0.1 History

- Electron (1897, J. J. Thomson, plum pudding)
- Nucleus/Hydrogen=Proton (Rutherford scattering α into gold)
- Bohr Model (1914, Bohr)
- Neutron (also in Nucleus) (1932, Chadwick)

Photon:
- $\hbar$ (1900, Planck, ultraviolet catastrophe)
- Quantitization (1905, Einstein, Photoelectric effect)
- $\Delta\lambda = \frac{h}{mc}(1-\cos\theta)$ (1923, A. H. Compton, Compton Scattering)

• Short range strong force/potential ($\frac{-e^{-r/a}}{r}$) = Heavy boson $\sim 300m_e$ = <u>Meson</u>(2 quark, Middle-weight) (1934, Yukawa)
∗ Lepton (light-weight), Baryon (3 quark, Heavy-weight) ∗ Found in cosmic rays (1937, twice)
• Wrong lifetime and lighter $\rightarrow \exists$ Pion(upp. atmos., strong force meson) + <u>Muon</u>(sea level, lepton) (1947, Powell+co.)
∗ $\pi \rightarrow \mu + \{\nu\}$ (energy never varied)

• Antimatter from Dirac Equation (1927, Dirac) ∗ Positron (1931, Anderson)
• Feynman-Stuckelberg formulation of antiparticles (1940s, Feynman+Stuckelberg)
∗ $p^- + \bar{n}$ (1955+1956, Berkely Bevatron)

• Theory Neutrino: Electrons in beta decay varied in energy meant <u>extra particle</u>, $n \rightarrow p^+ + \underline{e^- + \bar{\nu}_{\{e\}}}$ (1930, Pauli idea; 1933, Fermi theory)
∗ $\pi \rightarrow \mu + \nu_{\{\mu\}}$ ∗ $\mu^- \rightarrow e^- + \underline{\bar{\nu}_{\{e\}} + \nu_{\{\nu\}}}$ (1948?, Powell)
• Lepton Number Conservation Idea (1953, Konopinski-Mahmoud)
• Neutrinos (mid-1950s, Cowan-Rheines, Savannah River Nuclear Reactor, tank of water, $\bar{\nu}_{\{e\}} + p^+ \rightarrow n + e^+$)
•* $\nu \neq^* \bar{\nu}$ (late-1950s, Davis-Harmer, never saw $\bar{\nu}_{\{e\}} + n \rightarrow p^+ + e^-$ *<u>but could be because of spin-state diff.</u>)
• μ, e (Leption Gen) Num. Cons. Idea (late-1950s, many)
∗ μ, e (Lepton Gen) Num. Cons. Experiment (1962, Lederman-Schwartz-Steinberger-et. al, Brookhaven, 44 ft steel wall, $\nu_\mu + p^+ \neq n + e^+$)

• Kaons, ?? $\rightarrow \mu^+ + \mu^-$ (Dec 1947, Rochester-Butler, Cosmic rays on lead plate in cloud chamber)
• θ^+ and τ^+ are the same K^+

Gell-Mann/Okubo (Baryon octet) : $2[m_N + m_\Xi] = m_\Sigma + 3m_\Lambda$
Coleman-Glashow : $\Sigma^+ - \Sigma^- = p - n + \Xi^0 - \Xi^-$

Gell-Mann/Okubo (Meson nonet) : $2[m_K^2 + m_{\overline{K}}^2] = m_\pi^2 + 3m_\eta^2$
Coleman-Glashow : $\Sigma^+ - \Sigma^- = p - n + \Xi^0 - \Xi^-$

Particle Strangeness : • $S_\Delta = S_\pi = 0$ • $S_\Sigma = -1$ • $S_\Xi = -2$ • $S_\Omega = -3$ • $S_K = \pm 1$

Baryon Octet / Pseudoscalar Meson, $s = \frac{1}{2}/0$:

$$\begin{array}{ccccc} & n_{ddu} & & p_{uud} & \\ \Sigma_{dds} & & \Sigma, \boxed{\Lambda} & & \Sigma_{uu\bar{s}} \\ & \Xi_{dss} & & \Xi_{uss} & \end{array} \qquad \begin{array}{ccccc} & K^0_{d\bar{s}} & & K^+_{u\bar{s}} & \\ \pi^-_{d\bar{u}} & & \pi^0, \boxed{\eta'}, \eta & & \pi^+_{-u\bar{d}} \\ & K^-_{\bar{u}s} & & \overline{K}^0_{-\bar{d}s} & \end{array}$$

Baryon Decuplet / Vector Meson, $s = \frac{3}{2}/1$:

$$\begin{array}{ccccccc} \Delta & & \Delta^0 & & \Delta^+ & & \Delta^{++} \\ & \Sigma^* & & \Sigma^{*0} & & \Sigma^* & \\ & & \Xi^* & & \Xi^{*0} & & \\ & & & \Omega^- & & & \end{array} \qquad \begin{array}{ccccc} & K^{*0} & & K^{*+} & \\ \rho^- & & \rho^0, \omega, \phi & & \rho^+ \\ & K^{*-} & & \overline{K}^{*0} & \end{array}$$

Scalar $s = \vec{a}\cdot\vec{b} = (-\vec{a})\cdot(-\vec{b}) = Ps$ Vector $v = -(-\vec{a}) = -Pv$

Pseudovector $w = \vec{a}\times\vec{b} = (-\vec{a})\times(-\vec{b}) = Pw$ Pseudoscalar $t = \vec{a}\times\vec{b}\cdot\vec{c} = -(-\vec{a})\times(-\vec{b})\cdot(-\vec{c}) = -Pt$

Quark Masses :

$m_u = m_d = \cancel{308}$

$m_s = \cancel{483}$

$m_c = 1250$

$m_b = 4500$

Baryon Octuplet / Pseudoscalar Meson Masses (MeV/c^2) :

$m_n = 939.57$ $m_p = 938.27$ $\overline{m_\pi} = 138.04$

$\overline{m_\Lambda} = 1115.7$ $\overline{m_\Sigma} = 1190.5$ $\overline{m_K} = 495.67$

$\overline{m_\Xi} = 1318.1$ $m_\eta = 547.3$

Baryon Decuplet / Vector Meson Masses (MeV/c^2) :

$\overline{m_\Delta} = 1232$ $m_\Omega = 1672$ $m_\rho = 776$

$\overline{m_{\Sigma^*}} = 1385$ $m_\phi = 1020$ $m_\omega = 783$

$\overline{m_{\Xi^*}} = 1533$ $m_{K^*} = 894$

Isospin, I, and I_3 : $|I\ I_3\rangle$ ($SU(2)$ rules like spin), $\boxed{\text{Conserved under Strong} = \text{Strong Invariant, rotat. } I_{1,2,3}\text{-space}}$

* $I_{n/p} = \frac{1}{2}$ * $I_\pi = 1$ * $I_\Sigma = 1,\ I_\Lambda = 0$ * $I_\Delta = \frac{3}{2}$
- row mult. $= 2I+1$
- $(U,C,T)\to 1,\ (D,S,B)\to -1$
- highest $Q \to I_3 = I$
- $I_3 = \frac{U+D}{2}$

$$\boxed{\begin{aligned} \bullet\ Q &\sim I_3 + \tfrac{1}{2}(B_{aryon} + S_{trange}) \\ &= \tfrac{1}{2}\big[B_{aryon} + (U+C+T) + (D+S+B)\big] \\ &= \tfrac{2}{3}(U+C+T) + \tfrac{1}{3}(D+S+B) \end{aligned}}$$

$\boxed{\bullet\ u = |\tfrac{1}{2}\tfrac{1}{2}\rangle \quad \bullet\ d = |\tfrac{1}{2} - \tfrac{1}{2}\rangle \quad \bullet\ \bar{d} = -|\tfrac{1}{2}\tfrac{1}{2}\rangle \quad \bullet\ \bar{u} = |\tfrac{1}{2} - \tfrac{1}{2}\rangle}$ $\Rightarrow$ * $\boxed{p/n = |\tfrac{1}{2} \pm \tfrac{1}{2}\rangle}$ (see baryons)

* Deuteron, $d = pn$ $\boxed{|00\rangle = \frac{1}{\sqrt{2}}(|pn\rangle - |np\rangle) = \sum pn}$

$|1\ 1\rangle = |pp\rangle$ (exper. dne)

$|1\ 0\rangle = \sum pn \sim |pn\rangle + |np\rangle$

$|1\ -1\rangle = |nn\rangle$ (exper. dne)

* ^{4}He, $\alpha = ppnn$ $\boxed{|00\rangle = \sum ppnn}$

$|1\ 1\rangle = \sum pppn$ (exper.? dne)

$|1\ 0\rangle = \sum ppnn$

$|1\ -1\rangle = \sum pnnn$ (exper.? dne)

$|2\ 2\rangle = |pppp\rangle$

$|2\ 1\rangle = \sum pppn$ (exper.? dne)

$|2\ 0\rangle = \sum ppnn$

$|2\ -1\rangle = \sum pppn$ (exper.? dne)

$|2\ -2\rangle = |nnnn\rangle$

Heavy Mesons : $\boxed{M \approx m_1 + m_2 + E_n/c^2}$

* $V \approx -\frac{k}{r} + r\,\boxed{F_0}_{\,const}$ (estimate)

Light Mesons (u,d,s)
- $l=0,\ s=0$ (pseudoscalar)
- $l=0,\ s=1$ (vector)

* (pseudoscalar meson is "good SU(3) symmetry")

Meson Spin ψ $(2\otimes 2 = 3\oplus 1)$:

$\boxed{\text{Symmetric (vec)}}$

$|1\ 1\rangle = (\uparrow\uparrow)$

$|1\ 0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$

$|1\ -1\rangle = (\downarrow\downarrow)$

$\boxed{\text{Antisymmetric (pseu)}}$

$|0\ 0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$

Meson Color $\psi^{sing.}_{\cancel{antisym.}}$: $\boxed{\frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})}$

Meson Flavour ψ $(3\otimes 3 = 8\oplus 1)$:

$\boxed{\cancel{\text{Symmetric}}}$

$|1\ 1\rangle_{\pi,\rho} = -u\bar{d}$

$|1\ 0\rangle_{\pi^0,\rho^0} = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$

$|1\ -1\rangle_{\pi,\rho} = d\bar{u}$

$SU(2)$ isospin singlet $\rightarrow$

$\boxed{\cancel{\text{Antisymmetric}}}$

$|0\ 0\rangle_\omega = \frac{1}{\sqrt{2}}(\underline{u\bar{u} + d\bar{d}})$

$|0\ 0\rangle_\eta = \frac{1}{\sqrt{6}}(\underline{u\bar{u} + d\bar{d}} - 2s\bar{s})$

$|0\ 0\rangle_{\eta'} = \frac{1}{\sqrt{3}}(\underline{u\bar{u} + d\bar{d}} + s\bar{s})$

$|0\ 0\rangle_\phi = \frac{1}{\sqrt{2}}(s\bar{s})$

Meson Ground State (symm.) $\Rightarrow$ $\boxed{\psi_{spin}\psi_{flavor} = \psi_{antisym.}}$

- $\boxed{M \approx m_1 + m_2 + A_{exper}\frac{S_1\cdot S_2}{m_1 m_2}}$

* $M_\eta \approx \frac{M_{u\bar{u}} + M_{d\bar{d}}}{6} + \frac{2M_{s\bar{s}}}{3}$ * $M_{\eta'} =$ bad

- $\langle u\uparrow|u\uparrow\rangle = 1,\ \langle u\uparrow|u\downarrow\rangle = 0$
- Magnetic Moment : $\boxed{\mu_x = g\frac{qL}{2m} = \frac{q\hbar}{2m} = -\mu_{\bar{x}}}$

$$\boxed{\begin{aligned} \mu_{x\bar{y}} &= \langle\psi|(\mu_1+\mu_2)_z\,|x\bar{y}\otimes jm\rangle \\ &= \langle\psi|\mu_1 S_1 + \mu_2 S_2\,|x\bar{y}\otimes jm\rangle/\tfrac{\hbar}{2} \end{aligned}}$$

Example : $\psi_{\rho^+} = (-u\bar{d})(\uparrow\uparrow) = (u\uparrow -\bar{d}\uparrow)$

- $[\mu_1 S_1 + \mu_2 S_2]\,|u\uparrow\rangle_1|{-\bar{d}}\uparrow\rangle_2/\frac{\hbar}{2}$

$= [\mu_u + \mu_{\bar{d}}]\,|\rho^+\uparrow\rangle = \underline{[\mu_u - \mu_d]}|\rho^+\uparrow\rangle$

- $\mu|\rho^-\uparrow\rangle = \mu_{\rho^-}|\rho^-\uparrow\rangle = -\underline{\mu_{\rho^+}}|\rho^-\uparrow\rangle$

Baryon Spin ψ $(2\otimes 2\otimes 2 = 4\oplus 2\oplus 2)$: $\boxed{\text{1-3 Anti.}}$ $|\ \rangle_{13} = |\ \rangle_{12} + |\ \rangle_{23}$, $\boxed{s_1 + s_3 = 0}$

$\boxed{\text{Fully Symmetric}}$, $\boxed{\forall ij,\ s_i + s_j = 1}$

$|\tfrac{3}{2}\tfrac{3}{2}\rangle = (\uparrow\uparrow\uparrow)$

$|\tfrac{3}{2}\tfrac{1}{2}\rangle = \tfrac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \uparrow\downarrow\downarrow)$

$|\tfrac{3}{2}\tfrac{-1}{2}\rangle = \tfrac{1}{\sqrt{3}}(\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow + \uparrow\downarrow\downarrow)$

$|\tfrac{3}{2}\tfrac{-3}{2}\rangle = (\downarrow\downarrow\downarrow)$

$\boxed{\text{1-2 Antisymmetric}}$ $\boxed{s_1 + s_2 = 0}$

$\boxed{|\tfrac{1}{2}\tfrac{1}{2}\rangle = \tfrac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\uparrow}$

$|\tfrac{1}{2}\tfrac{-1}{2}\rangle = \tfrac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow$

$\boxed{\text{2-3 Antisymmetric}}$ $\boxed{s_2 + s_3 = 0}$

$|\tfrac{1}{2}\tfrac{1}{2}\rangle = \tfrac{1}{\sqrt{2}}\uparrow(\uparrow\downarrow - \downarrow\uparrow)$

$|\tfrac{1}{2}\tfrac{-1}{2}\rangle = \tfrac{1}{\sqrt{2}}\downarrow(\uparrow\downarrow - \downarrow\uparrow)$

- $\langle\psi_{12}|\psi_{23}\rangle = -\tfrac{1}{2}$
- $\langle\psi_{23}|\psi_{31}\rangle = \tfrac{1}{2}$
- $\langle\psi_{31}|\psi_{12}\rangle = \tfrac{1}{2}$

Baryon Flavor ψ $(3\otimes 3\otimes 3 = 10\oplus 8\oplus 8\oplus 1)$: $\boxed{\text{Fully Antisymmetric}}$ $\psi_A = \frac{1}{\sqrt{6}}(uds - usd + dsu - dus + sud - sdu)$

$\boxed{\text{1-2-3 Fully Symmetric}}$

ddd $\qquad \frac{1}{\sqrt{3}}(ddu + dud + udd)$ $\qquad \frac{1}{\sqrt{3}}(uud + udu + duu)$ $\qquad uuu$

$\frac{1}{\sqrt{3}}(dds + dsd + sdd)$ $\qquad \frac{1}{\sqrt{6}}(uds + usd + dus + dsu + sud + sdu)$ $\qquad \frac{1}{\sqrt{3}}(uus + usu + suu)$

$\frac{1}{\sqrt{3}}(dss + sds + ssd)$ $\qquad \frac{1}{\sqrt{3}}(uss + sus + ssu)$

uuu

$\boxed{\text{1-2 Antisymmetric}}$ $\boxed{*\ T_{23}\psi_{12} = \psi_{23} \quad *\ T_{31}\psi_{12} = -\psi_{23}}$

$|\tfrac{1}{2}-\tfrac{1}{2}\rangle = \frac{1}{\sqrt{2}}(ud - du)d$ $\qquad \boxed{p = |\tfrac{1}{2}\tfrac{1}{2}\rangle = \frac{1}{\sqrt{2}}(ud - du)u}$

$|1-1\rangle = \frac{(ds-sd)d}{\sqrt{2}}$ $\qquad |10\rangle = \Sigma^0 = \frac{1}{2}[(us - su)d + (ds - sd)u]$ $\qquad |11\rangle = \frac{(us-su)u}{\sqrt{2}}$

$\frac{1}{\sqrt{12}}[2(ud - du)s + (us - su)d - (ds - sd)u]$

$|\tfrac{1}{2}-\tfrac{1}{2}\rangle = \frac{1}{\sqrt{2}}(ds - sd)s$ $\qquad |\tfrac{1}{2}\tfrac{1}{2}\rangle = \frac{1}{\sqrt{2}}(us - su)s$

$\boxed{\text{2-3 Antisymmetric}}$ $\boxed{*\ T_{12}\psi_{23} = \psi_{31} \quad *\ T_{31}\psi_{23} = -\psi_{12}}$

$|\tfrac{1}{2}-\tfrac{1}{2}\rangle = \frac{1}{\sqrt{2}}d(ud - du)$ $\qquad |\tfrac{1}{2}\tfrac{1}{2}\rangle = \frac{1}{\sqrt{2}}u(ud - du)$

$|1-1\rangle = \frac{d(ds-sd)}{\sqrt{2}}$ $\qquad |10\rangle = \Sigma^0 = \frac{1}{2}[d(us - su) + u(ds - sd)]$ $\qquad |11\rangle = \frac{u(us-su)}{\sqrt{2}}$

$\frac{1}{\sqrt{12}}[2s(ud - du) + d(us - su) - u(ds - sd)]$

$|\tfrac{1}{2}-\tfrac{1}{2}\rangle = \frac{1}{\sqrt{2}}s(ds - sd)$ $\qquad |\tfrac{1}{2}\tfrac{1}{2}\rangle = \frac{1}{\sqrt{2}}s(us - su)$

- $1\cdot n^0_{23} = I_- p^+_{23} = \frac{1}{\sqrt{2}}[I_-uud - I_-udu + uI_-ud - uI_-du + uuI_-d - udI_-u] = \frac{1}{\sqrt{2}}\left[dud - ddu + \cancel{udd - u0u + uu0 - udd}\right]$
- $\sqrt{2}\Sigma^0_{23} = I_-\Sigma^+_{23} = \frac{1}{\sqrt{2}}[I_-uus - I_-usu + uI_-us - uI_-su + uuI_-s - usI_-u] = \frac{1}{\sqrt{2}}[dus - dsu + uds - \cancel{u0u + uu0} - usd]$
  - $|00\rangle = \Lambda^0_{23} = Au[ds - sd] + Bd[us - su] + Cs[ud - du]$, $(\Lambda\cdot\Sigma^0 = 0)$, $(\Lambda\cdot\psi_A = 0) \Rightarrow (A, B, C) = \frac{(-1,1,2)}{\sqrt{12}}$

$\boxed{\text{1-3 Anti.}}$ $\psi_{13} = \psi_{12} + \psi_{23}$ $\qquad$ Baryon Color ψ : $\boxed{\psi(\text{Color})_{antisymm.\ singlet} = \frac{1}{\sqrt{6}}(rgb - rbg + gbr - grb + brg - bgr)}$

Baryon Ground state (symm.) $\Rightarrow$ $\boxed{\psi_{spin}\psi_{flavour} = \psi_{symm.}}$ $*$ Decuplet : $\boxed{\psi_{symm.} = |\tfrac{3}{2}\,x\rangle\,\psi_{123}}$, $\cancel{\psi_{anti.} = |\tfrac{3}{2}\,x\rangle\,\psi_A}$

$*$ Octet : $\boxed{\begin{aligned}\psi_{symm.} &= \tfrac{\sqrt{2}}{3}\big(\,|\ \rangle_{12}\psi_{12} + |\ \rangle_{23}\psi_{23} + |\ \rangle_{13}\psi_{13}\,\big)\\ &= \tfrac{\sqrt{2}}{3}\big(\,2|\ \rangle_{12}\psi_{12} + 2|\ \rangle_{23}\psi_{23} + |\ \rangle_{12}\psi_{23} + |\ \rangle_{23}\psi_{12}\,\big)\end{aligned}}$, $\cancel{\psi_{anti.} = |\ \rangle_{12}(\psi_{23} + \psi_{31}) + \ldots \quad *\ (\mu(\psi_p^{anti.}) < 1;\ \mu(\psi_p^{symm.}) > 1)}$

- $\langle u\uparrow|u\uparrow\rangle = 1,\ \langle u\uparrow|u\downarrow\rangle = 0$
- Magnetic Moment : $\boxed{\begin{aligned}\mu_{xyz} &= \langle\psi|\,(\mu_1 + \mu_2 + \mu_3)_z\,|xyz\otimes jm\rangle\\ &= \langle\psi|\,\mu_1 S_1 + \mu_2 S_2 + \mu_3 S_3\,|xyz\otimes jm\rangle/\tfrac{\hbar}{2}\end{aligned}}$

  $\boxed{\mu = g\frac{qL}{2m} = \frac{q\hbar}{2m}}$

Example : $(udu)(\uparrow\uparrow\downarrow) = (u\uparrow d\uparrow u\downarrow)$

- $[\mu_1 S_1 + \mu_2 S_2 + \mu_3 S_3]\,|u\uparrow\rangle_1 |d\uparrow\rangle_2 |u\downarrow\rangle_3$

  $= [\mu_u + \mu_d - \mu_u]\,|u\uparrow\rangle_1 |d\uparrow\rangle_2 |u\downarrow\rangle_3/\tfrac{\hbar}{2}$

- Mass : $\boxed{M \approx m_1 + m_2 + m_3 + A'_{exper.}\left[\frac{S_1\cdot S_2}{m_1 m_2} + \frac{S_2\cdot S_3}{m_2 m_3} + \frac{S_1\cdot S_3}{m_1 m_3}\right]}$ $\quad * \ \boxed{m_u \approx m_d}$ (!!!!!)

* $(xxx), [m_u \approx m_d \Rightarrow \Delta]: \ J^2 = \frac{15}{4}\hbar^2 = S_1^2 + S_2^2 + S_3^2 + 2(\underline{S_1\cdot S_2 + S_2\cdot S_3 + S_3\cdot S_1})$

* $(\psi_{123}) \Rightarrow \forall ij, \ s_i + s_j = 1: \ (S_i + S_j)^2 = 2\hbar^2 = S_i^2 + S_j^2 + 2\underline{S_i\cdot S_j}$

* $\Sigma \in |1x\rangle^{symm.}_{iso.} \Rightarrow |\tfrac{3}{2}x\rangle^{symm.}_{spin}: \ (S_u + S_d)^2 = 2\hbar^2 = S_u^2 + S_d^2 + 2\underline{S_u\cdot S_d}; \ \underline{S_u\cdot S_3 + S_d + S_3} = \sum S_i\cdot S_j - S_u\cdot S_d$

* $\Lambda \in |00\rangle^{antisymm.}_{iso.} \Rightarrow |\tfrac{1}{2}x\rangle^{antisymm.}_{spin}: \ (S_u + S_d)^2 = 0 = S_u^2 + S_d^2 + 2\underline{S_u\cdot S_d}: \ \underline{S_u\cdot S_3 + S_d + S_3} = \ldots$

* $\Xi_{s-s \text{ symm.}} \in (ssx) \Rightarrow |1x\rangle^{symm.}_{ss\,spin}: \ (S_{s1} + S_{s2})^2 = 2\hbar^2 = \ldots; \ J^2 = \cancel{\tfrac{3}{4}\hbar^2} = (S_{s1} + S_{s2})^2 + \underline{2(S_{s1} + S_{s2})\cdot S_x} + \cancel{S_x^2}$

Parity, $P^2 = 1$, $\boxed{\text{Conserved under Strong+EM = Invariant under Reflections}}$

- $P_{\text{Fermion}} \neq P_{\overline{\text{Fermion}}}$ $\quad$ • $P_{\text{Quark}} = 1 \neq -1 = P_{\overline{\text{Quark}}}$

* $(xyz): \ \boxed{p = (+1)^3 = 1}$ $\quad * \ (x\bar{y}): \ p = (+1)(-1) * \underline{(-1)^l} = \boxed{(-1)^{l+1}}$ (l orb. ang. mom.; l=0 for 1,2 nonet)

- $P_{\text{Boson}} = P_{\overline{\text{Boson}}}$ $\quad$ • $A \to B + C$ $\quad$ • ν- "Always" Left-Handed $\quad * \ m_\nu \neq 0 \ \to \ \exists$ right handed ν

* $\boxed{\gamma: \ s = 1, \ p = -1, \ \text{vec.}}$ $\quad \boxed{p_a = p_b \cdot p_c \cdot (-1)^l}$ $\quad$ • $\bar{\nu}$- "Always" Right-Handed

"Charge" Conjugation (charge, quantum num., not spin/mass/etc.) : $C|p\rangle = |\bar{p}\rangle$, $\boxed{\text{Conserved under Strong+EM}}$

- *Eigenstate* : $C^2 = 1 \ \Rightarrow \ C|q\rangle = \pm|q\rangle = |\bar{q}\rangle \ \Rightarrow \ \langle q|q\rangle = \langle \bar{q}|\bar{q}\rangle$

* $\gamma: \ \boxed{c = -1}$ $\quad * \ |\tfrac{1}{2}\rangle \otimes |\overline{\tfrac{1}{2}}\rangle: \ \boxed{c = (-1)^{l+s}} \ \Rightarrow$ Central Meson $(x\bar{x}): \ l = 0$, $s = 0$ (sca.), $\boxed{c = +1}$; $= 1$ (vec.), $\boxed{c = -1}$

G-Parity : $G = CR_2, \ R_2 = e^{i\pi I_2}$: "$R_2|I_3\rangle = |-I_3\rangle$" e.g. "$G|\pi^+\rangle = C|\pi^-\rangle = |\pi^+\rangle$" (not exactly)

- Isospin+Charge $\to$ $\boxed{\text{Conserved under Strong}}$ $\quad * \ G|I \ I_3\rangle_{\text{no S meson}} \overset{??}{=} C\underline{R_2|I\ 0\rangle} \overset{??}{=} C\underline{(-1)^I|I\ 0\rangle} = (-1)^I c_0$ $\quad$ (??)

- *Eigenstate* : $\underline{x, y \in \{u, d, \bar{u}, \bar{d}\} \ (\pi \text{ or } \eta = d\bar{d})} \ \Rightarrow \ G(x\bar{y}) = (x\bar{y})$, $\boxed{g = (-1)^I c_0} \ \to \ \underline{g_\pi = (-1)^1 * 1 = -1}$

$\underline{K^0 \ \leftrightarrows \ \overline{K}^0}$ $\quad$ $\underline{A \leftrightarrows B \ \text{ Req:}}$ $\boxed{\bullet\ m_a = m_a \ (B = \overline{A}) \quad \bullet\ B(ary)_a = B_b \ (A = x\bar{y}) \quad \bullet\ Q_a = Q_b \ (Q_x + Q_{\bar{y}} = 0) \quad * \ \text{note, heavy vec. mes. decay too fast}}$

- $CP|K^0\rangle = -|\overline{K}^0\rangle$, $CP|\overline{K}^0\rangle = -|K^0\rangle$ $\Rightarrow$ $|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K}^0\rangle\right)$, $|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K}^0\rangle\right)$ $\Rightarrow$ $CP|K_1\rangle = |K_1\rangle$, $CP|K_2\rangle = -|K_2\rangle$ $\Rightarrow$

(cp=1)	$\lvert K_1\rangle \to 2\pi$	(3π poss. if $l \neq 0, \pi^+\pi^-\pi^0$)
(cp=-1)	$\lvert K_2\rangle \to 3\pi$	(decays $> 2\pi$; $\underline{\cancel{CP} \to \exists 2\pi}$)

* $|K^0\rangle = \frac{1}{\sqrt{2}}\left(|K_1\rangle + |K_2\rangle\right)$ $\quad * \ C|K_{1,2}\rangle = -|K_{1,2}\rangle$

- $\cancel{CP}$ for K_2 $\underline{\text{(1964, Fitch-Cronin)}}$ $\to$ $\boxed{|K_L\rangle \equiv \frac{1}{\sqrt{1+|\epsilon|^2}}\left(|K_2\rangle + \epsilon|K_1\rangle\right)}$ $\quad$ * CKM matrix accounts for CP viol.; predicts 3 gen.

* $\underline{|K_L\rangle \to \pi^+ + e^- + \bar{\nu}_e \ \text{ very slightly more likely than } \ \pi^- + e^+ + \nu_e}$ (also true for B^0-system)

Strong Decay of $(xyz)_{10} \to (abc)_9 + (uv)_9$

* $\boxed{\text{decays in} \sim 10^{-23}\ \text{s}}$

* $\boxed{\text{Conserves } C \text{ and } S}$

- $\Delta^{++}_{uuu} \to p^+_{uud} + \pi^+_{\bar{d}u}$
- $\Delta^-_{ddd} \to n^0_{ddu} + \pi^-_{\bar{u}d}$
  $\to \cancel{\Sigma^-_{dds} + K^0_{\bar{s}d}}$

- $\Sigma^{*+}_{uus} \to \pi^0_{u\bar{u}} + \Sigma^+_{uus}$ / $\pi^+_{u\bar{d}} + \underline{\Sigma^0_{dus}}$ / $\pi^+_{u\bar{d}} + \underline{\Lambda^0_{dus}}$
  $\to K^+_{u\bar{s}} + \cancel{\Xi^0_{sus} / p^+_{uud} + \overline{K}^0_{\bar{d}s}}$ / $\Sigma^+_{uus} + \eta'_{\bar{s}s}$
- $\Xi^{*-}_{dss} \to \pi^-_{d\bar{u}} + \Xi^0_{uss}$ / $\cancel{\eta^0_{d\bar{d}}(?) + \Xi^-_{dss}}$ / $\cancel{\underline{\Lambda^0_{dsu}} + K^-_{\bar{u}s}}$
  $\to \underline{\Sigma^0_{dsu}} + \cancel{K^-_{\bar{u}s} / \Sigma^-_{dsd} + \overline{K}^0_{\bar{d}s}}$ / $\Xi^-_{dss} + \eta'_{\bar{s}s}$

Elastic		Inelastic	
Ruther. Scatter., $v \ll c$:	$q_1 + q_2 \to q_1 + q_2$	Pair Annhilation :	$e + e^+ \to \gamma + \gamma$
Mott Scattering, $q_2 \gg q_1$:	$q_1 + q_2 \to q_1 + q_2$	Pair Production :	$\gamma + \gamma \to e + e^+$
Møller Scattering :	$e + e \to e + e$	Delbruck Scattering?? :	$\gamma + \gamma \to \gamma + \gamma$
Bhabha Scattering :	$e + e^+ \to e + e^+$		
Compton Scattering :	$\gamma + e \to \gamma + e$		

- $\underline{\text{OZI suppressed + no other opt. = (strong?) Decay} \sim 10^{-20}\text{s}}$

$\phi_{s\bar{s}} \to 2K_{\text{-}s}$ $\underline{\text{(more mass than } 3\pi)}$
$\to 3\pi$ (OZI suppressed)
$\to 2\pi$ (parity viol.)

$\psi_{c\bar{c}} \to 2D_{\text{-}c}$ (more mass than ψ)
$\to 3\pi$ $\underline{\text{(OZI suppressed)}}$
$\to 2\pi$ (parity viol.)

$\Upsilon_{b\bar{b}} \to 2B_{\text{-}b}$ (more mass than Υ)
$\to 3\pi$ $\underline{\text{(OZI suppressed)}}$
$\to 2\pi$ (parity viol.)

- $n^{2s+1}l_j$ Charmonium
  - S states, $l = 0$: $(s = 0 \to \eta_c)$, $(s = 1 \to \phi_c)$
  - P states, $l = 1$: $(s = 0, 1, 2 \to \chi_c)$
  - $n = 1, 2$ long-lived (OZI)
  - $n \geq 3$ ("quasi-bound") short-lived ($>$ OZI threshold)

Bottomium
- similar to others

Particle [Mean] Lifetimes, τ : $\boxed{\frac{N(t)}{N(0)} = e^{-\frac{t}{\tau}}}$

$\tau_\mu = 2.2\text{E-6 s}$
$\tau_{\pi^-} = 2.6\text{E-8 s}$

- Decay Rate, $\Gamma = \frac{1}{\tau}$

Fermi's Golden Rule, Decay $(m_1 \to m_2 + ... + m_n)$:

$$\begin{aligned}\Gamma &= \frac{S}{2\hbar m_1} \int |M|^2 \cdot (2\pi)^4 \delta^4(P_1 - [P_2 + ...P_n]) \times \prod_{j=2}^{n} 2\pi\delta(P_j^2 - m_j^2c^2) \cdot \theta(P_j^0) \cdot \frac{d^4 P_j}{(2\pi)^4} \\ &= \frac{S}{2\hbar m_1} \int |M|^2 \cdot (2\pi)^4 \delta^4(P_1 - [P_2 + ...P_n]) \times \prod_{j=2}^{n} \frac{1}{2\sqrt{p_j^2 + m^2c^2}} \cdot \frac{d^3 p_j}{(2\pi)^3}\end{aligned}$$

$*$ $(m_1 \to m_2 + m_3)$: $\boxed{\Gamma = \frac{1}{16\pi^2} \frac{S}{2\hbar m_1} \int |M|^2 \cdot \frac{\delta^4(P_1 - [P_2 + P_3])}{\sqrt{p_2^2 + m_2^2c^2}\sqrt{p_3^2 + m_3^2c^2}} \cdot d^3p_2 \, d^3p_3}$

$*$ $(m_1 \to m_2 + m_3)$ $(p_1 = 0)$: $\boxed{\Gamma = \frac{S|p|}{8\pi\hbar m_1^2 c}|M|^2 \quad , \quad |p| = |p_2| = |p_3|}$ (see scattering examples)

Differential (Scattering) Cross Section :

$\boxed{\frac{d\sigma}{d\Omega} = \left| \frac{b}{\sin\theta} \frac{db}{d\theta} \right|}$, $(\phi + \Delta\phi)(b + \Delta b)^2 - \phi b^2 = \Delta\sigma$, $b\Big|$ $/\theta$

- Barn : $1 \text{ b} = \text{E-24 cm}^2 = \text{E-28 M}^2$

Luminosity, L : $\boxed{L\frac{d\sigma}{d\Omega} = \frac{dN}{d\Omega}}$

Scattering Example : $V = k/r^2$

$\boxed{\theta = \pi - \phi}$, $L = bmv_0 = mr^2\dot{\phi}$,

$E = \frac{1}{2}mv_0^2 = \frac{1}{2}m\dot{r}^2 + \frac{L^2}{2mr^2} + \frac{k}{r^2}$

$\dot{r} = \frac{dr}{d\phi}\dot{\phi}$, $u \equiv \frac{1}{r}$ $\Rightarrow$ $\frac{dr}{d\phi} = \frac{dr}{du}\dot{u}$

$$\frac{1}{2}m\dot{r}^2 = E - \frac{L^2}{2mr^2} - \frac{k}{r^2} \;\Rightarrow\; r_{m[in]} = \sqrt{\frac{L^2/2m + k}{E}}$$

$$\sqrt{\frac{1}{2}m}\frac{dr}{d\phi}L = mr^2\sqrt{E - \frac{Er_m^2}{r^2}} \;\Rightarrow\; \sqrt{\frac{1}{2}m}\left|\cancel{\frac{-1}{u^2}}\right|\frac{du}{d\phi}L = m\cancel{\frac{1}{u^2}}\sqrt{E - Er_m^2u^2}$$

$$\sqrt{2mE} \cdot \phi = 2L\int_0^{u_m} \frac{u_m du}{\sqrt{u_m^2 - u^2}} = 2Lu_m\frac{\pi}{2} \;\Rightarrow\; \phi = \frac{\pi L u_m}{\sqrt{2mE}} = \boxed{\frac{\pi L}{\sqrt{L^2 + 2mk}}}$$

Fermi's Golden Rule, 2 Scattering $(m_1 + m_2 \to m_3 + ... + m_n)$:

$$\sigma = \frac{S\hbar^2}{4\sqrt{(P_1 \cdot P_2)^2 - (m_1c)^2(m_2c)^2}} \int |M|^2 \cdot (2\pi)^4 \delta^4(P_1 + P_2 - [P_3 + ...P_n]) \times \prod_{j=3}^{n} 2\pi\delta(P_j^2 - m_j^2c^2) \cdot \theta(P_j^0) \cdot \frac{d^4P_j}{(2\pi)^4}$$

$$= \frac{S\hbar^2}{4\sqrt{(P_1 \cdot P_2)^2 - (m_1c)^2(m_2c)^2}} \int |M|^2 \cdot (2\pi)^4 \delta^4(P_1 + P_2 - [P_3 + ...P_n]) \times \prod_{j=3}^{n} \frac{1}{2\sqrt{p_j^2 + m_j^2c^2}} \cdot \frac{d^3p_j}{(2\pi)^3}$$

$*$ $(m_1 + m_2 \to m_3 + m_4)$ $(p_1 + p_2 = 0)$:

$$\sigma = \frac{1}{16\pi^2} \frac{S\hbar^2 c}{4(E_1 + E_2)|p_1|} \int |M|^2 \cdot \frac{\delta^4(P_1 + P_2 - [P_3 + P_4])}{\sqrt{p_3^2 + m_3^2c^2}\sqrt{p_4^2 + m_4^2c^2}} \cdot d^3p_3\, d^3p_4$$

$$= A \iint_0^\infty |M|^2 \cdot \frac{\delta\big([E_1 + E_2]/c - \boxed{u(p_3)}\,\big)\cancel{\delta^3(p_3 + p_4)}}{\sqrt{p_3^2 + m_3^2c^2}\sqrt{p_{\cancel{4}3}^2 + m_4^2c^2}} \cdot \cancel{d^3p_4}\,\cancel{d^3p_3}\, p_3^2\, d|p_3|\, d\Omega$$

$$= A \iint_{m_3c + m_4c}^\infty |M|^2 \cdot \delta([E_1 + E_2]/c - u) \frac{p_3}{u}\, du\, d\Omega$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S|M|^2}{(E_1 + E_2)^2} \frac{|p_f|}{|p_i|} \quad , \quad |p_f| = |p_3| = |p_4|,\ |p_i| = |p_1| = |p_2| \quad E_f = E_i$$

$*$ $(m_1 + m_2 \to m_3 + m_4)$ $(p_2 = 0)$ $(m_3 = 0,\ m_4 = 0)$:

$$\sigma = \frac{1}{16\pi^2} \frac{S\hbar^2}{4m_2|p_1|c} \int |M|^2 \cdot \frac{\delta^4(P_1 + P_2 - [P_3 + P_4])}{\sqrt{p_3^2}\sqrt{p_4^2}} \cdot d^3p_3\, d^3p_4$$

$$= A \int |M|^2 \frac{\delta\big(E_1/c + m_2c - |p_3| - |p_1 - p_3|\big)\cancel{\delta^3(p_1 - p_3 - p_4)}}{|p_3|\sqrt{p_1^2 + p_3^2 - 2|p_1||p_3|\cos\phi_{p_3p_3}}} \cdot \cancel{d^3p_3}\,\cancel{d^3p_4}\, p_3^2\, d|p_3|\, d\Omega$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar}{8\pi}\right)^2 \frac{S|M|^2}{m_2|p_1|c} \cdot \frac{|p_3|}{|p_1 - p_3|} \frac{1}{\left|1 + \frac{2|p_3| - 2|p_1|\cos\phi}{2|p_1 - p_3|}\right|}$$

$$= \left(\frac{\hbar}{8\pi}\right)^2 \frac{S|M|^2}{m_2|p_1|} \frac{|p_3|}{E_1 + m_2c^2 - |p_1|c\cos\phi} \quad , \quad |p_3| : (E_1 + m_2 - |p_3|)^2 = p_1^2 + p_3^2 - |p_1||p_3|\cos\phi$$

$(m_1 + m_2 \to m_3 + m_4)$

$(p_2 = 0)$

$(m_3 = m_1,\ m_4 = m_2)$

$*$ $\begin{vmatrix} p_1 & 0 \\ p_4 & p_3 \end{vmatrix} = \begin{vmatrix} E_1 & m_2 \\ E_4 & E_3 \end{vmatrix} + m_2^2 - m_1^2$:

For $(m_1 = 0)$

$\begin{vmatrix} \vec{p_1} & 0 \\ \vec{p_4} & \vec{p_3} \end{vmatrix} = \begin{vmatrix} |p_1| & m_2 \\ E_4 & |p_3| \end{vmatrix} + m_2^2$

$$\sigma = \frac{1}{16\pi^2} \frac{S\hbar^2}{4m_2|p_1|c} \int |M|^2 \frac{\delta^4(P_1 + P_2 - [P_3 + P_4])}{\sqrt{p_3^2 + m_1^2c^2}\sqrt{p_4^2 + m_2^2c^2}} \cdot d^3p_3\, d^3p_4$$

$$= A \int |M|^2 \frac{\delta\Big(E_1/c + m_2c - E_3/c - \sqrt{(p_1 - p_3)^2 + m_2^2}\Big)\cancel{\delta^3(p_1 - p_3 - p_4)\, d^3p_3}}{\sqrt{p_3^2 + m_1^2c^2}\sqrt{(p_1 - p_3)^2 + m_2^2c^2}} p_3^2\, d|p_3|\, d\Omega$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar}{8\pi}\right)^2 \frac{S|M|^2}{m_2|p_1|c} \cdot \frac{p_3^2c^2}{E_3E_4} \cdot \left|\frac{2|p_3|c}{2E_3} + \frac{2|p_3| - 2|p_1|\cos\phi_{13}}{2E_4/c}\right|^{-1}$$

$$= \left(\frac{\hbar}{8\pi}\right)^2 \frac{S|M|^2}{m_2|p_1|} \cdot \frac{p_3^2}{\Big[\cancel{|p_3|E_4 + |p_3|E_3} = |p_3|(E_1 + m_2c^2)\Big] - |p_1|E_3\cos\phi}$$

$$\frac{d\sigma}{d\Omega}_{m_1 = 0} = \left(\frac{\hbar}{8\pi}\right)^2 \frac{S|M|^2}{m_2E_1} \cdot \cancel{\frac{E_3}{E_1 + m_2c^2 - E_1\cos\phi}} \frac{E_3^2}{E_1m_2c^2}$$

$*$ $(m_1 + m_2 \to m_1 + m_2)$

$*$ $(p_2 = 0)$ $(p_4 \approx 0)$:

$$\sigma = \frac{1}{16\pi^2} \frac{S\hbar^2}{4m_2|p_1|c} \int |M|^2 \cdot \frac{\delta^4(P_1 + P_2 - [P_3 + P_4])}{\sqrt{p_3^2 + m_1^2c^2}\sqrt{p_4^2 + m_2^2c^2}} \cdot d^3p_3\, d^3p_4$$

$$= A \int |M|^2 \cdot \frac{\delta\Big(E_1/c + m_2c - E_3/c - \sqrt{(p_1 - p_3)^2 + m_2^2}\Big)\cancel{\delta^3(p_1 - p_3 - p_4)}\cancel{d^3p_4}}{\sqrt{p_3^2 + m_1^2c^2}\sqrt{(p_1 - p_3)^2_{\ll m_2c} + m_2^2c^2}} \cdot p_3^2 d|p_3| d\Omega$$

$$\frac{d\sigma}{d\Omega} \approx \left(\frac{\hbar}{8\pi}\right)^2 \frac{S|M|^2}{m_2^2c^2} \cancel{\frac{|p_3|}{|p_1|}} \frac{|p_3|c}{E_3} \left|\frac{2|p_3|c}{2|E_3|} + \cancel{\frac{2|p_3| + 2|p_1|\cos\theta}{2E_4/c_{\gg|p_1}}}\right|^{-1} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S|M|^2}{(\cancel{E_1} + E_2)^2} \cancel{\frac{|p_f|}{|p_i|}}$$

<u>Feynman Diagrams, Amplitude : $|M|^2$</u>

Dim M : $(mc)^{4-n}$

$(A = B + C)$

$* \ A + A = B_3 + B_4$

$* \ A + A = B_4 + B_3$:

$(m_b = m_c = 0)^*$

$\begin{vmatrix} p_1 & p_2 \\ p_4 & p_3 \end{vmatrix} = \begin{vmatrix} E_1 & E_2 \\ |p_4| & |p_3| \end{vmatrix}$

$$M_{34} = \frac{i(-ig)^2}{(2\pi)^4\delta^4(P\ldots)} \int \frac{i}{q_c^2 - m_c^2} \cdot \frac{(2\pi)^4\delta^4(P_1 + q_c - P_3) \cdot \frac{d^4 q_c}{(2\pi)^4}}{(2\pi)^4\delta^4(P_2 - q_c - P_4)}$$

$$M = M_{34} + M_{43} = \frac{g^2}{(P_4 - P_2)^2} + \frac{g^2}{(P_3 - P_2)^2} = \frac{g^2}{(P_1 - P_3)^2} + \frac{g^2}{(P_3 - P_2)^2}$$

$$\frac{d\sigma}{d\Omega}_{cm^*} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S|M|^2}{(2E_1)^2} \frac{|p_f|}{|p_i|}, \ M = \frac{2g^2(m_a^2 - 2E_i^2)}{(m_a^2 - 2E_i^2)^2 - (2|p_i|E_i\cos\phi)^2}$$

$$\frac{d\sigma}{d\Omega}_{lab^*} = \left(\frac{\hbar}{8\pi}\right)^2 \frac{S|M|^2|p_3|}{m_a|p_1|\left(E_1 + m_a c^2 - |p_1|c\cos\phi\right)}, \ M = \frac{g^2}{m_a^2 - 2[E_4 m_a = E_3(E_1 - |p_1|\cos\phi_{13})]} + \frac{g^2}{m_a^2 - 2E_3 m_a}$$

$$\text{(nonrel.)} \approx \left(\frac{\hbar}{8\pi}\right)^2 \frac{S|M|^2 m_a c}{m_a|m_a v_1|\left(m_a c^2 + m_a c^2\right)}, \ M = \frac{2g^2}{m_a^2 - 2m_a c^2 \cdot m_a}$$

$(A = B + C)$

$* \ A + B = B + A$

$* \ A + B = B + A$:

$(cm, \ m_a = m_b, \ m_c = 0)^*$

$(lab, \ m_b \gg m_a)^*$

$$M_{c|} = \frac{i(-ig)^2}{(2\pi)^4\cancel{\delta^4(\ldots)}} \frac{i}{q_c^2 - m_c^2} \cdot \frac{(2\pi)^{4*2}}{(2\pi)^4} \qquad (P_{A1} + q_c = P_{B2}; \ P_{B1} - q_c = P_{A2})$$

$$M_{c-} = \frac{i(-ig)^2}{(2\pi)^4} \frac{i}{q_c^2 - m_c^2} \cdot \frac{(2\pi)^{4*2}}{(2\pi)^4} \qquad (P_{A1} + P_{B1} = q_c = P_{B2} + P_{A2})$$

$$M = M_{c|} + M_{c-} = \frac{g^2}{(P_3 - P_1)^2} + \frac{g^2}{(P_2 + P_1)^2}$$

$$\frac{d\sigma}{d\Omega}_{cm^*} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S|M|^2}{(E_1 + E_2)^2} \frac{|p_f|}{|p_i|}, \ M = \frac{g^2}{-2p_1^2 - 2p_1^2\cos\phi} + \frac{g^2}{4E_1^2}$$

$$\frac{d\sigma}{d\Omega}_{lab^*} \approx \left(\frac{\hbar}{8\pi}\right)^2 \frac{S|M|^2}{m_2^2 c^2}, \ M \approx \frac{g^2/m_b^2}{m_a^2/m_b^2 + 1 - 2E_1/m_b} + \frac{g^2/m_b^2}{m_a^2/m_b^2 + 1 + 2E_1/m_b} \approx \frac{2g^2}{m_b^2}$$

0.2 Feynman QED

- Incoming Particle : $u, \overline{v}$; e_μ (number)
- $e^{\mp}$ Internal Progagator : $\dfrac{i(\not{q} + m)}{q^2 - m^2}$
- $(2\pi)^4\delta^4$ Vertex Factor : $ig_e\gamma^\mu$ $(g_e = \sqrt{4\pi\alpha})$
- Closed Fermion Loop : $-1 \cdot \mathrm{Tr}(\text{Loop})$
- Outgoing Particle : $\overline{u}, v$; e_μ^*
- γ Internal Progagator : $\dfrac{-ig_{\mu\nu}}{q^2}$
- <u>Antisymmetry of 2-interchangable fermion Diagrams</u>
- <u>Work Backwards in **Time** for **Fermions**</u>

$\boxed{\text{Casimir's Trick}}$: $\sum_{s_1,s_2} \left[\overline{(u/v)}_1\Gamma_1(u/v)_2\right]\left[\overline{(u/v)}_1\Gamma_2(u/v)_2\right]^{*=\dagger} = \mathrm{Tr}\left([\not{P}_1 \pm_v^u m_1]\Gamma_1[\not{P}_2 \pm_v^u m_2]\overline{\Gamma}_2\right) \equiv \boxed{\mathrm{Tr}\left(\not{p}_1^{\pm}\Gamma_1\not{p}_2^{\pm}\overline{\Gamma}_2\right)}$

<u>• $y^T M x = y^\mu M_{\mu\nu} x^\nu = M_{\mu\nu} x^\nu y^\mu = \mathrm{Tr}(Mxy^T)$</u>

• $\mathrm{Tr}(\not{p}_1^{\pm}\gamma^\mu\not{p}_2^{\pm}\gamma^\nu) = P_1 P_2 \mathrm{Tr}(\gamma^1\gamma^\mu\gamma^2\gamma^\nu) + 0(\pm\pm)m_1 m_2 \mathrm{Tr}(\gamma^\mu\gamma^\nu) = \boxed{4(P_1^\mu P_2^\nu - g^{\mu\nu}P_1 \cdot P_2 + P_1^\nu P_2^\mu) + (\pm\pm)4m_1 m_2 g^{\mu\nu}}$

<u>Compton, $e + \gamma \to e + \gamma$</u> : $M = \cancel{i}\cancel{\int}[\overline{u}^{(s_3)}\cancel{i}g_e\not{\epsilon}_4^*]\frac{\cancel{i}(\not{P}_1 + \not{P}_2 + m)}{(q = P_1 + P_2)^2 - m^2}[\not{\epsilon}_2\cancel{i}g_e u^{(s_1)}]\cancel{dq^4} + [\overline{u}^{(s_4)}g_e\not{\epsilon}_2]\frac{\not{P}_1 - \not{P}_3 + m}{(P_1 - P_3)^2 - m^2}[\not{\epsilon}_3^* g_e u^{(s_1)}]$

$\boxed{\text{Electron-Muon, } e+\mu\to e+\mu}$: $M = \cancel{\int}\cancel{\int}[\overline{u}^{(s_3)}\cancel{i}g_e\gamma^\mu u^{(s_1)}]\frac{-\cancel{i}g_{\mu\nu}}{(q=P_3-P_1)^2}[\overline{u}^{(s_4)}\cancel{i}g_e\gamma^\nu u^{(s_2)}]\cancel{dq^4}$

$CM,\ p=p_z,\ \underline{(\hat{p}\cdot\Sigma)u_+^{(s)} = u_+^{(s)}}$

(move back w/ -1/2 spin = helicity 1)

$$M = \frac{-g_e^2N_1N_2N_3N_4}{(P_3-P_1)^2}\begin{bmatrix} e^2 \\ \frac{-\sigma\cdot p_1}{E_e+m}e^2\end{bmatrix}^\dagger\gamma^0\gamma^\mu\begin{bmatrix} e^1 \\ \frac{\sigma\cdot p_1}{E_e+m}e^1\end{bmatrix}\begin{bmatrix} e^1 \\ \frac{\sigma\cdot p_1}{E_\mu+M}e^1\end{bmatrix}^\dagger\gamma^0\gamma_\mu\begin{bmatrix} e^2 \\ \frac{-\sigma\cdot p_1}{E_\mu+M}e^2\end{bmatrix}$$

$$= \frac{g_e^2}{(P_3-P_1)^2}\begin{bmatrix}\sqrt{E_e+m}e^2 \\ \sqrt{E_e-m}e^2\end{bmatrix}^\dagger\begin{bmatrix}0 & \sigma \\ \sigma & 0\end{bmatrix}^\mu\begin{bmatrix}\sqrt{E_e+m}e^1 \\ \sqrt{E_e-m}e^1\end{bmatrix}\begin{bmatrix}\sqrt{E_\mu+M}e^1 \\ \sqrt{E_\mu-M}e^1\end{bmatrix}^\dagger\begin{bmatrix}0 & \sigma \\ \sigma & 0\end{bmatrix}_\mu\begin{bmatrix}\sqrt{E_\mu+M}e^2 \\ \sqrt{E_\mu-M}e^2\end{bmatrix}$$

$$= \frac{g_e^2}{(P_3-P_1)^2}p^2\begin{bmatrix}e^2 \\ e^2\end{bmatrix}\cdot\begin{bmatrix}\sigma e^1 \\ \sigma e^1\end{bmatrix}^\mu\times\begin{bmatrix}e^1 \\ e^1\end{bmatrix}\cdot\begin{bmatrix}\sigma e^2 \\ \sigma e^2\end{bmatrix}_\mu = \frac{8g_e^2p^2}{(0,0,0,2p)^2} = \boxed{-2g_e^2}$$

High E $(p\gg m, M \to m, M=0)$

$P_1\cdot P_2 = P_3\cdot P_4$

$P_1\cdot P_3 = P_2\cdot P_4$

$P_1\cdot P_4 = P_2\cdot P_3$

$$\langle M\rangle^2_{spins} = \frac{8g_e^4}{4(P_1\cdot P_3)^2}\left[(P_1\cdot P_2)^2+(P_1\cdot P_4)^2\right] = 2g_e^4\frac{(2E_1E_2)^2+(E_1E_4\cancel{-p_1\cdot p_4}+p_1\cdot p_3)^2}{(E_1E_3-p_1\cdot p_3)^2}$$

$$= 2g_e^4\frac{4p_1^4+p_1^2p_3^2(1+\cos\theta_{13})^2}{p_1^2p_3^2(1-\cos\theta_{13})^2} = \boxed{\frac{4+(1+\cos\theta_{13})^2}{(1-\cos\theta_{13})^2}2g_e^4}$$

Møller, $2e\to 2e$ (see $e+\mu\to e+\mu$) :

$$M = -\frac{g_e^2}{(P_3-P_1)^2}[\overline{u}_3\gamma^\mu u_1][\overline{u}_4\gamma_\mu u_2] + \frac{g_e^2}{(P_4-P_1)^2}[\overline{u}_4\gamma^\mu u_1][\overline{u}_3\gamma_\mu u_2]$$

$$\langle M\rangle^2_{spins} = \langle M_{34}\rangle^2+\langle M_{43}\rangle^2+\langle M_{34}M_{43}^*\rangle+\langle M_{43}M_{34}^*\rangle$$

$$= \frac{A^2}{4}\mathrm{Tr}(\not{p}_3\gamma^\mu\not{p}_1\gamma^\nu)\mathrm{Tr}(\not{p}_4\gamma_\mu\not{p}_2\gamma_\nu) + \frac{B^2}{4}[34\to 43] - \frac{AB}{4}\mathrm{Tr}(\not{p}_3\gamma^\mu\not{p}_1\gamma^\nu\not{p}_4\gamma_\mu\not{p}_2\gamma_\nu) - \frac{BA}{4}[34\to 43]$$

High E $(p\gg m\to m=0)$

$P_1\cdot P_2 = P_3\cdot P_4$

$P_1\cdot P_3 = P_2\cdot P_4$

$P_1\cdot P_4 = P_2\cdot P_3$

$(P_1\cdot P_4+P_1\cdot P_3)^2 = P_1^2+(P_1\cdot P_2)^2$

$$\langle M\rangle^2_{spins} = \frac{4*2g_e^4}{(P_1-P_3)^4}\left[(P_1\cdot P_2)(P_3\cdot P_4)+(P_1\cdot P_4)(P_2\cdot P_3)\right] + \frac{B^2}{4}[34\to 43]$$

$$-\frac{AB}{4}\sum_{spins}[\overline{u}_3\gamma^\mu u_1][\overline{u}_4\gamma_\mu u_2][\overline{u}_1\gamma^\nu u_4][\overline{u}_2\gamma_\nu u_3] - \frac{BA}{4}[34\to 43]$$

$$= \frac{8(1,2)(3,4)}{4(1,3)^2}+\frac{8(1,4)(2,3)}{4(1,3)^2}+\frac{B^2}{4}[34\to 43] - \frac{-32g_e^4}{4(P_1-P_3)^2(P_1-P_4)^2}\left[2(P_1\cdot P_2)(P_3\cdot P_4)\right]$$

$$= g_e^4\left[\underline{\frac{2(1,2)^2(1,4)^2}{(1,3)^2(1,4)^2}}+\frac{2(1,4)^2(1,4)^2}{(1,3)^2(1,4)^2}+\underline{\frac{2(1,2)^2(1,3)^2}{(1,4)^2(1,3)^2}}+\frac{2(1,3)^2(1,3)^2}{(1,4)^2(1,3)^2}+\underline{\frac{16(1,2)^2(1,3)(1,4)}{4(1,3)^2(1,4)^2}}\right]$$

$$= \boxed{\frac{2g_e^4}{(P_1\cdot P_3)^2(P_1\cdot P_4)^2}\left[(P_1\cdot P_4)^4+(P_1\cdot P_3)^4+\underline{(P_1\cdot P_2)^4}\right]}$$

$$CM\ \langle M\rangle^2_{spins} = 2g_e^4\frac{(1+\cos\theta)^4+(1-\cos\theta)^4+2^4}{(1-\cos\theta)^2(1+\cos\theta)^2} = \boxed{\frac{4g_e^4(3+\cos^2\theta)^2}{\sin^4\theta}}$$

Pair Annihilation, $e^-e^+\to 2\gamma$: $M = \boxed{[\overline{v}_2g_e\not{\epsilon}_4^*]\frac{(\not{q}+m)}{(P_1-P_3)^2-m^2}[\not{\epsilon}_3^*g_eu_1]+[\overline{v}g_e\not{\epsilon}_3^*]\frac{\not{P}_1-\not{P}_4+m}{(P_1-P_4)^2-m^2}[\not{\epsilon}_4^*g_eu_1]}$

$$\ldots M_{(p_1=p_2=0)} = \underset{\uparrow\uparrow,\uparrow\downarrow,\downarrow\uparrow,\downarrow\downarrow\ :\ \overline{v}_2\gamma^0u_1=0}{\cancel{\frac{g_e^2}{m}\overline{v}_2(\vec{\epsilon}_3^*\cdot\vec{\epsilon}_4^*)\gamma^0u_1}} + \frac{g_e^2}{m}i\overline{v}_2(\vec{\epsilon}_3^*\times\vec{\epsilon}_4^*)_i\Sigma^i\gamma^3u_1 = \boxed{\frac{g_e^2}{m}i(\vec{\epsilon}_3^*\times\vec{\epsilon}_4^*)_i2m[e^{s_2}]^T\sigma^i[-\sigma^3]e^{s_1}}$$

$M_{\uparrow\uparrow} = -2g_e^2i^2\left[i(\vec{\epsilon}_3^*\times\vec{\epsilon}_4^*)_x-(\vec{\epsilon}_3^*\times\vec{\epsilon}_4^*)_y\right]\ :\ \epsilon^\uparrow\times\epsilon^\uparrow=0\ \Rightarrow\ M_{\uparrow\uparrow}=0$

$M_{\downarrow\downarrow} = -2g_e^2i^2\left[i(\vec{\epsilon}_3^*\times\vec{\epsilon}_4^*)_x+(\vec{\epsilon}_3^*\times\vec{\epsilon}_4^*)_y\right]\ :\ \epsilon^\downarrow\times\epsilon^\downarrow=0\ \Rightarrow\ M_{\downarrow\downarrow}=0$

$M_{\uparrow\downarrow} = -2g_e^2i(\vec{\epsilon}_3^*\times\vec{\epsilon}_4^*)_z = -2g_e^2i(\epsilon_+^*\times\epsilon_-^*)_z = -2g_e^2$

$M_{\downarrow\uparrow} = 2g_e^2i(\vec{\epsilon}_3^*\times\vec{\epsilon}_4^*)_z = 2g_e^2i(\epsilon_-^*\times\epsilon_+^*)_z = 2g_e^2$

- $\epsilon^\uparrow = \epsilon^+ = (-1,-i,0)/\sqrt{2}$
- $\epsilon^- = (1,-i,0)/\sqrt{2}$
- $e^-e^+\to 2\gamma$ is singlet only

* $e^-e^+\to 3\gamma$ is triplet only

$\boxed{\langle M\rangle^2 = 16g_e^4\ \text{(for all, since only singlet)}}$