Representation Theory 1 V4A3

So Murata

2024/2025 Winter Semester - Uni Bonn

Exercise 3

(iii)

Let $g \in SU(4)$ and $u \wedge v \in \bigwedge^2 \mathbb{C}^4$, we let

$$g(u \wedge v) = gu \wedge gv.$$

For $v \in \bigwedge^2 \mathbb{C}^4$ we define $v^* \in \mathbb{C}^4$ by

$$u \wedge v^* = \langle u, v \rangle e_1 \wedge e_2 \wedge e_3 \wedge e_4,$$

where $\langle \cdot, \cdot \rangle$ is the Hermitian inner product induced by standard Hermitian inner product on $\mathbb{C}^4$.

For $i, j, k, l \in \{1, 2, 3, 4\}$ and each of them is distinct, we have

$$b_{ij,kl} = \frac{1}{\sqrt{2}}(e_i \wedge e_j + e_k \wedge e_l), \quad \bar{b}_{ij,kl} = \frac{i}{\sqrt{2}}(e_i \wedge e_j - e_k \wedge e_l).$$

Then this forms a basis in $\bigwedge^+ \mathbb{C}^4$. Obviously $SU(4)$ preserves the inner product so maps into $O(6)$, and as $SU(4)$ is connected image lies in $SO(6)$. By the first equation, above, we have the homomorphism between $SU(4)$ and $SO(6)$.