## Sample lesson:

# Simple linear regression models in R

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# Lesson context

# UBC MDS Curriculum

# Sample lesson: Simple linear regression models in R

1/4 of the way into the course. Before this lesson, students will have learned the following:

- model notation in R
- one-way & two-way ANOVA
    - theory
    - how to do in R with `aov()`
    - how to do in R with `lm()` (including reference-treatment parameterization)
    - interaction effects
- simple ordinary least squares linear regression (theory)

slides and code available at:
https://github.com/ttimbers/UBC-stat-sample-lesson

# Sample Lesson

# Sample lesson: Simple linear regression models in R

## learning objectives:

By the end of this lesson students are expected to be able to:

- fit a simple linear model in R
- interpret the output of the simple linear model object in R
- use three functions from the broom package extract simple linear model object output

# Lesson Motivation

# The Data:

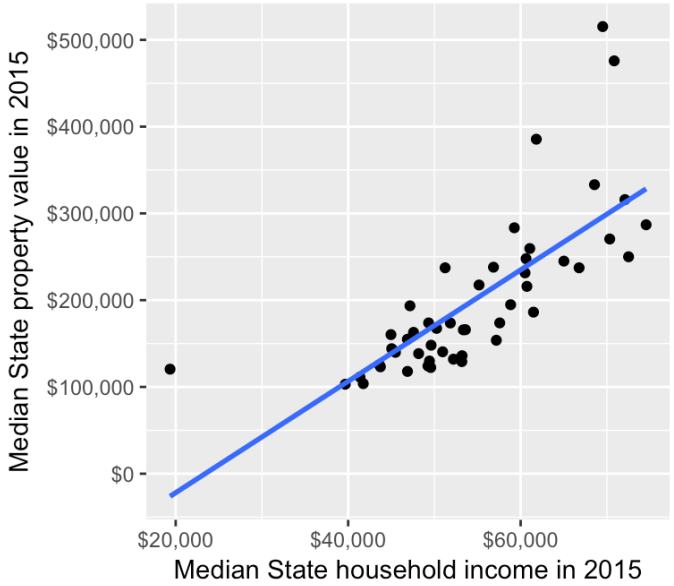
US property data from 2015 extracted from the Data USA API using Python
https://datausa.io/

| display_name | income | mean_commute_minutes | median_property_value | non_eng_speakers_pct | owner_occupied_housing_units | pop |
|---|---|---|---|---|---|---|
| **Texas** | 53207 | 24.5090 | 136000 | 0.3503480 | 0.622325 | 26538614 |
| **Pennsylvania** | 53599 | 25.2695 | 166000 | 0.1064820 | 0.692052 | 12779559 |
| **South Carolina** | 45483 | 23.0552 | 139900 | 0.0686766 | 0.685914 | 4777576 |
| **New Hampshire** | 66779 | 25.2973 | 237300 | 0.0786821 | 0.709609 | 1324201 |
| **Kansas** | 52205 | 18.2779 | 132000 | 0.1130530 | 0.666891 | 2892987 |
| **Hawaii** | 69515 | 25.5813 | 515300 | 0.2521790 | 0.569030 | 1406299 |

```
## [1] 52  7
```

# Simple linear regression:

US State property value as a function of income




$$Y = \text{State median property value}$$
$$X_1 = \text{income}$$

$$\text{State median property value} = \beta_0 + \beta_1 \, \text{income} + \varepsilon$$

# Syntax for linear regression in R


$$\text{State median property value} = \beta_0 + \beta_1 \, \text{income} + \varepsilon$$

```
prop_model <- lm(median_property_value ~ income, data = prop_data)
```

# Syntax for simple linear regression

create linear model object and view output in base R:

```
prop_model <- lm(median_property_value ~ income, data = prop_data)
summary(prop_model)
```

```
##
## Call:
## lm(formula = median_property_value ~ income, data = prop_data)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -65249 -36542  -6990   8003 219312
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.503e+05  4.393e+04  -3.422  0.00125 **
## income       6.420e+00  7.999e-01   8.026 1.52e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 58860 on 50 degrees of freedom
## Multiple R-squared:  0.563,  Adjusted R-squared:  0.5542
## F-statistic: 64.41 on 1 and 50 DF,  p-value: 1.517e-10
```

# Decoding R's output from `summary()`:

```
Call:
lm(formula = median_property_value ~ income, data = prop_data)

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   Min     1Q Median     3Q    Max
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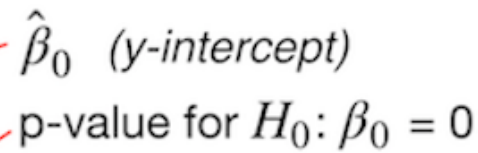

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$\hat{\beta}_1$ *(slope)*

p-value for $H_0: \beta_1 = 0$

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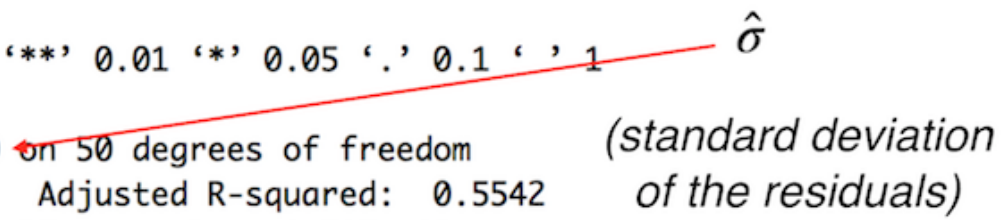

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```

*(proportion of variance in response explained by regression model )*

$R^2$

# Extracting output from model object in base R:

| model output | code |
|---|---|
| parameter/coefficient estimates ( $\beta_0$ & $\beta_1$ ) | `model_object$coefficients` |
| residuals | `model_object$residuals` |
| predicted/fitted values | `model_object$fitted.values` |
| p-values for coefficients | `summary(model_object)$coefficients[,4]` |
| $\sigma$ estimate | `summary(model_object)$sigma` |
| $R^2$ | `summary(model_object)$r.squared` |

Working with model output in base R,

the good, the bad and the ugly…

## The good…

- all the information you want is viewable
- this has been used for many many many years and thus should be familiar to most Data Scientists and Statisticians

## The bad…

- inconsistent syntax to extract model output
- model output is returned in a variety of forms, and is not tidy data

## The ugly…

- bizarre symbols in some column names ( `summary(model_object)$coefficient`)
- F-statistic p-value is never stored in memory and thus must be calculated by hand

`broom`:

A better way for working with model output in R

# `broom` for working with model output in R:

## The good…

- all the information you want is stored in memory, and easy to access
- consistent syntax
- no weird column names
- output is returned as data frames in tidy data format

## The bad…

- it's new, so not everyone is familiar with it

## The ugly…

- ???

# `broom` for working with model output in R:

| `broom` function | model output |
|---|---|
| `tidy(model_object)` | model parameters ( coefficients and p-values) |
| `augment(model_object)` | model data ( residuals and predicted values) |
| `glance(model_object)` | model quality, complexity and summaries ( $\sigma$ estimate and $R^2$) |

# Example output from `tidy()`

```
tidy(prop_model)
```

```
##          term      estimate    std.error statistic      p.value
## 1 (Intercept) -1.503050e+05 4.392739e+04 -3.421670 1.248896e-03
## 2      income  6.420101e+00 7.999334e-01  8.025795 1.517389e-10
```

# Example output from `augment()`

```
augment(prop_model)
```

```
##    median_property_value income   .fitted   .se.fit     .resid       .hat
## 1                 136000  53207 191289.28  8184.217 -55289.283 0.01933481
## 2                 166000  53599 193805.96  8167.204 -27805.963 0.01925451
## 3                 139900  45483 141700.42 10610.207  -1800.420 0.03249626
## 4                 237300  66779 278422.90 13107.757 -41122.898 0.04959552
## 5                 132000  52205 184856.34  8281.684 -52856.341 0.01979808
## 6                 515300  69515 295988.30 14882.799 219311.705 0.06393739
## 7                 162900  47583 155182.63  9624.070   7717.367 0.02673642
## 8                 193500  47169 152524.71  9803.561  40975.289 0.02774300
## 9                 160300  44963 138361.97 10880.682  21938.033 0.03417416
## 10                283400  59269 230207.94  9201.822  53192.063 0.02444181
## 11                153800  57181 216802.77  8559.790 -63002.765 0.02115007
## 12                237300  51243 178680.20  8446.070  58619.796 0.02059184
## 13                129200  53183 191135.20  8185.648 -61935.200 0.01934157
## 14                122400  49576 167977.89  8882.875 -45577.895 0.02277680
## 15                129900  49429 167034.14  8929.926 -37134.140 0.02301873
## 16                148100  49620 168260.38  8869.046 -20160.379 0.02270594
## 17                247800  60629 238939.27  9752.014   8860.725 0.02745202
## 18                159000  47507 154694.71  9656.419   4305.295 0.02691646
## 19                286900  74551 328319.93 18384.621 -41419.925 0.09756522
## 20                217500  55176 203930.46  8220.159  13569.538 0.01950501
## 21                259500  61062 241719.18  9945.789  17780.821 0.02855382
## 22                167500  50255 172337.14  8682.917  -4837.144 0.02176291
## 23                124200  49255 165917.04  8987.290 -41717.042 0.02331542
## 24                123200  43740 130510.18 11550.947  -7310.184 0.03851420
## 25                144100  45047 138901.26 10836.365   5198.744 0.03389635
## 26                173800  49331 166404.97  8962.014   7395.030 0.02318445
## 27                186200  61492 244479.82 10146.265 -58279.822 0.02971653
## 28                138400  48173 158970.49  9382.549 -20570.493 0.02541133
## 29                133200  52997 189941.06  8198.252 -56741.062 0.01940118
```

# Example output from `glance()`

```
glance(prop_model)
```

```
##   r.squared adj.r.squared    sigma statistic      p.value df    logLik
## 1 0.5629882      0.554248 58858.23  64.41339 1.517389e-10  2 -643.8752
##       AIC      BIC     deviance df.residual
## 1 1293.75 1299.604 173214534971          50
```

# Group challenge question:

https://tinyurl.com/UBC-stat-group-challenge

# What did we learn today?

# Where do we go from here?

- more complex linear regression models (multiple regression)
  - theory (least squares)
  - how to in R (revisit reference-treatment parameterization)
  - interaction effects in linear regression
- model diagnostics
- dealing with nonlinear terms

# Questions/Discussion