

Representation Theory 1 V4A3 Sheet 9

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1 Exercise 3

The faithfulness of the action is immediate from the fact that T is maximal abelian. Since \mathfrak{g}_α is of dimension 1, we will prove that $\text{Ad}(n) : \mathfrak{g}_\alpha \rightarrow \mathfrak{g}_\beta$ is an injection.

Let us note that we have for all $t \in T$,

$$X \in \mathfrak{g}_\alpha \Leftrightarrow \forall t \in \mathfrak{t}_{\mathbb{C}}, \text{ad}(t)X = \alpha(t)X.$$

Substituting $\text{Ad}(n)X$ for X we get

$$\text{ad}(t)X = \alpha(t) \text{Ad}(n)^{-1} \text{Ad}(n)X.$$

This is well-defined as $\text{Ad}(n)$ is an isomorphism and n is a normalizer of T .

Since the action is injective and the root system R is finite, there are at most $|R|!$ many ways of such actions as they represent permutations, we have that $|W|$ is also bounded by $|R|!$.