MATH 105A- Chapter 1- Section 1

Note 1

Math 105A

Numerical Analysis I

Math. Dept., UCI

PAPCO

Introduction.

In Math 105A, students learn to identify the types of problems that require numerical techniques for their solutions and see examples of the error propagation that can occur when numerical methods are applied.
The chosen applications clearly and concisely demonstrate how numerical techniques can be, and often must be, applied in real-life situations.
We will use the codes using MATLAB in Lab lectures.
Students should be familiar with MATLAB or Mathematica in order to solve HWs and taking the exames.

Chapter 1
Mathematical Preliminaries and Error Analysis

1.1. Review of Calculus

Limits and continuity

Def. 1.1. A function $f: X \subseteq \mathbb{R} \to \mathbb{R}$ has the limit L at x_0, say $\lim_{x \to x_0} f(x) = L$, if,

$\forall$ (for all) $\varepsilon > 0$, $\exists$ (there exists) $\delta > 0$ s.t. (such that)

$$|f(x) - L| < \varepsilon, \text{ whenever } x \in X \text{ and } 0 < |x - x_0| < \delta$$

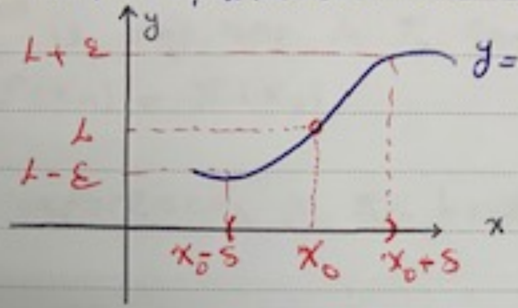

Fig. 1.1

Def. 1.2. Let $f: X \subseteq \mathbb{R} \to \mathbb{R}$ be defined on X and $x_0 \in X$. Then f is continuous at x_0 if

$$\lim_{x \to x_0} f(x) = f(x_0).$$

Remark 1. If f is cont. on the set X, then we denote $f \in C(X)$. If $X = \mathbb{R}$, then $f \in C(-\infty, \infty) \equiv C(\mathbb{R})$.

Def. 1.3. Let $\{x_n\}_{n=1}^{\infty}$ be an infinite sequence of real numbers. Then this seq. has the limit x (converges to x), if, $\forall\ \varepsilon > 0$ $\exists$ positive integer $N(\varepsilon)$ s.t. $|x_n - x| < \varepsilon$, whenever $n > N(\varepsilon)$. Here, we may write

$$\lim_{n \to \infty} x_n = x \quad \text{or} \quad x_n \to x \text{ as } n \to \infty.$$

Theorem 1.4. Let $f: X \to \mathbb{R}$ and $x_0 \in X$. Then the following statements are equivalent

a) f is cont. at x_0.

b) If $\{x_n\}_1^\infty$ is any seq. in X converging to x_0, then $\lim_{n\to\infty} f(x_n) = f(x_0)$.

Proof See the references in the book.

Remark 2. In this course the functions that we will discuss, will be assumed to be cont., because this is a minimal requirement for predictable behavior

Some symboles that we use in this course.

$\forall$: for all

$\exists$: there exists

$\exists!$: there exists only one

$|$: such that (or s.t.)

$\subset$: Proper subset

$\subseteq$: Proper subset or equal set

$\{x_n\}_{n=1}^\infty$: Infinite sequence

$f: X \subseteq \mathbb{R} \to \mathbb{R}$: function f defined on domain $X \subseteq \mathbb{R}$ to $\mathbb{R}$ (as range

$\in$: belongs or belong to

(a,b): Open set from a to b.

$[a,b]$: Closed set from a to b.

ε: absilon , δ: delta

$|\cdot|$: absolute value