

First Time Home Buyers Age Distribution: A Simple Model

1 Continuous Time Model

1.1 Environment

The economy is populated by households at different ages. We assume that households start working at the age of 20 and their wealth evolves according to the following diffusion process

$$dw_t^i = \mu dt + \sigma dB_t^i,$$

where $\exp(w_t^i)$ is the wealth of household i at age $20 + t$. B_t^i is a standard Brownian motion (which is independent across i 's). For simplicity, we assume that the household's initial wealth is equal to 1 so that $w_0 = 0$.

Denote the price of housing to be p_H . We assume this price is constant over time. Let θ be the fraction that must be paid when purchasing a house. Finally, we assume that whenever a household has enough wealth to pay the down payment and purchase a house it does so. Thus, the time in which a household purchases a house is given by

$$\tau^i = \inf_s \{s : w_s^i \geq \ln(\theta p_H)\}$$

We now turn to characterize the distribution τ^i 's in the economy as a function of the fundamental exogenous parameters (p_H , θ , μ , and σ).

1.2 First Time Home Buyers Age Distribution

In this section we characterize the distribution of τ^i . Notice that τ^i is simply a stopping time so that we can use known mathematical results regarding the distribution of stopping times.

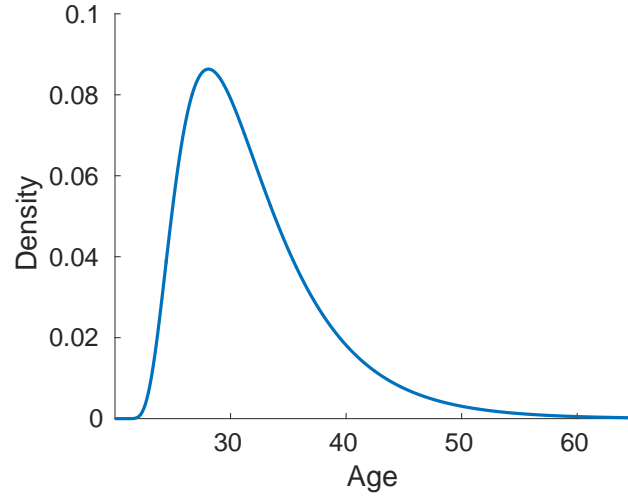
Proposition 1. The density of τ^i is given by

$$f_\tau(t) = \frac{\ln(\theta p_H)}{\sigma \sqrt{2\pi t^3}} \exp \left\{ -\frac{(\ln(\theta p_H) - \mu t)^2}{2\sigma^2 t} \right\}$$

Proof. A trivial modification of the proof provided here: <http://math.stackexchange.com/questions/1053294/density-of-first-hitting-time-of-brownian-motion-with-drift> \square

Calibration. Let $p_H = 10e$ and let $\theta = 0.1$. We shall calibrate μ to $1/12$ so that without the diffusion term it would take a household 12 years to purchase a house (such household would be 32 years old). Finally, we let $\sigma = 0.15$. Figure 1 displays the distribution of first time home buyers age given this calibration. The mode is at age 28, while the expected age of purchasing a house is at age 32.

Figure 1: Age Distribution



1.3 Comparative Statics

The 4 graphs in Figure 2 display the distribution varying the 4 exogenous parameters each at a time.

Figure 2: Age Distribution - Comparative Statics

