Projected d-DNNF Compilation for Feature Models

1. **What is this session about? (Bonus)**
   - What is a feature model?
2. **What you need to know about math?**
   - How to compute 42?
   - How to compute other numbers?
3. **Why should you care about this thesis?**
   - Is it novel?
   - Is it significant?
   - Is it sound?
   - Is it verifiable?
   - Is it clear?
   - (Is it great research?)
4. **More Details Wanted?**
   - Feature Models and Conjunctive Normal Forms
   - Feature-Model Slicing / Projection of Formulas
   - Model Counting
   - Projected Model Counting
   - Deterministic Decomposable Negation Normal Form (d-DNNF)
   - Slicing Implementation
   - d-DNNF Compilation
   - Heuristics
   - Heuristics: Dual Hypergraph
   - Projected d-DNNF Compilation
   - Integration and Optimization in D4
   - Dual Weighted Hypergraph Partitioning
   - Partial Resolution
   - Experimental Setup
   - Generated Projection for Feature Models (Table)

Projected d-DNNF Compilation for Feature Models

Master's Thesis by Jacob Loth, supervised by Chico Sundermann | Thomas Thüm | April 9, 2024

PADERBORN UNIVERSITY

The Boring Agenda Slide

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- Is it significant?
- Is it sound?
- Is it verifiable?
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The Boring Agenda Slide

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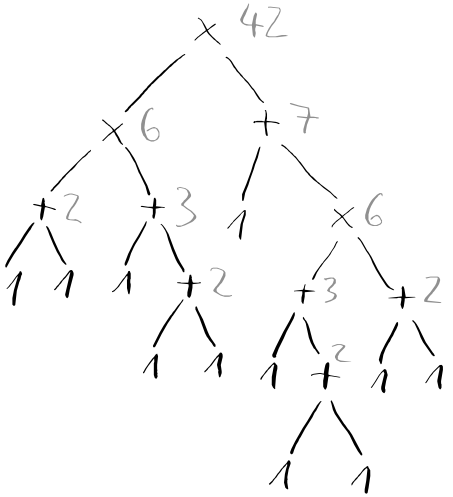
1. What is this session about? (Bonus)

What is a feature model?

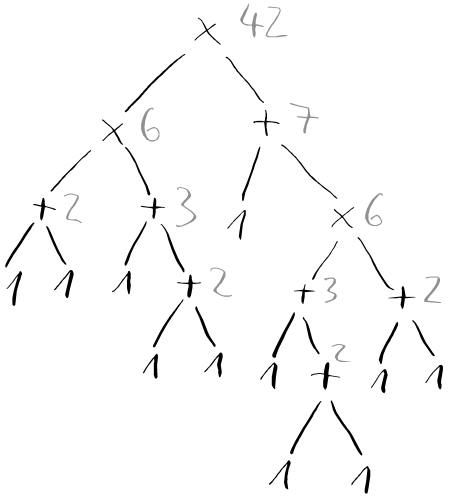
mockup: this slide shows a feature model with 42 valid configurations

2. What you need to know about math?

How to compute 42?



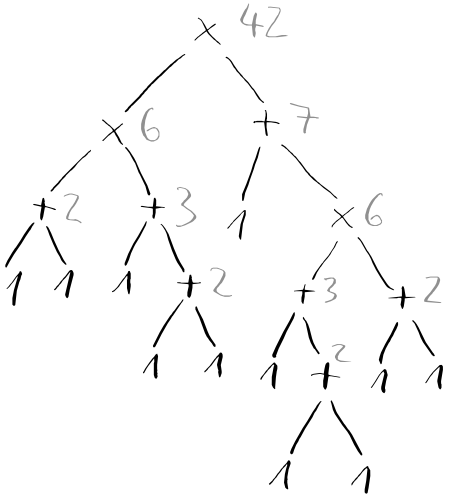
How to compute other numbers?



Queries

How many configurations of cars with Bluetooth?

How to compute other numbers?



Projections (Slices)

How many software configurations are there?

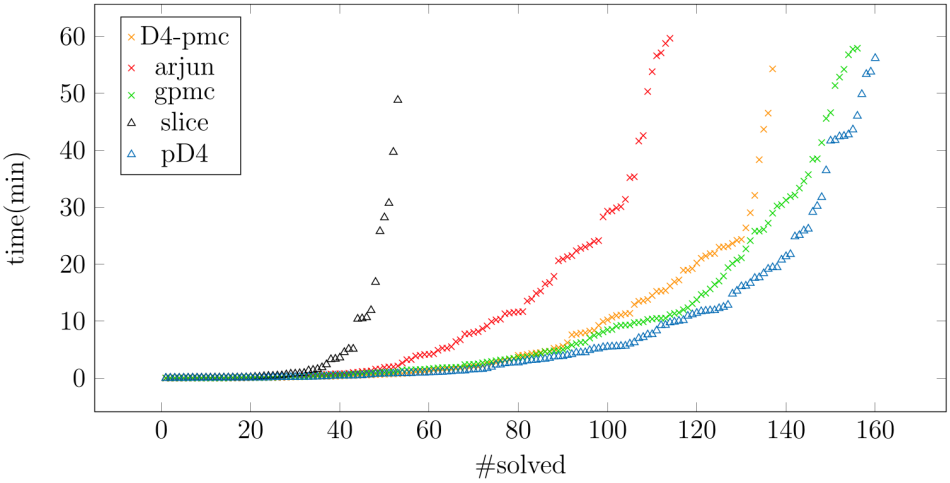
3. Why should you care about this thesis?

Is it novel?

Projection + Compilation = Projected Compilation

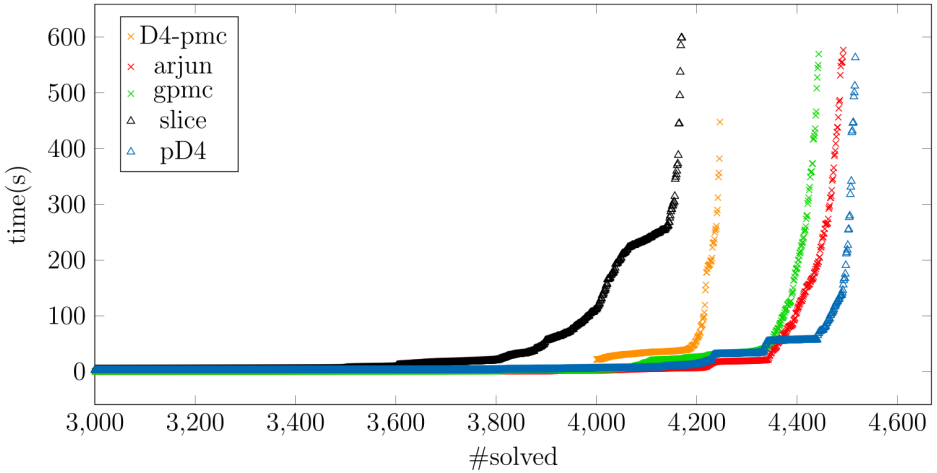
Is it significant?

Projected Model Counting Competition 2022



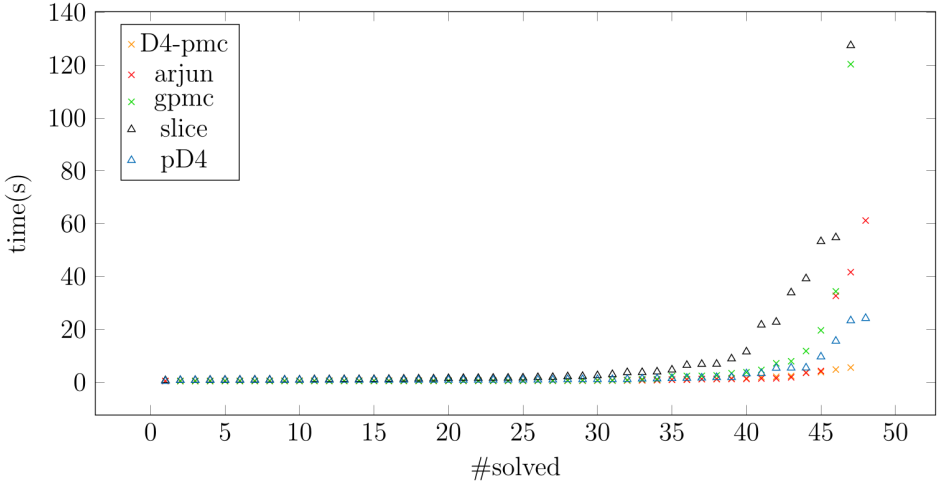
Is it sound?

Random Projections on Feature Models



Is it sound?

Real Projections from Automotive Industry



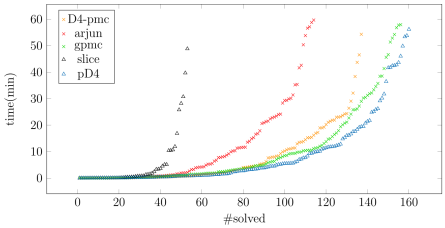
Is it verifiable?

mockup: this slide shows a screenshot from the github project / replication package / or point to the thesis? is it public?

Projected Compilation = Projection + Compilation

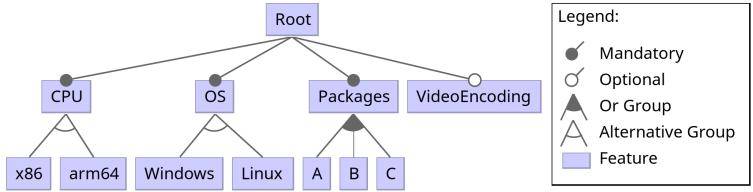
projected d-DNNF compilation ...

- is faster than projection / slicing (our goal)
- is faster than projected model counting (unexpected, due to many optimizations)
- is much faster for multiple queries (expected)
- produces d-DNNFs of comparable/smaller size
- is the first instance of projected compilation



4. More Details Wanted?

Feature Models and Conjunctive Normal Forms

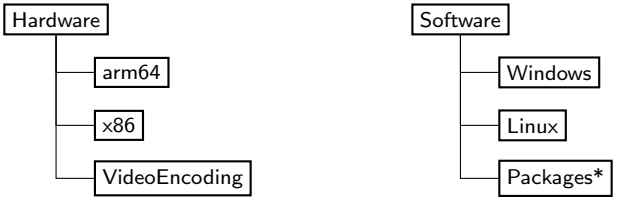


We can convert feature models to propositional formulas!

$$(x86 \wedge \neg arm64) \vee (\neg x86 \wedge arm64)...$$

Feature-Model Slicing / Projection of Formulas

Feature Model



Feature-Model Slicing / Projection of Formulas

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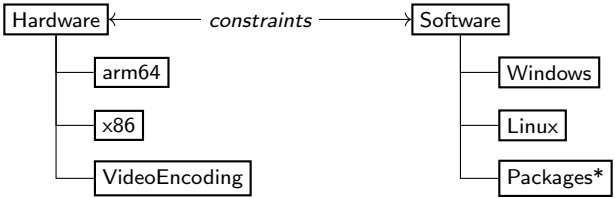
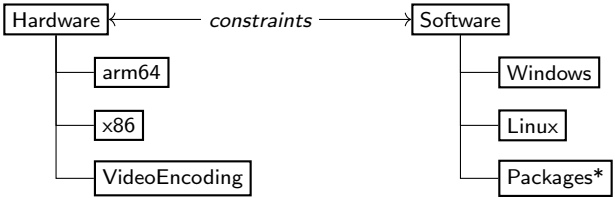


Feature-Model Slicing / Projection of Formulas

Feature Model

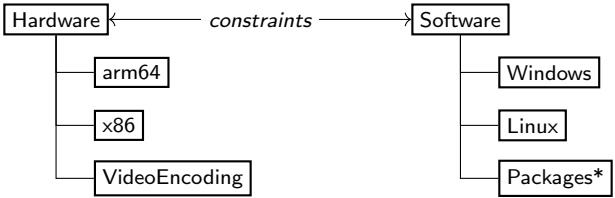


Problem

How many hardware configurations?

Feature-Model Slicing / Projection of Formulas

Feature Model



Problem

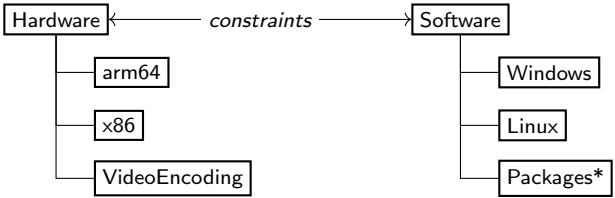
How many hardware configurations?

Transitive Constraints

$$\text{VideoEncoding} \implies \text{Windows}$$
$$\text{Windows} \implies \text{x86}$$

Feature-Model Slicing / Projection of Formulas

Feature Model



Problem

How many hardware configurations?

Transitive Constraints

VideoEncoding $\Longrightarrow$ Windows

Windows $\Longrightarrow$ x86

Sliced: VideoEncoding $\Longrightarrow$ x86

Model Counting

Problem

- How many hardware configurations?

Model Counting

Problem

- How many hardware configurations?
- Counting the number of satisfiable assignments of a propositional formula F. Denoted as $|F|$.

p	q	$F = a \wedge b$
1	1	1
1	0	0
0	1	0
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Model Counting

Problem

- How many hardware configurations?
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p	q	$F = a \wedge b$
1	1	1
1	0	0
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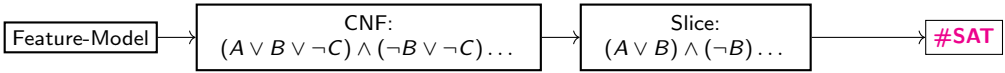
#SAT

Counts the number of solutions of a propositional formula.
Worst-case exponential complexity!

Projected Model Counting

Problem

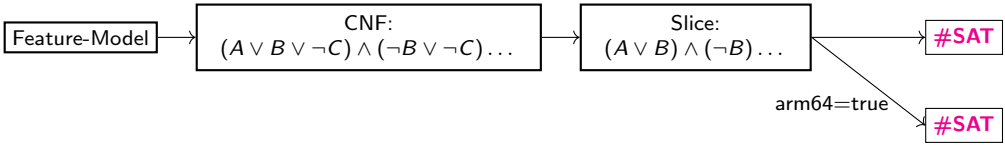
How many hardware configurations?



Projected Model Counting

Problem

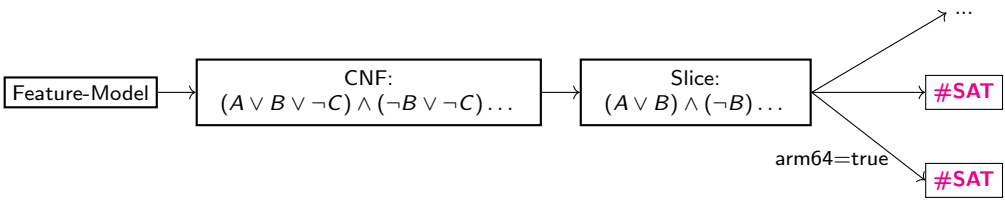
How many hardware configurations with arm64?



Projected Model Counting

Problem

How many hardware configurations with X ?



Projected Model Counting

Problem

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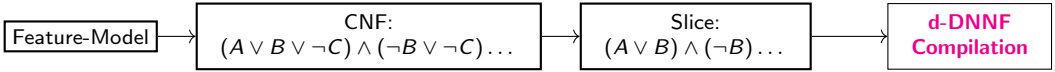

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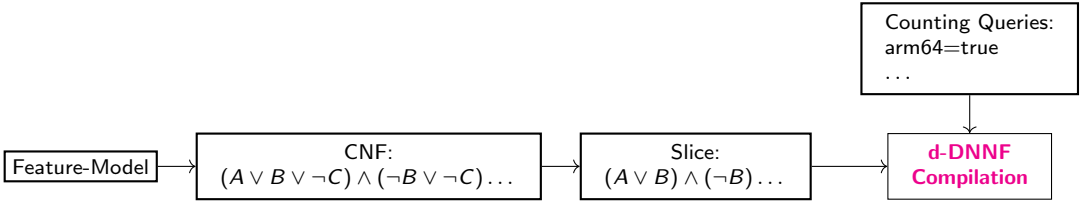

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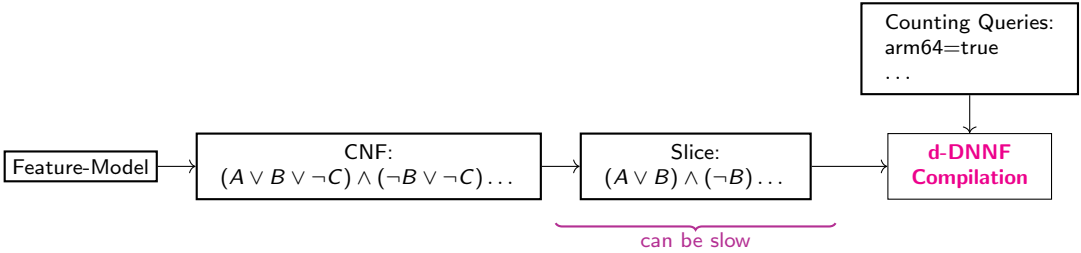

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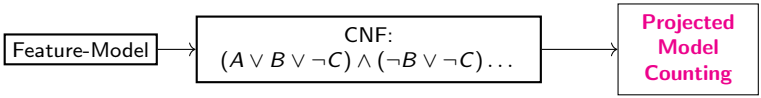

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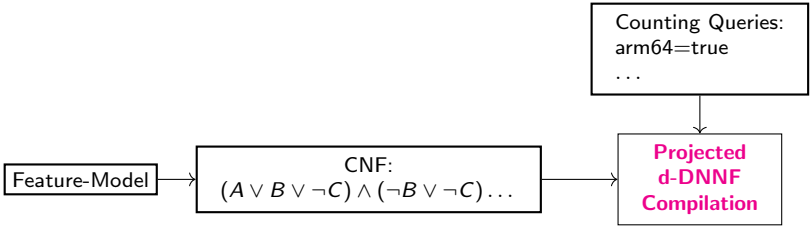
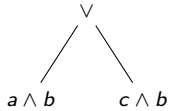


Deterministic Decomposable Negation Normal Form (d-DNNF)

Any propositional formula which is:

deterministic
Exclusive or-operators $F = A \vee B$
Never simultaneous $A = 1$ and $B = 1$
If-then-else
$|F| = |A| + |B|$



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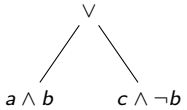

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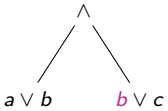
Decomposable
And-operands $F = A \wedge B$ never share variables
$|F| = |A| * |B|$



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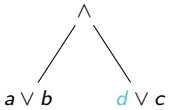

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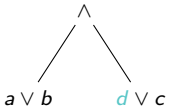
Negation **N**ormal **F**orm



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And-operands $F = A \wedge B$ never share variables
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Negation **N**ormal **F**orm

d-DNNF formulas allow linear-time model counting
d-DNNF compilation: CNF → d-DNNF
knowledge compilation: It's still just a formula

Slicing Implementation

[Comparing Algorithms for Efficient Feature-Model Slicing, Krieter et al.]

Resolve all clauses with v with all clauses with $\neg v$

Resolving Two Clauses

$$(\neg \mathit{VideoEncoding} \vee \mathit{Windows}), (x86 \vee \neg \mathit{Windows}) \rightarrow (\neg \mathit{VideoEncoding} \vee x86)$$

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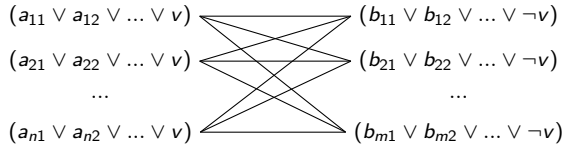
Resolving Many Clauses



Exponential clause count increase for multiple variables.

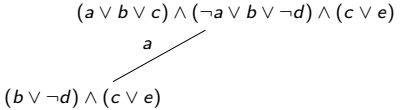
d-DNNF Compilation

DPLL

$$(a \vee b \vee c) \wedge (\neg a \vee b \vee \neg d) \wedge (c \vee e)$$

d-DNNF Compilation

DPLL



d-DNNF Compilation

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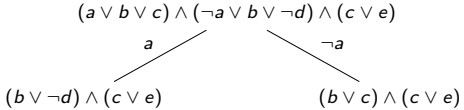

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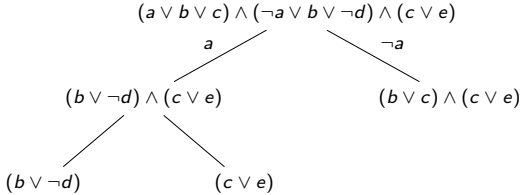

d-DNNF Compilation

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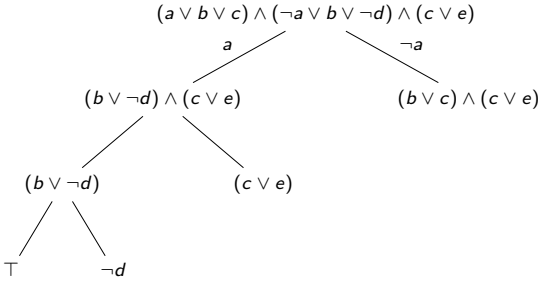

d-DNNF Compilation

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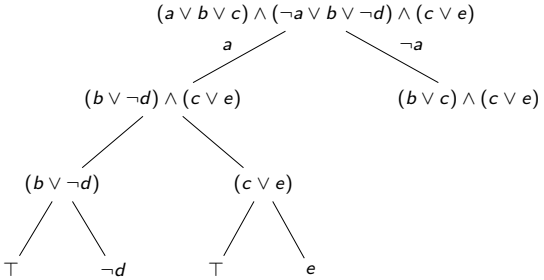

d-DNNF Compilation

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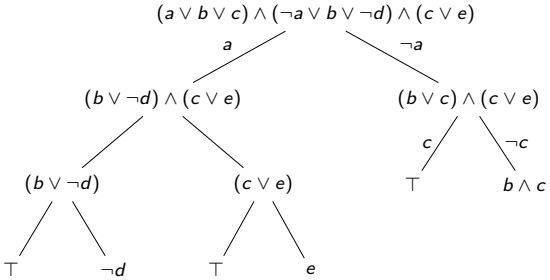

d-DNNF Compilation

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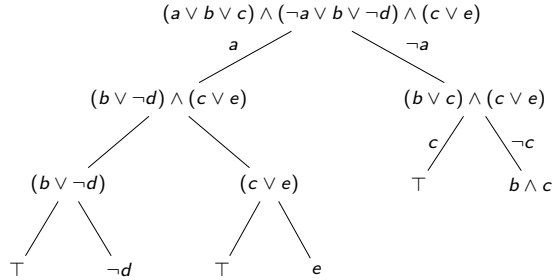
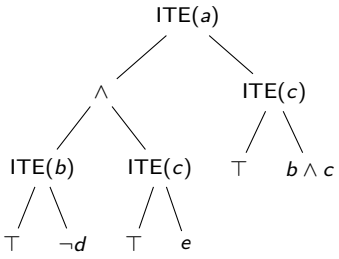


d-DNNF



ITE = If Then Else

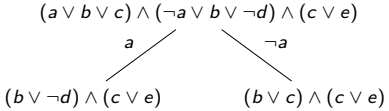
Heuristics

Vanilla DPLL is very slow

Heuristics

Vanilla DPLL is very slow

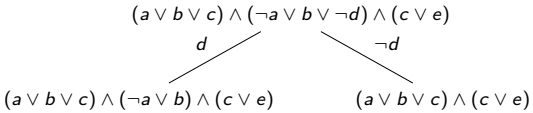
Variable Odering



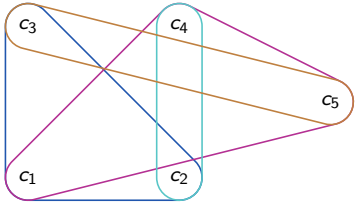
Heuristics

Vanilla DPLL is very slow

Variable Odering



Heuristics: Dual Hypergraph

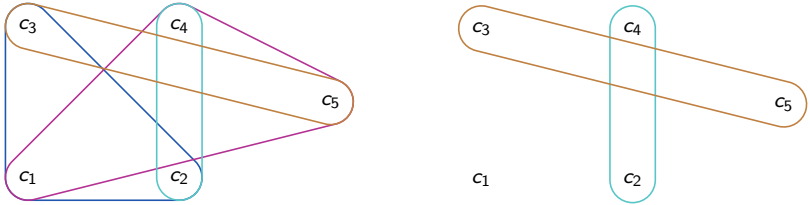


Construction

$F = \underset{c_1}{(a \vee b)} \wedge \underset{c_2}{(a \vee \neg c)} \wedge \underset{c_3}{(a \vee \neg d)} \wedge \underset{c_4}{(b \vee \neg c)} \wedge \underset{c_5}{(b \vee \neg d)}$

Split formula into independent sub-problems of roughly equal size.

Heuristics: Dual Hypergraph



Construction

$F = (\underbrace{a \vee b}_{c_1}) \wedge (\underbrace{a \vee \neg c}_{c_2}) \wedge (\underbrace{a \vee \neg d}_{c_3}) \wedge (\underbrace{b \vee \neg c}_{c_4}) \wedge (\underbrace{b \vee \neg d}_{c_5})$

Split formula into independent sub-problems of roughly equal size.

Projected d-DNNF Compilation

Concept

Projected variables $= a, b, d$
Sliced variables $= c, e$

$$(a \vee b \vee c) \wedge (\neg a \vee b \vee \neg d) \wedge (c \vee e)$$

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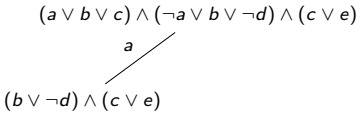

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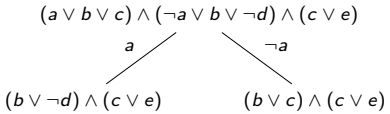

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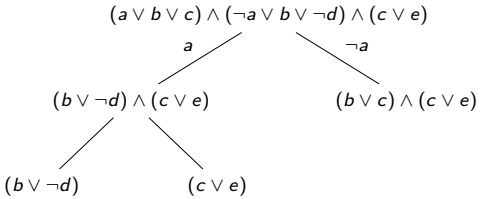

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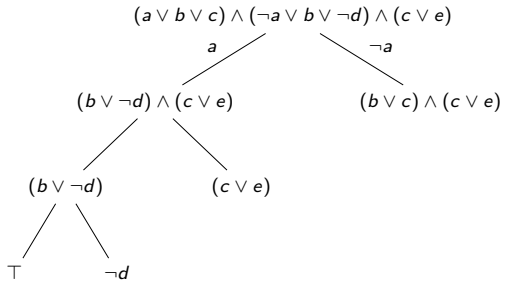

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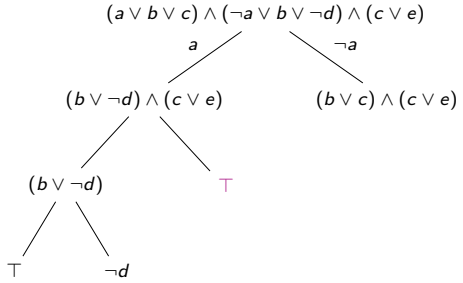

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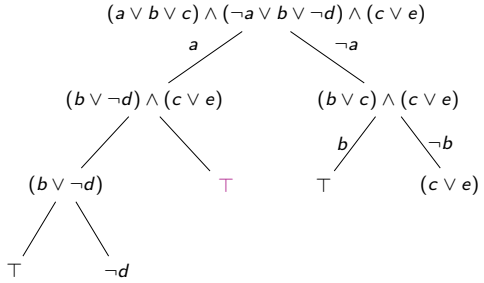

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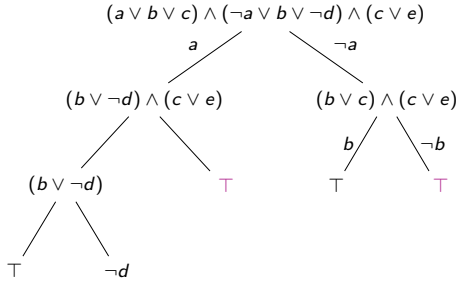
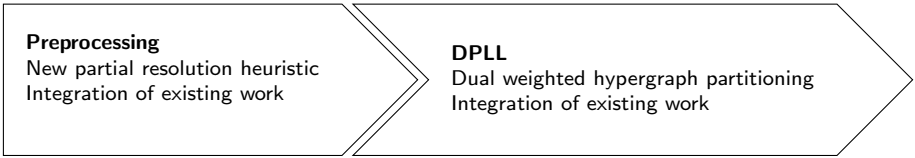
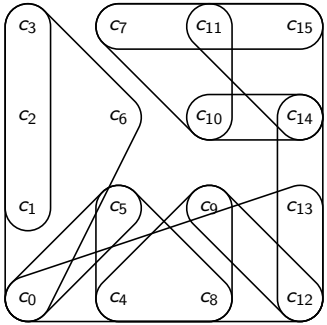


Integration and Optimization in D4

[An Improved Decision-DNNF Compiler, Lagniez et al.]



Dual Weighted Hypergraph Partitioning



$F = c_0 \wedge c_1 \wedge ... \wedge c_{15}$

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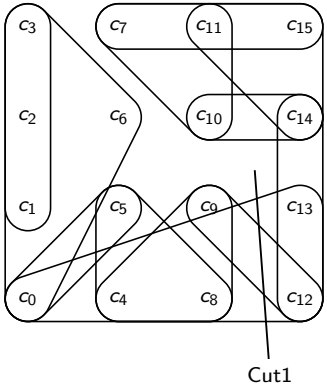

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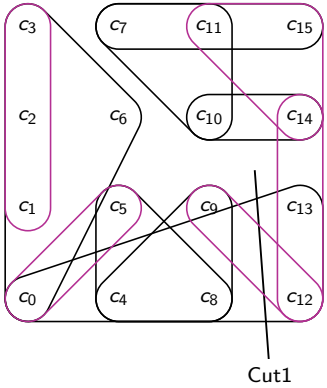

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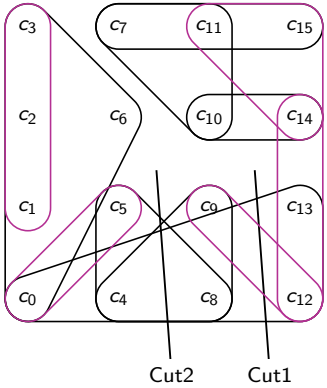

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Partial Resolution

Greedily resolve "easy" sliced variables until the clause count increases

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$F = (a \vee b \vee c) \wedge (\neg a \vee b \vee \neg d) \wedge (c \vee e)$

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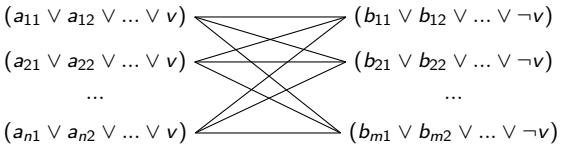

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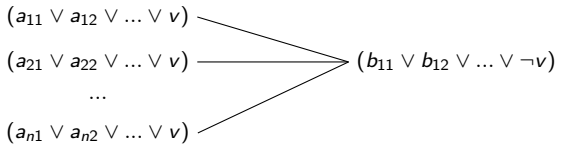

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Connectivity

$(\neg a \vee b \vee v), (a \vee b \vee \neg v) \rightarrow (\neg a \vee b \vee b \vee c) \equiv \top$

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Simpical Variable [1]: neighbors form a clique through clauses

[1] from GPMC source code

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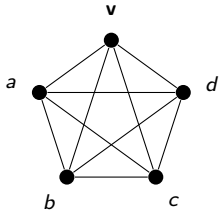


[1] from GPMC source code

Partial Resolution

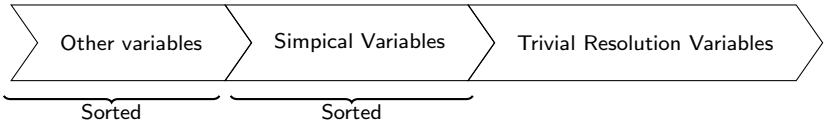
Greedily resolve "easy" sliced variables until the clause count increases

New Combined Heuristic

Group by:

1. Trivial resolution variables
2. Simpical variables
3. Other variables

Sort by: $v_p * v_n$ and average clause length



Experimental Setup

Solvers

- **pD4**: Our approach
- **slice**: Slicing followed by d-DNNF compilation
- **gpmc**: 1st place projected model counter in MC2022
- **D4-pmc**: 2nd place projected model counter in MC2022
- **arjun**: 3rd place projected model counter in MC2022

Data

- **Industrial Projection**: Real feature model slicing problems
- **Generated Projection**: Adding randomly selected projected variables to real feature models
- **MC2022**: Private+public instances from the MC2022 (many unknown sources...)

Questions

Compare runtime performance and d-DNNF size

Generated Projection for Feature Models (Table)

Model	d4-pmc	arjun	pD4	slice	gpmc
Smarch.2.6.32-2var	0	0	0	0	0
Smarch.2.6.28.6-icse11	0	0	0	0	0
Smarch.freetz	0	42	**65**	0	36
Smarch.buildroot	0	92	**100**	0	93
KConfig.linux-2.6.33.3	27	90	**96**	63	55
Smarch.embtoolkit	20	100	100	0	100
Smarch.freebsd-icse11	**100**	69	56	23	60
Smarch.uClinux-config	100	100	100	85	100
automotive02.automotive2_4	100	100	100	100	100