

Sample lesson:

Simple linear regression models in R

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Lesson context

UBC MDS Curriculum



UBC MDS Curriculum



DSCI 561 Regression I

Course Learning Outcomes

By the end of the course, students are expected to be able to:

- Fit and interpret a linear regression model.
- Identify whether a linear regression model is appropriate for a given dataset.
- Critique a specific regression model applied to a given dataset on the basis of both diagnostic plots and hypothesis tests.
- Specify and interpret interaction terms and nonlinear terms.

DSCI 561 Regression I

Reference Materials

- Faraway, Julian J. Linear Models with R, 2nd Edition. Chapman and Hall, 2014.
- broom package vignette in R

Sample lesson: Simple linear regression models in R

1/4 of the way into the course. Before this lesson, students will have learned the following:

- model notation in R
- one-way & two-way ANOVA
 - theory
 - how to do in R with `aov()`
 - how to do in R with `lm()` (including reference-treatment parameterization)
 - interaction effects
- linear regression via ordinary least squares (theory)

Sample lesson: Simple linear regression models in R

learning objectives:

By the end of this lesson students are expected to be able to:

- fit a simple linear model in R
- interpret the output of the simple linear model object in R
- use three functions from the broom package extract simple linear model object output

Lesson Motivation

slides and code available at:

<https://github.com/ttimbers/UBC-stat-sample-lesson>

Sample Lesson

The Data:

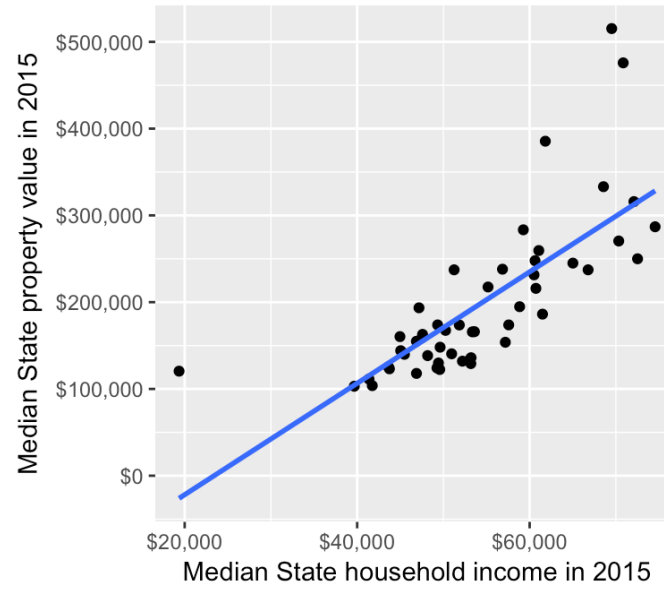
US property data from 2015 extracted from the Data USA API using Python
<https://datausa.io/>

display_name	income	mean_commute_minutes	median_property_value	non_eng_speakers_pct	owner_occupied_housing_units	pop
Texas	53207	24.5090	136000	0.3503480	0.622325	26538614
Pennsylvania	53599	25.2695	166000	0.1064820	0.692052	12779559
South Carolina	45483	23.0552	139900	0.0686766	0.685914	4777576
New Hampshire	66779	25.2973	237300	0.0786821	0.709609	1324201
Kansas	52205	18.2779	132000	0.1130530	0.666891	2892987
Hawaii	69515	25.5813	515300	0.2521790	0.569030	1406299

```
## [1] 52 7
```

Simple linear regression:

US State property value as a function of income



Y = State median property value

X_1 = income

$$\text{State median property value} = \beta_0 + \beta_1 \text{income} + \varepsilon$$

Model notation for linear regression in R

Simple linear regression:

Mathematical formula

$$Y = \beta_0 + \beta_1 X_1 + \varepsilon$$

model formula in R

`Y ~ X1`

More complex linear regression:

Mathematical formula

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \varepsilon$$

model formula in R

`Y ~ X1 + X2 + X3`

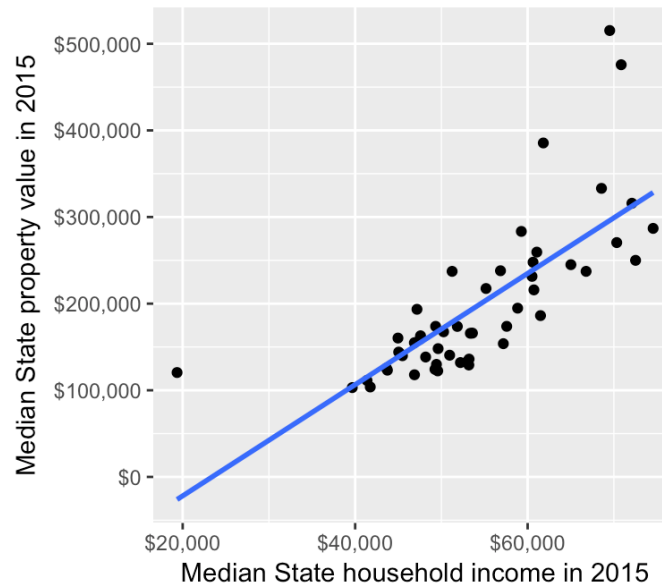
$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \varepsilon$$

`Y ~ X1 * X2`

Cheatsheet for model notation in R:

<http://faculty.chicagobooth.edu/richard.hahn/teaching/formulanotation.pdf>

Syntax for linear regression in R



$$\text{State median property value} = \beta_0 + \beta_1 \text{income} + \varepsilon$$

```
prop_model <- lm(median_property_value ~ income, data = prop_data)
```

Syntax for simple linear regression

create linear model object and view output in base R:

```
prop_model <- lm(median_property_value ~ income, data = prop_data)
summary(prop_model)
```

```
##
## Call:
## lm(formula = median_property_value ~ income, data = prop_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -65249 -36542  -6990   8003 219312
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.503e+05  4.393e+04  -3.422  0.00125 **
## income       6.420e+00  7.999e-01   8.026 1.52e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 58860 on 50 degrees of freedom
## Multiple R-squared:  0.563, Adjusted R-squared:  0.5542
## F-statistic: 64.41 on 1 and 50 DF, p-value: 1.517e-10
```

Decoding R's output from `summary()`:

Call:

```
lm(formula = median_property_value ~ income, data = prop_data)
```

Residuals:

Min	1Q	Median	3Q	Max
-65249	-36542	-6990	8003	219312

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-1.503e+05	4.393e+04	-3.422	0.00125 **
income	6.420e+00	7.999e-01	8.026	1.52e-10 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 58860 on 50 degrees of freedom

Multiple R-squared: 0.563, Adjusted R-squared: 0.5542

F-statistic: 64.41 on 1 and 50 DF, p-value: 1.517e-10

$\hat{\beta}_0$

$\hat{\beta}_1$

p-value for $H_0: \beta_0 = 0$

p-value for $H_0: \beta_1 = 0$

$\hat{\sigma}$

R^2

Extracting output from model object in base R:

model output

parameter/coefficient estimates (β_0 & β_1)

residuals

predicted/fitted values

p-values for coefficients

σ estimate

R^2

code

```
model_object$coefficients
```

```
model_object$residuals
```

```
model_object$fitted.values
```

```
summary(model_object)$coefficients[,4]
```

```
summary(model_object)$sigma
```

```
summary(model_object)$r.squared
```

Working with model output in base R,



the good, the bad and the ugly...

The good...

- all the information you want is viewable
- this has been used for many many many years and thus should be familiar to most Data Scientists and Statisticians

The bad...

- inconsistent syntax to extract model output
- model output is returned in a variety of forms, and is not tidy data

The ugly...

- bizarre symbols in some column names (`summary(model_object)$coefficient`)
- F-statistic p-value is never stored in memory and thus must be calculated by hand

broom:



A better way for working with model output in R

broom for working with model output in R:

The good...

- all the information you want is stored in memory, and easy to access
- consistent syntax
- no weird column names

The bad...

- it's new, so not everyone is familiar with it

The ugly...

- ???

broom for working with model output in R:

broom function

model output

tidy(model_object)

model parameters (coefficients and p-values)

augment(model_object)

model data (residuals and predicted values)

glance(model_object)

model quality, complexity and summaries (σ estimate and R^2)

Example output from `tidy()`

```
tidy(prop_model)
```

```
##           term      estimate  std.error statistic    p.value
## 1 (Intercept) -1.503050e+05  4.392739e+04  -3.421670  1.248896e-03
## 2      income   6.420101e+00  7.999334e-01   8.025795  1.517389e-10
```

Example output from `augment()`

```
augment(prop_model)
```

##	median_property_value	income	.fitted	.se.fit	.resid	.hat
## 1	136000	53207	191289.28	8184.217	-55289.283	0.01933481
## 2	166000	53599	193805.96	8167.204	-27805.963	0.01925451
## 3	139900	45483	141700.42	10610.207	-1800.420	0.03249626
## 4	237300	66779	278422.90	13107.757	-41122.898	0.04959552
## 5	132000	52205	184856.34	8281.684	-52856.341	0.01979808
## 6	515300	69515	295988.30	14882.799	219311.705	0.06393739
## 7	162900	47583	155182.63	9624.070	7717.367	0.02673642
## 8	193500	47169	152524.71	9803.561	40975.289	0.02774300
## 9	160300	44963	138361.97	10880.682	21938.033	0.03417416
## 10	283400	59269	230207.94	9201.822	53192.063	0.02444181
## 11	153800	57181	216802.77	8559.790	-63002.765	0.02115007
## 12	237300	51243	178680.20	8446.070	58619.796	0.02059184
## 13	129200	53183	191135.20	8185.648	-61935.200	0.01934157
## 14	122400	49576	167977.89	8882.875	-45577.895	0.02277680
## 15	129900	49429	167034.14	8929.926	-37134.140	0.02301873
## 16	148100	49620	168260.38	8869.046	-20160.379	0.02270594
## 17	247800	60629	238939.27	9752.014	8860.725	0.02745202
## 18	159000	47507	154694.71	9656.419	4305.295	0.02691646
## 19	286900	74551	328319.93	18384.621	-41419.925	0.09756522
## 20	217500	55176	203930.46	8220.159	13569.538	0.01950501
## 21	259500	61062	241719.18	9945.789	17780.821	0.02855382
## 22	167500	50255	172337.14	8682.917	-4837.144	0.02176291
## 23	124200	49255	165917.04	8987.290	-41717.042	0.02331542
## 24	123200	43740	130510.18	11550.947	-7310.184	0.03851420
## 25	144100	45047	138901.26	10836.365	5198.744	0.03389635
## 26	173800	49331	166404.97	8962.014	7395.030	0.02318445
## 27	186200	61492	244479.82	10146.265	-58279.822	0.02971653
## 28	138400	48173	158970.49	9382.549	-20570.493	0.02541133
## 29	133200	52997	189941.06	8198.252	-56741.062	0.01940118

Example output from `glance()`

```
glance(prop_model)
```

```
##   r.squared adj.r.squared   sigma statistic      p.value df    logLik
## 1 0.5629882      0.554248 58858.23  64.41339 1.517389e-10  2 -643.8752
##      AIC      BIC    deviance df.residual
## 1 1293.75 1299.604 173214534971          50
```

Group challenge question:

<https://tinyurl.com/UBC-stat-group-challenge>

What did we learn today?

Where do we go from here?

- finish simple linear regression
 - predicted values and confidence intervals
- more complex linear regression models (multiple regression)
 - theory (least squares)
 - how to in R (revisit reference-treatment parameterization)
 - interaction effects in linear regression
- model diagnostics
- dealing with nonlinear terms

Questions/Discussion