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% Thin Sectioned Essay

% LaTeX Template

% Version 1.0 (3/8/13)

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%----------------------------------------------------------------------------------------

% PACKAGES AND OTHER DOCUMENT CONFIGURATIONS

%----------------------------------------------------------------------------------------

\documentclass[a4paper, 11pt]{article} % Font size (can be 10pt, 11pt or 12pt) and paper size (remove a4paper for US letter paper)

\usepackage{hyperref}

\hypersetup{

colorlinks,

linkcolor={red!50!black},

citecolor={blue!50!black},

urlcolor={blue!80!black}

}

\usepackage[portuguese,english]{babel}

\usepackage[utf8]{inputenc}

%\usepackage{float}

\DeclareUnicodeCharacter{202D}{}

\DeclareUnicodeCharacter{202C}{}

%\usepackage{color} % for the notes

\usepackage{xcolor}

\usepackage[protrusion=true,expansion=true]{microtype} % Better typography

%\usepackage{graphicx} % Required for including pictures

%\usepackage{wrapfig} % Allows in-line images

%\usepackage{tocloft}

%\usepackage{multirow}

%\usepackage[titletoc]{appendix}

\usepackage[titletoc,title]{appendix}

%\AtBeginDocument{\renewcommand\appendixname{New Name}}

\usepackage{amsmath}

\usepackage{mathpazo} % Use the Palatino font

%\usepackage[T1]{fontenc} % Required for accented characters

%\linespread{1.05} % Change line spacing here, Palatino benefits from a slight increase by default

%\usepackage{etoolbox}

%\newtheorem{theorem}{Theorem}[section]

\newtheorem{theorem}{Theorem}

\newtheorem{theorem2}{Theorem}

\newtheorem{theorem3}{Theorem}

\newtheorem{lemma}[theorem]{Lemma}

\newtheorem{axiom}[theorem]{Axiom}

\newtheorem{proposition}[theorem]{Proposition}

\newtheorem{corollary}[theorem2]{Corollary}

\newtheorem{definition2}[theorem3]{Definition}

\newenvironment{proof}[1][Proof]{\begin{trivlist}

\item[\hskip \labelsep {\bfseries #1}]}{\end{trivlist}}

\newenvironment{definition}[1][Definition]{\begin{trivlist}

\item[\hskip \labelsep {\bfseries #1}]}{\end{trivlist}}

\newenvironment{example}[1][Example]{\begin{trivlist}

\item[\hskip \labelsep {\bfseries #1}]}{\end{trivlist}}

\newenvironment{remark}[1][Remark]{\begin{trivlist}

\item[\hskip \labelsep {\bfseries #1}]}{\end{trivlist}}

\newcommand{\qed}{\nobreak \ifvmode \relax \else

\ifdim\lastskip<1.5em \hskip-\lastskip

\hskip1.5em plus0em minus0.5em \fi \nobreak

\vrule height0.75em width0.5em depth0.25em\fi}

\makeatletter

\renewcommand\@biblabel[1]{\textbf{#1.}} % Change the square brackets for each bibliography item from '[1]' to '1.'

\renewcommand{\@listI}{\itemsep=0pt} % Reduce the space between items in the itemize and enumerate environments and the bibliography

\usepackage{epigraph}

%\pretocmd{\chapter}{\addtocontents{toc}{\protect\addvspace{5\p@}}}{}{}

%\pretocmd{\section}{\addtocontents{toc}{\protect\vspace{-4mm}}}{}{}

\renewcommand{\maketitle}{ % Customize the title - do not edit title and author name here, see the TITLE block below

\begin{flushright} % Right align

{\LARGE\@title} % Increase the font size of the title

\vspace{5pt} % Some vertical space between the title and author name

{\large\@author} % Author name

\\\@date % Date

\vspace{1pt} % Some vertical space between the author block and abstract

\end{flushright}

}

%----------------------------------------------------------------------------------------

% TITLE

%----------------------------------------------------------------------------------------

\title{\textbf{A simple model that explains why inequality is ubiquitous}\\ % Title

%a natural collective focus\\on the collective being} % Subtitle

} % Subtitle

\author{\textsc{Renato Fabbri, Osvaldo N. Oliveira Jr.} % Author

\\{\textit{São Carlos Institute of Physics, University of São Paulo, CP 369, 13560-970 São Carlos, SP, Brazil}}} % Institution

\date{\today} % Date

%----------------------------------------------------------------------------------------

\begin{document}

\maketitle % Print the title section

%----------------------------------------------------------------------------------------

% ABSTRACT AND KEYWORDS

%----------------------------------------------------------------------------------------

%\renewcommand{\abstractname}{Summary} % Uncomment to change the name of the abstract to something else

%

\begin{abstract}

Inequality has always been a crucial issue for human kind, particularly concerning the highly unequal distribution of wealth, which is at the root of major problems facing humanity, including extreme poverty and wars. A quantitative observation of inequality has become commonplace in recent years with increasing recognition that many natural and man-made systems can be represented as scale-free networks, whose distribution of connectivity obeys a power law. These networks may be generated by the preferential attachment for the nodes, within the so-called rich-gets-richer paradigm. In this letter we introduce a simple model that explains the ubiquity of inequality, based on three simple assumptions applied to a generic system with a large number of parts.

The first assumption is the diversity of the components, while the second is related to a uniform distribution of resources for individual components. In the third assumption, we keep the amount of each resource input to the system fixed. This implies that the more resources are allocated per component, the smaller is the number of such components in the system, thus conserving the amount of resources distributed. The second and third assumptions are conservations laws, with the resources being conserved in each input and across the component wealth. This can be geometrically described by the distribution of object sizes in a 3D space, where each dimension is assumed to be isotropic. Applying these assumptions to a generic system results in a power-law distribution, whose coefficient is the number of inputs that are independent from each other, i.e. the dimensionsionality of the allocated resources. Even though there is no restriction to the value of the coefficient,

in practice we observe that existing systems normally exhibit a coefficient between 1.5 and 3.0. With our simple model it is not possible to determine whether this limitation in the coefficient values arises from a fundamental principle, but we indicate reasonable hypotheses.

The assumptions in the model lead to a framework analogous to the laws of thermodynamics: conservation of resources and a time arrow pointing to inequality. Since these assumptions are easily justified based on established knowledge, the model proves unequivocally that inequality is ubiquitous. We also discuss ways to control this tendency to inequality,

which is analogous to a decrease in entropy in a closed system induced with an external action.

\end{abstract}

%

%{

%\selectlanguage{portuguese}

%\begin{abstract}

%

%\end{abstract}

%}

\hspace\*{3,6mm}\textit{Keywords:} power laws, fundamental theory, complex systems, complex networks, anthropological physics

%, statistics % Keywords

%\vspace{30pt} % Some vertical space between the abstract and first section

%----------------------------------------------------------------------------------------

% ESSAY BODY

%----------------------------------------------------------------------------------------

%\newpage

%\tableofcontents

%\vspace\*{1cm}

%{\bf This is a report on the newborn concept of \emph{anthropological physics}. Further efforts should contextualize, develop and correct theoretical nuances. The sharing of this naive text is a convenient step to the collective maturing and research.}

%\vspace\*{.6cm}

%\newpage

%\epigraph{A single dramatic incident involving a breach of privacy could produce a set of statutes, rules, and prohibitions that could strangle the nascent field of computational social science in its crib. What is necessary, now, is to produce a self-regulatory regime of procedures, technologies, and rules that reduce this risk but preserve most of the research potential.}{David Lazer, Alex (Sandy) Pentland, Lada Adamic, Sinan Aral, Albert Laszlo Barabasi, Devon Brewer, Nicholas Christakis, Noshir Contractor, James Fowler, Myron Gutmann, Tony Jebara, Gary King, Michael Macy, Deb Roy, and Marshall Van Alstyne~\cite{life}}

\section{Introduction}

Inequality has always been at the focus of social studies for the obvious importance for humanity and quality of life. In some respects, inequality can be regarded as asymmetry, thus being at the center stage of science since symmetry is regarded as fundamental to cognition

and knowledge in nearly all fields~\cite{deleuze,part}. In fact, the ubiquity of symmetry is appreciated so profoundly that human thinking is believed to present explicit, basic symmetry operations, and therefore the world is modelled with symmetry, regardless of its presence. In recent years, inequality in distributions has been addressed quantitatively, particularly in large systems represented as networks. Of particular importance are the scale-free networks [?], whose distribution of connectivity (number of edges per node) obeys a power law [?]. Significantly, a wide range of natural and man-made systems have been represented as networks with power-law distributions~\cite{newman} [?].

As a rule of thumb, the distribution of resources among (often self-interested) components tends to follow a power law, which includes distribution of human wealth, interactions, friendships;

connections among airports and synaptic count among neurons. Other examples of power-law distributions are associated with earthquake intensity and allometric relations of animal bodies, while the most canonical law examples seem to be Pareto and Zipf laws [ref.].

Some advocate that a more precise fitting can be obtained with superposition of a power law distribution and a Weibull distribution~\cite{powWeib}, but inequality remains.

Power laws are also prevailing in other phenomena and areas, which is not necessarily related to distribution of resources. For example, power laws govern perception, as indicated by the Webner-Fechner and Stevens laws [ref. ?]. In basic physics there are various power laws, e.g. in a Newtonian context force $F$ is related to distance $d$ through gravity

($F=G\frac{m\_1\;m\_2}{d^2}$)

with $\alpha=2$

and force is related to acceleration

($F=m\;a^1$)

with $\alpha=-1$.

Not surprisingly, there has been extensive body of research to explain power law behavior, with models for generating scale-free networks [?] and for dealing with broad classes of phenomena~\cite{part,pbook}. But to the best of our knowledge, a unified framework for interpreting any power law is lacking. This is what we do in this Letter, with the axiomatic and phenomenological description a simple model in Section~\ref{sec:form}. ????

Section~\ref{sec:par} exposes separately a geometric and a productive system

paradigms.

Special cases are considered in Section~\ref{sec:esp},

which is followed by concluding remarks.

The most elementary mathematical description

of power-law distributions is outlined in the

Appendix.

\section{Formalization}\label{sec:form}

% Definitions explicited beforehand:

% Complex System

% Component of a CS

\begin{definition2}

A complex system is taken as one in which the whole is more

than the sum of the individual parts.

\end{definition2}

Other definitions currently in use in the literature could also apply, since for our purposes the system is generic and can have any type of component.

%Given the broad range of definitions currently in use, it seems

%more profitable to claim the commonsense notion of

%a complex system to be ``a system in which the whole is more

%than the sum of the individual parts'', and start with:

\begin{definition2}

A {\bf resource} is anything that is used by a complex system to subsist and communicate.

\end{definition2}

In practice, a complex system is commonly delimited and specified through its components and the resources to keep it. The resources are not restricted to any type and can be, for example, geometrical volumes,

wave cycles or currency values.

\begin{definition2}

A {\bf resource-based system} is a complex system that has an underlying resource vital to the their components and their interdependent roles in the system.

\end{definition2}

%Specially in Game Theory, the components are often considered

%``self-interested''. This is not a required condition for the

%framework here presented.

%Even so, we understand that this formalization shed insight

%in the reasons why and the ways that self-interested agents

%organize themselves

%with extensive incidence of power laws.

\begin{definition2}

The {\bf component wealth} $k$ is the amount of resources (of a certain kind) allocated to a component.

\end{definition2}

We will use $p(k)$ to denote the fraction of components with component wealth $k$.

Likewise, the same notation $p(k)$ will denote the probability of choosing a component with an amount $k$ of resources.

The context should make it clear which is the appropriate meaning.

This correspondence among probability, frequency and relative count is

instrumental for the interpretative framework in this article,

which is conveniently embedded in the notation.

It is also useful to observe $k$ as a wave period $\lambda$.

Then, $p(k)$ is more immediately understood as

the frequency $f$.

Notice that $k$ is a measure of the resources,

not total resources (or total component wealth), which

might be correlated to $k$ and have $k$ as one of the resources.

\subsection{Propositions and corollaries}

% corollaries, lemmas, etc

\setcounter{theorem}{-1}

\begin{proposition}\label{prop:0}

There is diversity among the components of the resource-based system.

\end{proposition}

For a large set of components, there is hardly any way to avoid diversity. This could occur with location, size, age, etc. This can be regarded as both a statistical law, or even as a deeper truth.

The mere existence of two objects imposes diversity,

otherwise they would be both the same.

Proposition~\ref{prop:0} is important for the distribution of resources among components: if the latter are all equal, by definition they have the same component wealth.

\begin{proposition}\label{prop:2}

The allocation of resources in a resource-based system is uniform across the components. This is expressed as a uniform distribution $p\_U(k)$ of resources with respect to component wealth $k$.

\end{proposition}

That is, resources are allocated democratically with respect to component wealth values, which has two major consequences: 1) there is a maximum value for the wealth of a component;

2) the wealthier the components considered, the fewer they are.

In the analysis, components without resources allocated are discarded.

\begin{corollary}\label{prop:1}

The different resource inputs are combined in the $\alpha$ dimensions of the resources.

% The amount of each resource input to the system is fixed.

\end{corollary}

% If you observe one dimension k of an \alpha -dimensional resource,

% observe a C/k^\alpha distribution, regardless of which are

% the other dimensions: the \alpha-dimension relation

% of size and scale is preserved.

% If total resource = resource A x resource B ...

% they are all directly proportional to total resource

%Total resource is the product of resources in each dimension.

%

%Also, resources can be proportionally related.

This is important to consider resources with distinct dimensions, as the amounts of different resources may be incomparable. Let $\lambda\_1$ and $\lambda\_2$ be distinct resources, leading to a total

resource: $E=\widetilde{C\_1} . \lambda\_1 . \lambda\_2 \equiv \widetilde{C\_2}\lambda^{\alpha=2}$

with $\widetilde{C\_X}$ constant.

Note that $E=\widetilde{C\_1} . \lambda\_1 + \widetilde{C\_2}.\lambda\_2 \equiv \widetilde{C\_3}\lambda^{\alpha=1}$, that is: a resource resulting from the summation of other two unidimensional resources is an unidimensional resource and these resources are not of distinct dimensions.

For example, if the resources are $components$ and $time$,

a final resource $E=5\, components + 4h=9$

holds little if any information, while

$E= 5\, components \, . \, 4 h= 20\; components . hours$ is

a reasonable measure of resources in a canonical metric. (nao entendi?)

\begin{corollary}\label{prop:3}

The equilibrium state implied by Propositions~\ref{prop:0},~\ref{prop:2} and~\ref{prop:1} is characterized by a power-law distribution $p(k)$ of components with component wealth $k$.

\end{corollary}

From the propositions, the same amount $C$ of resources is allocated

across all quantities $k$ of resources allocated per component.

Therefore, the fraction $p(k)$ of components with component wealth $k$ is

$p(k)=\frac{C}{k^\alpha}$, where $\alpha$ is

the dimensionality of the resources compared to the dimension of $k$.

As the system continues to exist,

and resources are continually allocated, deviations from the equanimous

distribution $p\_U(k)$ of resources along component wealth $k$

tend to be transient.

It is worthnoting that $p\_U(k)$ is usually not made explicit in the literature, but only the power-law distribution $p(k)$ of components

with component wealth $k$.

%In other words,

%if there are $\alpha$ independent inputs,

%i.e. resources in $\alpha$ dimensions,

%each have a component wealth $k\_n$ distribution

%$p\_{k\_n}(k\_n)=C\_n.k\_n^{-1}$

%and the final and perceived distribution

%is the product of these functions

%%O = (R1\*R2\*… RN)/tN since a given input can be written as Ri/t.

%$p(k) = \prod\_1^{\alpha} C\_n.k\_n^{-1}\equiv C.k^{-\alpha}$.

\begin{corollary}\label{cor:2}

The extension of allocation is $[k\_L,k\_R]$ with $R\_L$ often 0 or 1 and $k\_R\approx C$.

\end{corollary}

This follows from Proposition~\ref{prop:2}.

If the allocation of resources is insensitive to component wealth,

it should sweep all possible values, which have a lower limit by being a positive quantity of resources.

Systems are normally considered as a set of components

Coupled by the resources,

thus $k\_L=0$ and $k\_R=1$ are reasonable, respectively. (não entendi?)

The distributions $p(k)$ and $p\_U(k)$ also have an upper limit as the component wealth reaches $k\_R \approx C$,

the total amount of resources uniformly distributed.

In empirical data, $k\_R$ can vary considerably, due to

nonlinearity of the resources scaling (most often $k\_R<C$) and

to self-interested agents (most often $k\_R>C$).

Power laws are reported to conduct empirical data sovereignly and

often are strict only for a

(broad) portion of component wealth range. (nao entendi?)

%\begin{corollary}

% For a system with a finite quantity of resources,

% the allocation is compact with a superior limit $k\_2$.

%\end{corollary}

\begin{corollary}

The upper limit $k\_R$ for the allocation of resources is an estimate of the amount of resources $C$ equally distributed along component wealth ($C\approx k\_2$).

\end{corollary}

This follows from Corollary~\ref{cor:2}. (acho que isso já foi mencionado no corolário anterior, e então fica repetitivo)

\begin{corollary}

$p(1)$ provides another estimate of the amount of resources $C$ equally distributed along component wealth ($C\approx N . p(1)$, where N is the number of components).

\end{corollary}

\begin{corollary}

An estimate for $C$ can also be found by $\alpha$, $k\_L$ and $k\_R$ through Equation~\ref{eq:con}.

\end{corollary}

\begin{corollary}

The dimensionality of the allocated resources is the scaling factor $\alpha$.

\end{corollary}

This last corollary follows from box counting or, most easily,

through wave-like reasoning about power laws, as explained in the next section.

\subsection{Phenomenological approach: power laws are consistent with the assumptions}\label{sec:phen}

% pure mathematical interpretation of the power law

One important step for checking the consistency of our model is to verify whether the assumptions above lead to any type of power law. The simple argument presented below taking wave propagation as example corresponds to a quasi-``if and only if''interpretation of power laws.

{\bf (1D case)} Consider a wave propagation with constant speed $v$

(suppose linear media with no dispersion). The number of oscillations $f$ per unit time is inversely proportional to the cycle length $\lambda$ (the period). In usual notation $f=\frac{v}{\lambda}$.

The constant speed $v$ implies a power law between

$f$ and $\lambda$ (with $\alpha=1$ and $C=v$).

This is the core insight, the general case that a power law can

be interpreted as resulting from a constant amount $C$ of

fundamental resources with dimensionality $\alpha$

being homogeneously distributed across

component wealth $k$ and

resulting in $p(k)$ of such components.

{\bf (2D case and beyond)} Now let $f=\frac{v=C}{\lambda\_1 . \lambda\_2}$, that is, the frequency of occurrence

(or the probability of choosing such event at random) go with the inverse of two periods while the speed is constant.

If $\lambda\_1==\lambda\_2==\lambda$, then $f=\frac{v}{\lambda^2}$.

In other words, the density given by an amount $v$

(or a quantity $C$ of resources) in a hypercube of

edge $\lambda$ (or component wealth $k$)

and $\alpha=2$ dimensions.

This same reasoning yields the relative count,

The fraction or probability $p(k)$ of components with such component wealth $k$ and corresponding hypercubes $k^\alpha$.

Let $C\_i$ be the amount of resources allocated at

specific resource costs, say $\lambda\_{1,i}$ and $\lambda\_{2,i}$ to strengthen the wave argumentation,

and assume linearity $\lambda\_{1,i}=c.\lambda\_{2,i}$

such that

$p(k\_i)=\frac{C}{C\_i}=\frac{C}{\lambda\_{1,i}.\lambda\_{2,i}}=

\frac{C}{c\lambda\_{2,i}^2}=\frac{\widetilde{C}}{\lambda\_{2,i}^2}\equiv\frac{v}{\lambda^2}$.

Therefore one can consider that we only access $\lambda\_1==\lambda\_2$ and a ``normalized resource C'' allocated by the environment uniformly across component wealth:

higher component wealth implies less numerous components. (não entendi?)

Na parte debaixo não entendi a argumentação. Como o que está em verde fecha a seção em que se procurava mostrar que as hipóteses levam a power laws.

The cases above were related to phenomena represented by power laws.

For power-law distributions involving random variables, we assume

random variables

$\lambda\_{1,i}$ and $\lambda\_{2,i}$ as presenting an uniform distribution of resources along component wealth

themselves:

$p\_{\lambda\_{1}}(\lambda\_{1,i})=\frac{C\_i}{\lambda\_{1,i}^{\alpha\_1}}$

and

$p\_{\lambda\_{2}}(\lambda\_{2,i})=\frac{C\_i}{\lambda\_{2,i}^{\alpha\_2}}$.

Consider de product distribution of the

$\lambda\_{3,i}^2=\lambda\_{1,i}.\lambda\_{2,i}$

variable:

\begin{equation}

p\_{\lambda\_3^2}(\lambda\_{3,i}^2)=C\_1.C\_2.\lambda\_3^{-2\alpha\_2}\frac{(R\_{1,R}-R\_{1,L})^{\alpha\_2-\alpha\_1}}{\alpha\_2-\alpha\_1}

=C\_1.C\_2.\lambda\_3^{-2\alpha\_1}\frac{(R\_{2,R}-R\_{2,L})^{\alpha\_1-\alpha\_2}}{\alpha\_1-\alpha\_2}

\end{equation}

where $R\_R$ and $R\_L$ are the upper and lower limits of the respective resources in the system considered.

Notice that $p\_{\lambda\_3^2}(\lambda\_{3,i}^2)=\frac{N\_i}{N}=p(\lambda\_{0,i})$,

where $p(\lambda\_{0,i}$ is the probability that a component has a measured

component wealth $\lambda\_{0,i}$ correlated to $\lambda\_{3,i}^2$.

The number $N\_i$ of components with component wealth $\lambda\_{3,i}^2$ and the total

number of components $N$ are conserved for $\lambda\_{0,i}$.

If assumed $\lambda\_{0,i} \propto \lambda\_{3,i}$, than

$p(k\_{0,i}) \propto k\_{0,i}^2$.

\section{Paradigmatic observations of the propositions}\label{sec:par}

In the following we discuss a few paradigmatic cases where the propositions can be applied to specific systems.

\subsection{Object sizes in a house}\label{sec:siz}

Take a measure unit $l$ for length and consider $m \in (0,1)$.

In a house, there are probably more objects of volume $\approx (m.l)^3$ than those of volume $\approx l^3$. We may assume that the chance $\rho$ of finding an object fitting an arbitrary cube of volume $\approx (m.l)^3$ is the same as that for an object with of volume $\approx l^3$.

Since the house has a fixed volume, $m^{-3}$ more of

the smaller cubes can be fitted and therefore there should be $m^{-3}$ more objects of such volume. The result is a power-law distribution of object volumes related to the length $l$ with $\alpha=3$:

$p(l)=C.l^{-3}$. Note that if the objects are mutually exclusive,

$\alpha$ is probably lower, as the number of smaller

cubes will decrease considerably.

This example gives a geometrical interpretation of the formalism presented in section ??. It also might be assumed true even if there is no isotropy and can be considered a ``best guess'' if total ignorance

about the system is assumed. Extreme choices of $l$ and $m$ (and of $dl$ and $dm=mdl$ for tolerance in size) will often exhibit spurious observations as the result of a bad fit of the scale for the analysis (nao entendi?).

\subsection{Workers in a factory}

In a factory where many tasks need to be performed, we may consider three types of inputs: number of workers $N$, working hours $W$

and efficiency $E$. This is similar to the canonical resource

$person.hour$ since Taylorism and taken to the extreme in Fordism [ref.?].

We assume that all concentrations $C.k^3=n.w.e$

of these resources might be found in working groups.

Assuming the tasks changes all the time,

all concentrations are important.

By Proposition~\ref{prop:2} they are equally important beforehand

in the sense that

resources will be spread uniformly over $[k\_L,k\_R]$

with $p\_U(k)=\frac{1}{k\_R-k\_L}$.

The number of such components, therefore,

decreases with $C.k^3=n.w.e$, the component wealth with three dimensions.

In other words,

$p(k)=\frac{C}{k^3}$, that is,

the fraction $p(k)$ of components (working groups) with component wealth $k$

decrease with $k^3$ and distributes the same

amount of resources $C=\frac{N.W.E}{k\_R-k\_L}$ along $k$. (não entendi as contas)

\section{Implications}\label{sec:esp}

The consequences of the interpretation of power laws presented in Section~\ref{sec:form} are severe for understanding and dealing with

diverse (if not almost all) phenomena. Selected cases are discussed in this section.

\subsection{Scale-free complex networks}

In a network, one has essentially $E$ edges and $N$ vertices.

Assuming linearity of resources:

\begin{equation}\label{eq:nre}

Resources=C\_1 N + C\_2 E \approx C\_2 E = C\_2 N \frac{E}{N} = C\_2 \frac{N \overline{k}}{2}

\end{equation}

\noindent where $C\_1$ and $C\_2$ are constants and $\overline{k}$ is

the average degree, i.e. the mean of the number of edges attached to each vertex.

As in the literature, nodes can be disregarded, since if they are not connected. Therefore:

\begin{equation}\label{eq:eqf}

p\_{E\_i}(k\_i)=\frac{C}{resources\_i}=\frac{C}{C\_2 E\_i}=\frac{C/C\_2}{N\_i \frac{E\_i}{N\_i}}=\frac{2C/C\_2}{N\_i k\_i} \equiv \frac{v}{\lambda\_1 \lambda\_2}

\end{equation}

Since $N\_i$ and $k\_i$ are both directly proportional to $E\_i$, which is the fundamental resource, one can factor out another constant and consider the special case where $\lambda=\lambda\_1=\lambda\_2$ and, consequently, $f=v/\lambda^2$. This is the case for a social network, the number of agents allocated $N\_i$ and the time each of them put (related to $k\_i$), are seen as the primary resource ($individuals . time$), and $\alpha\approx 2$ as shown by empirical evidence~\cite{newman}.

Also:

\begin{equation}\label{eq:eqf}

p\_{E\_i}(k\_i)=\frac{N\_i}{N}=\frac{2C/C\_2}{N\_i k\_i} \Rightarrow \frac{N\_i^2}{N^2}=2\frac{C}{N.C\_2}k\_i^{-1} \Rightarrow p\_{k\_i}(k\_i)\propto k\_i^{-\frac{1}{2}}

\end{equation}

\noindent so that if $p(k\_i)$ is observed only with respect to the degree,

$\alpha=0.5$,

which is far from empirical evidence because it only captures the distribution of one of the resources with respect to $k\_i$.

\subsection{Equanimous inequality}

Paradoxically, power laws, which are the

current utmost inequality paradigm,

follow from an equanimous consideration

of resource inputs

and exhibit other equanimous aspects:

\begin{itemize}

\item $p(k)=C.k^{-\alpha} \Rightarrow p(k).k^{\alpha}=C$, with C constant. That is, the amount $k^\alpha$ of resources per component times the amount of those components, which is the total ``instantaneous'' allocated resources, is constant $C$.

\item Each component participates in numerous other complex systems, potentially infinite, and should present a broad, if not complete, sweep of resources allocated to itself.

These resources are not necessarily of the same type. We assume that human systems, for example, present power-law distributions of knowledge $p\_k(k\_k)$ and of wealth $p\_w(k\_w)$, with potentially different (relative) amount $k$ of resources. At the same time, within a fixed type of resource, resources allocated vary in different systems. For instance, an individual tends to have many acquaintances (resource) in its own family, work and neighborhood, but fewer in circles of distant family members, partners and friends.

\item The total resource available to each component is potentially the same, but spread differently across systems. For example, human individuals form complex social systems with power-law distributions of relations. All the participants, by being humans, have the same amount of time (resource) available each day to engage in all the complex systems that are presented by the environment. One can even assume that each individual creates the same amount of relationships with the world each day, be them with other people, ideas, things, etc.

\end{itemize}

\subsection{Wealth distribution}

One manifestation of inequality through power laws

most fundamental in society is the discrepant wealth distributions worldwide. There are continuous efforts to deal with this issue,

usually advocating ways to minimize ``social inequality''.

Considering the framework presented within this letter:

\begin{itemize}

\item Such an inequality is a natural tendency that follows from Propositions~\ref{prop:0},~\ref{prop:2} and~\ref{prop:1} in accordance with the phenomenological analysis of Section~\ref{sec:phen}.

\item Deviations from a power-law distribution of wealth should require work.

Occasional deviations are part of the statistical aspect of the phenomena involved, but the maintenance of a pattern different from the power law derived from the resources distribution should require expenditure of energy.

\item Both deviations of power laws towards a more equanimous or towards a more inequanimous distribution should be ephemeral or require work.

\end{itemize}

In summary, power-law inequality seems inevitable,

and a consequence of a distribution of wealth equanimous and insensitive

along wealth allocated to each component.

This implies the necessity of ``work'' for equalization.

Also, we observe that the higher the $k\_L$ of equation~\ref{eq:pow}, the

higher all the probabilistic mass will be located, which

implies greater wealth of the wealthier, the hubs or the ``elites''.

In other words, the richer the least rich,

the richer the more rich. (não entendi?)

\subsection{When inequality is good}

Inequality is normally treated as a bad consequence, especially in social studies. There are cases, however, where inequality in distributions is highly beneficial. One such example is given by perception, which presents many psychophysical power-law relations between the

magnitude of the physical stimulus and the perceived

(subjective) quantity~\cite{pbook}. This enhances the utility of perception capability by broadening the spectrum.

The explanation is in the physical phenomenon itself. Consider a sound wave traveling with constant speed $v$. If the organism can capture wavelengths from $\lambda\_1$ to $\lambda\_2$, $f=\frac{v}{\lambda} \in [\frac{v}{\lambda\_2},\frac{v}{\lambda1}]$ follows

a power law with $\alpha=1$. As discussed with regard to Proposition~\ref{prop:2}, the power-law distribution

maximizes the component-wealth domain and,

in the present case, the reception of signals. In other words, power-law inequality maximized versatility.

The existence of power laws in perception

suggests a pertinence, and is regarded as such by literature.

This raises a fit, in advance, for complex systems with power laws

to be analysed as sensors (or meta-sensors). (nao entendi os significados de pertinence e fit na sentença). For example, an group interested in hiking can be understood as a meta-sensor about hiking, and involved a community to find current good places, equipment, people, proper behavior, etc. The power law, i.e. ``the scale-free trace'' to use

the complex network jargon, sweeps a wide

range of types of engagement (regarding the concentration of resources).

The higher the component wealth $k$ the higher the engagement,

but the lesser diversity is brought to the group

by the participant~\cite{tStable}:

as one allocates more resources (say time)

in one system,

it allocates less resources in outside systems. (não entendi a argumentação).

\section{Discussion}

A discussão poderia começar com algo mais genérico, e podemos rearranjar isso depois.

Our model does not require the components to be ``self-interested''.

This is often required in this context (por quem?), and

we understand that the formalization presented here shed insight

in the reasons why and the ways that self-interested agents

organize themselves with extensive incidence of power laws. These principles can be thought of as laws met in

a broad class of phenomena. The resulting axioms can be viewed as statistical tendencies that hold with such universality that they can be seen as laws that govern phenomena ubiquitously. Deviations from $p(k)$ tend to be transient or require effort, the expenditure of energy (as work in needed to reduce entropy), or be imposed by harsh conditions.

Power laws are very frequent in empirical data, which already grants its place among the study of natural phenomena.

Different explanations are given for the many cases where

they are found, with most common denominators being fields such as

fractals, chaos, networks and unifying models e.g. as given by the Theory of Self-Organized Criticality~\cite{part}. Cases in more traditional fields, such as in Newtonian mechanics with the gravitational force relation to distance with $\alpha=2$ (if masses are fixed), are usually not mentioned in the specialized literature. In other words: there seems not to be a unifying theory of why power laws express such ubiquitous spectrum of relationships.

If the framework in Section~\ref{sec:form} is valid for all cases where power laws are found, the consideration of power laws as tied to natural phenomena \emph{per se} goes a step further.

Phenomenologically we have a fit of the analysis. (o que quer dizer?)

That is: if there is a power law, the analysis developed in Section~\ref{sec:phen} holds and the power law relation can be regarded as an equanimous distribution of resources in $\alpha$ dimensions.

If the acting of the ``laws'' given by Propositions~\ref{prop:0},~\ref{prop:2} and~\ref{prop:1} is fundamentally what is taking place. We advocate that this framework deepens the understanding of potentially all power law

incidences. The core meaning seems to emanate, with the simplest formalism, from

the object sizes in the isotropic space shown in Section~\ref{sec:siz}.

That is a reasonable geometric abstraction for

one to grasp the power law inequality originated from

a uniform distribution

of fundamental resources through component wealth.

Also, relating power laws to the environment is the most effective

way we found to make explicit both the axiomatic

and the phenomenological backbones of power law ubiquity described in

Section~\ref{sec:form}.

We hypothesize that $\alpha \approx 2$ is due to two basic resources

input in any system: components and their time, energy or engagement.

All other resources end correlated to these two.

We also hypothesize that deviations from $\alpha \approx 2$ are due

to other resources less correlated to them

or to any other nonlinearities in the relationship of resources.

\section{Conclusions}

The presented framework

most importantly laid inequality ubiquitous

through power laws which follow from simple assumptions.

These assumptions can also be observed in any power-law incidence.

Therefore, the interpretation of diverse natural and human phenomena

are impacted and inequality is posed as a natural tendency.

Immediate consequences were therefore scrutinized case-by-case.

We hypothesize that

inequality should be minimizable by the expenditure of energy

and that the characteristic $\alpha=2$ is a consequence of

the existence of two types of independent resources.

Further work should link these findings to individual fields

and contextualize the general framework.

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% BIBLIOGRAPHY

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\begin{appendices}

\section\*{Appendix: Power laws}

A power law is a functional relationship between two quantities $p(k)$ and $k$ in the form:

\begin{equation}\label{eq:pow}

p(k) = Ck^{-\alpha}

\end{equation}

\noindent where $k\in [k\_L,k\_R]$

and $C$ is constant.

There are four degrees of freedom in four characteristic variables:

$\alpha$, $C$, $k\_L$ and $k\_R$.

Suppose also $p(k)$ normalized so that

$\int\_{k\_L}^{k\_R}p(k)=1$, i.e.

the power law is fit to represent a

probability density function.

Assuming idealized phenomena:

\begin{equation}\label{eq:con}

\int\_{k\_L}^{k\_R}p(k)=1\;\Rightarrow\;

C=\frac{1-\alpha}{k\_R^{1-\alpha}-k\_L^{1-\alpha}}

\end{equation}

The cumulative distribution function $P(k)$, the and median $m$, mean $\mu$ and variance $\sigma^2$ are:

%\begin{equation}

%\begin{aligned}\label{eq:md}

\begin{align}\label{eq:md}

P(k) &=\int\_{k\_L}^k p(\tilde{k})d\tilde{k}=\frac{C}{1-\alpha}(k^{1-\alpha}-k\_L^{1-\alpha})=

\frac{k^{1-\alpha}-k\_L^{1-\alpha}}{k\_R^{1-\alpha}-k\_L^{1-\alpha}}\nonumber\\

&\left ( m : \int\_{k\_L}^{m}p(k)dk=\int\_{m}^{k\_R}p(k)dk=\frac{1}{2} \right )\nonumber\\

m&=\sqrt[1-\alpha]{k\_L^{1-\alpha}+\frac{1-\alpha}{2C}}=

\sqrt[1-\alpha]{k\_R^{1-\alpha}-\frac{1-\alpha}{2C}}=

\sqrt[1-\alpha]{\frac{k\_R^{1-\alpha}-k\_L^{1-\alpha}}{2}}\\

\mu &= \int\_{k\_L}^{k\_R}kp(k)dk=C\;\frac{k\_R^{2-\alpha}-k\_L^{2-\alpha}}{2-\alpha}=\left(\frac{1-\alpha}{2-\alpha}\right)\left(\frac{k\_R^{2-\alpha}-k\_L^{2-\alpha}}{k\_R^{1-\alpha}-k\_L^{1-\alpha}}\right)\nonumber\\

\sigma^2 &= \left[ \int\_{k\_L}^{k\_R}k^2p(k)dk=C\;\frac{k\_R^{3-\alpha}-k\_L^{3-\alpha}}{3-\alpha} =\left(\frac{1-\alpha}{3-\alpha}\right)\left(\frac{k\_R^{3-\alpha}-k\_L^{3-\alpha}}{k\_R^{1-\alpha}-k\_L^{1-\alpha}}\right) \right] -\mu^2\nonumber

% m\_n&=\frac{1}{N}\sum\_{i=1}^N z\_i^n \nonumber\\

% R\_n&=|m\_n|\\

% \theta\_\mu&=Arg(m\_1) \nonumber \\

% \theta\_\mu'&=\frac{T}{2\pi} \theta\_\mu \nonumber

\end{align}

%\end{aligned}

%\end{equation}

Often, $k\_R\rightarrow \infty$ which implies: $\alpha>1$ as a condition for convergence of $p(k)$ and for a finite median $m$. Also,

the mean $\mu$ is finite only if $\alpha>2$, and the

variance $\sigma^2$ is finite only when $\alpha>3$:

\begin{align}\label{eq:md}

if \; k\_R \rightarrow \infty \Rightarrow & & \nonumber\\

& if\; \alpha>1 \;\Rightarrow & C = &\; \frac{\alpha-1}{k\_L^{1-\alpha}}\nonumber\\

& & P(k) =& 1-\left(\frac{k\_L}{k}\right)^{\alpha-1}\nonumber\\

& & m = & \sqrt[1-\alpha]{\frac{\alpha-1}{2C}}&=&\;\;k\_L.\sqrt[\alpha-1]{2}&\\

& if\; \alpha>2 \;\Rightarrow & \mu = & \;k\_L^{2-\alpha}\frac{C}{\alpha-2}&= &\;\;k\_L\frac{\alpha-1}{\alpha-2}& \nonumber\\

& if\; \alpha>3 \;\Rightarrow & \sigma^2 = & \;k\_L^{3-\alpha}\frac{C}{\alpha-3}-\mu^2&= &\;\;k\_L^2\frac{\alpha-1}{(\alpha-3)(\alpha-2)^2}&\nonumber

% \mu = \int\_{k\_L}^{k\_R}kp(k)dk=C\;\frac{k\_R^{2-\alpha}-k\_L^{2-\alpha}}{2-\alpha}\nonumber\\

% \sigma^2 \int\_{k\_L}^{k\_R}k^2p(k)dk -\mu^2 =C\;\frac{k\_R^{3-\alpha}-k\_L^{3-\alpha}}{3-\alpha} -\left[C\;\frac{k\_R^{2-\alpha}-k\_L^{2-\alpha}}{2-\alpha}\right]^2\\

% m : \int\_{k\_L}^{m}p(k)dk=\int\_{m}^{k\_R}p(k)dk=\frac{1}{2}

% m\_n&=\frac{1}{N}\sum\_{i=1}^N z\_i^n \nonumber\\

% R\_n&=|m\_n|\\

% \theta\_\mu&=Arg(m\_1) \nonumber \\

% \theta\_\mu'&=\frac{T}{2\pi} \theta\_\mu \nonumber

\end{align}

\end{appendices}

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Parte geral que poderá ser reaproveitada

Consider a generic problem in which a System ($S$) provides an Output ($O$) depending on the Input ($I$) it receives.

The following assumptions are established:

\begin{enumerate}

\item $S$ is made of a number of components that are not all equal among themselves.

That is to say, there is diversity in the components, in accordance with Proposition~\ref{prop:0}.

\item Distribution of resources is uniform with regard to the ``size'' of the components (component wealth) as in the geometric isotropic case of your house.

This reflects Proposition~\ref{prop:2}

\item There may be several inputs, but for each input the amount of resources furnished to the System can be considered the same

in accordance with Proposition~\ref{prop:1}.

\end{enumerate}

The Output ($O$)

is assumed to be the performance (or richness) in terms of the components of $S$.

If there are $\alpha$ types of independent inputs, i.e. resources in $\alpha$ dimensions,

the Output is the product of these functions should be

%O = (R1\*R2\*… RN)/tN since a given input can be written as Ri/t.

$O =C.k^{-\alpha}$, where $k$ is a one dimensional component wealth observation,

as shown in Section~\ref{sec:phen}.

The Output has therefore a power-law dependence on $k$ with coefficient $\alpha$.

Empirical values of $\alpha$

are normally between $1.5$ and $3$,

from which one infers that there are

fundamentally between two and three types

of independent inputs to both natural and artificial systems.