Data Analysis: Band Pass Filters

Signal Processing and Filtering Techniques for Data Visualization

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1 Introduction

This document details information about the filtering methods available in the gas data visualization dashboard.

2 Butterworth Filter

The Butterworth filter is a commonly used bandpass filter characterized by its maximally flat frequency response in the passband. It provides a smooth roll-off and no ripples, making it ideal for applications where a clean frequency response is required. The magnitude response of an nth-order Butterworth filter is given by:

$$|H(f)| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_c}\right)^{2n}}}$$

where f_c is the cutoff frequency and n is the filter order. This filter allows frequencies within a specified band to pass through with minimal attenuation while effectively suppressing frequencies outside this band.

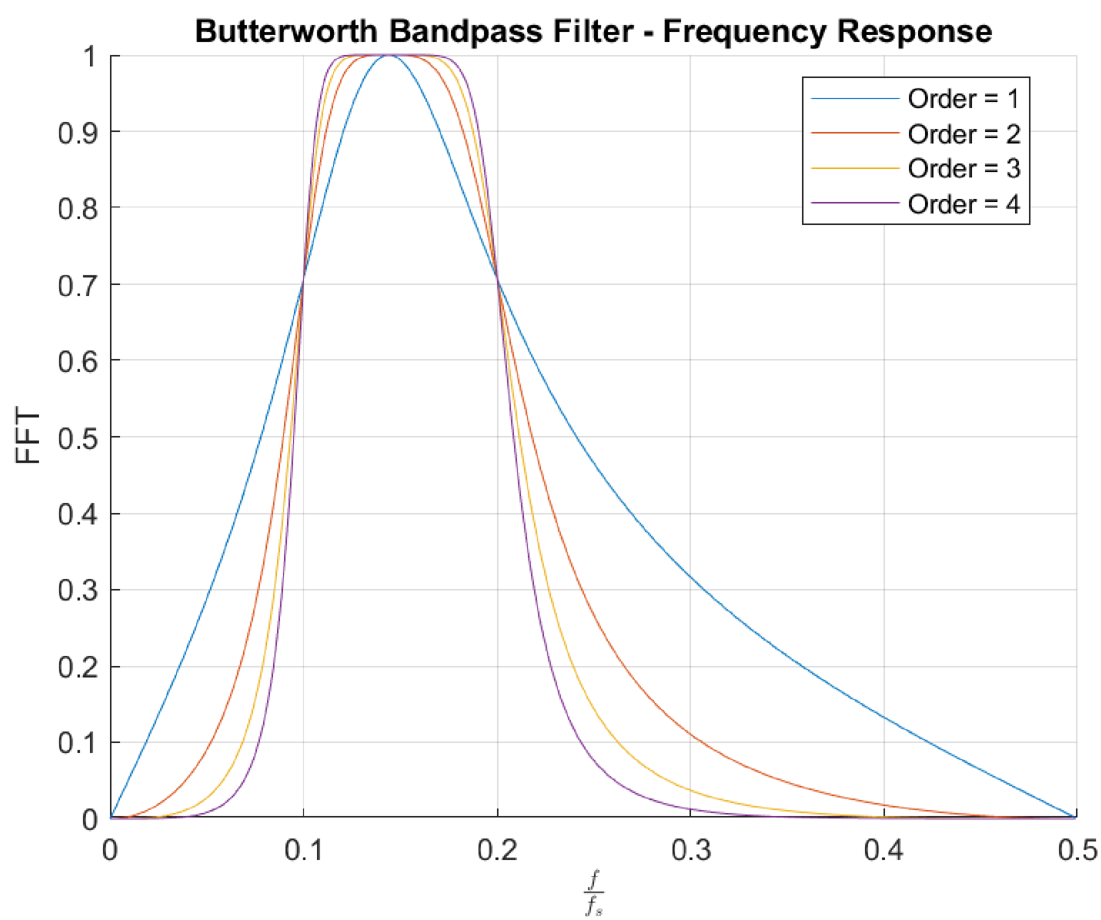

Figure 1: Frequency response of the Butterworth filter.

The filter used in the dashboard is a Butterworth filter of order 4, cutting the frequencies below 0.8 Hz and above 16 Hz.

3 Band Pass Filters Using Averaging Approach

The following filters were designed to mimic the processing performed in the analysis Excel sheet. All operate based on the following principle:

Given a signal $s(n)$ with $n \in [1, N]$, we first remove the DC component by subtracting the mean:

$$s_{\text{centered}}(n) = s(n) - \frac{1}{N}\sum_{n=1}^{N} s(n)$$

This step acts as a rudimentary high-pass filter to remove the zero-frequency component. We then apply an averaging filter, which acts as a low-pass filter:

$$s_{\text{filtered}}(n) = (s_{\text{centered}} * \frac{h}{A})(n)$$

where $h(n)$ is the impulse response (transfer function) of the averaging filter, and $A = \sum_{n \in Z} |h(n)|$ a normalization constant that guarantees

3.1 Moving Average

The moving average filter averages the signal over K points:

$$h(n) = \frac{1}{K}\sum_{k=0}^{K-1} \delta(n-k)$$

This results in a smoothing effect by averaging adjacent samples.

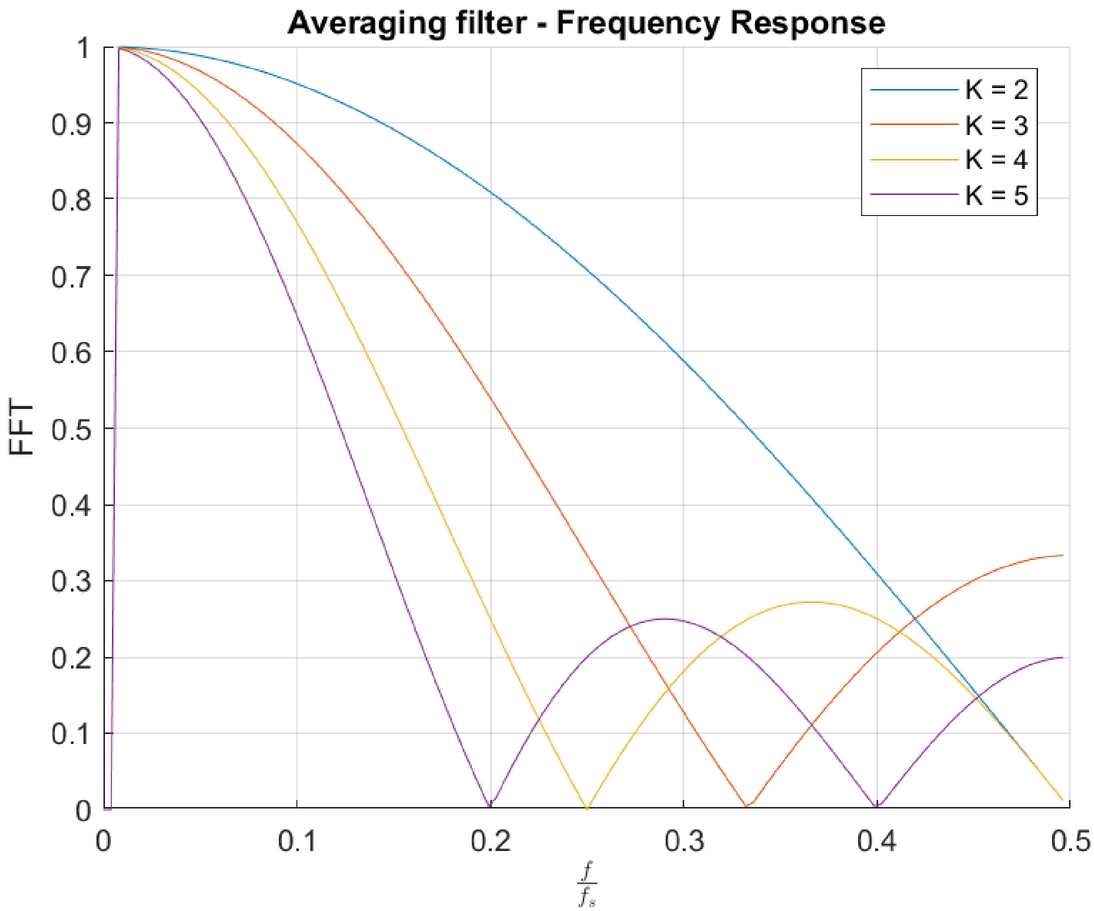

Figure 2: Frequency response of the moving average filter.

As shown in Figure 2, the relatively high secondary lobes introduce unwanted high-frequency components in the filtered signal. The frequency axis is normalized; to convert to Hertz, multiply by the sampling frequency f_s.

3.2 Weighted Moving Average

In this filter, each output point is a weighted average of its $2K+1$ neighbors (including itself). Weights α_k are introduced to reduce the amplitude of secondary lobes in the frequency response:

$$h(n) = \frac{1}{A} \sum_{k=-K}^{K} \alpha_k \delta(n-k)$$

where

$$\alpha_k = ReLU\left(2 - \cosh\left(\frac{k}{K}\right)\right)$$

and

$$A = \sum_{k=-K}^{K} \alpha_k = 4K - \frac{\sinh\left(\frac{1}{2K}+1\right)}{\sinh\left(\frac{1}{2K}\right)} + 2$$

is a normalization constant.

This choice gradually reduces the contribution of samples farther from the center, leading to a smoother and more localized impulse response.

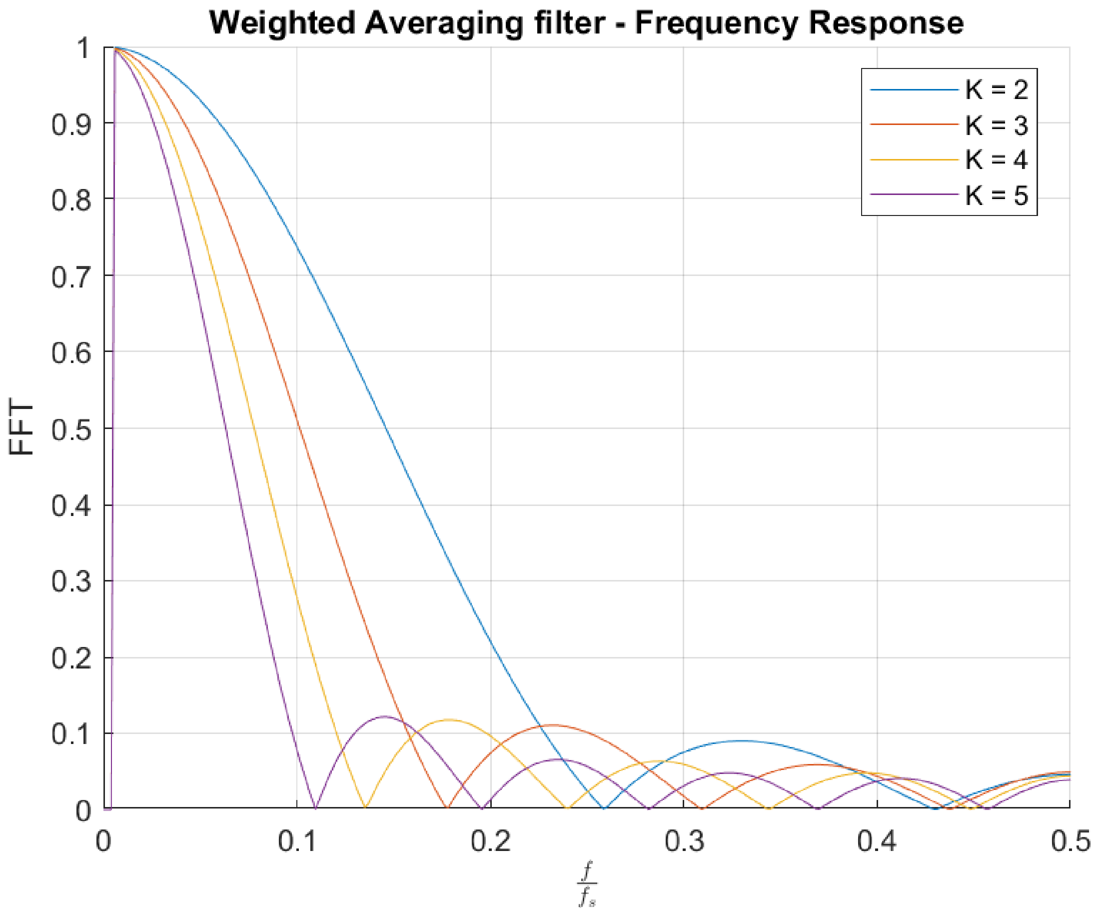

Figure 3: Frequency response of the weighted moving average filter.

3.3 Hamming Window Filter

This method involves convolving the signal with a Hamming window, yielding a filter whose secondary lobes are significantly reduced, thus minimizing unwanted high-frequency components:

$$h(n) = \frac{1}{A}\left[\alpha - \beta\cos\left(\frac{2\pi n}{K-1}\right)\right], \quad n \in [0, K-1], \quad \begin{cases}\alpha = 0.54\\ \beta = 0.46\end{cases}$$

where α and β are constants defining the Hamming window shape, and

$$A = \alpha K + \frac{\beta \sin\left(\frac{\pi K}{1-K}\right)}{\sin\left(\frac{\pi}{1-K}\right)}$$

is the normalization constant.

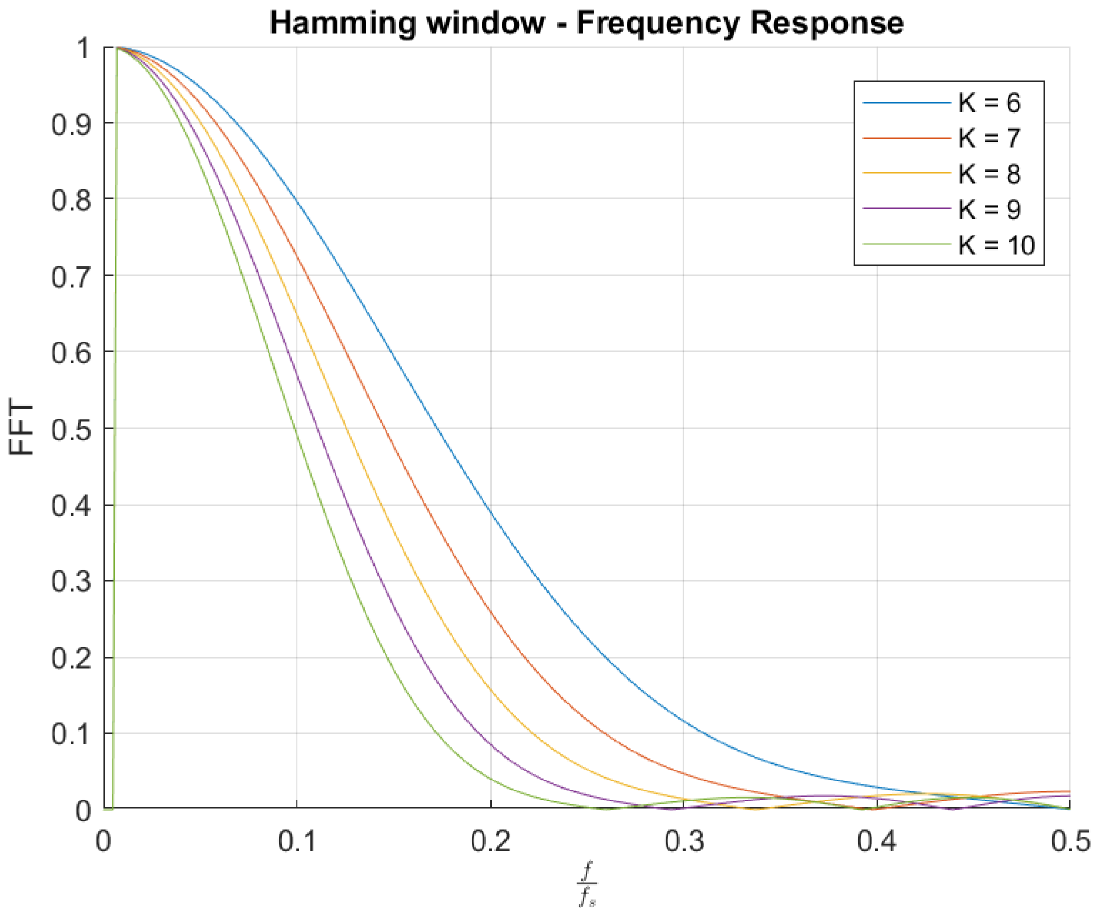

Figure 4: Frequency response of the Hamming window filter.