1 Finding Eigenvalues

1.1 Error Bound and Conditioning

$A + \Delta A = Q(D + \Delta D)Q^{-1}$

- $v = (\Delta\lambda I - D)^{-1}(\Delta D)v$
- $\|(\Delta\lambda I - D)^{-1}\|_2^{-1} \leq \|\Delta D\|_2$

$$|\Delta\lambda - \lambda_i| \leq \|Q(\Delta A)Q^{-1}\|_2$$

$\rightarrow$ $\boxed{|\Delta\lambda - \lambda_i| \leq \mathrm{cond}(Q)\ \|\Delta A\|_2}$

$(A + \Delta A)(x + \Delta x) = (\lambda + \Delta\lambda)(x + \Delta x)$

- $Ax = \lambda x\ ,\quad y^H A = \lambda y^H$
- $\underline{\lambda\text{ is simple}} \;\Rightarrow\; y^H x \neq 0 \quad (?)$
- $y^H \cancel{Ax} + \cancel{y^H A\Delta x} + y^H(\Delta A)x + y^H\cancel{(\Delta A)\Delta x}$
  $\approx y^H \cancel{\lambda x} + \cancel{y^H \lambda\Delta x} + y^H(\Delta\lambda)x + y^H\cancel{(\Delta\lambda)\Delta x}$

$\rightarrow$ $\boxed{|\Delta\lambda| \ \lessapprox\ \frac{\|y\|_2 \cdot \|x\|_2}{|y^H x|}\ \|\Delta A\|_2 \ = \ \frac{1}{\cos\theta}\ \|\Delta A\|_2}$

- $AA^\dagger = A^\dagger A \ \rightarrow\ \mathrm{cond}(A) = 1$
- Non-simple (multiple) eigenvalue is complicated:
- allows $y^H x = 0$, depends on eigenvalue spacings, vector angles, etc.
- Balancing can improve conditioning - diagonal rescaling