MA1200 Lecture 1 Self Notes

Polar Form

$\forall P(x,y), x \neq 0, r := \sqrt{x^2+y^2}, \theta := \arctan\left(\frac{y}{x}\right),$

	x > 0	**x < 0**
$y > 0$	1st quadrant => $0 + \theta$	2nd quadrant => $180^\circ - \theta$
$y < 0$	4th quadrant => $360^\circ - \theta$	3rd quadrant => $-180^\circ + \theta$

Parabola

Formulas

Equations	**Vertex**	**AoS**	**Focus**	F_l	**Directrix**
$(y-k)^2 = 4p(x-h)$	(h,k)	$y = k$	$(h+p, k)$	p	$x = k - p$
$(x-h)^2 = 4p(y-k)$	(h,k)	$x = h$	$(h, k+p)$	p	$y = k - p$
$y = ax^2 + bx + c$	$(-\frac{b}{2a}, \frac{4ac-b^2}{4a})$	$x = -\frac{b}{2a}$	$(h, k + \frac{1}{4a})$	$\frac{1}{4a}$	$y = \frac{4ac-b^2-1}{4a}$

Determining Equations based on Graphs

- Determine Concavity

Equations	$p > 0$	$p < 0$
$(y-k)^2 = 4p(x-h)$	"tails" point to the right	"tails" point to the left
$(x-h)^2 = 4p(y-k)$	"tails" point upwards	"tails" point down

- Find Vertex
- Find $\triangle = b^2 - 4ac$,

$\triangle > 0$	$\triangle = 0$	$\triangle < 0$
Two real roots	One real root	No real root

Ellipse

Formulas

Equation	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1,\ a > b$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1,\ a > b$
Eccentricity	$e = \frac{\sqrt{a^2-b^2}}{a}, a > b$	$e = \frac{\sqrt{a^2-b^2}}{a}, a > b$
Centre	(h, k)	(h, k)
Shape	Fat and short	Thin and long
Major Axis	$y = k$ (len: $2a$)	$x = h$ (len: $2a$)
Minor Axis	$x = h$ (len: $2b$)	$y = k$ (len: $2b$)
Vertices	$(h \pm a, 0)$	$(0, k \pm a)$
Co-Vertices	$(0, k \pm b)$	$(h \pm b, 0)$
Directrices	$x = h \pm \frac{a}{e}$	$y = k \pm \frac{a}{e}$
Foci	$(h \pm ae, k)$	$(h, k \pm ae)$

Extra Formulas

Area	$(\pi)(ab)$
Perimeter Approximation	$\pi(a+b)(1+\frac{p}{10+\sqrt{4-p}}), p=3(\frac{a-b}{a+b})^2$
Lantus Recta Length	$\frac{2b^2}{a}$
Semi-lantus Rectum Length	$\frac{b^2}{a}$
Parametric Form	$x=h+a\cos{(t)}, y=b+\sin{(t)}$

Deduction

$$\sqrt{(x-(-c))^2+y^2}+\sqrt{(x-(c))^2+y^2}=2a$$

$$\sqrt{(x+c)^2+y^2}+\sqrt{(x-c)^2+y^2}=2a$$

$$\sqrt{(x-c)^2+y^2}=2a-\sqrt{(x+c)^2+y^2}$$

$$(x-c)^2+y^2=4a^2-4a\sqrt{(x+c)^2+y^2}+((x+c)^2+y^2)$$

$$-4cx=4a^2-4a\sqrt{(x+c)^2+y^2}$$

$$4a\sqrt{(x+c)^2+y^2}=4a^2+4cx$$

$$\sqrt{a^2(x+c)^2+a^2y^2}=a^2+cx$$

$$a^2(x+c)^2+a^2y^2=(a^2+cx)^2$$

$$a^2x^2+2a^2cx+a^2c^2+a^2y^2=a^4+2a^2cx+c^2x^2$$

$$a^2x^2+a^2c^2+a^2y^2=a^4+c^2x^2$$

$$a^2x^2-c^2x^2+a^2y^2=a^4-a^2c^2$$

$$(a^2-c^2)x^2+a^2y^2=a^2(a^2-c^2)$$

let $b=\sqrt{a^2-c^2}$, then

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$\frac{(x)^2}{a^2} + \frac{(y)^2}{b^2} = 1,\ a > b$$