

A household is born with 1 unit of housing, $h_{-1} = 1$ and lives for two periods. A unit of house is worth p . The household must pay a fraction $(1 - \gamma)ph_{-1}$ in period 0, meaning that only a fraction γ of the house value was paid “yesterday” (normalization). The household has income w_0 and w_1 in periods 0 and 1 respectively. The agent only values housing in period 1, and consumption in both periods. The utility function is

$$u(c_0, c_1, v) = c_0 + c_1 + v(h_1)$$

The household’s budget constraints are

$$c_0 + \gamma ph_0 \mathcal{I}(h_0 \neq 1) + (1 - \gamma)p \leq p \mathcal{I}(h_0 \neq 1) + w_0 + T_0^b \mathcal{I}(h_0 > 1)$$

$$c_1 + ph_1 \mathcal{I}(h_1 \neq h_0) + (1 - \gamma)ph_0 \mathcal{I}(h_0 \neq 1) \leq ph_0 \mathcal{I}(h_1 \neq h_0) + w_1 + T_1^b \mathcal{I}(h_1 > 1)$$

If $h_0 = 1$ then the first one is

$$c_0 + (1 - \gamma)p \leq w_0 + T_0^b \mathcal{I}(h_0 > 1)$$

if $h_0 \neq 1$ then

$$c_0 + \gamma p(h_0 - 1) \leq w_0 + T_0^b \mathcal{I}(h_0 > 1).$$

The second if $h_1 = h_0$ is

$$c_1 + (1 - \gamma)ph_0 \mathcal{I}(h_0 \neq 1) \leq w_1 + T_1^b \mathcal{I}(h_1 > 1)$$

else

$$c_1 + p(h_1 - h_0) + (1 - \gamma)ph_0 \mathcal{I}(h_0 \neq 1) \leq w_1 + T_1^b \mathcal{I}(h_1 > 1)$$

the second is equal to the first when $h_1 = h_0$ therefore we can use the conditions

$$c_0 + \gamma ph_0 \mathcal{I}(h_0 \neq 1) + (1 - \gamma)p \leq p \mathcal{I}(h_0 \neq 1) + w_0 + T_0^b \mathcal{I}(h_0 > 1)$$

$$c_1 + p(h_1 - h_0) + (1 - \gamma)ph_0 \mathcal{I}(h_0 \neq 1) \leq w_1 + T_1^b \mathcal{I}(h_1 > 1)$$

These two bind, and hence we can write

$$\begin{aligned} C &\equiv c_0 + c_1 = p \mathcal{I}(h_0 \neq 1) + w_0 + T_0^b \mathcal{I}(h_0 > 1) - \gamma ph_0 \mathcal{I}(h_0 \neq 1) - (1 - \gamma)p \\ &\quad w_1 + T_1^b \mathcal{I}(h_1 > 1) - p(h_1 - h_0) - (1 - \gamma)ph_0 \mathcal{I}(h_0 \neq 1) \\ &= p(1 - h_0) \mathcal{I}(h_0 \neq 1) - p(h_1 - h_0) - (1 - \gamma)p + w_0 + w_1 + T_0^b \mathcal{I}(h_0 > 1) + T_1^b \mathcal{I}(h_1 > 1) \\ &= p(1 - h_0) - p(h_1 - h_0) - (1 - \gamma)p + w_0 + w_1 + T_0^b \mathcal{I}(h_0 > 1) + T_1^b \mathcal{I}(h_1 > 1) \\ &= p(1 - h_1) - (1 - \gamma)p + w_0 + w_1 + T_0^b \mathcal{I}(h_0 > 1) + T_1^b \mathcal{I}(h_1 > 1) \end{aligned}$$

On top of these two constraints, the household also has the collateral constraints

$$\gamma ph_0 \leq \gamma p + w_0 + T_0^c \Leftrightarrow h_0 \leq 1 + \frac{w_0 + T_0^c}{\gamma p}$$

$$\gamma ph_1 \leq \gamma ph_0 \mathcal{I}(h_0 \neq 1) + p \mathcal{I}(h_0 = 1) + w_1 + T_1^c \Leftrightarrow h_1 \leq h_0 \mathcal{I}(h_0 \neq 1) + \frac{\mathcal{I}(h_0 = 1)}{\gamma} + \frac{w_1 + T_1^c}{\gamma p}$$

This assymetry is due to the fact that if the household does not purchase a house in period 0 it will not be indebted to period $t + 1$. Kind of a consequence of this financial market...

The full problem

$$\begin{aligned} & \max_{C, h_0, h_1} C + v(h_1) \text{ s.t.o.} \\ C &= p(1 - h_1) - (1 - \gamma)p + w_0 + w_1 + T_0^b \mathcal{I}(h_0 > 1) + T_1^b \mathcal{I}(h_1 > 1) \\ h_0 &\leq 1 + \frac{w_0 + T_0^c}{\gamma p} \text{ if } h_0 \neq 1 \\ h_1 &\leq h_0 \mathcal{I}(h_0 \neq 1) + \frac{\mathcal{I}(h_0 = 1)}{\gamma} + \frac{w_1 + T_1^c}{\gamma p} \text{ if } h_1 \neq h_0 \end{aligned}$$

The unconstrained problem

$$\begin{aligned} & \max_{C, h_0, h_1} C + v(h_1) \text{ s.t.o.} \\ C &= p(1 - h_1) - (1 - \gamma)p + w_0 + w_1 + T_0^b \mathcal{I}(h_0 > 1) + T_1^b \mathcal{I}(h_1 > 1) \end{aligned}$$

The marginal condition is

$$v'(h_1) = p$$

This defines the unconstrained optimal level of housing

$$h^* = [v']^{-1}(p)$$

assume $h^* > 1$. Assume that $T_0^b, T_1^b \geq 0$, then this is the optimal choice.

An effectively unconstrained household at time 0 Suppose that for a household

$$1 + \frac{w_0 + T_0^c}{\gamma p} \geq h^* \Leftrightarrow w_0 \geq \bar{w} \equiv \gamma p(h^* - 1) - T_0^c.$$

Then the household chooses

$$\begin{aligned} h_0 &= h_1 = h^* \\ C &= p(1 - h^*) - (1 - \gamma)p + w_0 + w_1 + T_0^b + T_1^b \end{aligned}$$

Slightly constrained household at time 0 Consider instead households such that

$$w_0 < \bar{w}.$$

Define

$$\bar{h}_0 = 1 + \frac{w_0 + T_0^c}{\gamma p}$$

which is the maximum house that the household could buy. The household wants to get h_1 the closes it can to h^* , and since h_0 doesn't appear directly in the definition of C , conditional on moving to a new house the household chooses $h_0 = \bar{h}_0$. Then the problem becomes

$$\max_{C, h_0, h_1} C + v(h_1) \text{ s.t.o.}$$

$$C = p(1 - h_1) - (1 - \gamma)p + w_0 + w_1 + T_0^b \mathcal{I}(h_0 > 1) + T_1^b \mathcal{I}(h_1 > 1)$$

$$h_0 \in \{\bar{h}_0, 1\}$$

$$h_1 \leq \bar{h}_0 \mathcal{I}(h_0 \neq 1) + \frac{\mathcal{I}(h_0 = 1)}{\gamma} + \frac{w_1 + T_1^c}{\gamma p} \text{ if } h_1 \neq h_0$$

Claim. Suppose that

$$\bar{h}_0 \geq \frac{1}{\gamma}$$

then the household moves in period 0. (I am actually assuming that $h^* > 1/\gamma$).

This is simple. If moving makes us less constrained tomorrow and also gives the transfer today then it is strictly better to move.

Claim. Suppose that

$$\bar{h}_0 + \frac{w_1 + T_1^c}{\gamma p} \geq h^* \text{ and } \bar{h}_0 > 1$$

then it is also strictly better to move because the household gets the transfer and still goes to the optimal house.

If either

$$\bar{h}_0 \geq \frac{1}{\gamma} \text{ or } \bar{h}_0 + \frac{w_1 + T_1^c}{\gamma p} \geq h^*$$

then the household moves in period 0 and in period 1. These constraints can be written as

$$1 + \frac{w_0 + T_0^c}{\gamma p} \geq \frac{1}{\gamma} \Leftrightarrow w_0 \geq p(1 - \gamma) - T_0^c \Rightarrow \begin{cases} h_0 = 1 + \frac{w_0 + T_0^c}{\gamma p} \\ h_1 = \max \left\{ \bar{h}_0, \min \left\{ h^*, \bar{h}_0 + \frac{w_1 + T_1^c}{\gamma p} \right\} \right\} \end{cases}$$

$$w_0 > -T_0^c$$

$$1 + \frac{w_0 + T_0^c}{\gamma p} + \frac{w_1 + T_1^c}{\gamma p} \geq h^* \Leftrightarrow w_1 \geq \gamma p(h^* - 1) - T_0^c - T_1^c - w_0 \Rightarrow \begin{cases} h_0 = 1 + \frac{w_0 + T_0^c}{\gamma p} \\ h_1 = h^* \end{cases}$$

Very very constrained households Suppose that

$$w_0 \leq -T_0^c$$

These households never buy in period 0. They buy in period 1 if they can, so

$$h_0 = 1$$

$$\begin{aligned} h_1 &= \max \left\{ 1, \min \left\{ h^*, \frac{1}{\gamma} + \frac{w_1 + T_1^c}{\gamma p} \right\} \right\} \\ &= \begin{cases} 1 & \text{if } w_1 \leq \gamma \left(1 - \frac{1}{\gamma} \right) - T_1^c \\ \frac{1}{\gamma} + \frac{w_1 + T_1^c - 1}{\gamma p} & \text{if } w_1 < \gamma p \left(h^* - \frac{1}{\gamma} \right) - T_1^c \\ h^* & \text{if } w_1 > \gamma p \left(h^* - \frac{1}{\gamma} \right) - T_1^c \end{cases} \end{aligned}$$

Slightly more constrained households Suppose that

$$-T_0^c < w_0 < p(1 - \gamma) - T_0^c$$

and

$$w_1 < \gamma p(h^* - 1) - T_0^c - T_1^c - w_0$$

These are households that can buy houses in period 0 but its not necessarily optimal for them to do so, this is because by buying a house they become more constrained.

If there was no transfer in case they move houses in period 0 this would be simple. They would not buy a house, and in period 1 their decisions would be the usual.

Very constrained households Suppose that

$$w_1 \leq \gamma p \left(1 - \frac{1}{\gamma}\right) - T_1^c$$

If they don't buy in time 0 they cannot buy a house larger than 1 in time 1. However, they can still purchase a larger house in period 0

$$h_0 = 1 + \frac{w_0 + T_0^c}{\gamma p}$$

$$h_1 = h_0$$

With the transfer Assume that

$$w_1 > \gamma p \left(1 - \frac{1}{\gamma}\right) - T_1^c$$

which just means that if the household does not purchase a house at time 0 it can do so at time 1.

If the household buys a house it will buy

$$\bar{h}_0 = 1 + \frac{w_0 + T_0^c}{\gamma p} \in \left(1, \frac{1}{\gamma}\right)$$

and at time 1

$$\tilde{h}_1 = \bar{h}_0 + \max \left\{ 0, \frac{w_1 + T_1^c}{\gamma p} \right\} = 1 + \frac{w_0 + T_0^c}{\gamma p} + \max \left\{ 0, \frac{w_1 + T_1^c}{\gamma p} \right\}$$

With utility

$$V^p = p(1 - \tilde{h}_1) - (1 - \gamma)p + w_0 + w_1 + T_0^b + T_1^b + v(\tilde{h}_1)$$

Else, if the household does not buy in period 0 then

$$h_0 = 1$$

$$\hat{h}_1 = \min \left\{ h^*, \frac{1}{\gamma} + \frac{w_1 + T_1^c}{\gamma p} \right\} > \tilde{h}_1$$

With utility

$$V^n = p(1 - \hat{h}_1) - (1 - \gamma)p + w_0 + w_1 + T_1^b + v(\hat{h}_1)$$

Then the household purchases a house in period 1 if

$$V^p \geq V^n \Leftrightarrow p(\hat{h}_1 - \tilde{h}_1) + T_0^b \geq v(\hat{h}_1) - v(\tilde{h}_1)$$

Suppose that $h^* \leq \frac{1}{\gamma} + \frac{w_1 + T_1^c}{\gamma p} \Leftrightarrow w_1 \geq \gamma p \left(h^* - \frac{1}{\gamma} \right) - T_1^c$ Then

$$V^p \geq V^n \Leftrightarrow v(\tilde{h}_1) - p\tilde{h}_1 + T_0^b \geq v(h^*) - ph^*$$

There is a single $h_1^c < h^*$ such that

$$v(h_1^c) - ph_1^c + T_0^b = v(h^*) - ph^*,$$

Furthermore, the household will buy if they can afford more than this level of housing, and will not buy in period 0 if they can't.

Proof is simple: RHS is increasing in h_1^c because $h_1^c < h^*$ furthermore, as $h_1^c \rightarrow h^*$ then the RHS is strictly larger than the LHS. Assume that $h_1^c > 1$. This means that the household will buy if

$$1 + \frac{w_0 + T_0^c}{\gamma p} + \max \left\{ 0, \frac{w_1 + T_1^c}{\gamma p} \right\} \geq h_1^c \Leftrightarrow w_1 \geq \gamma p (h_1^c - 1) - T_0^c - T_1^c - w_0$$

Then if

$$w_1 \geq \gamma p \left(h^* - \frac{1}{\gamma} \right) - T_1^c \text{ and } w_1 \geq \gamma p (h_1^c - 1) - T_0^c - T_1^c - w_0$$

The household acquires

$$\begin{aligned} h_0 &= 1 + \frac{w_0 + T_0^c}{\gamma p} \\ h_1 &= h_0 + \frac{w_1 + T_1^c}{\gamma p} \end{aligned}$$

If instead

$$w_1 \geq \gamma p \left(h^* - \frac{1}{\gamma} \right) - T_1^c \text{ and } w_1 < \gamma p (h_1^c - 1) - T_0^c - T_1^c - w_0$$

Then

$$\begin{aligned} h_0 &= 1 \\ h_1 &= h^* \end{aligned}$$

Furthermore,

$$\frac{dh_1^c}{dT_0^b} (v'(h_1^c) - p) + 1 = 0 \Leftrightarrow \frac{dh_1^c}{dT_0^b} = -\frac{1}{v'(h_1^c) - p} < 0$$

Suppose that $w_1 \leq -T_1^c$ Then

$$\tilde{h}_1 = 1 + \frac{w_0 + T_0^c}{\gamma p}$$

and

$$\hat{h}_1 = \frac{1}{\gamma} + \frac{w_1 + T_1^c}{\gamma p}$$

The condition for optimality is

$$\begin{aligned} p(\hat{h}_1 - \tilde{h}_1) + T_0^b &\geq v(\hat{h}_1) - v(\tilde{h}_1) \\ v(\tilde{h}_1) - p\tilde{h}_1 + T_0^b &\geq v\left(\frac{1}{\gamma} + \frac{w_1 + T_1^c}{\gamma p}\right) - p\left(\frac{1}{\gamma} + \frac{w_1 + T_1^c}{\gamma p}\right) \\ v\left(1 + \frac{w_0 + T_0^c}{\gamma p}\right) - p\left(1 + \frac{w_0 + T_0^c}{\gamma p}\right) + T_0^b &\geq v\left(\frac{1}{\gamma} + \frac{w_1 + T_1^c}{\gamma p}\right) - p\left(\frac{1}{\gamma} + \frac{w_1 + T_1^c}{\gamma p}\right) \end{aligned}$$

This defines a relation, $w_1^d(w_0)$ such that if $w_1 < w_1^d(w_0)$ it is worthwhile to buy today, else if $w_1 > w_1^d(w_0)$ we should wait to tomorrow to buy, and

$$v\left(1 + \frac{w_0 + T_0^c}{\gamma p}\right) - p\left(1 + \frac{w_0 + T_0^c}{\gamma p}\right) + T_0^b = v\left(\frac{1}{\gamma} + \frac{w_1^d(w_0; T) + T_1^c}{\gamma p}\right) - p\left(\frac{1}{\gamma} + \frac{w_1^d(w_0; T) + T_1^c}{\gamma p}\right)$$

this relation is increasing in w_0 :

$$\begin{aligned} v'\left(1 + \frac{w_0 + T_0^c}{\gamma p}\right) - p &= \left(v'\left(\frac{1}{\gamma} + \frac{w_1^d(w_0) + T_1^c}{\gamma p}\right) - p\right) \frac{dw_1^d(w_0)}{dw_0} \\ \frac{dw_1^d(w_0)}{dw_0} &= \frac{v'\left(1 + \frac{w_0 + T_0^c}{\gamma p}\right) - p}{v'\left(\frac{1}{\gamma} + \frac{w_1^d(w_0) + T_1^c}{\gamma p}\right) - p} > 0 \end{aligned}$$

Finally suppose that $w_1 > -T_1^c$ **and** $w_1 < \gamma p\left(h^* - \frac{1}{\gamma}\right) - T_1^c$ If purchasing in period 0 then

$$\begin{aligned} h_0 &= 1 + \frac{w_0 + T_0^c}{\gamma p} \\ \bar{h}_1 &= h_0 + \frac{w_1 + T_1^c}{\gamma p} = 1 + \frac{w_0 + w_1 + T_1^c + T_0^c}{\gamma p} \end{aligned}$$

which implies that

$$V^p = p(1 - \bar{h}_1) - (1 - \gamma)p + w_0 + w_1 + T_0^b + T_1^b + v(\bar{h}_1)$$

else if not buying in period 0 we get

$$\begin{aligned} h_0 &= 1 \\ \hat{h}_1 &= \frac{1}{\gamma} + \frac{w_1 + T_1^c}{\gamma p} \end{aligned}$$

and

$$V^n = p(1 - \hat{h}_1) - (1 - \gamma)p + w_0 + w_1 + T_1^b + v(\hat{h}_1)$$

Note that $\bar{h}_1 < \hat{h}_1$. This implies that the household will buy a house in period 0 if

$$V^p \geq V^n \Leftrightarrow v(\bar{h}_1) - p\bar{h}_1 + T_0^b \geq v(\hat{h}_1) - p\hat{h}_1$$

$$v\left(1 + \frac{w_0 + w_1 + T_1^c + T_0^c}{\gamma p}\right) - p\left(1 + \frac{w_0 + w_1 + T_1^c + T_0^c}{\gamma p}\right) + T_0^b \geq v\left(\frac{1}{\gamma} + \frac{w_1 + T_1^c}{\gamma p}\right) - p\left(\frac{1}{\gamma} + \frac{w_1 + T_1^c}{\gamma p}\right)$$

Fix w_0 . The LHS grows faster than the RHS. There exists a single $\hat{w}_1(w_0)$ such that

$$v\left(1 + \frac{w_0 + \hat{w}_1(w_0) + T_1^c + T_0^c}{\gamma p}\right) - p\left(1 + \frac{w_0 + \hat{w}_1(w_0) + T_1^c + T_0^c}{\gamma p}\right) + T_0^b = v\left(\frac{1}{\gamma} + \frac{\hat{w}_1(w_0) + T_1^c}{\gamma p}\right) - p\left(\frac{1}{\gamma} + \frac{\hat{w}_1(w_0) + T_1^c}{\gamma p}\right)$$

and for $w_1 \geq \hat{w}_1(w_0)$ the household should buy, else it should not buy. Furthermore, useful things

$$\frac{1 + \hat{w}_1'(w_0)}{\gamma p} \left(v' \left(1 + \frac{w_0 + \hat{w}_1(w_0) + T_1^c + T_0^c}{\gamma p} \right) - p \right) = \frac{\hat{w}_1'(w_0)}{\gamma p} \left(v' \left(\frac{1}{\gamma} + \frac{\hat{w}_1(w_0) + T_1^c}{\gamma p} \right) - p \right)$$

$$\hat{w}_1'(w_0) = \frac{v' \left(\frac{1}{\gamma} + \frac{\hat{w}_1(w_0) + T_1^c}{\gamma p} \right) - v' \left(1 + \frac{w_0 + \hat{w}_1(w_0) + T_1^c + T_0^c}{\gamma p} \right)}{v' \left(1 + \frac{w_0 + \hat{w}_1(w_0) + T_1^c + T_0^c}{\gamma p} \right) - p} < 0$$

The transfers are complicating everything, so let me do it without the transfers...

Slightly more constrained households when $T_0^b = 0$ This case is nice because the household is not trading off getting a transfer in period zero versus having a larger house tomorrow. It is just about whatever maximizes the house level in period 1.

Households very constrained at time 1 Suppose that

$$w_1 \leq \gamma p \left(1 - \frac{1}{\gamma} \right) - T_1^c$$

If they don't buy in time 0 they cannot buy a house larger than 1 in time 1. However, they can still purchase a larger house in period 0

$$h_0 = 1 + \frac{w_0 + T_0^c}{\gamma p}$$

$$h_1 = h_0$$

Households not constrained at time 1 Suppose that

$$w_1 \geq \gamma p \left(h^* - \frac{1}{\gamma} \right) - T_1^c$$

this is the case in which the household can move to the optimal house at time 1 if it does not move in period 0

$$h_0 = 1$$

then

$$h_1 = h^*$$

which is optimal. Here I assume that households such that

$$w_1 \geq \gamma p(h^* - 1) - T_0^c - T_1^c - w_0$$

also buy at time 0. In fact they are indifferent, since their time 1 house is h^* in any case, but assume that if

$$w_1 \geq \gamma p(h^* - 1) - T_0^c - T_1^c - w_0$$

then

$$h_0 = 1 + \frac{w_0 + T_0^c}{\gamma p}$$

$$h_1 = h^*$$

Households slightly constrained at time 1 Suppose that

$$w_1 \geq -T_1^c$$

this is the case in which the household can move to a larger house than $1/\gamma$ if it does move in period 0, or larger than $h_0 = 1 + \frac{w_0 + T_0^c}{\gamma p}$, since we are looking at the region in which

$$1 + \frac{w_0 + T_0^c}{\gamma p} < \frac{1}{\gamma} \Leftrightarrow w_0 < \gamma p \left(\frac{1}{\gamma} - 1 \right) - T_0^c$$

then it is better not to change in period 0

$$h_0 = 1$$

$$h_1 = \frac{1}{\gamma} + \frac{w_1 + T_1^c}{\gamma p}$$

Finally the last case Suppose that

$$\gamma p \left(1 - \frac{1}{\gamma}\right) - T_1^c \leq w_1 < -T_1^c$$

then if the household buys a house in period 0 it gets

$$h_1 = h_0 = 1 + \frac{w_0 + T_0^c}{\gamma p}$$

If the household does not then

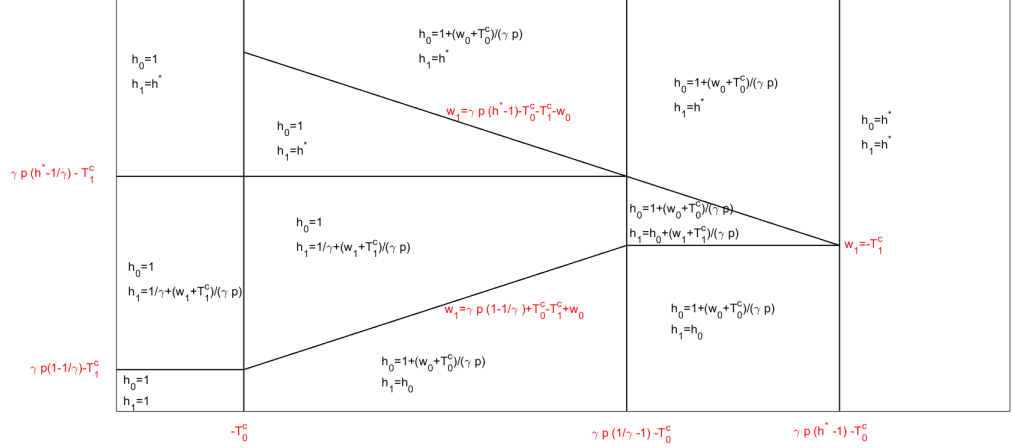
$$h_1 = \frac{1}{\gamma} + \frac{w_1 + T_1^c}{\gamma p}$$

Then the household buys in period 1 if

$$\frac{1}{\gamma} + \frac{w_1 + T_1^c}{\gamma p} \leq 1 + \frac{w_0 + T_0^c}{\gamma p} \Leftrightarrow w_1 \leq \gamma p \left(1 - \frac{1}{\gamma}\right) + T_0^c - T_1^c + w_0.$$

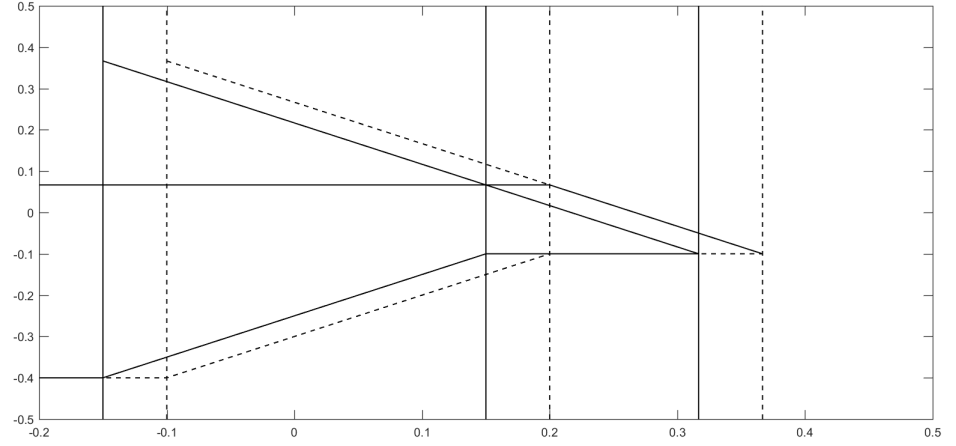
Summary without transfers

Without transfers we get the following table and figure



w_0	w_1	h_0	h_1
$(-\infty, -T_0^c)$	$(-\infty, \gamma p \left(1 - \frac{1}{\gamma}\right) - T_1^c)$	1	1
$(-\infty, -T_0^c)$	$\left(\gamma p \left(1 - \frac{1}{\gamma}\right) - T_1^c, \gamma p \left(h^* - \frac{1}{\gamma}\right) - T_1^c\right)$	1	$\frac{1}{\gamma} + \frac{w_1 + T_1^c}{\gamma p}$
$(-\infty, -T_0^c)$	$\left(\gamma p \left(h^* - \frac{1}{\gamma}\right) - T_1^c, +\infty\right)$	1	h^*
$\left(-T_0^c, \gamma p \left(\frac{1}{\gamma} - 1\right) - T_0^c\right)$	$(-\infty, \gamma p \left(1 - \frac{1}{\gamma}\right) - T_1^c)$	$1 + \frac{w_0 + T_0^c}{\gamma p}$	$1 + \frac{w_0 + T_0^c}{\gamma p}$
$\left(-T_0^c, \gamma p \left(\frac{1}{\gamma} - 1\right) - T_0^c\right)$	$\left(\gamma p \left(1 - \frac{1}{\gamma}\right) - T_1^c, \gamma p \left(1 - \frac{1}{\gamma}\right) + T_0^c - T_1^c + w_0\right)$	$1 + \frac{w_0 + T_0^c}{\gamma p}$	$1 + \frac{w_0 + T_0^c}{\gamma p}$
$\left(-T_0^c, \gamma p \left(\frac{1}{\gamma} - 1\right) - T_0^c\right)$	$\left(\gamma p \left(1 - \frac{1}{\gamma}\right) + T_0^c - T_1^c + w_0, -T_1^c\right)$	1	$\frac{1}{\gamma} + \frac{w_1 + T_1^c}{\gamma p}$
$\left(-T_0^c, \gamma p \left(\frac{1}{\gamma} - 1\right) - T_0^c\right)$	$(-T_1^c, \gamma p \left(h^* - \frac{1}{\gamma}\right) - T_1^c)$	1	$\frac{1}{\gamma} + \frac{w_1 + T_1^c}{\gamma p}$
$\left(-T_0^c, \gamma p \left(\frac{1}{\gamma} - 1\right) - T_0^c\right)$	$\left(\gamma p \left(h^* - \frac{1}{\gamma}\right) - T_1^c, \gamma p (h^* - 1) - T_0^c - T_1^c - w_0\right)$	1	h^*
$\left(-T_0^c, \gamma p \left(\frac{1}{\gamma} - 1\right) - T_0^c\right)$	$(\gamma p (h^* - 1) - T_0^c - T_1^c - w_0, +\infty)$	$1 + \frac{w_0 + T_0^c}{\gamma p}$	h^*
$\left(\gamma p \left(\frac{1}{\gamma} - 1\right) - T_0^c, \gamma p (h^* - 1) - T_0^c\right)$	$(-\infty, -T_1^c)$	$1 + \frac{w_0 + T_0^c}{\gamma p}$	$1 + \frac{w_0 + T_0^c}{\gamma p}$
$\left(\gamma p \left(\frac{1}{\gamma} - 1\right) - T_0^c, \gamma p (h^* - 1) - T_0^c\right)$	$(-T_1^c, \gamma p (h^* - 1) - T_0^c - T_1^c - w_0)$	$1 + \frac{w_0 + T_0^c}{\gamma p}$	$1 + \frac{w_0 + T_0^c}{\gamma p} + \frac{w_1 + T_1^c}{\gamma p}$
$\left(\gamma p \left(\frac{1}{\gamma} - 1\right) - T_0^c, \gamma p (h^* - 1) - T_0^c\right)$	$(\gamma p (h^* - 1) - T_0^c - T_1^c - w_0, +\infty)$	$1 + \frac{w_0 + T_0^c}{\gamma p}$	h^*
$(\gamma p (h^* - 1) - T_0^c, +\infty)$	$-$	h^*	h^*

Here goes a figure about what the change in T_0^c does to those thresholds.
Continuous lines are the new thresholds, after $T_0^c \uparrow$.



Summary with transfers

Without transfers we get the following table and figure where

$$w_1^d(w_0; T) : v \left(1 + \frac{w_0 + T_0^c}{\gamma p} \right) - p \left(1 + \frac{w_0 + T_0^c}{\gamma p} \right) + T_0^b = v \left(\frac{1}{\gamma} + \frac{w_1^d(w_0; T) + T_1^c}{\gamma p} \right) - p \left(\frac{1}{\gamma} + \frac{w_1^d(w_0; T) + T_1^c}{\gamma p} \right)$$

$$\begin{aligned} \hat{w}_1(w_0; T) : v \left(1 + \frac{w_0 + \hat{w}_1(w_0; T) + T_1^c + T_0^c}{\gamma p} \right) - p \left(1 + \frac{w_0 + \hat{w}_1(w_0; T) + T_1^c + T_0^c}{\gamma p} \right) + T_0^b \\ = v \left(\frac{1}{\gamma} + \frac{\hat{w}_1(w_0; T) + T_1^c}{\gamma p} \right) - p \left(\frac{1}{\gamma} + \frac{\hat{w}_1(w_0; T) + T_1^c}{\gamma p} \right) \end{aligned}$$

$$h_1^c(T_0^b) : v(h_1^c) - p h_1^c + T_0^b = v(h^*) - p h^*$$

w_0	w_1	h_0	h_1
$(-\infty, -T_0^c)$	$(-\infty, \gamma p(1 - \frac{1}{\gamma}) - T_1^c)$	1	1
$(-\infty, -T_0^c)$	$(\gamma p(1 - \frac{1}{\gamma}) - T_1^c, \gamma p(h^* - \frac{1}{\gamma}) - T_1^c)$	1	$\frac{1}{\gamma} + \frac{w_1 + T_1^c}{\gamma p}$
$(-\infty, -T_0^c)$	$(\gamma p(h^* - \frac{1}{\gamma}) - T_1^c, +\infty)$	1	h^*
$(-T_0^c, \gamma p(\frac{1}{\gamma} - 1) - T_0^c)$	$(-\infty, \gamma p(1 - \frac{1}{\gamma}) - T_1^c)$	$1 + \frac{w_0 + T_0^c}{\gamma p}$	$1 + \frac{w_0 + T_0^c}{\gamma p}$
$(-T_0^c, \gamma p(\frac{1}{\gamma} - 1) - T_0^c)$	$(\gamma p(1 - \frac{1}{\gamma}) - T_1^c, w_1^d(w_0; T))$	$1 + \frac{w_0 + T_0^c}{\gamma p}$	$1 + \frac{w_0 + T_0^c}{\gamma p}$
$(-T_0^c, \gamma p(\frac{1}{\gamma} - 1) - T_0^c)$	$(w_1^d(w_0; T), -T_1^c)$	1	$\frac{1}{\gamma} + \frac{w_1 + T_1^c}{\gamma p}$
$(-T_0^c, \gamma p(\frac{1}{\gamma} - 1) - T_0^c)$	$(-T_1^c, \hat{w}_1(w_0; T))$	1	$\frac{1}{\gamma} + \frac{w_1 + T_1^c}{\gamma p}$
$(-T_0^c, \gamma p(\frac{1}{\gamma} - 1) - T_0^c)$	$(\hat{w}_1(w_0; T), \gamma p(h^* - \frac{1}{\gamma}) - T_1^c)$	$1 + \frac{w_0 + T_0^c}{\gamma p}$	$1 + \frac{w_0 + T_0^c}{\gamma p} + \frac{w_1 + T_1^c}{\gamma p}$
$(-T_0^c, \gamma p(\frac{1}{\gamma} - 1) - T_0^c)$	$(\gamma p(h^* - \frac{1}{\gamma}) - T_1^c, \gamma p(h_1^c(T_0^b) - 1) - T_0^c - T_1^c - w_0)$	1	h^*
$(-T_0^c, \gamma p(\frac{1}{\gamma} - 1) - T_0^c)$	$(\gamma p(h_1^c(T_0^b) - 1) - T_0^c - T_1^c - w_0, \gamma p(h^* - 1) - T_0^c - T_1^c - w_0)$	$1 + \frac{w_0 + T_0^c}{\gamma p}$	$1 + \frac{w_1 + T_1^c}{\gamma p}$
$(-T_0^c, \gamma p(\frac{1}{\gamma} - 1) - T_0^c)$	$(\gamma p(h^* - 1) - T_0^c - T_1^c - w_0, +\infty)$	$1 + \frac{w_0 + T_0^c}{\gamma p}$	h^*
$(\gamma p(\frac{1}{\gamma} - 1) - T_0^c, \gamma p(h^* - 1) - T_0^c)$	$(-\infty, -T_1^c)$	$1 + \frac{w_0 + T_0^c}{\gamma p}$	$1 + \frac{w_0 + T_0^c}{\gamma p}$
$(\gamma p(\frac{1}{\gamma} - 1) - T_0^c, \gamma p(h^* - 1) - T_0^c)$	$(-T_1^c, \gamma p(h^* - 1) - T_0^c - T_1^c - w_0)$	$1 + \frac{w_0 + T_0^c}{\gamma p}$	$1 + \frac{w_0 + T_0^c}{\gamma p} + \frac{w_1 + T_1^c}{\gamma p}$
$(\gamma p(\frac{1}{\gamma} - 1) - T_0^c, \gamma p(h^* - 1) - T_0^c)$	$(\gamma p(h^* - 1) - T_0^c - T_1^c - w_0, +\infty)$	$1 + \frac{w_0 + T_0^c}{\gamma p}$	h^*
$(\gamma p(h^* - 1) - T_0^c, +\infty)$	$-$	h^*	h^*

Here goes a figure about what the change in T_0^c does to those thresholds.
Continuous lines are the new thresholds, after $T_0^c \uparrow$.