EGALITARIAN ASPECTS OF SCALE-FREE NETWORKS

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Abstract. *Scale-free networks are frequently described as the zenith of inequality and sometimes even pin-pointed as a natural cause of concentrations, including accumulation of resources in human society. Although coherent with theory and empirical data, there are at least three aspects of scale-free networks that are egalitarian. Using the social network metaphor (which is easier to grasp): 1) the presence of each agent (vertex or component) in diverse networks, while each agent has the same amount of resources (e.g. time) for engaging (establishing links or edges) with other individual agents, ideas or objects; 2) the constant change in the concentration of resources (in this case, links) of each agent; 3) the uniform distribution of resources with respect to the amount of resources per agent (the more resources per agent, the fewer the agents). We also consider the importance of vertices that are not hubs and overstatements on the importance of the hubs. The conclusions suggest the urge of research to better examine and model the importance of the vertices that are not hubs (i.e. peripheral and intermediary) and verify the need to better weight the emphasis current literature places in the association of inequality to the scale-free networks (and other phenomena in which power laws are found).*

Keywords: *Complex networks, Scale-free networks, Statistical physics, Egalitarianism, Erdös sectors*

1. INTRODUCTION

Price (1965) reported that citations networks had a heavy-tailed distribution following a power law. He also described a "cumulative advantage" that explains the power law (Price, 1976). This same networks with a power-law distribution of connectivity is nowadays called "scale-free" and the same explanation for the distribution is called "preferential attachment", as the result of a rediscovery of the property by Barabási & Albert (1999). This power-law rediscovery, and the rediscovery of small-world networks by Watts & Strogatz (1998), is usually considered the birth of the complex networks field, in the turn of the millennium. In the last decade and a half, most of the articles in physics report advances in complex networks.

Another interesting fact about the meaning of the field is related to the essence of these structures. Components (be them human agents or not), in diverse contexts, often exhibit a

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power-law distribution of activity. If their activity yields links, the power-law distribution of connectivity is one of the byproducts of activity. Indeed, connectivity and activity present high correlation in many of such scenarios (Fabbri, 2017; Fabbri et al., 2017; Fabbri & Oliveira, 2017).

The complex network literature stresses concentration and often revolves around the hub. More accurately, it glamorizes the most connected nodes as "both the strength and the weakness of scale-free networks" (Wikipedia, 2017; Newman, 2010), and places them as the most important vertexes. In this work, we argue that this is naive for a number of reasons. Therefore, this document is countercurrent with respect to available literature: not only the focus here is to observe scale-free networks egalitarian aspects, but also to dilute the hubs boast. No academic writing was found by the authors to expose this simple and pertinent content.

Section 2 presents a glimpse at standard theory related to power laws. Section 3 states the aspects of scale-free networks that are egalitarian. Section 4 considers also the hubs, intermediary and peripheral vertices and a probable unbalance current literature has in considering the importance of these sectors. Section 5 is dedicated to conclusions and future work.

2. CANONICAL BACKGROUND

Networks with a power-law distribution of connectivity (degree or strength[1]) are called *scale-free*. In other words, be $p(k)$ the probability that an arbitrary vertex has degree k, than, for a scale-free network, one can assume:

$$p(k) \sim k^{-\gamma} \tag{1}$$

where $\gamma > 1$ is constant and typically $\gamma \in [2, 3]$. This same distribution is called, under certain conditions, e.g. the Pareto distribution or the Zipf's law. In fact, they all report that nature is performing a distribution of resources in a way that can be modeled by power laws. The next sections exposes this content very briefly.

2.1 Zipf's law

This law is unique among the ones mentioned in this article because it relates the frequency of an element to its rank in an ordering where the most frequent comes first. More specifically, a word with rank r has the frequency (or probability of occurring):

$$p(r) \sim r^{-\gamma} \tag{2}$$

where $\gamma \approx 1$. If the definition given above is followed strictly, the law entails that:

$$p(r) = \frac{r^{-\gamma}}{\sum_{n=1}^{N} n^{-\gamma}} \tag{3}$$

where N is the vocabulary size. Mandelbrot proposed a shifted version of this law that follows more closely the distributions found in real languages:

$$p(r) \sim (r + s)^{-\gamma} \tag{4}$$

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[1]Degree of a vertex is the number of links connected to it. Strength of a vertex is the sum of the weight of the links connected to it.

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where $s \approx 2.7$.

There are many non-trivial considerations to be made about the Zipf's law but, overall, it seems to hold universally in all languages, including those that are extinct and untranslated (Piantadosi, 2014).

2.2 Pareto distribution

The Pareto distribution is defined in many ways throughout the literature. Wikipedia (2017), for example, defines the distribution through its survival function and presents 4 variants and a generalized form with an extra parameter. In its most basic form, the Pareto distribution can be regarded as having the probability density function:

$$p(k) = \frac{\gamma k_m^\gamma}{k^{\gamma+1}} \tag{5}$$

where k_m is the minimum value for k to yield $p(k) > 0$. This distribution is used to describe many observable phenomena, including in social, financial and geophysical theories.

2.3 Human perception: Steven's law

The relation between the magnitude of a physical stimulus and its perceived intensity is often given by the Steven's law:

$$S(I) = kI^\gamma \tag{6}$$

where $S(I)$ is the perceived intensity (or strength) of intensity I of stimulus type that entails γ and k is a proportionality constant that accounts for the arbitrary units used.

2.4 Many other laws

Notice that this last law (Steven's) is not a probability law as in the other cases above. In fact, many other phenomena are modeled as power laws, especially natural phenomena, such as Newton's laws of gravitation and inertia. This calls for a consideration of the expressive capabilities of power laws and for an honest inquiry of what is so special about these functions that can account for its ubiquity (Fabbri & Oliveira, 2017).

2.5 Scale-free networks

Now back to the main subject of this article. A scale-free network is one where the distribution of connectivity follows the Equation 2. These networks also present other non-trivial properties, such as the 'small-world-ness', high clusterization and abundant motifs (Newman, 2010). Most interesting for this discussion, these networks are not isolated entities, are not static and the power-law distribution is directly related to an uniform distribution as will be exposed in the next section.

3. THREE EGALITARIAN ASPECTS

This is the most important section of this article. We strived to keep everything as simple as possible to allow both this document to be reasonable to a large audience and for the reader to

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promptly grasp the contents. The kernel of our subject is: given the ubiquity of power-laws, and that they arise from an uniform distribution, and that, in the context of social networks, these structures are not isolated nor static, what should be the emphasis given to the association of scale-free networks to inequality? In the same context: does it explain the ubiquity of inequality but states, at the same time, paradoxically, that equality is also prevalent? We should examine each of the egalitarian aspects of scale-free networks:

- according to the exposition in Fabbri & Oliveira (2017), a power-law is entailed by a uniform distribution. The same quantity C of resources is distributed uniformly across all concentrations of resources: $\forall k : p_U(k) = C \Rightarrow p(k) = C.k^{-\gamma}$ where γ is the dimensionality of the resource. In other words, the uniform distribution of a γ-dimensional resource result is a power law with coefficient γ. The more resources is allocated per component (e.g. per agent), the less numbered are the components.

- The networks are not static! Some of them might be (e.g. those yield by bone cavities) but social networks, which are at the center of our discussion, are not. Social networks are in constant change and, in fact, stable hubs were only observed in very small networks (Barabási et al., 2007). Although the overall distribution is somewhat stable (or invariant) in the power-law outline, the participants move constantly between the periphery to intermediary and hubs sectors (Fabbri, 2017; Fabbri et al., 2017; Fabbri, 2014). Thus, the scale-free (social) networks are egalitarian in the sense that the participants present only transient concentrations of resources.

- The networks are not isolated! This has deep consequences for social networks. For example, you, the reader, are a hub in some nucleus of your own family. Some friend of yours is a peripheral in your family because he/she only knows you. In the same way, everyone is a hub, a peripheral and an intermediary[2] at the same time, but in a (vast) number of different networks.

4. EXALTATION OF HUBS AND DELUSIONS OF GRANDEUR

The hubs, defined as the nodes with the greatest number of links in a scale-free network, are reported in many studies as being the most relevant components. In fact, selective attacks to the hubs (generally modeled as removing them) cause the networks to loose many of the properties that make scale-free networks appealing: the low average-distance between nodes, the fast transmission of information and other sorts of transportation (Newman, 2010). Nevertheless, aging of hubs is a recognized phenomenon, i.e. the descent of the most connected hubs is observed and even expected (Barabási, 2013). Also, diseases spread rapidly by means of the hubs and there are many other important negative contributions of the hubs and important positive contributions of the other vertices. In social networks terms, often used for such discussions:

- Hubs are but a few vertexes, which are replaced constantly.

- Hubs present the most trivial behavior: they interact as much as possible, in every situation, with every other agent. That is a prerequisite for they to be hubs.

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[2]A sound method for deriving the hubs, intermediary and peripheral sectors, by comparing the real network against the Erdös-Rényi model, is described in Fabbri (2017) and Fabbri et al. (2017).

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- Hubs tend to present corrupt behavior, as they apply huge amounts of time and energy (resources) on the network where it is a hub and frequently depend on such activity for basic provision. This might also be seen as a tendency of the hub to have a committed attitude, but given that hubs are also ubiquitous in non-professional networks, this might be understood as corrupt behavior by the non-warned.

- World input to the network is done mostly by peripheral and intermediary agents, as they do not apply all their energy on the network.

- Authorities are often intermediaries or less active hubs, especially in cases where they deliver quality, not quantity. This can be noticed by the fact that authorities usually exploit their authority in order to achieve their goals by means of the dedication of other agents.

- Network structure is yield by intermediary vertices (Fabbri et al., 2017), as they are the only with non-trivial behavior: peripheral vertexes interact only a few times; hubs interact everywhere possible.

In summary, the constant exaltation of the hubs in current complex networks literature seems to be related to the fact that the field is still incipient. A psychologist might also add that it is related to delusions of grandeur which affect a great fraction of our society and thus leaks into the scientific tradition. A social scientist might add that there is a constant pressure performed by the more powerful and rich groups to legitimate themselves and the inequality as a natural tendency. Anyway, there is an evident consonance between the constant ennoblement of the hubs, the stress on the association of scale-free networks and inequality, and our observation that these associations should be considered with parsimony.

5. CONCLUSIONS AND FURTHER WORK

The observations made in this brief article have direct consequences for the purposes of the mathematical and agent-based models of complex networks. How can our models reflect the diversity input by peripherals and the greater authority held by the intermediary sector or the less connected hubs? Considering our current scientific literature: should one build a survey of the excesses about inequality and exaltation of hubs in recent complex networks books, article and software? More fundamentally, are the issues reported in this article sound and relevant? Given that the complex networks field is recent and that it is as driven by social constructs and conveniences as any knowledge field, we need to choose a very lucid path in 1) asking more questions than stating fresh conclusions as facts; 2) establishing a deeper dialog with the traditions that have studied these phenomena for centuries in advance, such as social scientists and philosophers.

Further work should involve attempts to answer the questions and follow the recommendations above, further confirm or refute the arguments in this documents, and:

- develop models to reflect the fundamental roles performed by nodes beyond hubs (i.e. peripherals and intermediary).

- Deepen the model exposed in Fabbri & Oliveira (2017) and submit it to peer review processes in order to understand the extent of its validity.

- Study the dynamics of individual agents in social networks with respect to the concentration of links, as suggested in Fabbri (2017); Fabbri et al. (2017); Fabbri (2014).

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- Study the integration of the networks, maybe by using the database in Fabbri & Oliveira (2016) which contains a number of networks with shared participants.

- Deepen the absorption of the literature and the partnership with academics which produce knowledge within the complex networks field, both to better understand the convictions and to spread our concerns.

- Consider an antithesis: the formalism of institutionalized posts might impose both a less dynamic variation of participant roles and a less extreme concentration of resources.

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