Distances between histograms

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This document presents reference values for a distance metric derived from the Kolmogorov-Smirnov statistical test. Each measure is a distance between two histograms. The sections are self-explanatory on deriving benchmarks by comparing samples from usual distributions and on exemplifying the power of the acquired knowledge.

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I. INTRODUCTION

Be $F_{1,n}$ and $F_{2,n'}$ two empirical cumulative distributions, where n and n' are the number of observations on each sample. The two-sample Kolmogorov-Smirnov test rejects the null hypothesis (that the histograms are the outcome of the same underlying distribution) if:

$$D_{n,n'} > c(\alpha)\sqrt{\frac{n+n'}{nn'}} \tag{1}$$

where $D_{n,n'} = sup_x[F_{1,n} - F_{2,n'}]$ and $c(\alpha)$ are related to the critical region α by:

α	0.1	0.05	0.025	0.01	0.005	0.001
$c(\alpha)$	1.22	1.36	1.48	1.63	1.73	1.95

If distributions are drawn from empirical data, $D_{n,n'}$ is given as are n and n'. All terms in equation 1 are positive and $c(\alpha)$ can be isolated:

$$c(\alpha) < \frac{D_{n,n'}}{\sqrt{\frac{n+n'}{nn'}}} = c'(\alpha) \tag{2}$$

When $c'(\alpha)$ is high, low values of α favor rejecting the null hypothesis. For example, when $c'(\alpha)$ is greater than ≈ 1.7, one might assume that $F_{1,n}$ and $F_{2,n'}$ are outcomes of different distributions. More importantly for us is that $c'(\alpha)$ is a measure of distance between both distributions[1]. The main contribution of the following sections is the explicit display of reference values from which one might derive knowledge from collections of empirical measures of $c'(\alpha)$ or even a single value of $c'(\alpha)$.

II. REFERENCES THROUGH SIMULATIONS

III. REFERENCES IN EMPIRICAL DATA

IV. CONCLUSIONS

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[1] R. Chicheportiche and J.-P. Bouchaud, "Weighted kolmogorov-smirnov test: Accounting for the tails," Physical Review E **86**, 041115 (2012).

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