

Parallelized reservoir computing scheme and results

Libraries such as **TensorFlow** and **PyTorch** fundamentally rely on **backpropagation** (gradient descent), and modifying that core algorithm is **VERY DIFFICULT** at the user level!!

⇒ I take a **math-first** approach to model design.

My research topic (prediction for parallelized data)

- Forecasting of **stock price** data and **weather** data, focusing on small-size datasets (less than 10,000 samples).

Architecture selection

Reservoir computing performs best for the small-size datasets.

- See my GitHub repository, “**compare-LSTM-GRU-Reservoir**” .

Data structure for my research: at $u.shape[0]$ fixed observation points generate $u.shape[0]$ -elements time series data.

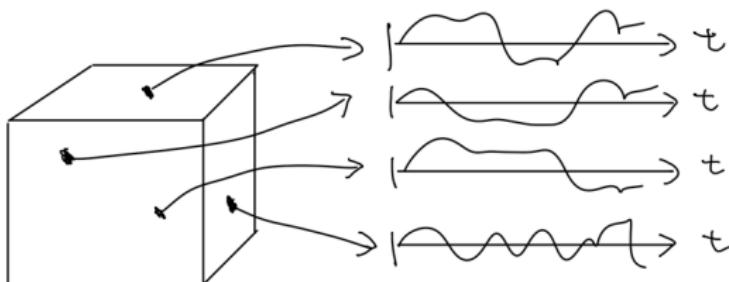


Figure: This case $u.shape[0] = 4$

Case 1: the set of time series data contains rich spatial information

- Apply FFT (fast Fourier transform)
Example: in periodic box turbulence obtained from DNS
- Focus on time evolution of the **amplitudes**. See Nakai-Saiki (2018).
- Spatial filters (e.g. low-pass, band-pass) are appropriate.

Case 2: the set of time series data contains little spatial information

- Filters in the spatial direction may not be appropriate.
- Time filtering can still be applied
but need assumption: underlying systems generating each time series are considered to be nearly identical.

case 3: Intermediate case

- Graph Neural Networks might be one of them?

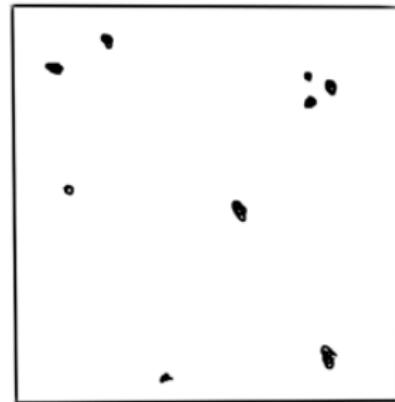
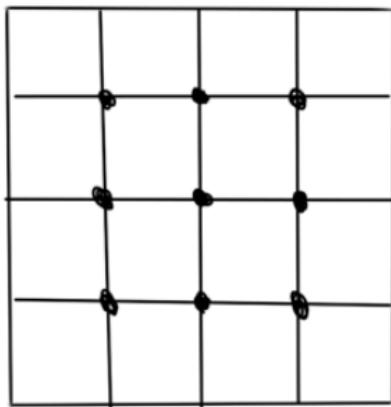


Figure: Left: suitable for FFT, RIGHT: try to apply GNN in the future

Focus on the case 2

- Wind-speed data seems having little spatial information
- Stock data clearly do not have such spatial information

Overview of the Learning Scheme (Case 2)

Parallelized data can be written as:

$$\{u_{j,t}\}_{j \in [0, 1, \dots, u.shape[0]-1], t \in [0, 1, \dots, u.shape[1]-1]}$$

- the number of discrete data points: $u.shape[1]$
- the number of time series data: $u.shape[0]$

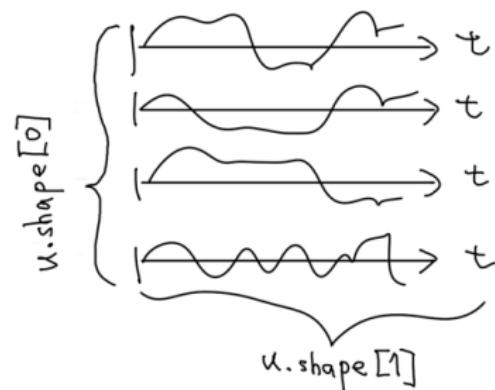


Figure: Image of data shape

Delay coordinate (see Nakai-Saiki 2021)

Rewrite the data in a form of dim -dimensional vector as the following:

$$U_{j,t} = \begin{pmatrix} u_{j,t} \\ u_{j,t-lag} \\ \vdots \\ u_{j,t-(dim-1)lag} \end{pmatrix}.$$

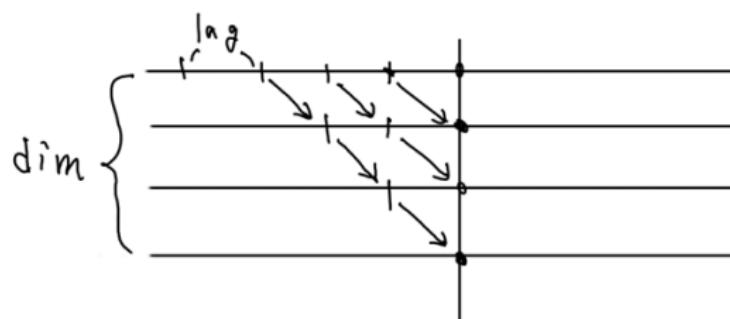


Figure: Image of $U_{j,t}$

Overview of our ML architecture

- M : the number of learnable parameters
- $\{w_i\}_{i=1}^M$: learnable parameters (reservoir's W_{in} , W , W_{out})

For a function $F_j : \mathbb{R}^M \times \mathbb{R}^{dim} \rightarrow \mathbb{R}^{dim}$, the prediction value $\hat{U}_{j,t+1}$ is

$$\hat{U}_{j,t+1} = F_j \left(\{w_i\}_{i=0}^{M-1}, \{U_{j,t}\}_{j=0}^{u.shape[0]-1} \right).$$

We then select the $\{w_i\}_i$ that minimizes the following energy:

$$\sum_{j=0}^{u.shape[0]-1} \sum_{t=0}^{T_{train}-1} |\hat{U}_{j,t+1} - U_{j,t+1}|^2$$

(we omit regularization term here). Assume that **underlying systems are almost identical**, then F_j can be unified into a single F independent of j :

$$\hat{U}_{j,t+1} = F \left(\{w_i\}_{i=0}^{M-1}, U_{j,t} \right).$$

This significantly simplifies the learning model.

Reservoir Computing (overview)

Trainable parameters

- Input weight matrix: $W^{in} \in \mathbb{R}^{N_x \times dim}$
- Recurrent weight matrix: $W \in \mathbb{R}^{N_x \times N_x}$
- Output weight matrix: $W^{out} \in \mathbb{R}^{dim \times N_x}$

Substitute data $U_{j,t} \in \mathbb{R}^{dim}$ and feature vector $x_{j,t} \in \mathbb{R}^{N_x}$

Output predicted value $\hat{U}_{j,t+1} \in \mathbb{R}^{dim}$ and feature vector $x_{j,t+1} \in \mathbb{R}^{N_x}$

$$(U_{j,t}, x_{j,t}) \mapsto (\hat{U}_{j,t+1}, x_{j,t+1}),$$

$$\begin{cases} x_{j,t+1} &= (1 - \alpha)x_{j,t} + \alpha \tanh(W^{in}U_{j,t} + Wx_{j,t}), \\ \hat{U}_{j,t+1} &= W^{out}x_{j,t+1}. \end{cases}$$

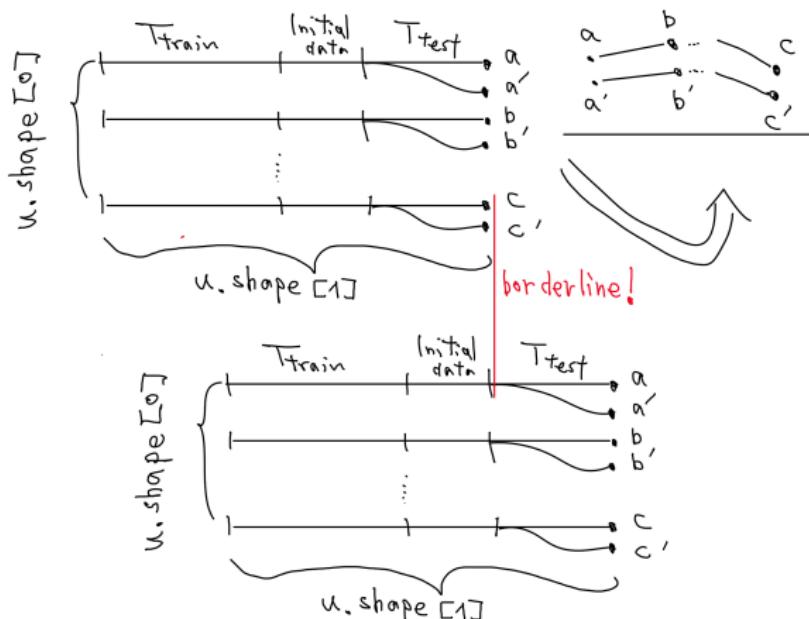
$\alpha \in (0, 1]$ is called the leaking rate, which is similar to the well-known concept of residual connections.

Conceptual diagram of the learning scheme

The scheme has two stages:

- Model selection
- Generalization performance evaluation

Each of them are including a training phase



Training phase

We select W^{out} that minimizes the following E :

Evaluation function

$$E = \frac{1}{2} \sum_{j=0}^{u.shape[0]-1} \sum_{t=0}^{T_{train}-1} |U_{j,t} - W^{out}x_t|^2 + \frac{\beta}{2} \sum_{i=0}^{dim-1} \sum_{j=0}^{N_x-1} |W_{ij}^{out}|^2.$$

- $\beta > 0$ is a regularization parameter
- T_{train} is the number of training data points
- due to convexity, W^{out} can be solved analytically

Model selection phase

- We forecast T_{test} steps ahead
- The initial value for model selection is $U_{j, T_{train}}$
- $x_{j, T_{train}}$ is already obtained during the training phase

Based on these, the next step is estimated as follows:

$$\begin{cases} x_{j, T_{train}+1} &= (1 - \alpha)x_{j, T_{train}} + \alpha \tanh(W^{in}U_{j, T_{train}} + Wx_{j, T_{train}}), \\ \hat{U}_{j, T_{train}+1} &= W^{out}x_{j, T_{train}+1}. \end{cases}$$

The resulting $\hat{U}_{j,t}, x_{j,t}$ are then sequentially fed into the following recurrence relation to generate the predicted data:

$$\begin{cases} x_{j,t+1} &= (1 - \alpha)x_{j,t} + \alpha \tanh(W^{in}\hat{U}_{j,t} + Wx_{j,t}), \\ \hat{U}_{j,t+1} &= W^{out}x_{j,t+1} \end{cases}$$

for $t = T_{train} + 1, \dots, T_{train} + T_{test}$.

Bayesian optimization phase

Apply **Bayesian opt (Optuna)** using the prediction data $\hat{U}_{j, T_{train}+T_{test}}$.
More specifically, minimize the following MAE-based evaluation function:

Evaluation function

$$\frac{1}{u.shape[0]} \sum_{j=0}^{u.shape[0]-1} |\hat{U}_{j, T_{train}+T_{test}} - U_{j, T_{train}+T_{test}}|$$

in the selection of the following parameters:

dim, lag, α , W , W^{in} .

Generalization Performance: re-training Phase

After selecting dim, lag, α , W , and W^{in} in the previous phase, we implement a new test to evaluate **generalization performance**. We **re-select** W^{out} that minimizes the following E :

Evaluation function

$$E = \frac{1}{2} \sum_{j=0}^{u.shape[0]-1} \sum_{t=T_{test}}^{T_{train}+T_{test}-1} |U_{j,t} - W^{out}x_{j,t}|^2 + \frac{\beta}{2} \sum_{i=0}^{dim-1} \sum_{j=0}^{N_x-1} |W_{ij}^{out}|^2.$$

- We evaluate the generalization performance from $T_{train} + T_{test}$
- The initial value is $U_{j,T_{train}+T_{test}}$
- $x_{j,T_{train}+T_{test}}$ is already obtained during the re-training phase

Generalization Performance: evaluation phase

We generate the next step as follows.

$$\begin{cases} x_{j, T_{train}+T_{test}+1} &= (1 - \alpha)x_{j, T_{train}+T_{test}} \\ &\quad + \alpha \tanh(W^{in}U_{j, T_{train}+T_{test}} + Wx_{j, T_{train}+T_{test}}), \\ \hat{U}_{j, T_{train}+T_{test}+1} &= W^{out}x_{j, T_{train}+T_{test}+1}. \end{cases}$$

The $(\hat{U}_{j,t}, x_{j,t})$ obtained in this manner are then sequentially fed into the following recurrence relation to generate the prediction data:

$$\begin{cases} x_{j,t+1} &= (1 - \alpha)x_{j,t} + \alpha \tanh(W^{in}\hat{U}_{j,t} + Wx_{j,t}), \\ \hat{U}_{j,t+1} &= W^{out}x_{j,t+1} \end{cases}$$

for $t = T_{train} + T_{test} + 1, \dots, T_{train} + 2T_{test}$. Then, finally, the generalization performance is evaluated using the following MAE:

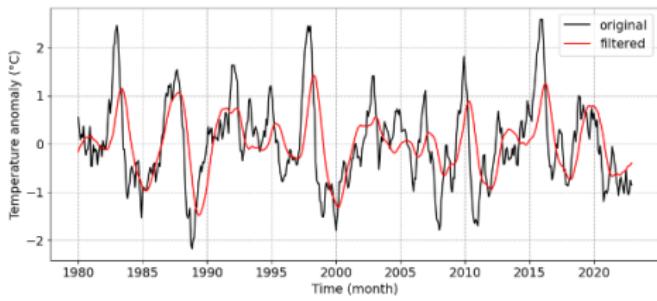
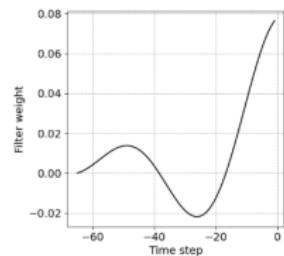
Evaluation function

$$\frac{1}{u.shape[0]} \sum_{j=0}^{u.shape[0]-1} |\hat{U}_{j, T_{train}+2T_{test}} - U_{j, T_{train}+2T_{test}}|$$

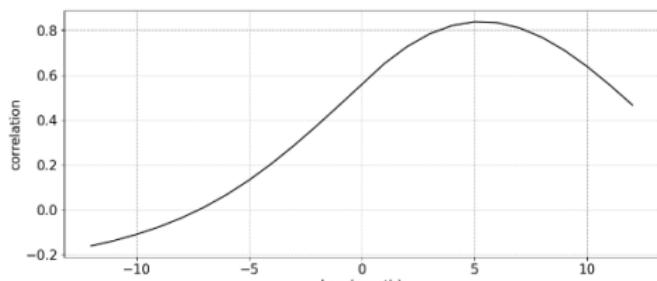
ML Results

El Nino prediction (Jinno-Mitsui-Nakai-Saiki-Y 2025)

Nino 3.4 (this time series expresses the strength of El Nino)



(a)



(b)

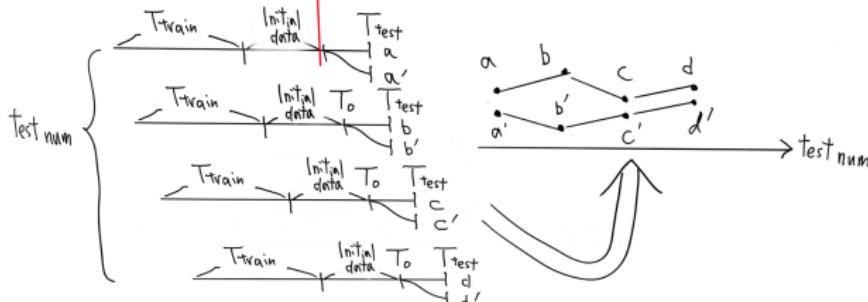
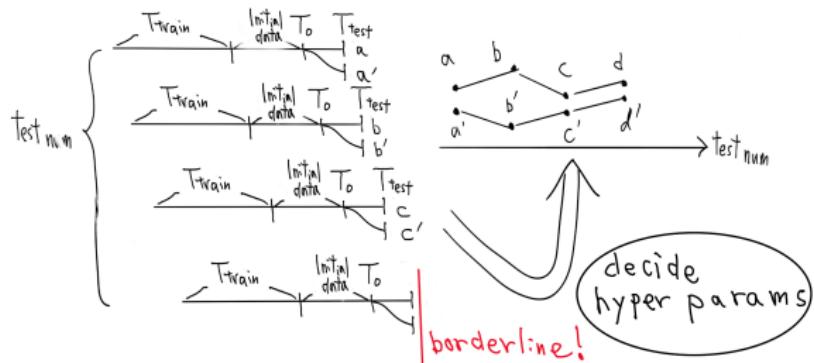
Figure: data-driven filter

(a): original (black) and filtered (red)

(b): Lag correlation between original and filtered : correlation > 0.8

How to evaluate the model?

Pearson correlation coefficient (PCC) between real data and prediction

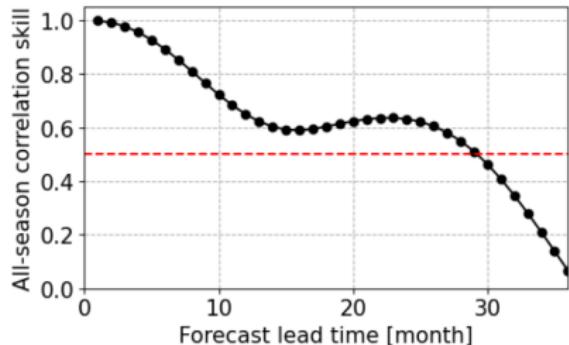


Generalization performance evaluation: $T_{test} = 1, 2, \dots$

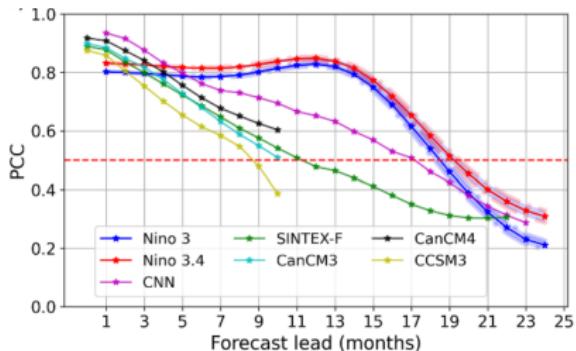
Threshold of the correlation is 0.5 in this competition!!



Result (El Nino)



Prediction ($29 - 5(\text{lag}) = 24$)

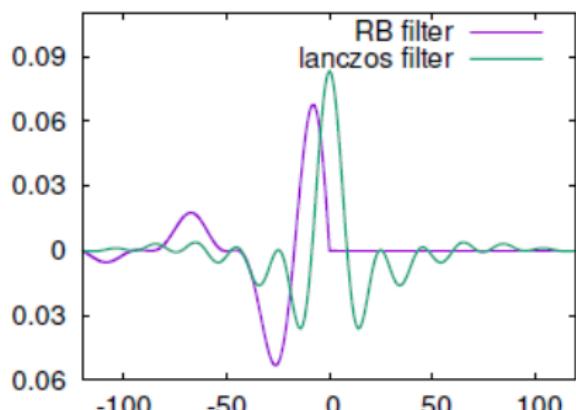


previous results (the best is 19
but incorporating future values)

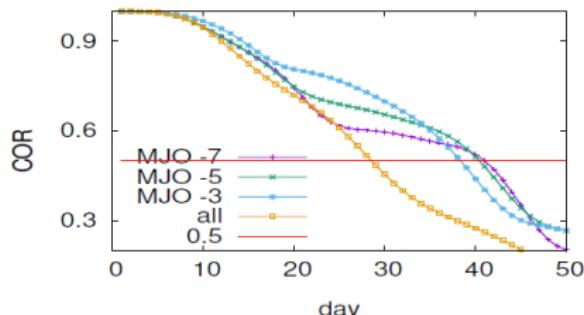
- Hassanibesheli-Kurths-Boers (2022) red curve: They employed the usual Littlewood-Paley decomposition, so it incorporate future values. Thus their result CANNOT directly turn into a real-time forecasting.

Result (MJO prediction)

Nakai-Suematsu-Miura-Y-Takasuka-Jinno-Saiki (arXiv)



Lag (weighted center): -7.9 days



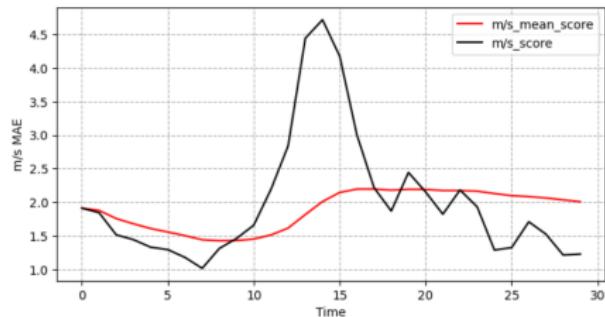
Prediction with RB filter
(29-8(lag)=**21**)

Even when using state-of-the-art physical models with high computational cost, the prediction accuracy is only on the order of about 20 days!

Result (windspeed in Tokyo region)

- start date = "2024-Jan-01" end date = "2024-Feb-15"
- predict 3 hours ahead ($T_{test} = 3$)
- most recent 1,000 hours of data are used to train(=: T_{train})
- $dim = 4$, $lag = 1$: fixed
- Thirty independent trials were performed using a rolling-window validation scheme

Result MAE: 2.01 m/s (data-driven filter is applied: correlation 0.83, Jinno-Mitsui-Nakai-Saiki-Yoneda 2025)



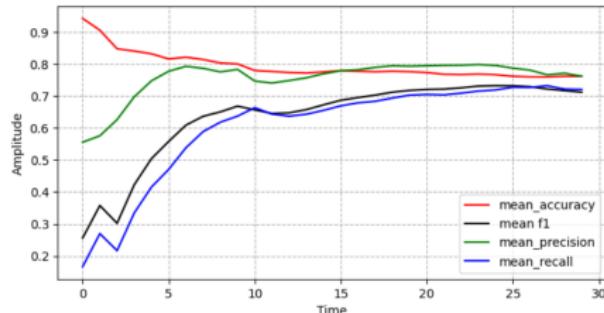
Cheng and Steenburgh (2005) CIRP, WRF (physical models)

MAE: 1.6 ~ 1.9 m/s, but unclear whether a filter is applied...

Result (stock S&P500)

- start date = "2024-Jan-01" end date = "2025-May-10"
- predict 1 day ahead ($T_{test} = 1$)
- most recent 60 days of data are used to train(=: T_{train})
- $dim = 2$, $lag = 1$: fixed
- Thirty independent trials were performed using a rolling-window validation scheme

Result Accuracy 0.762, F1 0.712, Recall 0.721 (data-driven filter is applied: correlation 0.985, Jinno-Mitsui-Nakai-Saiki-Yoneda 2025)



Gil, Duhamel-Sebline, McCarren (2024) xLSTM-TS

Accuracy 0.709, F1 0.730, Recall 0.768, wavelet filter is applied.

Thank you!