In-class Exercises: Projection and Minimal Basis

1. Suppose we have these FDs: : $S = \{ABE \rightarrow CF, DF \rightarrow BD, C \rightarrow DF, E \rightarrow A, AF \rightarrow B\}$

Project the FDs onto: L = CDEF

Attributes to take all subsets X of:	Closure of the subset	
C D E F	X^+	Functional dependencies inferred

Solution:

С	D	Е	F	closure	FDs
\checkmark				$C^+ = CDFBD$	C o DF
	√			$D^+ = D$	nothing
		✓		$E^+ = EA$	nothing
			✓	$F^+ = F$	nothing
\checkmark	\checkmark			$CD^+ = CDFB$	nothing, since $CD \to DF$ is weaker than $C \to DF$ which we have already
\checkmark		✓		$CE^+ = CEDFAB$	nothing, since $CE \to DF$ is weaker than $C \to DF$ which we have already
\checkmark			✓	$CF^+ = CFDB$	nothing, since $CF \to D$ is weaker than $C \to DF$ which we have already
	✓	√		$DE^+ = DEA$	nothing
	✓		✓	$DF^+ = DFB$	nothing
		√	✓	$EF^+ = EFABCD$	$EF \to CD$
\checkmark	√	√		$CDE^+ = CDEF$	nothing, since $CDE \to F$ is weaker than $C \to DF$ which we have already
\checkmark	✓		✓	$CDF^+ = CDFB$	nothing
\checkmark		√	✓	since EF is a key, supersets of EF can only yield FDs that are weaker than ones we have.	
	✓	√	✓	since EF is a key, supersets of EF can only yield FDs that are weaker than ones we have.	

Final answer: The projection of S onto L is $C \to DF$, $EF \to CD$.

2.	Find a minimal basis for this set of FDs: $S = \{ABF \rightarrow G, \ BC \rightarrow H, \ BCH \rightarrow EG, \ BE \rightarrow GH\}.$
	Solution:
	Step 1: Split the RHSs to get our initial set of FDs. S1:



(b)
$$BC \to H$$

(c)
$$BCH \rightarrow E$$

(d)
$$BCH \rightarrow G$$

(e)
$$BE \to G$$

(f)
$$BE \to H$$
.

Step 2: For each FD, try to reduce the LHS:

- (a) $A^+ = A, B^+ = B, F^+ = F$. In fact, no singleton LHS yields anything. $AB^+ = AB, AF^+ = AF$, and $BF^+ = BF$, so none of them yields G either. We cannot reduce the LHS of this FD.
- (b) Since this FD has only two attributes on the LHS, and no singleton LHS yields anything, we cannot reduce the LHS of this FD.
- (c) Since no singleton LHS yields anything, we need only consider LHSs with two or more attributes. We only have three to begin with, so that leaves LHSs with two attributes. $BC^+ = BCHEG$. So we can reduce the LHS of this FD, yielding the new FD: $BC \to E$.
- (d) By the same argument, we can reduce this FD to: $BC \to G$.
- (e) Since no singleton LHS yields anything, we cannot reduce the LHS of this FD.
- (f) Since no singleton LHS yields anything, we cannot reduce the LHS of this FD.

Our new set of FDs, let's call it S2, is

(a)
$$ABF \rightarrow G$$

(b)
$$BC \rightarrow H$$

(c)
$$BC \rightarrow E$$

(d)
$$BC \to G$$

(e)
$$BE \to G$$

(f)
$$BE \to H$$
.

Step 3: Try to eliminate each FD.

- (a) $ABF_{S2-(a)}^+ = ABF$. We need this FD.
- (b) $BC_{S2-(b)}^+ = BCEG\underline{H}$. We can remove this FD.

(c)
$$BC_{S2-\{(b),(c)\}}^+ = BCG$$
. We need this FD.

- (d) $BC_{S2-\{(b),(d)\}}^+ = BCE\underline{G}H$. We can remove this FD.
- (e) $BE_{S2-\{(b),(d),(e)\}}^+ = BEH$. We need this FD.
- (f) $BE_{S2-\{(b),(d),(f)\}}^+ = BEG$. We need this FD.

Our final set of FDs is:

(a)
$$ABF \rightarrow G$$

(b)
$$BC \rightarrow E$$

(c)
$$BE \to G$$

(d)
$$BE \rightarrow H$$
.