# Representation Theory 1 V4A3 Exercise Sheet 2 Problem 3

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## (1)

Let $\alpha(t) = \Phi_v(t+s, p)$ and $\beta(t) = \Phi_v(t, \Phi_v(s, p))$. Then for a fixed $s$ and $t = 0$, we have

$$\alpha(0) = \Phi_v(s, p) = \Phi_v(0, \Phi_v(s, p)) = \beta(0).$$

Also we have for some neighborhood,

$$\frac{d\alpha}{dt}(t) = v(\alpha(t)), \quad \frac{d\beta}{dt}(t) = v(\beta(t)).$$

By the uniqueness of $c_{p,v}$, we derive that $\alpha = \beta$.

The above also proves for $s$, since $s$ can be picked arbitrary and they are equal at $t = 0$.

## (2)

Let $\beta(t) = R_p \Phi_v(t, 1)$ and $\alpha(t) = R_p \beta(t)$ then $\alpha(0) = p$. We need to show that $v(\alpha(t)) = dR_{\alpha(t)}(1)X$. Indeed we have

$$dR_{pq}(1) = d(R_q \circ R_p)(1) = dR_q(R_p(1))dR_p(1).$$

Using this we derive

$$\begin{aligned}
(\alpha)'(t) &= dR_p(\beta(t))dR_{\beta(t)}(1)X \\
&= dR_p(R_{\beta(t)}1)dR_{\beta(t)}(1)X \\
&= dR_{\beta(t)p}(1)X \\
&= dR_{\alpha(t)}(1)X.
\end{aligned}$$

By the uniqueness, we get $\alpha(t) = \Phi_v(t, p)$.