# Three Dimensional Intercriteria Analysis over Intuitionistic Fuzzy Data

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**Abstract.** In the paper is extended two dimensional intercriteria analysis over intuitionistic fuzzy data to three dimensional and will be discussed possibility for application of this analysis as an illustration of the application of the intercriteria analysis.

**Keywords:** Intercriteria analysis  $\cdot$  Intuitionistic fuzzy index matrix  $\cdot$  Intuitionistic fuzzy pair

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## 1 Introduction

The concept of intercriteria analysis was introduced in [4,7]. The intercriteria analysis is based on the apparatus of the Index Matrices (IMs, [4]) and of Intuitionistic Fuzzy Sets (IFSs, [3]).

The paper is a continuation of the papers [1,6,7,9-13,16] and we for the first time discuss the possibility, the data, that will be processed by three dimensional intercriteria analysis, to be intuitionistic fuzzy pairs (IFP, see [8]) or more general intuitionistic fuzzy data, saved in 3D-intuitionistic fuzzy index matrix (3D-IFIM, [19]).

### 2 Basic Definitions

### 2.1 Short Notes on Intuitionistic Fuzzy Pairs

Let us started with some remarks on Intuitionistic Fuzzy Logic from [3,8]. The IFP is an object with the form  $\langle a,b\rangle$ , where  $a,b\in[0,1]$  and  $a+b\leq 1$ , that

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is used as an evaluation of some object or process. Its components (a and b) are interpreted as degrees of membership and non-membership, or degrees of validity and non-validity, or degree of correctness and non-correctness, etc. Let us have two IFPs  $x = \langle a, b \rangle$  and  $y = \langle c, d \rangle$ .

Let us have two IFPs  $x=\langle a,b\rangle$  and  $y=\langle c,d\rangle.$  In [8] were defined following operations:

and relations

$$x < y$$
 iff  $a < c$  and  $b > d$   
 $x > y$  iff  $a > c$  and  $b < d$   
 $x \ge y$  iff  $a \ge c$  and  $b \le d$   
 $x \le y$  iff  $a \le c$  and  $b \ge d$   
 $x = y$  iff  $a = c$  and  $b = d$ .

### 2.2 Short Remarks on Index Matrices

The concept of Index Matrix (IM) was discussed in a series of papers and collected in [4].

**Definition of 3D-Extended Index Matrix (3D-EIM).** Let  $\mathcal{X}$  be a fixed set of objects (real numbers, numbers 0 or 1, logical variables, propositions or predicates, intuitionistic fuzzy pairs (IFPs), function and etc.). Let  $\mathcal{I}$  be a fixed sets of indices and

$$\mathcal{I}^n = \{\langle i_1, i_2, ..., i_n \rangle | (\forall j : 1 \le j \le n) (i_j \in \mathcal{I}) \} \text{ and } \mathcal{I}^* = \bigcup_{1 \le n \le \infty} \mathcal{I}^n.$$

By 3D-extended IM (3D-EIM) [4,17], with index sets K, L and H ( $K, L, H \subset I^*$ ) and elements from the set  $\mathcal{X}$  we denote the object :

$$[K,L,H,\{a_{k_i,l_j,h_g}\}] = \begin{cases} \frac{h_g & l_1 & \dots & l_j & \dots & l_n \\ k_1 & a_{k_1,l_1,h_g} & \vdots & a_{k_1,l_j,h_g} & \dots & a_{k_1,l_n,h_g} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ k_i & a_{k_i,l_1,h_g} & \dots & a_{k_i,l_j,h_g} & \dots & a_{k_i,l_n,h_g} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ k_m & a_{k_m,l_1,h_g} & \dots & a_{k_m,l_j,h_g} & \dots & a_{k_m,l_n,h_g} \end{cases} \mid h_g \in H$$

where  $K = \{k_1, k_2, \dots, k_m\}$ ,  $L = \{l_1, l_2, \dots, l_n\}$ ,  $H = \{h_1, h_2, \dots, h_f\}$ , and for  $1 \le i \le m$ ,  $1 \le j \le n$ ,  $1 \le g \le f$ :  $a_{k_i, l_j, h_g} \in \mathcal{X}$ .

In [4,17,19], different operations, relations and operators are defined over IMs. For the needs of the present research, we will introduce the definitions of some of them. With  $3D-EIM_{\mathcal{R}}$  we denote the set of all 3D-EIMs with elements real numbers, with  $3D-EIM_{\{0,1\}}$  – the set of all (0,1)-3D-EIMs with elements 0 or 1, with  $3D-EIM_{\mathcal{P}}$  – the set of all 3D-EIMs with elements – predicates and , with  $3D-EIM_{\mathcal{T}PP}$  – the set of all 3D-EIMs with elements – IFPs.

We can define the evaluation function V that juxtaposes to this 3D-EIM a new one with elements – IFPs  $\langle \mu, \nu \rangle$ , where  $\mu, \nu, \mu + \nu \in [0, 1]$ . The new IM, called Intuitionistic Fuzzy IM (IFIM), contains the evaluations of the variables, propositions, etc., i.e., it has the form

$$\begin{split} V([K,L,H,\{a_{k_{i},l_{j},h_{g}}\}]) &= [K,L,H,\{V(a_{k_{i},l_{j},h_{g}})\}] \\ &= [K,L,H,\{\langle \mu_{k_{i},l_{j},h_{g}},\nu_{k_{i},l_{j},h_{g}}\rangle\}] \\ &= \left\{ \begin{aligned} \frac{h_{g} \in H & l_{1} & \dots & l_{n} \\ \hline k_{1} & \langle \mu_{k_{1},l_{1},h_{g}},\nu_{k_{1},l_{1},h_{g}}\rangle & \dots & \langle \mu_{k_{1},l_{n},h_{g}},\nu_{k_{1},l_{n},h_{g}}\rangle \\ \vdots & \vdots & \dots & \vdots \\ k_{i} & \langle \mu_{k_{i},l_{1},h_{g}},\nu_{k_{i},l_{1},h_{g}}\rangle & \dots & \langle \mu_{k_{i},l_{n},h_{g}},\nu_{k_{i},l_{n},h_{g}}\rangle \\ \vdots & \vdots & \dots & \vdots \\ k_{m} & \langle \mu_{k_{m},l_{1},h_{g}},\nu_{k_{m},l_{1},h_{g}}\rangle & \dots & \langle \mu_{k_{m},l_{n},h_{g}},\nu_{k_{m},l_{n},h_{g}}\rangle \end{aligned} \right\}, \end{split}$$

where for every  $1 \le i \le m, 1 \le j \le n, \ 1 \le g \le f$ :  $V(a_{k_i,l_j,h_g}) = \langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g}, \mu_{k_i,l_j,h_g} + \nu_{k_i,l_j,h_g} \le 1$ .

**Aggregation Operations over 3D-EIM.** Let the 3D-EIM  $A = [K, L, H, \{a_{k_i, l_j, h_g}\}]$  be given, where  $K, L, H \subset \mathcal{I}^*$ , and let  $k_0 \notin K, l_0 \notin L, h_0 \notin H$ . Let  $\circ : \mathcal{X} \times \mathcal{X} \longrightarrow \mathcal{X}$  and  $* : \mathcal{X} \times \mathcal{X} \longrightarrow \mathcal{X}$ .

Let

$$\circ \in \begin{cases} \{\text{``+'', ``\times'', ``average'', ``max'', ``min''}\}, & \text{if } A \in 3D - EIM_{\mathcal{R}}, \\ \{\text{``max'', ``min''}\}, & \text{if } A \in 3D - EIM_{\{0,1\}} \\ \{\text{``\wedge'', ``\vee''}\}, & \text{if } A \in 3D - EIM_{\mathcal{P}} \\ & \text{or } A \in 3D - EIM_{\mathcal{I}FP} \end{cases}$$

In the case of  $3D-EIM_{\mathcal{I}FP}$ , in aggregation operations can participate aggregating pair operations  $(\circ, *)$  whose elements are applied respectively on the first and second element of IFP, where

$$(\circ, *) \in \{(min, max)(min, average), (min, min), (average, average), (average, min), (max, min)\}.$$

Therefore when  $A \in 3D - EIM_{\mathcal{I}FP}$ , operations " $(\circ, *)$ " are defined for the intuitionistic fuzzy pairs  $\langle a, b \rangle$  and  $\langle c, d \rangle$ , elements of A by

$$\langle a,b\rangle(\circ,*)\langle c,d\rangle=\langle \circ(a,c),*(b,d)\rangle.$$

In all other cases, we use only one operation  $(\circ)$ .

In [18] were defined aggregation operations. We will recall the following definition:

# $(\circ) - \alpha_H$ -aggregation

$$\alpha_{(H,\circ)}(A,h_0) = \begin{cases} \frac{l_j & h_0}{k_1} & \circ & a_{k_1,l_1,h_g} \\ k_2 & \circ & a_{k_2,l_2,h_g} & | l_j \in L \end{cases}$$

$$\vdots & \vdots \\ k_m & \circ & a_{k_m,l_n,h_g} \\ 1 \leq g \leq f \end{cases}$$

$$= \frac{l_1}{k_1} \quad \frac{l_2}{1 \leq g \leq f} \quad \frac{l_2}{1 \leq g$$

# 3 Three Dimensional Intercriteria Analysis Applied over Intuitionistic Fuzzy Data

In this section we extended two-dimensional intercriteria analysis from [12] applied over intuitionistic fuzzy data to three dimensional.

Let us have the set of objects  $O=\{O_1,O_2,...,O_n\}$  that must be evaluated by criteria from the set  $C=\{C_1,C_2,...,C_m\}$  in the index  $h_g\in H$  for  $1\leq g\leq f$ , where H is the third fixed scale and  $h_g$  is its element. For example, index set H can be interpreted as a time-scale and its elements  $h_g$  – as time-moments.

Let us have an 3D-EIM

where for every  $p, q \ (1 \le p \le m, \ 1 \le q \le n)$ :

- (1)  $C_p$  is a criterion, taking part in the evaluation,
- (2)  $O_q$  is an object, being evaluated.
- (3)  $a_{C_p,O_q,h_g}$  is a variable, formula or  $a_{C_p,O_q,h_g} = \langle \alpha_{C_p,O_q,h_g}, \beta_{C_p,O_q,h_g} \rangle$  is an IFP, that is comparable about relation R with the other a-objects, so that for each i,j,k,g:  $R(a_{C_k,O_i,h_g}, a_{C_k,O_j,h_g})$  is defined. Let  $\overline{R}$  be the dual relation of R in the sense that if R is satisfied, then  $\overline{R}$  is not satisfied and vice versa. For example, if "R" is the relation "<", then  $\overline{R}$  is the relation ">", and vice versa.

For each index  $h_g$   $(1 \le g \le f)$  let  $S_{k,l,g}^{\mu}$  be the number of cases in which

$$\langle \alpha_{C_k,O_i,h_q}, \beta_{C_k,O_i,h_q} \rangle \leq \langle \alpha_{C_k,O_i,h_q}, \beta_{C_k,O_i,h_q} \rangle$$

and

$$\langle \alpha_{C_l,O_i,h_g}, \beta_{C_l,O_i,h_g} \rangle \le \langle \alpha_{C_l,O_j,h_g}, \beta_{C_l,O_j,h_g} \rangle,$$

or

$$\left<\alpha_{C_k,O_i,h_g},\beta_{C_k,O_i,h_g}\right> \geq \left<\alpha_{C_k,O_j,h_g},\beta_{C_k,O_j,h_g}\right>$$

and

$$\langle \alpha_{C_l,O_i,h_q}, \beta_{C_l,O_i,h_q} \rangle \ge \langle \alpha_{C_l,O_i,h_q}, \beta_{C_l,O_i,h_q} \rangle$$

are simultaneously satisfied.

Let  $S_{k,l,q}^{\nu}$  be the number of cases in which

$$\langle \alpha_{C_k,O_i,h_q}, \beta_{C_k,O_i,h_q} \rangle \ge \langle \alpha_{C_k,O_i,h_q}, \beta_{C_k,O_i,h_q} \rangle$$

and

$$\langle \alpha_{C_l,O_i,h_g},\beta_{C_l,O_i,h_g} \rangle \leq \langle \alpha_{C_l,O_j,h_g},\beta_{C_l,O_j,h_g} \rangle,$$

or

$$\left<\alpha_{C_k,O_i,h_g},\beta_{C_k,O_i,h_g}\right> \leq \left<\alpha_{C_k,O_j,h_g},\beta_{C_k,O_j,h_g}\right>$$

and

$$\langle \alpha_{C_l,O_i,h_q}, \beta_{C_l,O_i,h_q} \rangle \ge \langle \alpha_{C_l,O_i,h_q}, \beta_{C_l,O_i,h_q} \rangle$$

are simultaneously satisfied. We can see, that  $S_{k,l,g}^{\mu} + S_{k,l,g}^{\nu} \leq \frac{n(n-1)}{2}$  for each g, so that  $1 \leq g \leq f$ .

Now, for every k, l, g, such that  $1 \le k < l \le m, n \ge 2$  and g is fixed, we define

$$\mu_{C_k,C_l} = 2 \frac{S_{k,l}^{\mu}}{n(n-1)}, \quad \nu_{C_k,C_l} = 2 \frac{S_{k,l}^{\nu}}{n(n-1)}.$$

Hence,

$$\mu_{C_k,C_l,h_g} + \nu_{C_k,C_l,h_g} = 2\frac{S_{k,l,g}^{\mu}}{n(n-1)} + 2\frac{S_{k,l,g}^{\nu}}{n(n-1)} \leq 1.$$

Therefore,  $\langle \mu_{C_k,C_l,h_g}, \nu_{C_k,C_l,h_g} \rangle$  is an IFP.

Now, we can construct the IM

$$R = \left\{ \begin{array}{c|ccc} h_g & C_1 & \cdots & C_m \\ \hline C_1 & \langle \mu_{C_1, C_1, h_g}, \nu_{C_1, C_1, h_g} \rangle & \cdots & \langle \mu_{C_1, C_m, h_g}, \nu_{C_1, C_m, h_g} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ C_m & \langle \mu_{C_m, C_1, h_g}, \nu_{C_m, C_1, h_g} \rangle & \cdots & \langle \mu_{C_m, C_m, h_g}, \nu_{C_m, C_m, h_g} \rangle \end{array} \right. | h_g \in H$$

that determines the degrees of correspondence between criteria  $C_1, \ldots, C_m$ .

Let apply aggregation operation to the 3D-IM  $R = [K, K, H, \{a_{k_i, l_j, h_g}\}]$   $(K, H \subset \mathcal{I}^*)$  and let  $h_0 \notin H$ . Let  $\circ : \mathcal{X} \times \mathcal{X} \longrightarrow \mathcal{X}$  and  $* : \mathcal{X} \times \mathcal{X} \longrightarrow \mathcal{X}$ . Let

$$\langle \circ, * \rangle \in \{\langle min, max \rangle, \langle max, min \rangle, \langle average, average \rangle\}.$$

Follow [18] we used aggregation operations as follows:

# $(\circ) - \alpha_H$ -aggregation

$$\alpha_{(H,\circ)}(R,h_0) = \begin{cases} \frac{k_i & h_0}{k_1} & o & a_{k_1,k_1,h_g} \\ k_1 & o & a_{k_2,k_2,h_g} \\ k_2 & o & a_{k_2,k_2,h_g} & k_i \in K \\ 1 \le g \le f & \vdots & \vdots \\ k_m & o & a_{k_m,k_m,h_g} \\ 1 \le g \le f & \end{cases}$$

Therefore, finally, R obtains the form

$$R = \begin{cases} h_g & C_1 & \cdots \\ C_1 & \langle \underset{1 \leq g \leq f}{\circ} \mu_{C_1, C_1, h_g}, \underset{1 \leq g \leq f}{*} \nu_{C_1, C_1, h_g} \rangle & \cdots \\ \vdots & \vdots & \ddots \\ C_m & \langle \underset{1 \leq g \leq f}{\circ} \mu_{C_m, C_1, h_g}, \underset{1 \leq g \leq f}{*} \nu_{C_m, C_1, h_g} \rangle & \cdots \end{cases}$$

$$\begin{array}{c|c}
 & C_m \\
 & \cdots & \langle \underset{1 \leq g \leq f}{\circ} \mu_{C_1, C_m, h_g}, \underset{1 \leq g \leq f}{*} \nu_{C_1, C_m, h_g} \rangle \\
 & \cdots & \vdots \\
 & \cdots & \langle \underset{1 \leq g \leq f}{\circ} \mu_{C_m, C_m, h_g}, \underset{1 \leq g \leq f}{*} \nu_{C_m, C_m, h_g} \rangle
\end{array}$$

where  $\langle \circ, * \rangle \in \{\langle min, max \rangle, \langle max, min \rangle, \langle average, average \rangle\}.$ 

If the pair  $\langle \circ, * \rangle = \langle min, max \rangle$  is used in this aggregation operation, then we obtain pessimistic forecast of intercriteria correlation coefficient values. With pair  $\langle \circ, * \rangle = \langle max, min \rangle$ , then optimistic evaluations are acquired. With pair  $\langle \circ, * \rangle = \langle average, average \rangle$ , we obtain the averaged estimate of the intercriteria correlation coefficients.

## 4 Conclusion

In the presented research, a three dimensional intercriteria analysis over intuitionistic fuzzy data is discussed. Intercriteria Analysis can be applied over intuitionistic fuzzy data to determine possible correlations between the pairs of criteria. In a next research of the authors, the above described constructions will be extend to the case of 3-dimensional multilayer IMs and will be applied to practical data.

In future, follow [14,15] for the Kendall rank correlation coefficient between two IFSs, we will present another approach to 3-dimensional intercriteria analysis applied over intuitionistic fuzzy data.

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