A simple model that explains why inequality is inevitable and ubiquitous

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Abstract

Inequality has always been a crucial issue for human kind, particularly concerning the highly unequal distribution of wealth, which is at the root of major problems facing humanity, including extreme poverty and wars. A quantitative observation of inequality has become commonplace in recent years with the discovery that many natural as well as man-made systems can be represented as scale-free networks, whose distribution of connectivity obeys a power law. These networks may be generated by the preferential attachment for the nodes, within the so-called rich-gets-richer paradigm. In this letter we introduce a simple model that explains the ubiquity of inequality, based on simple assumptions applied to a generic system. The main assumption is that the system does not distinguish component cost in allocating resources. This implies an uniform distribution of resources along component cost: the more resources are allocated per component, less components with such cost the system presents, with the conservation of the amount of resources distributed through cost sweep. This can be geometrically described by the distribution of object sizes in a 3D space, where each dimension is assumed to be isotropic. Applying these assumptions to a generic system results in a power-law distribution, whose coefficient is the number of inputs that are independent from each other, i.e. the dimensionsionality of the allocated resources. Even though there is no restriction to the value of the coefficient, in practice we observe that existing systems normally exhibit a coefficient between 1.5 and 3.0. With our simple model it is not possible to determine whether this limitation in the coefficient values arises from a fundamental principle, but we indicate reasonable hypotheses. The assumptions in the model are analogous to the first and second law of thermodynamics: conservation of resources and a time arrow pointing to inequality. Since these assumptions are easily justified based on established knowledge, the model proves unequivocally that inequality is ubiquitous. We also discuss ways to control this tendency to inequality, which is actually analogous to a decrease in entropy in a closed system induced with an external action.

Keywords: power laws, fundamental theory, complex systems, complex networks, anthropological physics

1 Introduction to inequality in power laws such as Zipf or Pareto

A power law is a functional relationship between two quantities $P(k)$ and k in the form:

$$P(k) = Ck^{-\alpha} \tag{1}$$

where $k \in [k_L, k_R]$ and C is constant. Assuming idealized phenomena:

$$C = \frac{1-\alpha}{k_R^{1-\alpha} - k_L^{1-\alpha}} \tag{2}$$

Often, $k_R \to \infty$ which implies $\alpha > 1$ as a condition for convergence of $P(k)$. In such cases, the power law has a well-defined mean only if $\alpha > 1$, a finite variance only when $\alpha > 2$. Well-defined skewness and kurtosis are restricted to the cases where $\alpha > 3$ and $\alpha > 4$.

In nearly all systems, power laws are observed through both theory and empirical data. Of special interest in the last decades are the scale-free complex networks, the basic characteristic of which is a power law distribution of connectivity (number of edges per node). Power laws also govern perception, as exposed by the general Webner-Fechner and Stevens laws. As a rule of thumb, the distribution of resources among (often self-interested) components tends to present power laws, which includes distribution of human wealth, interactions, friendships; airport connections, neuronal synaptic count. Some advocate about a better fit and theoretical backbone for the superimposition of a power law distribution and a Weibull distribution [**?**]. Most canonical examples in literature seem to be earthquake intensity and allometric relations of animal bodies, most canonical laws seem to be Pareto and Zipf laws.

2 Examples

2.1 Workers in a factory

Let us consider a generic problem in which a System (S) provides an Output (O) depending on the Input (I) it receives. The following assumptions are established. 1) The System S is made of a number of components that are not all equal to each other. That is to say, there is diversity in the nature of the components. 2) There may be several inputs, but for each input the amount of resources furnished to the System is fixed, as in a conservation law. 3) Distribution should be uniform with regard to the "size" of the component as in a geometric case where space is considered as isotropic.

There is no assumption for the Output (O), which is taken as to mean the performance (or richness) in terms of the components of S. Now assuming that there are N types of input, and for the sake of the argument, all of them have a time dependence (with 1/t), according to assumption 2) above. The Output is the product of the functions of these N inputs. O = (R1*R2*... RN)/tN since a given input can be written as Ri/t. The Output has therefore a power-law dependence on t with coefficient N. Now considering the values of N observed in practice (from many examples of power-law dependences), which is normally between 1.5 (2?) and 3, one infers that there are at least two types (?) of independent inputs and at maximum 3 independent inputs. Let us illustrate with a hypothetical case that may facilitate understanding the concepts. A piece of work is to be done in a company. What sort of resources can be established as inputs? We assume three inputs: number of workers, working hours and efficiency. We recall that all resources should be fixed and that there is diversity in the components. Then, first the total number of workers available are divided into groups of different sizes, Continuar exemplo ????

Falta resumir a literatura que mostra coeficiente entre 1.5 e 3. Mencionar casos em que é maior que 3. Incluir exemplos em que a distribuição uniforme se dá, mesmo que sejam empíricos. Lembro que você tinha isso para lista de e-mails, e acho que outros exemplos com maior apelo para a física precisariam ser incluídos.

2.2 Object sizes in your house or elsewhere

Que estah interessado na sua casa em objetos de dois tamanhos: um quase do tamanho de um cubo unitario (seja qual for a unidade q vc escolher); e outro quase do tamanho l (l inteiro e maior q um). Em cada cubo unitario ha rho de chance de ter um objeto com o tamanho almejado. Nos cubos de lado l tb. Dado um cubo de lado n (l divide n) ha, em media, rho . $(n/l)^3$ objetos de volume l^3 e $\rho.n^3$ objetos de volume 1^3. Ou seja, a frequencia de objetos com volume X eh inversamente proporcional ao volume X.

3 Formalization

3.1 Axioms and immediate consequences

3.2 Phenomenological approach

Let k denote the amount of edges allocated by each vertex, its degree. We know that scale-free networks follows a power law distribution of degree p(k), in the form $p(k) = C/k^{-\alpha}$ with C a constant. Our understanding is that this distribution stems from an equanimous distribution of resources in α dimensions, in a manner that resembles wave theory.

For a moment, regard $p(k)$ as the frequency of occurance f. Regard k as the amount of resource, now the time period λ. If there is no change of medium, the wave travels at a constant rate $v = \lambda.f$. Recal $p(k) = C/k^\alpha$. Accordingly, we can observe $f = v/\lambda$ and that, in this case, $v = C$ and $\alpha = 1$.

Now let $f = v/(\lambda_1.\lambda_2)$, that is, the frequency of occurance (or the probability of choosing such event at random) go with the inverse of two periods.

3.3 Scale free complex networks

We advocate that this is the similar case of that where edges reflect the resources allocated by individuals. If $f = v/resources_i = v/E_i = v/(N_i.(E_i/N_i)) = 2v/(N_i.k_i) \equiv v/(\lambda_1.\lambda_2)$. As N_i and k_i are both directly proportional to E_i, which is the primary resource, one can factor out another constant and consider the special case where $\lambda = \lambda_1 = \lambda_2$ and, consequently, $f = v/\lambda^2$. E.g. in a social network, the number of agents allocated and the time each of them put, are seen as the primary resource (individual . time).

Questions: *) the range of degree covered by scale-free networks is maximum, as do our perception, which also follows power laws. How far can we consider scale-free complex networks to be meta-sensors that captures and processes signals about the very reason of existance of the meta-sensor?

Theorem 1: every scale-free network with distribution of degree $p(k) = C/k^\alpha$ can be understood as having an equanimous distribution of resources in α dimensions. Corolary: if $p(E_i) = C/E_i$, with $\alpha = 1$. (might have to consider only edges with vertices of other connectivity, i.e. discard edges between vertices with same degree.)

4 Conclusions

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