

Chulalongkorn University

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```
1 Contest
2 Data structures
                                                               1
3 Tree
4 Flows
Contest (1)
template.hpp
<bits/stdc++.h>, <ext/pb_ds/assoc_container.hpp>, <ext/pb_ds/tree_policy.hpp> 1ac233,
using namespace std;
using namespace gnu pbds;
using 11 = long long;
using db = long double;
using vi = vector<int>;
using v1 = vector<11>;
using vd = vector<db>;
using pii = pair<int,int>;
using pll = pair<11,11>;
using pdd = pair<db, db>;
const int INF=0x3fffffff;
const ll LINF=0x1ffffffffffffffff;
const db DINF=numeric limits<db>::infinitv();
const db EPS=1e-9;
const db PI=acos(db(-1));
template < class T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
mt19937 rng(chrono::steady_clock::now().time_since_epoch().
    count());
mt19937_64 rng64(chrono::steady_clock::now().time_since_epoch()
    .count());
.bashrc
alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++17 \
  -fsanitize=undefined,address'
xmodmap -e 'clear lock' -e 'keycode 66=less greater' \#caps = \diamondsuit
.vimrc
set cin aw ai is ts=4 sw=4 tm=50 nu noeb bg=dark ru cul
sy on | im jk <esc> | im kj <esc> | no;:
" Select region and then type : Hash to hash your selection.
" Useful for verifying that there aren't mistypes.
ca Hash w !cpp -dD -P -fpreprocessed \| tr -d '[:space:]' \
\| md5sum \| cut -c-6
hash.sh
# Hashes a file, ignoring all whitespace and comments. Use for
# verifying that code was correctly typed.
cpp -dD -P -fpreprocessed | tr -d '[:space:]' | md5sum | cut -c-6
troubleshoot.txt
                                                          52 lines
```

Write a few simple test cases if sample is not enough.

Are time limits close? If so, generate max cases.

Is the memory usage fine?

```
Could anything overflow?
Make sure to submit the right file.
Wrong answer:
Print your solution! Print debug output, as well.
Are you clearing all data structures between test cases?
Can your algorithm handle the whole range of input?
Read the full problem statement again.
Do you handle all corner cases correctly?
Have you understood the problem correctly?
Any uninitialized variables?
Any overflows?
Confusing N and M, i and j, etc.?
Are you sure your algorithm works?
What special cases have you not thought of?
Are you sure the STL functions you use work as you think?
Add some assertions, maybe resubmit.
Create some testcases to run your algorithm on.
Go through the algorithm for a simple case.
Go through this list again.
Explain your algorithm to a teammate.
Ask the teammate to look at your code.
Go for a small walk, e.g. to the toilet.
Is your output format correct? (including whitespace)
Rewrite your solution from the start or let a teammate do it.
Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).
Time limit exceeded:
Do you have any possible infinite loops?
What is the complexity of your algorithm?
Are you copying a lot of unnecessary data? (References)
How big is the input and output? (consider scanf)
Avoid vector, map. (use arrays/unordered_map)
What do your teammates think about your algorithm?
Memory limit exceeded:
What is the max amount of memory your algorithm should need?
Are you clearing all data structures between test cases?
Data structures (2)
HashMap.h
Description: Hash map with mostly the same API as unordered_map, but
~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if
provided).
                                                     d77092, 7 lines
#include <bits/extc++.h>
// To use most bits rather than just the lowest ones:
struct chash { // large odd number for C
```

**const** uint64\_t C = 11(4e18 \* acos(0)) | 71;

LineContainer.hpp

"src/contest/template.hpp"

Time:  $\mathcal{O}(\log N)$ 

Description: Line Container (Minimize).

11 operator()(11 x) const { return \_\_builtin\_bswap64(x\*C); }

\_\_gnu\_pbds::gp\_hash\_table<11, int, chash> h({}, {}, {}, {}, {1<<16});

24a5c4, 37 lines

```
Line(ll p):m(0),c(0),p(p){}
    bool operator<(const Line &o)const{</pre>
        return querymode?p<o.p:m>o.m;
};
bool Line::querymode=false;
struct LineContainer:multiset<Line>{
    ll div(ll a, ll b) {
        return a/b-((a^b)<0&&a%b);
    bool isect(iterator x, iterator y) {
        if (y==end()) return x->p=LINF, false;
        if(x->m==y->m)x->p=x->c<=y->c?LINF:-LINF;
        else x->p=div(x->c-y->c,y->m-x->m);
        return x->p>=y->p;
    void add(ll m,ll c){
        auto x=insert(Line(m,c)), y=next(x);
        while(isect(x,y))y=erase(y);
        if((y=x)!=begin()\&\&isect(--x,y))isect(x,erase(y));
        while ((y=x)!=begin() && (--x)->p>=y->p) is ect (x,erase(y));
    ll get(ll x){
        if(empty())return LINF;
        Line::querymode=true;
        auto l=lower_bound(Line(x));
        Line::querymode=false;
        return 1->m*x+1->c;
};
Description: A short self-balancing tree. It acts as a sequential container
with log-time splits/joins, and is easy to augment with additional data.
Time: \mathcal{O}(\log N)
struct Node {
  Node *1 = 0, *r = 0;
  int val, y, c = 1;
  Node(int val) : val(val), y(rand()) {}
  void recalc();
int cnt(Node* n) { return n ? n->c : 0; }
void Node::recalc() { c = cnt(1) + cnt(r) + 1; }
template < class F > void each (Node * n, F f) {
  if (n) { each(n->1, f); f(n->val); each(n->r, f); }
pair<Node*, Node*> split(Node* n, int k) {
  if (!n) return {};
  if (cnt(n->1) >= k) { // "n-> val >= k" for lower_bound(k)}
    auto [L,R] = split(n->1, k);
    n->1 = R:
    n->recalc();
    return {L, n};
    auto [L,R] = split(n\rightarrow r, k - cnt(n\rightarrow 1) - 1); // and just "k"
    n->r = L;
    n->recalc();
```

struct Line{

11 m,c;

mutable 11 p;

static bool querymode;

Line(ll m, ll c):m(m), c(c), p(0) {}

return {n, R};

```
Node* merge(Node* 1, Node* r) {
  if (!1) return r;
  if (!r) return 1;
  if (1->y > r->y) {
    1->r = merge(1->r, r);
    return 1->recalc(), 1;
    r->1 = merge(1, r->1);
    return r->recalc(), r;
Node* ins(Node* t, Node* n, int pos) {
  auto [l,r] = split(t, pos);
  return merge(merge(1, n), r);
// Example application: move the range (l, r) to index k
void move(Node*& t, int 1, int r, int k) {
  Node *a, *b, *c;
  tie(a,b) = split(t, 1); tie(b,c) = split(b, r - 1);
  if (k \le 1) t = merge(ins(a, b, k), c);
  else t = merge(a, ins(c, b, k - r));
SegmentTreeBeats.hpp
Description: Segment Tree Beats
"src/contest/template.hpp"
                                                     bc2a26, 134 lines
struct SegmentTreeBeats{
    struct Node{
        11 sum, add;
        11 mn, mn2, fn;
        11 mx, mx2, fx;
        Node(){
            sum=add=fn=fx=0,mn=mn2=LINF,mx=mx2=-LINF;
        Node(ll v){
            sum=mn=mx=v,add=0,mn2=LINF,mx2=-LINF,fn=fx=1;
        friend Node operator+(const Node &1,const Node &r) {
            Node res:
            res.sum=1.sum+r.sum;
            res.add=0:
            if(1.mx>r.mx){
                res.mx=1.mx,res.fx=1.fx;
                res.mx2=max(1.mx2,r.mx);
            }else if(r.mx>1.mx){
                res.mx=r.mx,res.fx=r.fx;
                res.mx2=max(r.mx2,1.mx);
            }else{
                res.mx=1.mx,res.fx=1.fx+r.fx;
                res.mx2=max(1.mx2,r.mx2);
            if(1.mn<r.mn){
                res.mn=1.mn,res.fn=1.fn;
                res.mn2=min(1.mn2,r.mn);
            }else if(r.mn<1.mn){</pre>
                res.mn=r.mn,res.fn=r.fn;
                res.mn2=min(r.mn2,1.mn);
                res.mn=1.mn,res.fn=1.fn+r.fn;
                res.mn2=min(1.mn2, r.mn2);
            return res;
```

```
void apply(int 1,int r,ll v){
         sum+=(r-1+1)*v;
        mx+=v, mx2+=v;
        mn+=v, mn2+=v;
        add+=v:
    void chmin(ll v){
        if (v>=mx) return;
        sum+=(v-mx)*fx;
        if (mn==mx) mn=v;
        if (mn2==mx)mn2=v;
        mx=v;
    void chmax(ll v) {
        if (v<=mn) return;</pre>
        sum+=(v-mn)*fn;
        if (mx==mn) mx=v;
        if (mx2==mn) mx2=v;
        mn=v:
};
int n;
vector<Node> t;
SegmentTreeBeats(){}
SegmentTreeBeats(int n) {init(n, [&] (int) {return 0;});}
template < class F>
SegmentTreeBeats(int n,const F &f) {init(n,f);}
template < class F>
void init(int _n,const F &f) {
    int s=1;
    while(s<n*2)s<<=1;
    t.assign(s,Node());
    build(f);
template < class F>
void build(int 1,int r,int i,const F &f) {
    if(l==r)return void(t[i]=f(l));
    int m = (1+r)/2;
    build(1, m, i*2, f);
    build(m+1, r, i*2+1, f);
    pull(i);
void pull(int i) {
    t[i]=t[i*2]+t[i*2+1];
void push(int 1, int r, int i) {
    int m = (1+r)/2;
    t[i*2].apply(l,m,t[i].add);
   t[i*2+1].apply(m+1,r,t[i].add);
    t[i*2].chmin(t[i].mx);
   t[i*2+1].chmin(t[i].mx);
   t[i*2].chmax(t[i].mn);
    t[i*2+1].chmax(t[i].mn);
    t[i].add=0;
void range_add(int 1,int r,int i,int x,int y,11 v){
    if (y<1 | | r<x) return;</pre>
    if (x<=l&&r<=y) return t[i].apply(l,r,v);</pre>
    int m = (1+r)/2;
    push(l,r,i);
    range_add(1, m, i*2, x, y, v);
    range_add(m+1, r, i*2+1, x, y, v);
    pull(i);
void range_chmin(int 1,int r,int i,int x,int y,ll v){
    if (y<1 | | r<x | | t [i] .mx<=v) return;
    if (x<=1&&r<=y&&t[i].mx2<v) return t[i].chmin(v);</pre>
    int m = (1+r)/2;
```

```
push(l,r,i);
        range_chmin(1, m, i * 2, x, y, v);
        range chmin(m+1,r,i*2+1,x,v,v);
        pull(i);
    void range_chmax(int 1,int r,int i,int x,int y,11 v){
        if (y<1 | | r<x | |t[i].mn>=v) return;
        if (x<=1&&r<=y&&t[i].mn2>v) return t[i].chmax(v);
        int m=(1+r)/2;
        push(l,r,i);
        range_chmax(1, m, i * 2, x, y, v);
        range_chmax(m+1, r, i*2+1, x, y, v);
        pull(i);
    11 query(int 1,int r,int i,int x,int y) {
        if(y<1||r<x)return 0;
        if(x<=l&&r<=y) return t[i].sum;</pre>
        int m = (1+r)/2;
        push(l,r,i);
        return query (1, m, i * 2, x, y) +query (m+1, r, i * 2+1, x, y);
    template < class F >
    void build(const F &f) {build(0, n-1, 1, f);}
    void range_add(int x,int y,ll v) {range_add(0,n-1,1,x,y,v);}
    void range_chmin(int x,int y,ll v) {range_chmin(0,n-1,1,x,y,
    void range_chmax(int x,int y,ll v) {range_chmax(0,n-1,1,x,y,
    11 query(int x,int y) {return query(0,n-1,1,x,y);}
};
Tree (3)
LinkCutTree.hpp
Description: Link Cut Tree (1-indexed)
"src/contest/template.hpp"
                                                        38324f. 78 lines
template<int N.class T>
struct LinkCutTree{
    int ch[N][2],par[N],lz[N],rev[N];
    T val[N], sum[N], rsum[N];
    void toggle(int v) {
        if(!v)return;
        swap(ch[v][0],ch[v][1]);
        swap(sum[v],rsum[v]);
        rev[v]^=1;
    void push(int v) {
        if(!v||!rev[v])return;
        toggle(ch[v][0]);
        toggle(ch[v][1]);
        rev[v]=0;
    void pull(int v) {
        if(!v)return;
        sum[v] = sum[ch[v][0]] + val[v] + sum[ch[v][1]];
        rsum[v]=rsum[ch[v][0]]+val[v]+rsum[ch[v][1]];
```

**return** ch[par[v]][0]!=v&&ch[par[v]][1]!=v;

bool is root(int v){

return ch[par[v]][1]==v;

int u=par[v],g=par[u];

ch[u][x]=ch[v][!x];

**if**(!is\_root(u))ch[q][pos(u)]=v;

bool pos(int v) {

void rotate(int v) {

bool x=pos(v);

```
if(ch[u][x])par[ch[u][x]]=u;
        ch[v][!x]=u,par[u]=v,par[v]=q;
        pull(u),pull(v);
    void splay(int v) {
        if(!v)return;
        for (push (v);!is_root (v); rotate (v)) {
            int u=par[v];
            if(is_root(u))push(u),push(v);
            else push (par[u]), push (u), push (v), rotate (pos(u) ==
                 pos(v)?u:v);
    void access(int v) {
        for(int u=v,c=0;u;u=par[u]){
            splay(u);
            ch[u][1]=c;
            pull(c=u);
        splay(v);
    void evert(int v) {
        access(v),toggle(v);
    void link(int u,int v){
        evert(u);
        access(v);
        par[u]=v;
    void cut(int u,int v){
        evert(u);
        access(v);
        assert (par[u] == v);
        ch[v][0]=par[u]=0;
        pull(v);
    T aggregate(int u,int v) {
        evert(u);
        access(v);
        return sum[v];
    void set(int u,T v){
        evert(u);
        val[u]=v;
        pull(u);
};
StaticTopTree.hpp
Description: Static Top Tree.
"src/contest/template.hpp"
                                                       cfe8ea, 198 lines
template<class G>
struct StaticTopTree{
    using P = pair<int,int>;
    enum Type{Compress, Rake, AddEdge, AddVertex, Vertex};
    int n, root;
    G &adj;
    vector<int> hv, fa, lch, rch, par;
    vector<Type> type;
    StaticTopTree(G &adj):adj(adj) {build();}
    int dfs(int u) {
        int s=1, mx=0;
        for(auto v:adj[u]){
            if (v==fa[u]) continue;
            fa[v]=u;
            int t=dfs(v);
            if (t>mx) mx=t, hv[u]=v;
            s+=t;
```

```
return s:
void build() {
    n=hld.n;
    hv=fa=lch=rch=par=vector<int>(n,-1);
    type.assign(n,Compress);
    dfs(0,-1);
    root=compress(0).second;
int add(int i,int l,int r,Type t) {
    if(i==-1){
        i=n++;
        lch.emplace_back(1);
        rch.emplace_back(r);
        par.emplace_back(-1);
        type.emplace_back(t);
    }else{
        lch[i]=1,rch[i]=r,type[i]=t;
    if(1!=-1)par[1]=i;
    if(r!=-1)par[r]=i;
    return i;
pair<int, int> merge(vector<pair<int, int>> a, Type t) {
    if(a. size()==1)return \ a[0];
    int tot=0;
    vector < pair < int, int >> l, r;
    for(auto [i,s]:a)tot+=s;
    for(auto [i,s]:a)
         (tot>s?l:r).emplace\_back(i,s);
         tot = s * 2;
    auto [i, si]=merge(l, t);
    auto [j, sj] = merge(r, t);
    return \{add(-1,i,j,t), si+sj\};
P compress(int i) {
    vector<P> a{add_vertex(i)};
    auto work=[&](){
        auto [sj,j]=a.back();
        a.pop_back();
        auto [si,i]=a.back();
        a.back() = \{\max(si, sj) + 1, add(-1, i, j, Compress)\};
    while(hv[i]!=-1){
        a.emplace_back(add_vertex(i=hv[i]));
        while(true) {
             if(a.size()>=3&&(a.end()[-3].first==a.end()
                  [-2].first||a.end()[-3].first<=a.back().
                 P tmp=a.back();
                 a.pop_back();
                 work();
                 a.emplace_back(tmp);
             }else if(a.size()>=2&&a.end()[-2].first<=a.back
                  ().first){
                 work();
            }else break;
    while (a.size()>=2) work();
    return a[0];
P rake(int i) {
    priority_queue<P, vector<P>, greater<P>> pq;
    for(int j:adj[i])if(j!=fa[i]&&j!=hld.hv[i])pq.emplace(
         add_edge(j));
```

```
while (pq.size()>=2) {
            auto [si,i]=pq.top();pq.pop();
            auto [sj,j]=pq.top();pq.pop();
            pq.emplace(max(si, sj)+1,add(-1, i, j, Rake));
        return pq.empty()?make_pair(0,-1):pq.top();
    P add_edge(int i) {
        auto [sj,j]=compress(i);
        return {sj+1,add(-1,j,-1,AddEdge)};
    P add vertex(int i) {
        auto [sj,j]=rake(i);
        return {sj+1,add(i,j,-1,j==-1?Vertex:AddVertex)};
};
struct TreeDPf
    struct Path {
        static Path unit();
    struct Points
        static Point unit();
    static Path compress(Path l, Path r);
    static Point rake(Point l, Point r);
    static Point add_edge(Path p);
    static Path add_vertex(Point p, int u);
    static Path vertex(int u);
*/
template < class HLD, class TreeDP>
struct StaticTopTreeRerootingDP{
    using Path = typename TreeDP::Path;
    using Point = typename TreeDP::Point;
    StaticTopTree<HLD> stt;
    vector<Path> path,rpath;
    vector<Point> point;
    StaticTopTreeRerootingDP(HLD &hld):stt(hld){
        int n=stt.n;
        path.resize(n);
        point.resize(n);
        rpath.resize(n);
        dfs(stt.root);
    void _update(int u) {
        if(stt.type[u] == stt.Vertex) {
            path[u]=rpath[u]=TreeDP::vertex(u);
        }else if(stt.type[u] == stt.Compress) {
            path[u]=TreeDP::compress(path[stt.lch[u]],path[stt.
                 rch[u]]);
            rpath[u]=TreeDP::compress(rpath[stt.rch[u]],rpath[
                 stt.lch[u]]);
        }else if(stt.type[u] == stt.Rake) {
            point[u] = TreeDP::rake(point[stt.lch[u]], point[stt.
                 rch[u]]);
        }else if(stt.type[u] == stt.AddEdge) {
            point[u]=TreeDP::add_edge(path[stt.lch[u]]);
        }else{
            path[u]=rpath[u]=TreeDP::add_vertex(point[stt.lch[u]
                 ]],u);
    void dfs(int u){
        if (u==-1) return;
        dfs(stt.lch[u]);
        dfs(stt.rch[u]);
```

3

## Dinic BinaryOptimization

```
_update(u);
    void update(int u) {
        for(;u!=-1;u=stt.par[u])_update(u);
   Path query_all(){
        return path[stt.root];
   Path query_subtree(int u) {
       Path res=path[u];
        while(true) {
            int p=stt.par[u];
            if (p==-1||stt.type[p]!=stt.Compress)break;
            if (stt.lch[p] == u) res=TreeDP::compress(path[stt.rch[
                 p]],res);
        return res;
   Path query_reroot(int u) {
        auto rec=[&](auto &&rec,int u)->Point {
            int p=stt.par[u];
            Path below=Path::unit(),above=Path::unit();
            while (p! =-1&&stt.type[p] == stt.Compress) {
                int l=stt.lch[p],r=stt.rch[p];
                if(l==u)below=TreeDP::compress(below,path[r]);
                else above=TreeDP::compress(above,rpath[1]);
                p=stt.par[u];
            if(p!=-1){
                p=stt.par[u];
                Point sum=Point::unit();
                while(stt.type[p] == stt.Rake) {
                     int l=stt.lch[p],r=stt.rch[p];
                     sum=TreeDP::rake(sum,u==r?point[1]:point[r
                         ]);
                     u=p;
                     p=stt.par[u];
                sum=TreeDP::rake(sum, rec(rec,p));
                above=TreeDP::compress(above, TreeDP::add_vertex
                     (sum,p));
            return TreeDP::rake(TreeDP::add_edge(below),TreeDP
                 ::add_edge(above));
        };
        Point res=rec(rec,u);
        if(stt.type[u] == stt.AddVertex) {
            res=TreeDP::rake(res,point[stt.lch[u]]);
        return TreeDP::add vertex(res,u);
Description: Dinic's Algorithm for finding the maximum flow.
Time: \mathcal{O}(VE \log U) where U is the maximum flow.
"src/contest/template.hpp"
    static constexpr T INF=numeric_limits<T>::max()/2;
    struct Edge{
```

```
Flows (4)
```

## Dinic.hpp

template<class T,bool directed=true,bool scaling=true> struct Dinic{ int to; T flow, cap; Edge(int \_to,T \_cap):to(\_to),flow(0),cap(\_cap){}

```
T remain() {return cap-flow;}
};
int n,s,t;
T U;
vector<Edge> e;
vector<vector<int>> q;
vector<int> ptr,lv;
bool calculated:
T max_flow;
Dinic(){}
Dinic(int n, int s, int t) {init(n, s, t);}
void init(int _n,int _s,int _t){
    n=_n, s=_s, t=_t;
    U=0;
    e.clear();
    g.assign(n,{});
    calculated=false;
void add_edge(int from, int to, T cap) {
    assert(0<=from&&from<n&&0<=to&&to<n);
    g[from].emplace_back(e.size());
    e.emplace_back(to,cap);
    g[to].emplace_back(e.size());
    e.emplace_back(from,directed?0:cap);
    U=max(U,cap);
bool bfs(T scale) {
    lv.assign(n,-1);
    vector<int> q{s};
    lv[s]=0;
    for(int i=0;i<(int)q.size();i++){</pre>
        int u=q[i];
        for(int j:g[u]){
            int v=e[j].to;
            if(lv[v] ==-1&&e[j].remain()>=scale) {
                 q.emplace_back(v);
                 lv[v] = lv[u] + 1;
    return lv[t]!=-1;
T dfs(int u,int t,T f) {
    if (u==t||f==0) return f;
    for(int &i=ptr[u];i<(int)q[u].size();i++){</pre>
        int j=q[u][i];
        int v=e[i].to;
        if(lv[v] == lv[u]+1) {
            T res=dfs(v,t,min(f,e[j].remain()));
                 e[j].flow+=res;
                 e[j^1].flow-=res;
                 return res;
    return 0;
    if(calculated)return max flow;
    calculated=true;
    max flow=0;
    for(T scale=scaling?1LL<<(63-__builtin_clzll(U)):1LL;</pre>
         scale>0;scale>>=1){
        while (bfs(scale)) {
            ptr.assign(n,0);
            while(true) {
                T f=dfs(s,t,INF);
                 if (f==0) break;
```

```
max_flow+=f;
         return max_flow;
    pair<T, vector<int>> cut() {
         flow();
         vector<int> res(n);
         for (int i=0; i<n; i++) res[i] = (lv[i] ==-1);</pre>
         return {max_flow, res};
};
BinaryOptimization.hpp
Description: Binary Optimization.
                                            minimize \kappa + \sum_{i} \theta_{i}(x_{i}) +
\sum_{i < j} \phi_{ij}(x_i, x_j) + \sum_{i < j < k} \psi_{ijk}(x_i, x_j, x_k) where x_i \in \{0, 1\} and \phi_{ij}, \psi_{ijk}
are submodular functions. a set function f is submodular if f(S) + f(T) \ge
f(S \cap T) + f(S \cup T) for all S, T. \phi_{ij}(0,1) + \phi_{ij}(1,0) \ge \phi_{ij}(1,1) + \phi_{ij}(0,0).
"src/flows/Dinic.hpp"
template<class T,bool minimize=true>
struct BinaryOptimization{
     static constexpr T INF=numeric_limits<T>::max()/2;
     int n,s,t,buf;
     map<pair<int,int>,T> edges;
     \label{eq:binaryOptimization} \mbox{BinaryOptimization(int } \mbox{\_n):} \mbox{$n(\_n)$, $s(n)$, $t(n+1)$, buf(n+2)$, base}
     void add edge(int u,int v,T w){
         assert (w \ge 0);
         if (u==v | |w==0) return;
         auto &e=edges[{u,v}];
         e=min(e+w,INF);
    void add0(T w){
         base+=w;
     void _add1(int i,T a,T b) {
         if(a<=b){
              add0(a);
              add_edge(s,i,b-a);
         }else{
              add0(b);
              add_edge(i,t,a-b);
     void add1(int i,T x0,T x1){
         assert(0<=i&&i<n);
         if (!minimize) x0=-x0, x1=-x1;
         \_add1(i, x0, x1);
    void _add2(int i,int j,T a,T b,T c,T d) {
         assert (b+c>=a+d);
         add0(a);
         _add1(i,0,c-a);
         _add1(j,0,d-c);
         add_edge(i,j,b+c-a-d);
    void add2(int i,int j,T x00,T x01,T x10,T x11) {
         assert(i!=j&&0<=i&&i<n&&0<=j&&j<n);
         if (!minimize) x00=-x00, x01=-x01, x10=-x10, x11=-x11;
         _{add2}(i, j, x00, x01, x10, x11);
    void _add3(int i,int j,int k,T a,T b,T c,T d,T e,T f,T g,T
         T p=a+d+f+g-b-c-e-h;
         if(p>=0){
              add0(a);
```

};

## KaryOptimization MinCostFlow

```
_add1(i,0,f-b);
             _add1(j,0,g-e);
             add1(k,0,d-c);
             _add2(i,j,0,c+e-a-g,0,0);
             _add2(i,k,0,0,b+e-a-f,0);
             \_add2(j,k,0,b+c-a-d,0,0);
             int u=buf++;
             add0(-p);
             add_edge(i,u,p);
             add_edge(j,u,p);
             add_edge(k,u,p);
             add_edge(u,t,p);
        }else{
             add0(h);
             _add1(i,c-g,0);
             _add1(j,b-d,0);
             _{add1(k,e-f,0)};
             \_add2(i, j, 0, 0, d+f-b-h, 0);
             _{add2}(i,k,0,d+g-c-h,0,0);
             _{add2}(j,k,0,0,f+g-e-h,0);
             int u=buf++;
             add0(p);
             add_edge(s,u,-p);
             add_edge(u,i,-p);
             add_edge(u,j,-p);
             add_edge(u, k, -p);
    void add3(int i,int j,int k,T x000,T x001,T x010,T x011,T
         x100, T x101, T x110, T x111) {
        assert (i!=j&&j!=k&&k!=i&&0<=i&&i<n&&0<=j&&j<n&&0<=k&k<
        if(!minimize){
             x000=-x000, x001=-x001, x010=-x010, x011=-x011;
             x100=-x100, x101=-x101, x110=-x110, x111=-x111;
         _{add3}(i,j,k,x000,x001,x010,x011,x100,x101,x110,x111);
    pair<T, vector<int>> solve() {
        Dinic<T> dinic(buf,s,t);
        for(auto &[p,w]:edges){
             auto [u,v]=p;
             dinic.add_edge(u,v,w);
        auto [ans,cut]=dinic.cut();
        ans+=base;
        ans=min(ans, INF);
        cut.resize(n);
         return {minimize?ans:-ans,cut};
KaryOptimization.hpp
Description: k-ary Optimization. minimize \kappa + \sum_{i} \theta_{i}(x_{i}) +
\sum_{i < j} \phi_{ij}(x_i, x_j) where x_i \in \{0, 1, \dots, k-1\} and \phi_{i,j} is monge. A func-
tion f is monge if f(a,c) + f(b,d) \leq f(a,d) + f(b,c) for all a < b and
c < d. \phi_{ij}(x-1,y) + \phi_{ij}(x,y+1) \le \phi_{ij}(x-1,y+1) + \phi_{ij}(x,y).
\phi_{ij}(x,y) + \phi_{ij}(x-1,y+1) - \phi_{ij}(x-1,y) - \phi_{ij}(x,y+1) \ge 0.
                                                          5139ce, 88 lines
"src/flows/Dinic.hop"
template < class T, bool minimize = true >
struct K_aryOptimization{
    static constexpr T INF=numeric_limits<T>::max()/2;
    int n,s,t,buf;
    T base:
    vector<int> ks;
    vector<vector<int>> id;
    map<pair<int,int>,T> edges;
    K_aryOptimization(int n, int k) {init(vector<int>(n,k));}
```

```
K_aryOptimization(const vector<int> &_ks) {init(_ks);}
void init(const vector<int> &_ks) {
    ks= ks;
    n=ks.size();
    s=0, t=1, buf=2;
    base=0:
    id.clear();
    edges.clear();
    for(auto &k:ks) {
         assert (k>=1):
        vector<int> a(k+1);
        a[0]=s, a[k]=t;
        for (int i=1; i < k; i++) a[i] = buf++;</pre>
        id.emplace_back(a);
        for(int i=2;i<k;i++)add_edge(a[i],a[i-1],INF);</pre>
void add_edge(int u,int v,T w) {
    assert (w \ge 0);
    if (u==v | |w==0) return;
    auto &e=edges[{u,v}];
    e=min(e+w,INF);
void add0(T w) {
    base+=w;
void _add1(int i, vector<T> cost) {
    add0(cost[0]);
    for(int j=1; j<ks[i]; j++) {</pre>
        T x=cost[j]-cost[j-1];
        if(x>0)add_edge(id[i][j],t,x);
         if (x<0) add0 (x), add_edge(s,id[i][j],-x);</pre>
void add1(int i, vector<T> cost) {
    assert (0<=i&&i<n&& (int) cost.size() ==ks[i]);
    if(!minimize) for(auto &x:cost) x=-x;
    _add1(i,cost);
void _add2(int i,int j,vector<vector<T>> cost){
    int h=ks[i], w=ks[j];
    add1(j,cost[0]);
    for (int x=h-1; x>=0; x--) for (int y=0; y< w; y++) cost [x] [y] -=
    vector<T> a(h);
    for (int x=0; x<h; x++) a[x] = cost[x][w-1];</pre>
    for (int x=0; x<h; x++) for (int y=0; y<w; y++) cost [x][y]==a[x
    for (int x=1; x<h; x++) {</pre>
         for (int y=0; y<w-1; y++) {</pre>
             T = cost[x][y] + cost[x-1][y+1] - cost[x-1][y] - cost
                  [x][y+1];
             assert (w>=0); // monge
             add_edge(id[i][x],id[j][y+1],w);
void add2(int i,int j,vector<vector<T>> cost){
    assert(0<=i&&i<n&&0<=j&&j<n&&i!=j);
    assert((int)cost.size()==ks[i]);
    for(auto &v:cost)assert((int)v.size()==ks[j]);
    if(!minimize)for(auto &v:cost)for(auto &x:v)x=-x;
    _add2(i, j, cost);
pair<T, vector<int>> solve() {
    Dinic<T> dinic(buf,s,t);
    for(auto &[p,w]:edges){
         auto [u,v]=p;
```

```
dinic.add edge(u,v,w);
        auto [val,cut]=dinic.cut();
        val+=base:
        if(!minimize) val=-val;
        vector<int> ans(n);
        for (int i=0; i < n; i++) {</pre>
             ans[i]=ks[i]-1;
             for(int j=1; j<ks[i]; j++) ans[i] -=cut[id[i][j]];</pre>
        return {val,ans};
};
MinCostFlow.hpp
Description: minimum-cost flow algorithm.
Time: \mathcal{O}(FE \log V) where F is max flow.
"src/contest/template.hpp"
template<class F,class C>
struct MinCostFlow{
    struct Edge{
        int to;
        F flow, cap;
         Edge(int _to,F _cap,C _cost):to(_to),flow(0),cap(_cap),
             cost(cost){}
        F getcap(){
             return cap-flow;
    };
    int n;
    vector<Edge> e;
    vector<vector<int>> adi;
    vector<C> pot, dist;
    vector<int> pre;
    bool neg:
    const F FINF=numeric limits<F>::max()/2;
    const C CINF=numeric limits<C>::max()/2;
    MinCostFlow() {}
    MinCostFlow(int _n) {
        init(_n);
    void init(int _n){
        n=n:
        e.clear();
        adj.assign(n,{});
        neg=false;
    void addEdge(int u,int v,F cap,C cost){
        adj[u].emplace_back(e.size());
        e.emplace_back(v,cap,cost);
        adj[v].emplace_back(e.size());
        e.emplace_back(u,0,-cost);
        if (cost<0) neg=true;</pre>
    bool dijkstra(int s, int t) {
        using P = pair<C,int>;
        dist.assign(n,CINF);
        pre.assign(n,-1);
        priority_queue<P, vector<P>, greater<P>> pq;
        dist[s]=0;
        pq.emplace(0,s);
        while(!pq.empty()){
             auto [d,u]=pq.top();
             pq.pop();
             if (dist[u] < d) continue;</pre>
             for(int i:adj[u]){
                 int v=e[i].to;
                 C ndist=d+pot[u]-pot[v]+e[i].cost;
```

```
if(e[i].getcap()>0&&dist[v]>ndist){
                     pre[v]=i;
                     dist[v]=ndist;
                     pq.emplace(ndist,v);
        return dist[t] < CINF;</pre>
    pair<F,C> flow(int s,int t) {
        F flow=0;
        C cost=0;
        pot.assign(n,0);
        if (neg) for (int t=0; t<n; t++) for (int i=0; i<e.size(); i++)</pre>
             if(e[i].getcap()>0){
             int u=e[i^1].to, v=e[i].to;
             pot[v]=min(pot[v],pot[u]+e[i].cost);
        } // Bellman-Ford
        while(dijkstra(s,t)){
             for(int i=0;i<n;i++)pot[i]+=dist[i];</pre>
             F aug=FINF;
             for(int u=t;u!=s;u=e[pre[u]^1].to){
                 aug=min(aug,e[pre[u]].getcap());
             } // find bottleneck
             for(int u=t;u!=s;u=e[pre[u]^1].to){
                 e[pre[u]].flow+=aug;
                 e[pre[u]^1].flow-=aug;
             } // push flow
             flow+=aug;
             cost+=aug*pot[t];
        return {flow,cost};
};
HopcroftKarp.hpp
Description: Fast bipartite matching algorithm.
Time: \mathcal{O}\left(E\sqrt{V}\right)
"src/contest/template.hpp"
                                                        456024, 52 lines
struct HopcroftKarp{
    int n,m;
    vector<int> match, lv, ptr;
    vector<vector<int>> adj;
    HopcroftKarp(){}
    HopcroftKarp(int _n,int _m){init(_n,_m);}
    void init(int _n,int _m) {
        n=_n, m=_m;
        adj.assign(n+m, vector<int>{});
    void add_edge(int u,int v) {
        adj[u].emplace_back(v+n);
    void bfs() {
        lv.assign(n,-1);
        queue<int> q;
        for (int i=0; i<n; i++) if (match[i] ==-1) {</pre>
            lv[i]=0;
             q.emplace(i);
        while(!q.empty()){
             int u=q.front();
             q.pop();
             for(int v:adj[u])if(match[v]!=-1&&lv[match[v]]==-1)
                 lv[match[v]]=lv[u]+1;
                 q.emplace(match[v]);
```

```
bool dfs(int u) {
        for(int &i=ptr[u];i<adj[u].size();i++){</pre>
             int v=adj[u][i];
             if (match[v] ==-1|| (lv[match[v]] == lv[u] +1 &&dfs (match[
                  v]))){
                 match[u]=v, match[v]=u;
                 return true;
        return false;
    int max_matching(){
        int ans=0,cnt=0;
        match.assign(n+m,-1);
        do{
             ptr.assign(n,0);
            bfs();
             cnt=0;
             for (int i=0; i<n; i++) if (match[i] ==-1&&dfs(i)) cnt++;</pre>
             ans+=cnt;
        }while(cnt);
        return ans;
};
```