

CS171 Mini Project Report

We first tested the methods alone to get a sense of where to start.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
Methods	E1 time	E1 bt	E2 time	E2 bt	E3 time	E3 bt	E4 time	E4 bt	E5 time	E5 bt						avg(t)	avg(bt)	t/bt
Baseline	1.2622	1556	1.5488	2405	140.2655	170925	12.2649	14897	0.4041	460						31.1491	38049	0.0008
FC (Thomas)	0.0800	6	0.1062	9	0.5320	69	0.1811	7	0.0530	5						0.1904	19	0.0099
ACP	0.0429	3	0.0829	3	0.3139	29	0.1272	4	0.0448	4						0.1224	9	0.0142
MRV (T)	0.2142	197	0.2636	263	0.5678	654	0.5114	479	0.2071	181						0.3528	355	0.0010
DH (T)	0.6005	710	3.0872	5348	7.0240	10428	4.4413	5843	0.5196	649						3.1345	4596	0.0007
LCV (T)	1.1371	1387	1.0612	1284	130.6983	153325	12.1733	15024	0.2214	217						29.0583	34247	0.0008
NKP	0.0159	3	0.0597	5	0.2273	25	0.0276	1	0.0104	4						0.0682	8	0.0090
NKT	0.1315	3	0.5905	10	1.5614	17	0.2484	0	0.0934	4						0.5250	7	0.0772

Using LCV alone only resulted in a minor speed boost since there was no consistency check or heuristic to take advantage of it. All of the other methods lowered solve time – with most of them being two orders of magnitude faster.

Easy problems

Testing was split up into 3 parts: combinations with Naked Pairs, with Naked Triples, and with both Naked Pairs and Triples.

With Naked Pairs:

Methods	E1 time	E1 bt	E2 time	E2 bt	E3 time	E3 bt	E4 time	E4 bt	E5 time	E5 bt						avg(t)	avg(bt)	t/bt
NKP MRV	0.0190	4	0.0703	7	0.1295	11	0.0316	3	0.0123	2						0.0526	5	0.0097
NKP DH	0.0252	8	0.0761	4	0.1425	12	0.0398	4	0.0151	3						0.0598	6	0.0096
NKP MRV LCV	0.0199	0	0.0764	10	0.1628	49	0.0339	0	0.0147	0						0.0615	12	0.0052
NKP LCV	0.0156	0	0.0324	0	0.2686	147	0.0616	20	0.0108	0						0.0778	33	0.0023
NKP ACP MRV LCV	0.0409	0	0.0970	1	0.2645	22	0.0609	0	0.0313	0						0.0989	5	0.0215
NKP FC DH	0.0458	8	0.1415	4	0.1998	6	0.0824	4	0.0408	3						0.1021	5	0.0204
NKP FC MRV	0.0493	4	0.1356	8	0.2368	8	0.0675	3	0.0387	3						0.1056	5	0.0203
NKP DH LCV	0.0373	10	0.0793	10	0.3799	247	0.0495	10	0.0118	0						0.1116	55	0.0020
NKP ACP DH	0.0899	8	0.1537	4	0.1889	6	0.0958	4	0.0376	2						0.1132	5	0.0236
NKP ACP	0.0451	3	0.0978	3	0.3213	16	0.0618	0	0.0446	4						0.1141	5	0.0219
NKP FC	0.0436	3	0.0970	5	0.3625	20	0.0613	1	0.0313	4						0.1192	7	0.0181
NKP FC LCV	0.0320	0	0.0555	0	0.3849	75	0.0991	20	0.0384	0						0.1220	19	0.0064
NKP ACP LCV	0.0348	0	0.0589	0	0.3865	39	0.1027	2	0.0305	0						0.1227	8	0.0150
NKP FC MRV LCV	0.0370	0	0.1216	10	0.3623	31	0.0614	0	0.0312	0						0.1227	8	0.0150
NKP ACP MRV	0.0792	3	0.1435	6	0.2955	9	0.0903	3	0.0402	2						0.1297	5	0.0282
NKP ACP DH LCV	0.0690	1	0.1716	10	0.3283	24	0.0683	1	0.0312	0						0.1337	7	0.0186
NKP FC DH LCV	0.0415	1	0.1763	10	0.5548	69	0.0653	1	0.0421	1						0.1760	16	0.0107

The combinations with Naked Pairs performed the best, with the slowest combination (NKP FC DH LCV) averaging .176s per problem. Unexpectedly, ACP performed about the same as FC, in that they both did below average overall – the fastest combos did not involve consistency checks at all. Another common factor between the bottom half was the use of LCV. It seemed to only slow the solver down for the easy problems.

The relatively simple combination of Naked Pairs and Minimum remaining values heuristic ended up being the fastest overall for the easy problems.

With Naked Triples:

Methods	E1 time	E1 bt	E2 time	E2 bt	E3 time	E3 bt	E4 time	E4 bt	E5 time	E5 bt		avg(t)	avg(bt)	t/bt
NKT ACP DH LCV	0.0724	1	0.3490	1	0.5527	2	0.2195	1	0.0330	0		0.2453	1	0.2453
NKT ACP DH	0.0993	8	0.3538	1	0.4861	0	0.2561	4	0.0439	2		0.2478	3	0.0826
NKT ACP	0.0446	3	0.2150	3	1.0066	10	0.1851	0	0.0415	4		0.2986	4	0.0746
NKT FC DH	0.1177	8	0.3370	1	0.5058	1	0.4525	5	0.0873	2		0.3001	3	0.0883
NKT ACP MRV LCV	0.0437	0	0.5110	1	0.7240	12	0.1985	0	0.0337	0		0.3022	3	0.1162
NKT ACP LCV	0.0362	0	0.1744	0	1.0914	39	0.2226	2	0.0296	0		0.3109	8	0.0379
NKT ACP MRV	0.0706	3	0.5154	7	0.8033	9	0.2149	3	0.0434	2		0.3295	5	0.0686
NKT DH	0.2147	8	0.3518	1	0.5105	0	0.5178	7	0.1221	3		0.3434	4	0.0904
NKT FC MRV	0.1192	5	0.6167	7	0.7429	11	0.3039	4	0.0877	2		0.3741	6	0.0645
NKT MRV	0.1658	4	0.6953	7	0.7437	9	0.2594	4	0.1063	2		0.3941	5	0.0758
NKT FC MRV LCV	0.1136	0	0.6424	10	0.8536	30	0.2984	1	0.0862	0		0.3988	8	0.0486
NKT MRV LCV	0.1528	0	0.7154	10	0.8253	30	0.2628	0	0.0979	0		0.4108	8	0.0514
NKT DH LCV	0.2054	10	0.3949	10	0.8143	47	0.5647	28	0.1235	10		0.4205	21	0.0200
NKT FC DH LCV	0.1155	1	0.4258	10	1.0445	47	0.5155	10	0.0873	0		0.4377	14	0.0322
NKT FC	0.1417	3	0.3488	4	1.5944	15	0.2505	1	0.0804	4		0.4832	5	0.0895
NKT FC LCV	0.1435	0	0.2269	1	1.8158	119	0.5074	20	0.0815	0		0.5550	28	0.0198
NKT LCV	0.1553	0	0.2572	0	1.7055	128	0.5447	20	0.1126	0		0.5551	30	0.0188

The fastest Naked Triples combo was slower than the slowest Naked Pairs combo. The implementation of Naked Triples is similar in complexity to Naked Pairs, so the difference likely has to do with the frequency of finding a Naked Pair vs. a Naked Triple to exploit on the game board – you’re way more likely to find a Naked Pair than a Naked Triple.

ACP performed very well with Naked Triples, in contrast to the last section with Naked Pairs. The ACP algorithm probably had a lot more pruning to do – pruning that was probably done by Naked Pairs before, resulting in wasted CPU cycles iterating through the variables and checking the arcs.

With both Naked Pairs and Triples:

Methods	E1 time	E1 bt	E2 time	E2 bt	E3 time	E3 bt	E4 time	E4 bt	E5 time	E5 bt		avg(t)	avg(bt)	t/bt
NKPT ACP MRV LCV	0.0379	0	0.0716	1	0.2272	23	0.1270	2	0.0305	0		0.0988	5	0.0190
NKPT ACP DH	0.0903	8	0.0896	1	0.1255	5	0.1942	17	0.0387	2		0.1076	7	0.0163
NKPT ACP	0.0468	3	0.0968	3	0.3303	29	0.1298	4	0.0415	4		0.1291	9	0.0150
NKPT ACP MRV	0.0760	3	0.1336	6	0.2429	13	0.1722	6	0.0435	2		0.1336	6	0.0223
NKPT ACP LCV	0.0352	0	0.0464	0	0.5127	84	0.2005	13	0.0292	0		0.1648	19	0.0085
NKPT FC DH	0.1005	11	0.1443	2	0.2079	16	0.3301	27	0.0694	3		0.1704	12	0.0144
NKPT ACP DH LCV	0.0610	1	0.2034	19	0.2316	14	0.3338	55	0.0307	0		0.1721	18	0.0097
NKPT FC	0.0948	6	0.1286	9	0.5407	69	0.1830	7	0.0475	5		0.1989	19	0.0104
NKPT FC MRV	0.1210	6	0.1603	11	0.4107	29	0.2592	10	0.0636	4		0.2030	12	0.0169
NKPT FC LCV	0.0773	1	0.0959	1	0.6473	184	0.3085	41	0.0435	0		0.2345	45	0.0052
NKPT MRV LCV	0.1099	0	0.1471	46	0.6550	644	0.2192	119	0.1023	0		0.2467	162	0.0015
NKPT FC MRV LCV	0.1001	0	0.1428	10	0.6898	95	0.2417	11	0.0594	0		0.2468	23	0.0106
NKPT FC DH LCV	0.0903	2	0.4536	74	0.5407	113	0.6051	100	0.0591	0		0.3497	58	0.0061
NKPT MRV	0.2155	197	0.3065	263	0.5864	654	0.4876	479	0.2090	181		0.3610	355	0.0010
NKPT LCV	1.1687	1387	1.1634	1284								1.1661	1336	0.0009
NKPT DH LCV	0.5440	534	2.3137	3446								1.4289	1990	0.0007
NKPT	1.2497	1156	1.6980	2405								1.4738	1781	0.0008
NKPT DH	0.6161	710	3.4401	5348								2.0281	3029	0.0007

Using both heuristics resulted in performance somewhere between Pairs and Triples. Naked Triples may be more of a boon in bigger boards, where there are more opportunities to find triples (though the same can be said of pairs).

Of note, most of the combinations without a consistency check were dropped after testing problem 2 due to relatively low performance and high backtracking. Without either FC or ACP, the values were basically being chosen at random and then walked back if a constraint was violated (which was very often the case).

Medium problems

With Naked Pairs:

Methods	M1 time	M1 bt	M2 time	M2 bt	M3 time	M3 bt	M4 time	M4 bt	M5 time	M5 bt	avg(t)	avg(bt)	t/bt
NKP MRV	0.2895	5	1.3502	3	1.1958	4	2.3776	6	1.5715	32	1.3569	10	0.1357
NKP ACP MRV	0.5482	4	1.9230	3	1.9020	4	3.1610	5	2.1885	32	1.9446	10	0.2026
NKP FC MRV	0.4942	4	2.0137	3	1.8729	4	3.1413	5	2.6853	36	2.0415	10	0.1963
NKP ACP DH LCV	0.4783	1	2.0647	1	17.5062	1131	2.8406	1	5.2357	330	5.6251	293	0.0192
NKP DH LCV	0.7631	181	1.8329	121	20.7155	5489	2.2074	37	3.3718	834	5.7781	1332	0.0043
NKP MRV LCV	0.3202	13	2.3124	170	8.9669	2098	2.5568	0	22.0797	5189	7.2472	1494	0.0049
NKP ACP MRV LCV	0.4703	1	3.6997	110	2.0055	15	3.3050	0	46.5227	3221	11.2006	669	0.0167
NKP ACP DH	0.5594	4	3.0875	27	1.9974	10	69.4767	1814	1.4444	32	15.3131	377	0.0406
NKP FC DH LCV	0.4897	13	2.4534	13	68.7315	3497	3.0186	13	9.1613	528	16.7709	813	0.0206
NKP FC MRV LCV	0.7687	13	5.5120	170	25.1902	1568	3.5118	0	74.8620	4675	21.9689	1285	0.0171
NKP DH	0.4825	19	46.5842	2535	1.6032	12	164.0743	4176	1.2941	50	42.8077	1358	0.0315
NKP FC	2.1698	53	40.4324	2090	8.5651	157	155.9125	3578	22.0645	512	45.8288	1278	0.0359
NKP FC DH	0.5431	5	47.0129	774	2.3655	13	294.8597	4188	1.7628	37	69.3088	1003	0.0691
NKP ACP	0.6621	14	396.6731	14176	4.5522	103	62.7581	1970	9.4090	273	94.8109	3307	0.0287
NKP ACP LCV	0.8946	40	6.0051	310	5.6621	313	600.0000	timeout			153.1404	221	0.6929
NKP LCV	1.4014	557	4.6815	800	4.5726	759	600.0000	timeout			152.6639	705	0.2164
NKP FC LCV	3.7847	341	10.8123	490	7.9703	589	600.0000	timeout			155.6418	473	0.3288

Once again, simple is best – Naked Pairs and MRV. It solved the Medium problems the fastest while at the same time having the least amount of backtracks. Adding ACP and FC only served to slow it down, probably for the same reason as in the Easy problems.

The Medium problems showed the real value of having either MRV or DH – 3 combinations without them timed out on problem 4 and the rest were middling. The game board is getting big enough to where going down a less optimal path in the search tree is extremely costly, while the price of running MRV is minimal.

With Naked Triples:

Methods	M1 time	M1 bt	M2 time	M2 bt	M3 time	M3 bt	M4 time	M4 bt	M5 time	M5 bt	avg(t)	avg(bt)	t/bt
NKT ACP DH	3.4028	9	26.8538	59	9.5512	4	18.8164	11	3.5232	11	12.4295	19	0.6611
NKT ACP MRV	25.9128	72	11.8796	3	11.8288	5	17.5413	4	27.0229	72	18.8371	31	0.6038
NKT FC MRV	28.7994	74	11.9833	3	12.2903	5	17.8463	4	34.1192	78	21.0077	33	0.6405
NKT MRV	36.3916	100	11.6715	3	11.4011	5	16.5756	5	35.0653	82	22.2210	39	0.5698
NKT ACP DH LCV	3.5127	15	12.9551	2	90.7840	1266	16.3694	5	3.3969	4	25.4036	258	0.0983
NKT DH	6.2473	21	122.0275	311							64.1374	166	0.3864
NKT FC DH	5.0181	15	132.9720	341							68.9951	178	0.3876
NKT ACP MRV LCV	37.5452	474									37.5452	474	0.0792
NKT FC MRV LCV	42.8743	566									42.8743	566	0.0757
NKT MRV LCV	52.5330	871									52.5330	871	0.0603
NKT ACP	119.5750	538									119.5750	538	0.2223
NKT ACP LCV	152.0648	2490									152.0648	2490	0.0611

The combinations using LCV went up exponentially in time and backtracks, so they were dropped. It could be that the calculation of LCV is bogging things down but LCV is also forcing a lot of backtracks compared to the default `getValuesInOrder`, which simply returns the same values but in a different order. Perhaps ACP and MRV are so good at doing their jobs that `getValuesInOrder` often returns a smaller list of possible values to choose from, reducing the number of backtracks induced.

With both Naked Pairs and Triples:

Methods	M1 time	M1 bt	M2 time	M2 bt	M3 time	M3 bt	M4 time	M4 bt	M5 time	M5 bt	avg(t)	avg(bt)	t/bt
NKPT ACP MRV	17.7569	868	1.3713	29	1.1172	15	1.1915	5	10.5451	500	6.3964	283	0.0226
NKPT ACP DH LCV	2.3876	150	1.0215	14	62.4138	4821	1.0747	2	3.9778	270	14.1751	1051	0.0135
NKPT ACP DH	0.9404	33	29.3920	1615	2.6225	93	38.2712	1835	1.9354	84	14.6323	732	0.0200
NKPT FC MRV	41.1500	1593	2.7536	53	1.6696	20	1.5256	9	29.9451	1091	15.4088	553	0.0279
NKPT ACP MRV LCV	55.5227	5155	0.9032	2	9.5036	759	1.2338	0	89.3812	7691	31.3089	2721	0.0115
NKPT FC DH	7.9431	308	63.4118	2373	6.6407	178	124.5059	3723	6.6250	241	41.8253	1365	0.0307
NKPT FC DH LCV	23.6278	1699	2.1327	62	189.5798	10861					71.7801	4207	0.0171
NKPT ACP	31.7177	1719	591.1350	30466							311.4264	16093	0.0194
NKPT FC	60.7012	2947	600.0000	timeout							330.3506	2947	0.1121
NKPT ACP LCV	92.7995	8780									92.7995	8780	0.0106
NKPT FC MRV LCV	100.4333	9403									100.4333	9403	0.0107
NKPT FC LCV	133.4750	15632									133.4750	15632	0.0085

NKPT ACP MRV is the fastest of the combined heuristic combinations, but looking at the Naked Pairs only section reveals that that same combo without NKT ran for 1.9446 s and produced only 10 backtracks on average, so the Naked Triples heuristic is still not showing its worth. Adding NKT to the mix only seems to produce more backtracks without a proper return in speed.

Hard problems

With Naked Pairs:

Methods	H1 time	H1 bt	H2 time	H2 bt	H3 time	H3 bt	H4 time	H4 bt	H5 time	H5 bt	avg(t)	avg(bt)	t/bt
NKP ACP MRV	53.8148	25	2.5248	9	166.8019	1616	33.5181	287	600.0000	timeout	171.3319	484	0.3538
NKP MRV	50.4662	39	1.5592	9	179.7388	2503	26.1876	332	600.0000	timeout	171.5904	721	0.2381
NKP MRV LCV	47.0231	436	1.8853	83	53.9798	6470	600.0000	timeout			175.7220	2330	0.0754
NKP ACP MRV LCV	52.7106	148	3.1509	51	72.2973	2246	600.0000	timeout			182.0397	815	0.2234
NKP FC MRV LCV	62.9741	436	4.8869	67	237.7187	6070	600.0000	timeout			226.3949	2191	0.1033
NKP FC MRV	55.9967	29	2.8499	9	333.4382	2442	600.0000	timeout			248.0712	827	0.3001
NKP ACP DH	74.3070	202	19.8356	202	31.9093	289	600.0000	timeout			181.5129	231	0.7858
NKP DH	137.1404	349	22.1573	366	600.0000	timeout					253.0992	358	0.7080
NKP ACP DH LCV	120.4379	1774	51.4470	1484	600.0000	timeout					257.2949	1629	0.1579
NKP FC DH	155.2916	347	54.1691	423	600.0000	timeout					269.8202	385	0.7008
NKP DH LCV	600.0000	timeout									600.0000	#DIV/0!	#DIV/0!
NKP FC DH LCV	600.0000	timeout									600.0000	#DIV/0!	#DIV/0!
NKP FC	600.0000	timeout									600.0000	#DIV/0!	#DIV/0!
NKP ACP	600.0000	timeout									600.0000	#DIV/0!	#DIV/0!

While Naked Pair combinations were king in Easy and Medium, every combo timed out (>600s) in Hard. Two combos got past 1-4 before timing out on Hard: NKP ACP MRV and NKP MRV. With the NKT only falling behind in previous difficulties, the tests for combos with both heuristics were done next.

With both Naked Pairs and Triples:

Methods	H1 time	H1 bt	H2 time	H2 bt	H3 time	H3 bt	H4 time	H4 bt	H5 time	H5 bt	avg(t)	avg(bt)	t/bt
NKPT ACP DH	33.3640	575	37.1223	719	388.8117	8164	600.0000	timeout			153.0993	3153	0.0486
NKPT ACP MRV LCV	16.2152	214	24.4127	919	462.5999	17067	600.0000	timeout			275.8070	6067	0.0455
NKPT ACP MRV	12.9334	45	7.0101	122	600.0000	timeout					206.6478	84	2.4748
NKPT FC MRV	13.2122	51	19.4894	290	600.0000	timeout					210.9005	171	1.2370
NKPT FC DH	29.2405	607	182.3859	2735	600.0000	timeout					270.5421	1671	0.1619
NKPT ACP DH LCV	600.0000	timeout									600.0000	#DIV/0!	#DIV/0!

With more possible values, it seemed like the Naked Triples were starting to become a factor – H1 times are faster than NKP only combos. However, none of the combined methods made it past test 3 without timing out.