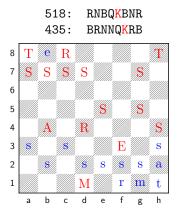
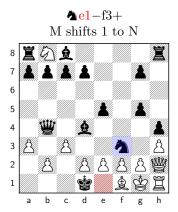
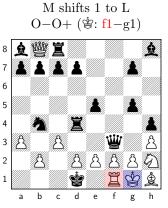
Retro Chess Puzzle Solution

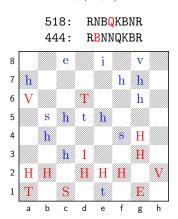
These are retrograde chess rebus puzzles, as described in, e.g., Jeff Coakley's Year of the Rebus and Chess Mysteries in a Retro World. The two armies are represented by CAPS and lowercase letters, with the same letter representing the same piece type (regardless of case). The first piece (going left-to-right) on rank 1 is assigned to the CAPS side. There should be only one assignment so that the position could be legally reached from the starting position, plus it is often possible to say something about the last move (and occasionally several more), using retrograde analysis.

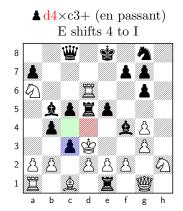


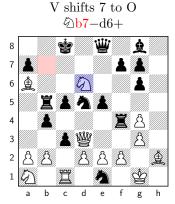




"Masters": $\mathbf{S} = \hat{\mathbb{A}}$, so CAPS is black. $\mathbf{M} = \hat{\mathbb{B}}$, not \mathbf{A} or \mathbf{E} due to impossible double check from pawns. White is missing three pieces, one of which is the dark-squared $\hat{\mathbb{A}}$, which never moved and was captured on its original square (c1 or a1), another is \mathbb{Z} , which never got past its pawns, and $\hat{\mathbb{A}}$ was captured on g5/g6 by $\hat{\mathbb{A}}$. So, in standard chess (518), $\mathbf{R} = \hat{\mathbb{A}}$ (trapped) and $\mathbf{E} = \hat{\mathbb{A}}$ (jumped to b8), while $\mathbf{T} = \mathbb{Z}$ and $\mathbf{A} = \hat{\mathbb{B}}$. Could only come from e1. For extraction, with e1 as the relevant square, we have $e \to \hat{\mathbb{B}} \to \mathbf{m}$, and shift_{right}(M,1) = N. In Chess 960 variant 435, similarly, $\mathbf{T} = \hat{\mathbb{A}}$ and $\mathbf{R} = \mathbb{Z}$ (\mathbb{Z} can't reach h2), while $\mathbf{E} = \hat{\mathbb{B}}$ (no check), and $\mathbf{A} = \hat{\mathbb{A}}$. $\mathbb{Z}f1$ could only have come via castling. For extraction, this is the only diagram where the same letter is shifted for both solutions, and we have shift_{left}(M,1) = L.



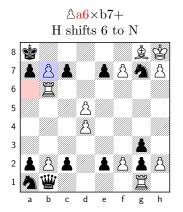


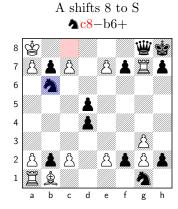


"Thieves": H=\(\delta\), CAPS is white. All missing pieces are accounted for as pawn captures, so:

- 518: $\mathbf{S} = \mathbb{Z}$ and $\mathbf{T} = \mathbb{Z}$ (can't escape), so $\mathbf{V} = \mathbb{Z}$. Then $\mathbf{I} = \mathbb{Z}$ and $\mathbf{E} = \mathbb{Z}$ (otherwise impossible check from $\mathbb{Z}e1$). Double check must be via *en passant* capture on c3.
- 444: $\mathbf{V} = \mathbf{1} \pmod{\mathbf{S}}$ (can't escape $\mathbf{A} f 7 g 7 g 6$), so $\mathbf{E} = \mathbf{1} \pmod{\mathbf{S}}$ and $\mathbf{I} = \mathbf{1} \pmod{\mathbf{S}}$ (otherwise both sides in check or impossible double check). Then $\mathbf{T} = \mathbf{1} \pmod{\mathbf{S}}$ Double check from $\mathbf{1} \pmod{\mathbf{1}}$ and $\mathbf{1} \pmod{\mathbf{1}}$ define $\mathbf{1} \pmod{\mathbf{1}}$ and $\mathbf{1} \pmod{\mathbf{1}}$ and $\mathbf{1} \pmod{\mathbf{1}}$ define $\mathbf{1} \pmod{\mathbf{1}}$ and $\mathbf{1} \pmod{\mathbf{1}}$ define $\mathbf{1} \pmod{\mathbf{1}}$ and $\mathbf{1} \pmod{\mathbf{1}}$ define $\mathbf{1} \pmod{\mathbf{1}$ define $\mathbf{1} \pmod{\mathbf{1}}$ define $\mathbf{1} \pmod{\mathbf{1}$ define $\mathbf{1} \pmod{\mathbf{1}}$ define $\mathbf{1} \pmod{\mathbf{1}$ define $\mathbf{1} \pmod{\mathbf{1}}$ define $\mathbf{1} \pmod{\mathbf{1}}$ define $\mathbf{1} \pmod{\mathbf{1}}$

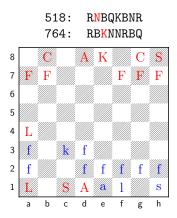


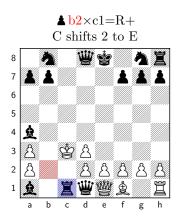


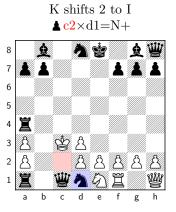


"Chairs": $S=\mathring{\otimes}$ (only one appearing exactly once for each side), $I=\mathring{\triangle}$ (all others appear on a back rank), all missing pieces accounted for by pawn captures: CAPS captured left, lowercase captured right. C and H are Ξ and $\mathring{\triangle}$ (in some order), as they appear twice for a side on same-color squares.

518: CAPS is black (otherwise $\triangleq f1$ not captured by \triangleq). $\triangleq b7$ check had to be a capture from a6. To avoid impossible double check, $\mathbf{H} = \mathbb{Z}$ and $\mathbf{R} = \triangleq$, so $\mathbf{C} = \lozenge$ and $\mathbf{A} = \stackrel{\text{\tiny{def}}}{=}$.





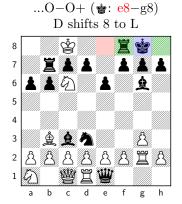


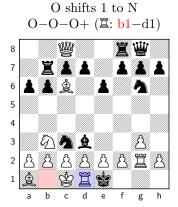
"Flacks": $\mathbf{F} = \triangle$ (all others appear on a back rank) and CAPS is black. $\mathbf{K} = \mathbf{\dot{\Xi}}$ (only one appearing exactly once for each side). Black is missing two pieces, both accounted for by \triangle captures. White is missing four pieces.

518: **L**= $\hat{\mathbb{Z}}$ (impossible squares to be on as **A**, **C**, or **S**). **S** is not $\stackrel{\text{def}}{\mathbb{Z}}$ (h1 unreachable), **A** is not $\stackrel{\text{def}}{\mathbb{Z}}$ (triple check). Now, **S** is also not $\stackrel{\text{def}}{\mathbb{Z}}$ due to **S** on h1 and counting captures¹. So, **S**= $\stackrel{\text{def}}{\mathbb{Z}}$, $A=\stackrel{\text{def}}{\mathbb{Z}}$, and **C**= $\stackrel{\text{def}}{\mathbb{Z}}$, with double check via promotion to $\stackrel{\text{def}}{\mathbb{Z}}$.

764: $\mathbf{C} = \underline{\hat{\mathbf{a}}}$ and $\mathbf{S} = \underline{\underline{\mathbf{w}}}$ (g8 and h8 trapped). So, $\mathbf{L} = \underline{\underline{\mathbf{u}}}$ (otherwise illegal double check) and $\mathbf{A} = \overline{\underline{\mathbf{v}}}$ (three captures suffice here), with double check via promotion to $\underline{\mathbf{a}}$.



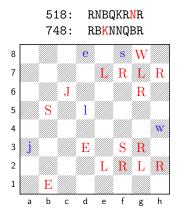


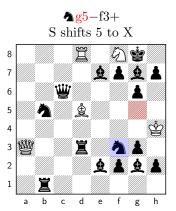


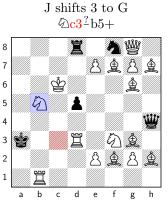
"Doctors": **T**=\text{\ti}\text{\texi}\text{\text{\text{\text{\text{\texi{\text{\texi}\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\texi

518: **R**=② (魚a1 not possible) and **C**=魚, while **S**=豐 and **D**=蛰 (both sides in check otherwise). **Z**f8 arrived via castling.

755: $\mathbf{D}=\stackrel{\text{de}}{=}$ and $\mathbf{S}=\stackrel{\text{de}}{=}$ (impossible rook check otherwise), while $\mathbf{R}=\stackrel{\text{de}}{=}$ (both sides in check otherwise) and $\mathbf{C}=\stackrel{\text{de}}{=}$. $\mathbb{Z}d1$ arrived via castling.







"Jewelers": CAPS has all 16 pieces and multiple promoted. \Dotsin can be \Dotsin or \Dotsin , while \Dotsin could technically be \Dotsin , \Dotsin , or \Dotsin , but it's straightforward to tell it's not \Dotsin . So, \Dotsin and \Dotsin and \Dotsin in some order. Also, \Dotsin can't be \Dotsin or \Dotsin , as both are on the same-color squares, which means \Dotsin and \Dotsin and \Dotsin , in some order, while \Dotsin and \Dotsin otherwise both sides are in check).

518: **L**=& (otherwise &f1 or &f8 could never have moved) and **R**=&; **J**= $\&gaige^*$ and **W**= $\&gaige^*$ (otherwise triple check). Finally, CAPS is black (otherwise triple check), and we have double check from $\&gaige^*$ and $\&gaige^*$

748: $\mathbf{R} = \mathbf{\hat{g}}$ (otherwise $\mathbf{\hat{g}}g1$ or $\mathbf{\hat{g}}g8$ could never have moved) and $\mathbf{L} = \mathbf{\hat{G}}$; $\mathbf{W} = \mathbf{\hat{g}}$ and $\mathbf{J} = \mathbf{\hat{g}}$ (otherwise impossible double check). Finally, CAPS is white (otherwise both sides in check), and we have double check from $\mathbf{\hat{G}}b5$ and $\mathbf{\tilde{g}}d3$.