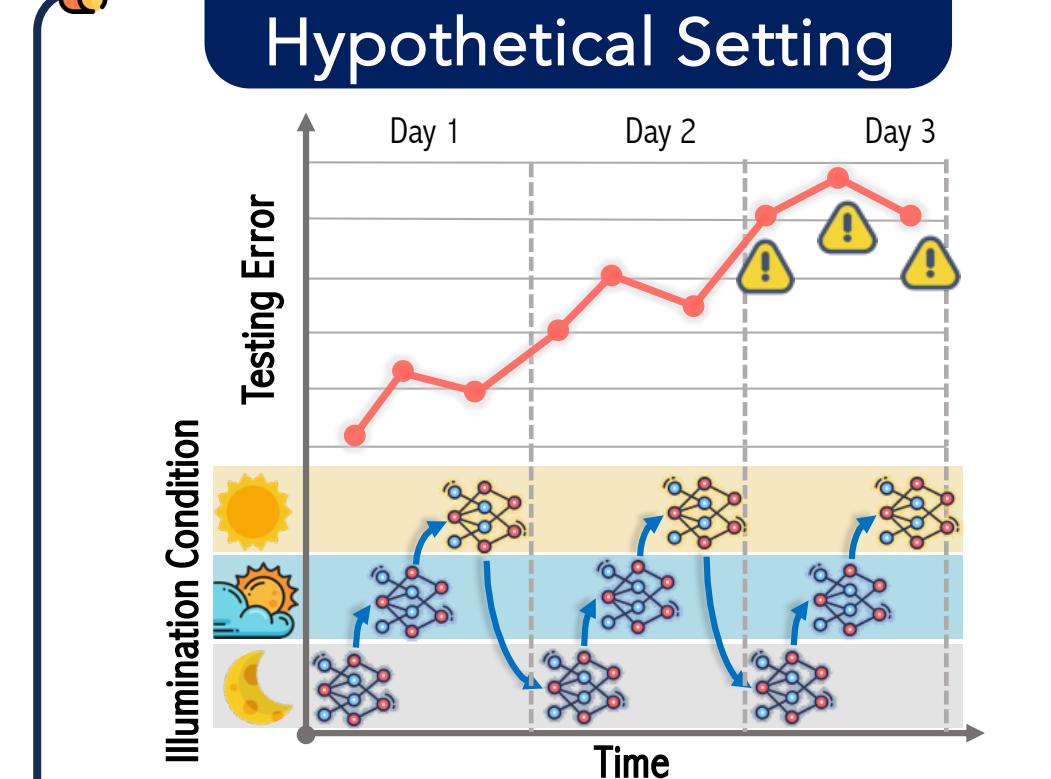


INTRODUCTION

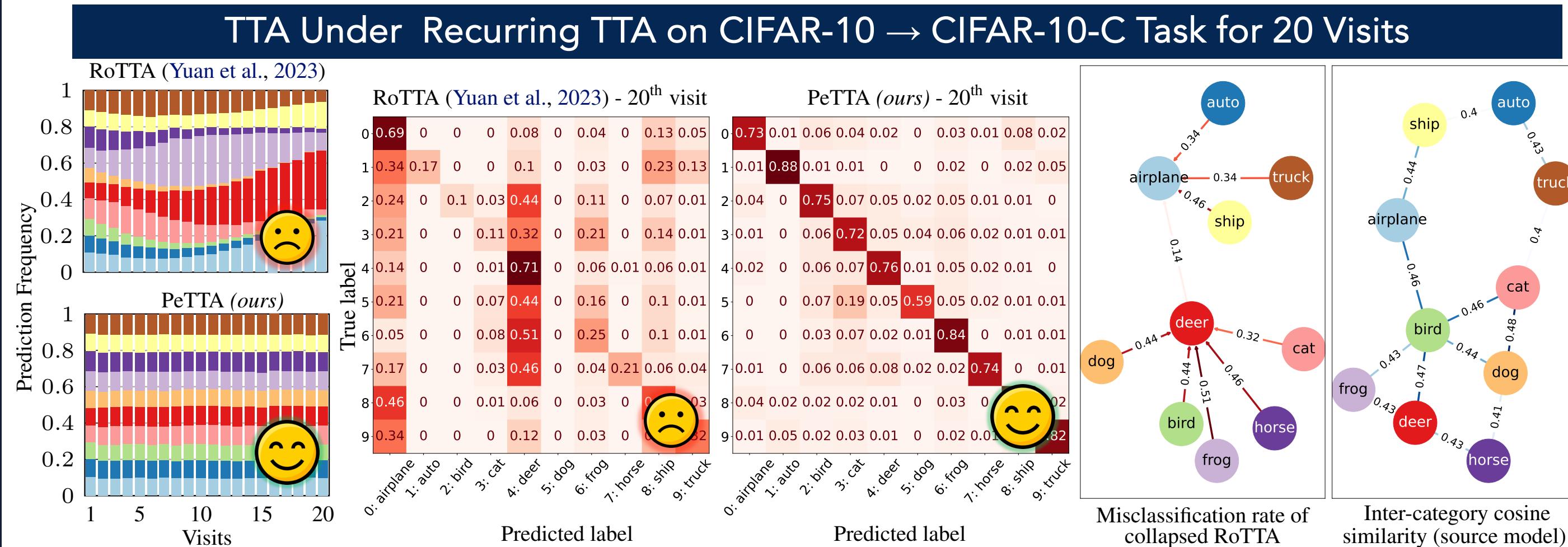
Continual Test-time Adaptation (TTA) operates on an ML classifier $f_t: \mathcal{X} \rightarrow \mathcal{Y}$, parameterized by $\theta_t \in \Theta$ gradually changing over time. The model explores an online stream of testing data $X_t \sim P_t$ for adapting itself $f_{t-1} \rightarrow f_t$ (self-supervised learning) before predicting $\hat{Y}_t = f_t(X_t)$.

Does the model adaptability persist after a long time adapting to multiple data shifts?

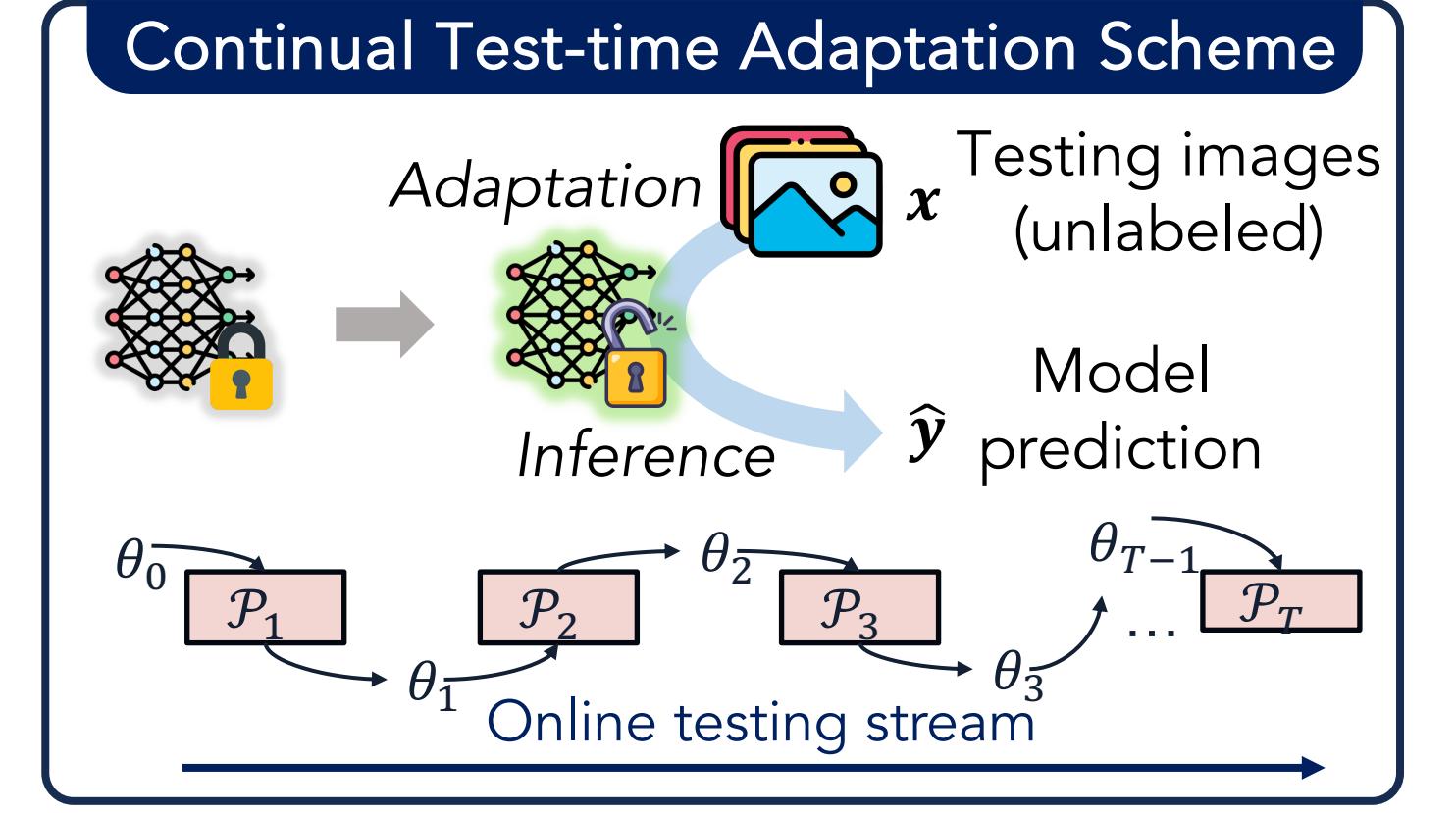


- In practice, testing environments may *change recurrently*.
- Preserving adaptability when visiting the same testing condition is *not guaranteed*.

Recurring Test-time Adaptation: $\mathcal{P}_1 \rightarrow \mathcal{P}_2 \rightarrow \dots \rightarrow \mathcal{P}_D \rightarrow \dots \rightarrow \mathcal{P}_1 \rightarrow \mathcal{P}_2 \rightarrow \dots \rightarrow \mathcal{P}_D$



- (a) Histogram of model PeTTA achieves a persisting performance while RoTTA degrades.
 (b) Confusion matrix at the last visit (c) Force-directed graph showing (left) the most prone to misclassification; (right) similar categories tend to be easily collapsed.



ε-PERTURBED GAUSSIAN MIXTURE MODEL CLASSIFIER (ε-GMMC)

ε-GMMC - a simple yet representative failure case of TTA for theoretical analysis

Setting: A simplified continual TTA process

- Let $p_{y,t} = \Pr(Y_t = y); \hat{p}_{y,t} = \Pr(\hat{Y}_t = y)$.
- Binary classification $\mathcal{X} \times \mathcal{Y} = \mathbb{R} \times \{0,1\}$.
- Underlying distribution follows a mixture of 2 Gaussian: $P_t(x, y) = p_{y,t} \mathcal{N}(x; \mu_y, \sigma_y^2)$.

Main Task: predicting X_t was sampled from cluster 0 or 1 (negative or positive).

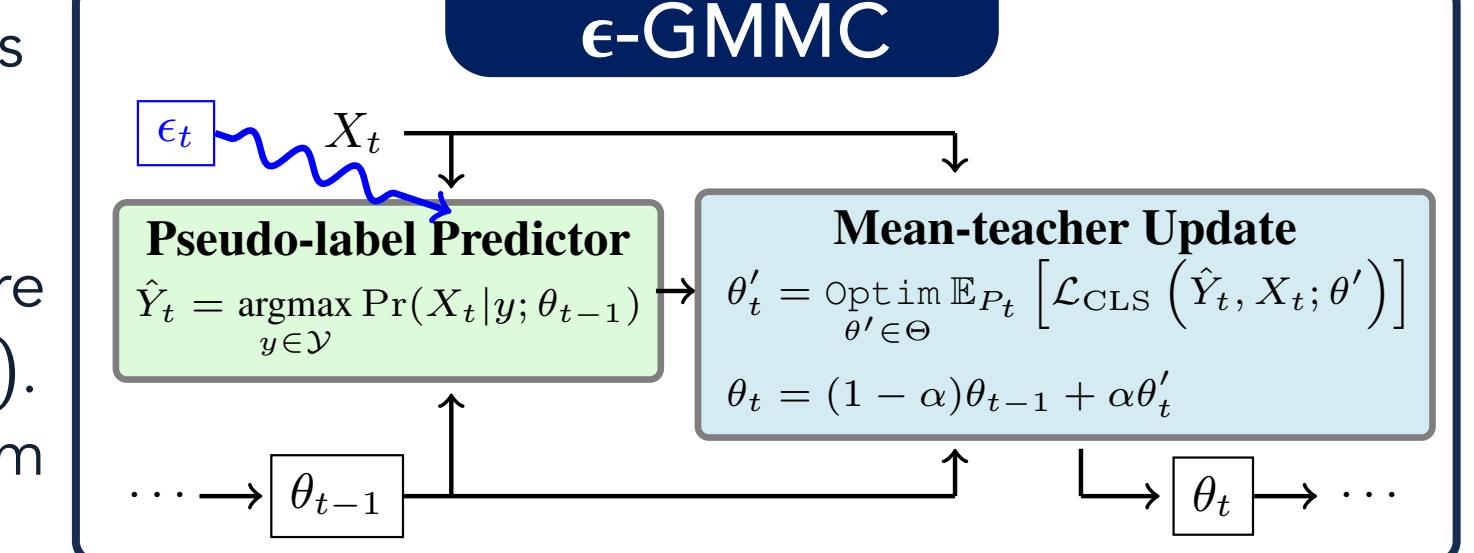
A Mathematical Definition of Model Collapse

Definition 1 (Model Collapse). A model is said to be collapsed from step $\tau \in \mathcal{T}, \tau < \infty$ if there exists a non-empty subset of categories $\tilde{\mathcal{Y}} \subseteq \mathcal{Y}$ such that $\Pr\{\hat{Y}_t \in \tilde{\mathcal{Y}}\} > 0$ but the marginal $\Pr\{\hat{Y}_t \in \tilde{\mathcal{Y}}\}$ converges to zero in probability:

$$\lim_{t \rightarrow \tau} \Pr\{\hat{Y}_t \in \tilde{\mathcal{Y}}\} = 0.$$

Factors contributing to the model collapse:

- (i) **Data-dependent factors:** the prior data distribution (p_0), the nature difference between two categories ($|\mu_0 - \mu_1|$) from the dataset.
- (ii) **Algorithm-dependent factors:** update rate (α), the false negative rate at each step (ε_t).



ε-GMMC performs 2 main steps:

- Predicting pseudo-labels (\hat{Y}_t).
- Updating with mean teacher model.

Key Idea: The predictor is perturbed for retaining a **false negative rate (FNR)** of $\varepsilon_t = \Pr\{\hat{Y}_t = 1 | \hat{Y}_t = 0\}$ to simulate undesirable effects of the testing stream in TTA, making model prone to collapse.

Assumption 1 (Static Data Stream). The marginal distribution of the true label follows the same Bernoulli distribution $\text{Ber}(p_0): p_{0,t} = p_0, (p_{1,t} = p_1 = 1 - p_0), \forall t \in \mathcal{T}$.

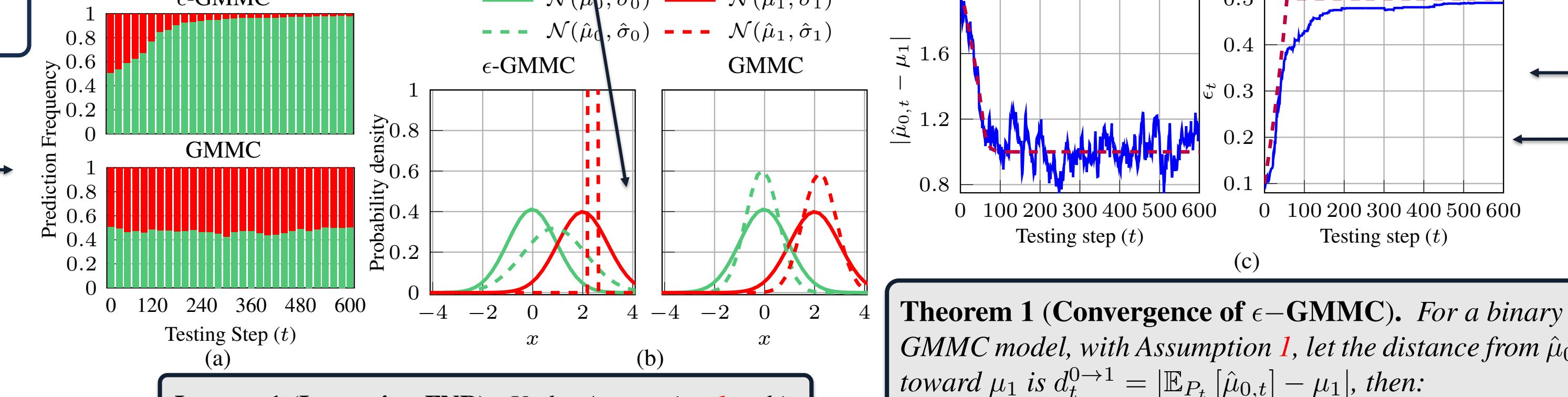
Lemma 2 (ε-GMMC After Collapsing). For a binary ε-GMMC model, with Assumption 1, if $\lim_{t \rightarrow \tau} \hat{p}_{1,t} = 0$ (collapsing), the cluster 0 in GMMC converges in distribution to a single-cluster GMMC with parameters:

$$\mathcal{N}(\hat{\mu}_{0,t}, \hat{\sigma}_{0,t}^2) \xrightarrow{d} \mathcal{N}(p_0 \mu_0 + p_1 \mu_1, p_0 p_0^2 + p_1 p_1^2 + p_0 p_1 (\mu_0 - \mu_1)^2).$$

Corollary 1 (A Condition for ε-GMMC Collapse). With fixed $p_0, \alpha, \mu_0, \mu_1$, ε-GMMC is collapsed if there exists a sequence of $\{\varepsilon_t\}_{\tau - \Delta_\tau}^\tau$ ($\tau \geq \Delta_\tau > 0$) such that:

$$p_1 \geq \varepsilon_t > 1 - \frac{d_{t-1}^{0 \rightarrow 1}}{|\mu_0 - \mu_1|}, \quad t \in [\tau - \Delta_\tau, \tau].$$

Numerical Simulation — **Theoretical Result**



Lemma 1 (Increasing FNR). Under Assumption 1, a binary ε-GMMC would collapsed (Def. 1) with $\lim_{t \rightarrow \tau} \hat{p}_{1,t} = 0$ (or $\lim_{t \rightarrow \tau} \hat{p}_{0,t} = 1$, equivalently) if and only if $\lim_{t \rightarrow \tau} \varepsilon_t = p_1$.

Theorem 1 (Convergence of ε-GMMC). For a binary ε-GMMC model, with Assumption 1, let the distance from $\hat{\mu}_{0,t}$ toward μ_1 is $d_t^{0 \rightarrow 1} = |\mathbb{E}_{P_t} [\hat{\mu}_{0,t}] - \mu_1|$, then:

$$d_t^{0 \rightarrow 1} - d_{t-1}^{0 \rightarrow 1} \leq \alpha \cdot p_0 \cdot \left(|\mu_0 - \mu_1| - \frac{d_{t-1}^{0 \rightarrow 1}}{1 - \varepsilon_t} \right).$$

PERSISTENT TEST-TIME ADAPTATION (PeTTA)

Key Idea: Striking a balance between **adaptation** and **preventing model collapse**

With ϕ_{θ_t} is the deep feature extractor of f_t , let $\mathbf{z} = \phi_{\theta_t}(\mathbf{x})$. Keeping track of a collection of the running mean of feature vector \mathbf{z} : $\{\hat{\mu}_t^y\}_{y \in \mathcal{Y}}$ in which $\hat{\mu}_t^y$ is exponential moving average updated with vector \mathbf{z} if $f_t(\mathbf{x}) = y$.

Persistent TTA

(2) Adaptive Learning Rate α_t and Regularization λ_t (1) Sensing the divergence from θ_0

$$\gamma_t^y = 1 - \exp \left(-(\hat{\mu}_t^y - \mu_0^y)^T (\Sigma_0^y)^{-1} (\hat{\mu}_t^y - \mu_0^y) \right)$$

μ_0^t, Σ_0^t are pre-computed on the source distribution

$$\tilde{\gamma}_t = \frac{1}{|\mathcal{Y}_t|} \sum_{y \in \mathcal{Y}_t} \gamma_t^y, \quad \hat{Y}_t = \{\hat{Y}_t^{(i)} | i = 1, \dots, N_t\}$$

$$\lambda_t = \tilde{\gamma}_t \cdot \lambda_0, \quad \alpha_t = (1 - \tilde{\gamma}_t) \cdot \alpha_0,$$

$$\theta'_t = \text{Optim} \mathbb{E}_{\theta' \in \Theta} [\mathcal{L}_{\text{CLS}} (\hat{Y}_t, X_t; \theta') + \mathcal{L}_{\text{AL}} (X_t; \theta')] + \lambda_t \mathcal{R}(\theta')$$

$$\theta_t = (1 - \alpha_t) \theta_{t-1} + \alpha_t \theta'_t.$$

$$\mathcal{L}_{\text{AL}} (X_t; \theta) = - \sum_{y \in \mathcal{Y}} \Pr(y | X_t; \theta_0) \log \Pr(y | X_t; \theta)$$

EXPERIMENTAL RESULTS

Average classification error on the task ImageNet → ImageNet-C for 20 recurring TTA visits.

Method	Recurring TTA visit	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Avg
Source																						82.0
LAME (Boudiaf et al., 2022)																						80.9
CoTTA (Wang et al., 2022)	98.6	99.1	99.4	99.4	99.5	99.5	99.5	99.5	99.6	99.6	99.6	99.6	99.6	99.6	99.6	99.6	99.6	99.7	99.7	99.7	99.5	
RMT (Döbler et al., 2022)	72.3	71.0	69.9	69.1	68.8	68.5	68.4	68.3	70.0	70.2	70.1	70.2	72.8	76.8	75.6	75.1	75.1	75.2	74.8	74.7	71.8	
MECTA (Hong et al., 2023)	77.2	82.8	86.1	87.9	88.9	89.4	89.8	89.9	90.0	90.4	90.6	90.7	90.8	90.9	90.9	90.8	90.8	90.7	90.7	90.8	90.9	
RoTTA (Yuan et al., 2023)	68.3	62.1	61.8	64.5	68.4	75.4	82.7	95.1	95.8	96.6	97.1	97.9	98.3	98.7	99.0	99.1	99.3	99.4	99.5	99.6	87.9	
RDumb (Press et al., 2023)	72.2	73.0	73.2	72.8	72.2	72.8	73.3	72.7	73.1	72.1	72.0	72.7	73.3	73.1	72.6	72.6	73.1	73.1	73.1	73.1	72.8	
PeTTA (ours) ^(*)	65.3	61.7	59.8	59.1	59.4	59.6	59.3	59.3	59.4	60.0	60.3	61.0	60.7	60.4	60.2	60.5	60.2	60.5	60.2	60.5	60.5	

Does model reset help? A comparison with a reset-based approach at different frequencies.

Reset Every	Recurring TTA visit	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Avg
T = 1000	72.2	73.0	73.2	72.8	72.8	73.3	72.7	71.9	73.0	73.2	73.1	72.0	72.7	73.3	73.1	72.1	72.6	73.3	73.1	72.8	72.8	
T = 5000	70.2	70.8	71.6	72.1	72.4	72.6	72.9	73.1	73.2	73.6	73.7	73.9	74.0	74.2	74.3	74.1	73.8	73.5	73.7	71.9	71.8	
PeTTA (ours) ^(*)	65.3	61.7	59.8	59.1	59.4	59.6																