CS 131 – Problem Set 2
Problems must be submitted by February 7, 2022 at 11:59pm, on Gradescope.

See instructions on PS1 for how and what to submit.

Reminder: At the top of each problem you submit, write either the text "Sources consulted: none" or a list of all sources consulted other than zyBook, lecture notes, and office hours. **This is required.** (Examples of things that should be listed if consulted: a classmate, a tutor, a friend, a website, a textbook other than zyBook etc.) One point will be deducted from every question for which this list is omitted.

Problem 1. (25 points)

a)

    zyBook 1.4.2b (3 points) (1.4.2a will be solved in lab)
    zyBook 1.4.5bc (6 points) (1.4.5ad are revealed)
    zyBook 1.4.6b (3 points) (1.4.6a is revealed)

b)

    zyBook 1.5.4bd (8 points) (1.5.4ac will be solved in lab)
    zyBook 1.5.5a (5 points)

Problem 2. (20 points) Prove the following problems using the laws of propositional logic (not truth tables). And then, implement your proof in Python using the following notations.

	Logical Operators					
Propositional Logic	conjunction	disjunction	negation	xor	if (implication)	iff (bi-implication)
Python	and	or	not	!=	<=	==

You must submit your handwritten (typed) proof in the written portion and four files named `p2a.py`, `p2b.py`, `p2c.py` and `p2d.py` in the code portion. (see Lab 2 Problem 2 for similar examples)

a) $P \leftrightarrow Q \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$

Hint: To prove using the laws of propositional logic, you must start from one of the propositions (either is fine) and find all equivalencies of that proposition (by applying the laws of propositional logic) to reach to the other proposition.

b) $\left((P \rightarrow Q) \vee (P \rightarrow R) \right) \rightarrow \left(P \rightarrow (Q \vee R) \right)$ is a tautology.

Hint: To prove using the laws of propositional logic, you must start with the entire proposition and find all equivalencies of that proposition (by applying the laws of propositional logic) to reach to a proposition which is always **True**.

c) $\neg \left((P \rightarrow Q) \vee (Q \rightarrow R) \right)$ is a contradiction.

Hint: To prove using the laws of propositional logic, you must start with the entire proposition and find all equivalencies of that proposition (by applying the laws of propositional logic) to reach to a proposition which is always **False**.

d) Let $M =$ "I am motivated", $S =$ "I study", and $P =$ "I pass". Prove that the following three statements cannot be simultaneously true (i.e. conjunction of them is equivalent to false) using the laws of propositional logic.

1. If I am motivated, I study

2. If I study, I pass

3. It is not true that if I am motivated, I pass

Problem 3. (15 points) (To start this problem, you need to wait till after Lab 2)

Continuing with Lab 2 Problem 1. Make sure to redo that problem first.

a) Bob's blindfold was then removed and he was asked the same question and he replied, "No." Can Chloe deduce the color of her hat based on Alice and Bob's answers?

Hint: Chloe learns some extra information from Bob's answer when his blindfold is removed. What can Chloe figure out about r_3 (the color of her own hat) using all of the above information? Note that Chloe should be thinking from Bob's point of view.

b) After all, can Alice or Bob figure out the color of their hats? If not, how many options are there for the color of their hats? What are the options?

Problem 4. (40 points) (To start this problem, you need to wait till after Lab 2)

Recall Lab 2 Problem 3:

Assume that you are the main character in a fantasy movie who is looking for a happy endless life. You meet a magician who gives you two boxes with different signs on. The sign on the first box reads "you'll live forever by choosing this box, and you'll have a miserable life by choosing the other box"; and the sign on the second box reads "you'll live forever by choosing one of these boxes, and you'll have a miserable life by choosing the other box."

You've successfully translated the above inscriptions (see Lab 2 Problem 3).

> **Solution for Lab 2 will be posted on BB on Wednesday 2/2 at 2:30 pm.**

You are excitedly about to open up the first box to earn your immortal life. However, before doing so the magician tells you: "Only one box tells the truth while the other tells a lie!" Remembering all the rules of logic taught to you by the wise sage Vahid, you take a minute to ponder the implications of the magician's statement. Do the following parts to deduce which box is the right one for you to find your immortal life, and which will lead to misery.

a) (7 points) Using the variables you made in Lab 2 (use the variable names in the posted solution for Lab 2) , Write down a compound proposition for the magician statement. You can use only AND, OR, NOT or XOR operators.

b) (3 points) Now, make a Boolean expression out of the compound proposition in part a. You can use only $\cdot$, $+$, NOT or XOR operators.

c) (15 points) Simplify the Boolean expression in part b using the laws of Boolean Algebra.

Hint1: You are asked to use Boolean expression to avoid difficulties of propositional logic such as writing the rules. You can treat Boolean expressions as Algebraic equations. So, in your computations use $=$ instead of $\equiv$. Also, do not forget that $0 + p = p$, $0 \cdot p = 0$, $1 \cdot p = p$ and $1 + p = 1$. Here, p is a Boolean variable, 1 means TRUE and 0 means FALSE.

Hint2: Your final answer should be equivalent to "live forever by choosing box 2 AND live miserably by choosing box 1 AND NOT (both live forever by choosing box 1 AND live miserably by choosing box 2)". Your job is figuring out how.

d) (5 points) Using your result from part c and the fact that the magician tells the truth, which box should you choose to get an immortal life? Explain your reasoning in English.

e) (5 points) Now that you've deduced which box lends you an immortal life, which inscription tells the truth and which one is a lie?

f) (5 points) As you're about to open the correct box, it occurs to you that you had just been assuming that immortality and a miserable life were mutually exclusive! Is that a correct assumption based on the information you've been given so far? If its not correct, what are the implications? Explain your reasoning and whether you choose to proceed to open the box!