

Bocconi University, 20236 - Time Series Analysis

Assignment 3: HMMs with R

Due by April 9, 2024

Hidden Markov Models with R

This assignment is about Hidden Markov Models (HMMs) with **R**. We use the **R** package *depmixS4*. **Look at the material for LAB 2, posted on BBoard.**

The assignment consists in part (a) of the exercise below. The proposal of an extension to a covariate-dependent transition matrix in the HMM is a suggestion, in case you are interested, but it is not mandatory, neither you will be penalized in any way if you don't go into that.

Shortly, we will also provide an introduction to your final project - so that you can start working on it, without being overwhelmed by work at the end of the course!

Exercise.

The dataset provided in the file `data_assHMM.csv` (posted on BBoard) provides monthly data including 10 years Italian government bond's interest rate, inflation represented by the Harmonised Index of Consumer Prices (HICP) and default ratings assigned by the agencies Moody's and Fitch, in the investment grade range, i.e. from Aaa/AAA to Baa3-/BBB-. The data set collects data for the period January 1997 to July 2019, and it has been built mainly using OECD data.

- **(a)** Let us focus on the nominal interest rate for the 10 years Italian government bond. In fact, you may want to consider the *real* interest rate, calculated from the HICP.

Plot the data and comment *briefly* if and why a HMM could be a reasonable model.

Let us indeed use a Hidden Markov Model, with 3 states (representing, say, boom (i.e. less risky, lower interest rates), recession (high risk, high interest rates) and a stable path), and Gaussian emission distributions, with state-dependent mean and variance.

Provide the MLEs of the unknown parameters of the model (and their standard errors). Comment *briefly*.

Find the optimal state sequence ("decoding") and plot it, comparing it with the data.

- **(b) – Optional.** HMMs are particularly useful for time series that present change points. However, one may want to go further, trying to improve *prediction* of a possible change point through available covariates. To this aim, one may use non-homogeneous HMMs, allowing the transition matrix to depend on covariates. A reference is Zucchini, W., MacDonald, I-L. and Langrock, R. (2016) *Hidden Markov Models for Time Series: an introduction using R*. Chapman and Hall/CRC; and the **R** package *depmixS4* allows this extension, see Visser and Speekenbrink (2010), *Journal of Statistical Software* – both references are posted on BBoard.

You may want to explore this more general class of HMMs for the data under study here.