Surfactant flooding: scouting calculations

$$f(S^*,C) = \frac{(S^*)^{n_w}}{(S^*)^{n_w} + M(C)(1-S^*)^{n_o}}$$

$$M(C) = \frac{\mu_w}{k^e_{rw}} \frac{k^e_{ro}}{\mu_o} \qquad S^* = \frac{S_w - S_{cw}}{1 - S_{cw} - S_{orw}}$$

What if S_{cw}, S_{orw}, n_w and n_o are also dependent on C

$$\frac{\partial f}{\partial C}(S^*(C),C) = \frac{\partial f}{\partial S^*}\frac{\partial S^*}{\partial C} + \frac{\partial f}{\partial C}$$

$$\frac{\partial S^*}{\partial C} = \frac{-S'_{cw}}{1-S_{cw}-S_{orw}} + \frac{(S_w - S_{cw})}{(1-S_{cw}-S_{orw})^2}(S'_{cw} + S'_{orw})$$

$$= \frac{-S'_{cw}(1-S_{cw}-S_{orw}) + (S_w-S_{cw})(S'_{cw}+S'_{orw})}{(1-S_{cw}-S_{orw})^2}$$

$$= \frac{(S_w-(1-S_{orw}))S'_{cw} + (S_w - S_{cw})S'_{orw}}{(1-S_{cw}-S_{orw})^2}$$

$$\frac{\partial f}{\partial C} = \frac{n'_w \log(S^*)(S^*)^{n_w}}{(\quad)} - \frac{(S^*)^{n_w}}{(\quad)^2}\cdot\Big\{ n'_w \log(S^*)(S^*)^{n_w}$$

$$+ M'(C)(1-S^*)^{n_o} + n'_o \log(1-S^*)M(C)(1-S^*)^{n_o}\Big\}$$

$$= \frac{-M'(C)(S^*)^{n_w}(1-S^*)^{n_o} + M(C)(S^*)^{n_w}(1-S^*)^{n_o}\left[n'_w \log(S^*) - n'_o \log(1-S^*)\right]}{\left((S^*)^{n_w} + M(C)(1-S^*)^{n_o}\right)^2}$$

$$M'(C) = \overbrace{\frac{\mu'_w}{k^e_{rw}}\frac{k^o_{rw}}{\mu_o}}^{>0} + \overbrace{\frac{\mu_w k'^o_{rw}}{k^e_{rw}\mu_o}}^{>0} \overbrace{- \frac{\mu_w k^o_{rw}\, k'^e_{rw}}{(k^e_{rw})^2 \mu_o}}^{<0}$$