}{\phi f_y \mu} = \frac{23.6}{0.75\,(60)\,1.4} = 0.37 \text{ in.}^2\text{/per effective width of 20.5 in.}$$ — *11.7.4.1*

where $\mu = 1.4$ — *11.7.4.3*

6. Check for punching shear (Eq. (3)) (c) 15-5

$V_u \leq 4\phi\sqrt{f'_c}\,(W + 2L + 2d_f)\,d_f$

$W = L = 4.5$ in.

$d_f \approx 10$ in. (assumed)

$$4\phi\sqrt{f'_c}\,(3W + 2d_f)\,d_f = 4 \times 0.75 \times \sqrt{5000}\,[(3 \times 4.5) + (2 \times 10)] \times 10/1000$$

$$= 71.1 \text{ kips} > 23.6 \text{ kips}$$

Example 15.3 (cont'd)	Calculations and Discussion	Code Reference

7. Determine reinforcement to resist direct tension A_n. Unless special provisions are made to reduce direct tension, N_u should be taken not less than $0.2V_u$ to account for unexpected forces due to restrained long-time deformation of the supported member, or other causes. When the beam ledge is designed to resist specific horizontal forces, the bearing plate should be welded to the tension reinforcement A_{sc}. *11.9.3.4*

$N_u = 0.2V_u = 0.2\,(23.6) = 4.7$ kips

$$A_n = \frac{N_u}{\phi f_y} = \frac{4.7}{0.75\,(60)} = 0.10 \text{ in.}^2\text{/per effective width of } 30.75 \text{ in. } (0.003 \text{ in.}^2\text{/in.})$$

8. Determine flexural reinforcement A_f.

$M_u = V_u a_f + N_u\,(h - d) = 23.6\,(5.25) + 4.7\,(12 - 10.75) = 129.8$ in.-kips

Find A_f using conventional flexural design methods. For beam ledges, Ref. 15.5 recommends to use $j_u d = 0.8d$. *11.9.3.3*

$\phi = 0.75$ *11.9.3.1*

$$A_f = \frac{129.8}{0.75\,(60)\,(0.8 \times 10.75)} = 0.34 \text{ in.}^2\text{/per 30.75 in. width} = 0.011 \text{ in.}^2\text{/in.}$$

9. Determine primary tension reinforcement A_{sc}. *11.9.3.5*

$$\left(\frac{2}{3}\right)A_{vf} = \left(\frac{2}{3}\right)0.37 = 0.25 \text{ in.}^2\text{/per 20.5 in. width} = 0.012 \text{ in.}^2\text{/in.}$$

$$A_{sc} = \left(\frac{2}{3}\right)A_{vf} + A_n = 0.012 + 0.003 = 0.015 \text{ in.}^2\text{/in. (governs)}$$

$$A_{sc} = A_f + A_n = 0.011 + 0.003 = 0.014 \text{ in.}^2\text{/in.}$$

Check $A_{sc(min)} = 0.04\left(\frac{f'_c}{f_y}\right)d$ per in. width *11.9.5*

$$= 0.04\left(\frac{5}{60}\right)10.75 = 0.036 \text{ in.}^2\text{/in.} > 0.015 \text{ in.}^2\text{/in.}$$

For typical shallow ledge members, minimum A_{sc} by 11.9.5 will almost always govern.

10. Determine shear reinforcement A_h.

$A_h = 0.5\,(A_{sc} - A_n) = 0.5\,(0.036 - 0.003) = 0.017$ in.^{2}/in. *11.9.4*

11. Determine final size and spacing of ledge reinforcement.

For A_{sc} = 0.036 in.^{2}/in.:

Try No. 5 bars (A = 0.31 in.^{2})

$$s_{max} = \frac{0.31}{0.036} = 8.6 \text{ in.}$$

Use No. 5 @ 8 in.

A_h = 0.017 in.^{2}/in. For ease of constructability, provide reinforcement A_h at same spacing of 8 in.

Provide No. 4 (A = 0.2 in.^{2}) @ 8 in. within 2/3d adjacent to A_{sc}.

12. Check required area of hanger reinforcement.

For strength (Eq. (4)):

$$A_v = \frac{V_u s}{\phi f_y S}$$

For s = 8 in. and S = 48 in. ,,k-20'3   Stews @ 48" o.c

$$A_v = \frac{23.6 \times 8}{0.75 \times 60 \times 48} = 0.09 \text{ in.}^2$$

For serviceability (Eq. (5)):

$$A_v = \frac{V}{0.5f_y} \times \frac{s}{(W + 3a_v)}$$

V = 11.1 + 6.4 = 17.5 kips   D.L+L.L (사용시)

$W + 3a_v$ = 4.5 + (3 × 4) = 16.5 in.

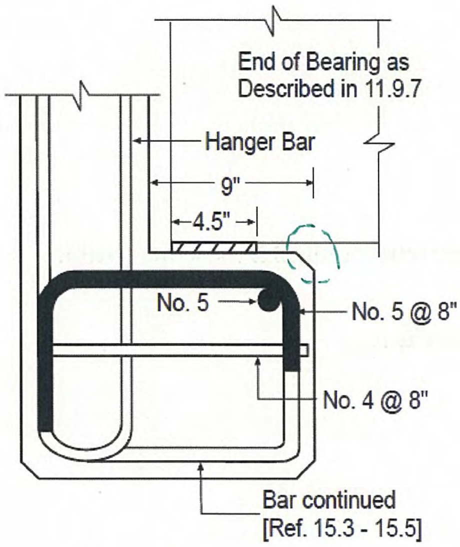
$$A_v = \frac{17.5}{0.5 \times 60} \times \frac{8}{16.5} = 0.28 \text{ in.}^2 \quad \text{(governs)}$$

No. 5 hanger bars @ 8 in. are required

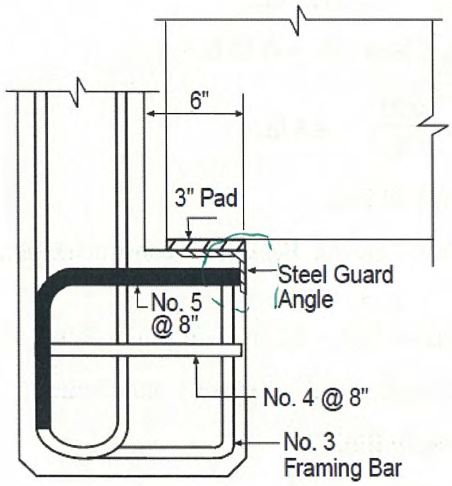
Sufficient stirrups for combined shear and torsion must be provided for global effects in the ledge beam. (See Refs. 15.5 and 15.6)

13. Reinforcement Details

In accordance with 11.9.7, bearing area (4.5 in. pad) must not extend beyond straight portion of beam ledge reinforcement, nor beyond inside edge of transverse anchor bar. With a 4.5 in. bearing pad, this requires that the width of ledge be increased to 9 in. as shown below. Alternately, a 6 in. ledge with a 3 in. medium strength pad (1500 psi) and the ledge reinforcement welded to an armor angle would satisfy the intent of 11.9.7.



9 in. Ledge Detail



6 in. Ledge Detail (Alternate)