Representation Theory 1 V4A3 Sheet 3 Exercise 1

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1.

By the property of the exponential map we have

$$\underline{\mathrm{Ad}}(g) \circ \exp_H = \exp_H \circ \mathrm{Ad}(g).$$

We have $\exp_H(\mathfrak{h})$ generates H. Thus $\underline{\mathrm{Ad}}(g) \circ \exp_H(\mathfrak{h})$ generates gHg^{-1}. With the equation above, we conclude the statement.

2.

If H is a normal subgroup then by the previous problem, we derive that

$$\mathrm{Ad}(g)(\mathfrak{h}) = \mathfrak{h}.$$