CS101 Algorithms and Data Structures

Array and Linked List

Textbook Ch 10.2

Outline

- List ADT
- Array
- Linked list
- Doubly linked list
- Node-based storage with arrays

Ex1 compute the summation for a polynomial at a fixed value x.

$$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_{n-1} x^{n-1} + a_n x^n$$

```
double fpoly1 ( int n, double a[ ], double x )
{ int i;
  double p = a[0];
  for (i = 1; i <=n; i++)
       p += (a[i] * pow( x, i) );
  return p;
}
```

$$f(x) = a_0 + x(a_1 + x(a_2 + \cdots x(a_{n-1} + x(a_n)) \cdots))$$

```
double fpoly2 ( int n, double a[ ], double x )
{ int i;
  double p = a[n];
  for (i = n; i > 0; i-- )
       p = a[i-1]  + x* p;
  return p;
}
```

Representation of polynomial coefficients $\boldsymbol{a_n}$

$$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_{n-1} x^{n-1} + a_n x^n$$

```
double fpoly1 ( int n, double a[ ], double x )
{ int i;
  double p = a[0];
  for (i = 1; i <=n; i++)
       p += (a[i] * pow( x, i) );
  return p;
}
```

- **Method 1: array**

$$f(x) = 4x^5 - 3x^2 + 1$$

$a[i]$	1	0	-3	0	0	4	⋯
Array indices	0	1	2	3	4	5	⋯

Problem?

Discussion 1

How to present coefficients for $\boldsymbol{f(x) = 4 + 3x^{2001}}$?

$a[i]$	~~4~~	~~0~~	~~0~~	~~0~~	~~…~~	~~…~~	~~3~~
Array indices	~~0~~	~~0~~	~~2~~	~~3~~	~~…~~	~~…~~	~~2001~~

Method 2: structure array

- For each non-zero term, need to know two components: the coefficient $\boldsymbol{a_i}$, the index no. $\boldsymbol{i}$.
- We can use a structure array ($\boldsymbol{a_i}$, $\boldsymbol{i}$).
- Ex:

$$P_1(x) = 3x^{100} + 10x^{50}+15 \quad \& \quad P_2(x) = 4x^{1000} + 30x^{60} + 5$$

$\boldsymbol{a}[\boldsymbol{i}]$	3	10	15	
Expon index i	100	50	0	
Array indices	0	1	2	$\cdots$

$\boldsymbol{a}[\boldsymbol{i}]$	4	30	5	
Expon index i	100	60	0	
Array indices	0	1	2	$\cdots$

Store the coefficients in descent order of exponential index.

Addition of Two Polynomials?

$$P_1(x) = 3x^{100} + 10x^{50} + 15 \quad \& \quad P_2(x) = 4x^{1000} + 30x^{60} + 5$$

$\boldsymbol{a}[\boldsymbol{i}]$	3	10	15	
Expon index i	100	50	0	
Array indices	0	1	2	⋯

4	30	5	
100	60	0	
0	1	2	⋯

Addition of Two Polynomials?

$P_1(x) = 3x^{100} + 10x^{50} + 15$ & $P_2(x) = 4x^{1000} + 30x^{60} + 5$

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$\boldsymbol{a}[\boldsymbol{i}]$	4	30	5	
Expon index i	100	60	0	
Array indices	0	1	2	⋯

$\boldsymbol{a}[\boldsymbol{i}]$								
Expon index i								
Array indices	0	1	2	3	4	5	6	⋯

Addition of Two Polynomials?

$$P_1(x) = 3x^{100} + 10x^{50} + 15 \quad \& \quad P_2(x) = 4x^{1000} + 30x^{60} + 5$$

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Expon index i	100	60	0	
Array indices	0	1	2	⋯

$\boldsymbol{a}[\boldsymbol{i}]$	7							
Expon index i	100							
Array indices	0	1	2	3	4	5	6	⋯

Addition of Two Polynomials?

$$P_1(x) = 3x^{100} + 10x^{50} + 15 \quad \& \quad P_2(x) = 4x^{1000} + 30x^{60} + 5$$

$\boldsymbol{a[i]}$	3	10	15	
Expon index i	100	50	0	
Array indices	0	1	2	···

$\boldsymbol{a[i]}$	4	30	5	
Expon index i	100	60	0	
Array indices	0	1	2	···

$\boldsymbol{a[i]}$	7							
Expon index i	100							
Array indices	0	1	2	3	4	5	6	···

Addition of Two Polynomials?

$P_1(x) = 3x^{100} + 10x^{50} + 15$ & $P_2(x) = 4x^{1000} + 30x^{60} + 5$

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Expon index i	100	50	0	
Array indices	0	1	2	⋯

4	30	5	
100	60	0	
0	1	2	⋯

$\boldsymbol{a}[\boldsymbol{i}]$	7	30						
Expon index i	100	60						
Array indices	0	1	2	3	4	5	6	⋯

Addition of Two Polynomials?

$P_1(x) = 3x^{100} + 10x^{50} + 15$ & $P_2(x) = 4x^{1000} + 30x^{60} + 5$

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Expon index i	100	50	0	
Array indices	0	1	2	⋯

$\boldsymbol{a[i]}$	4	30	5	
Expon index i	100	60	0	
Array indices	0	1	2	⋯

$\boldsymbol{a[i]}$	7	30						
Expon index i	100	60						
Array indices	0	1	2	3	4	5	6	⋯

Addition of Two Polynomials?

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Expon index i	100	50	0	
Array indices	0	1	2	⋯

$\boldsymbol{a}[\boldsymbol{i}]$	4	30	5	
Expon index i	100	60	0	
Array indices	0	1	2	⋯

$\boldsymbol{a}[\boldsymbol{i}]$	7	30	10					
Expon index i	100	60	50					
Array indices	0	1	2	3	4	5	6	⋯

Addition of Two Polynomials?

$$P_1(x) = 3x^{100} + 10x^{50} + 15 \quad \& \quad P_2(x) = 4x^{1000} + 30x^{60} + 5$$

$\boldsymbol{a[i]}$	3	10	15	
Expon index i	100	50	0	
Array indices	0	1	2	⋯

4	30	5	
100	60	0	
0	1	2	⋯

$\boldsymbol{a[i]}$	7	30	10					
Expon index i	100	60	50					
Array indices	0	1	2	3	4	5	6	⋯

Addition of Two Polynomials?

$P_1(x) = 3x^{100} + 10x^{50} + 15$ & $P_2(x) = 4x^{1000} + 30x^{60} + 5$

$\boldsymbol{a[i]}$	3	10	15	
Expon index i	100	50	0	
Array indices	0	1	2	⋯

$\boldsymbol{a[i]}$	4	30	5	
Expon index i	100	60	0	
Array indices	0	1	2	⋯

$\boldsymbol{a[i]}$	7	30	10	20				
Expon index i	100	60	50	0				
Array indices	0	1	2	3	4	5	6	⋯

Addition of Two Polynomials?

$P_1(x) = 3x^{100} + 10x^{50} + 15$ & $P_2(x) = 4x^{1000} + 30x^{60} + 5$

$\boldsymbol{a}[\boldsymbol{i}]$	3	10	15	
Expon index i	100	50	0	
Array indices	0	1	2	⋯

$\boldsymbol{a}[\boldsymbol{i}]$	4	30	5	
Expon index i	100	60	0	
Array indices	0	1	2	⋯

$\boldsymbol{a}[\boldsymbol{i}]$	7	30	10	20				
Expon index i	100	60	50	0				
Array indices	0	1	2	3	4	5	6	⋯

$$P_3(x) = P_1(x) + P_2(x) = 7x^{100} + 30x^{60} + 10x^{50} + 20$$

Can we store the coefficients in an increase order of exponential index?

Outline

- List ADT
- Array
- Linked list
- Doubly linked list
- Node-based storage with arrays

List ADT

An Abstract List (or List ADT) is linearly ordered data (with same data type)

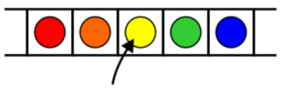
$$(\ A_1 \ \ A_2 \ \ldots \ A_{n-1} \ \ A_n \ )$$

- The number of elements in the List denotes the length of the List.
- When there is no element it is an empty List.
- The beginning of a List is called the List head; the end of a List is called the the List tail.
- The same value may occur more than once.

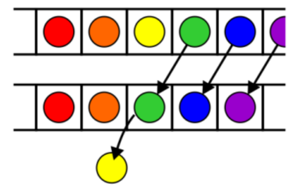
Operations

Operations at the k^{th} entry of the list include:

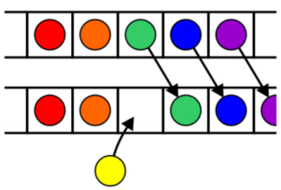
Access to the object



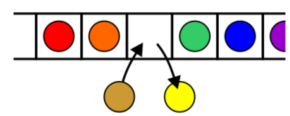
Erasing an object



Insertion of a new object

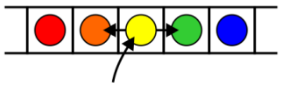


Replacement of the object



Operations

Given access to the k^{th} object, gain access to either the previous or next object



Given two abstract lists, we may want to

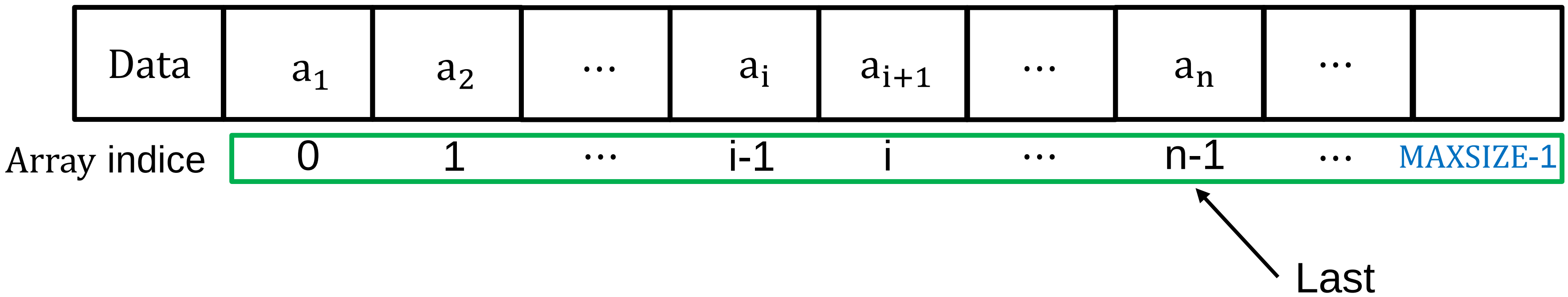
- Concatenate the two lists
- Determine if one is a sub-list of the other

Operations

- For $L \in List$, i denotes the indice, $X \in ElementType$, the basic opertions includes but not limited to:
  - $ListEmpty()$: initialize an empty list.
  - $ElementType\ FindKth(int\ K, List\ L)$: find the K_{th} element and return it.
  - $int\ Find(ElementType\ X, int\ I, List\ L)$:find the location for X.
  - $void\ Insert(ElementType\ X, int\ i, List\ L)$: insert a new element before the i_{th} element.
  - $void\ Delete(int\ i, List\ L)$: delete the i_{th} element.
  - $int\ Length(List\ L)$: return the length of a list.

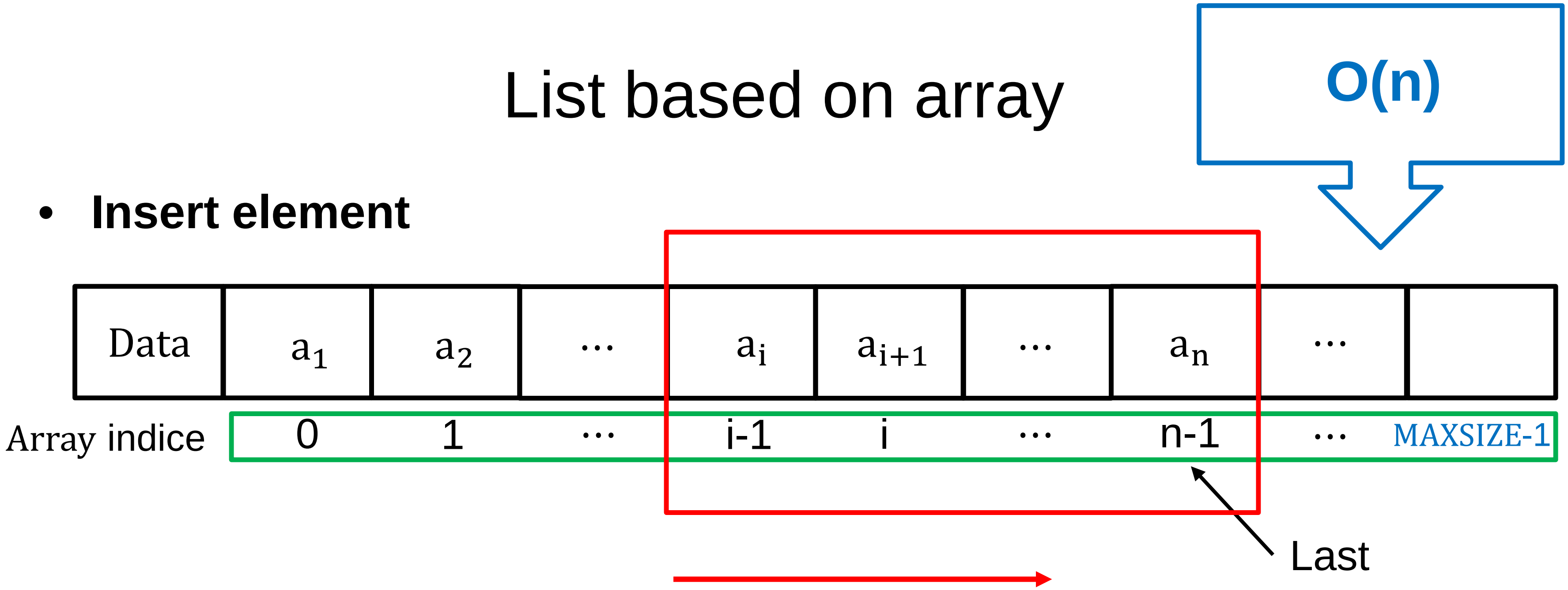
  … …

List based on array

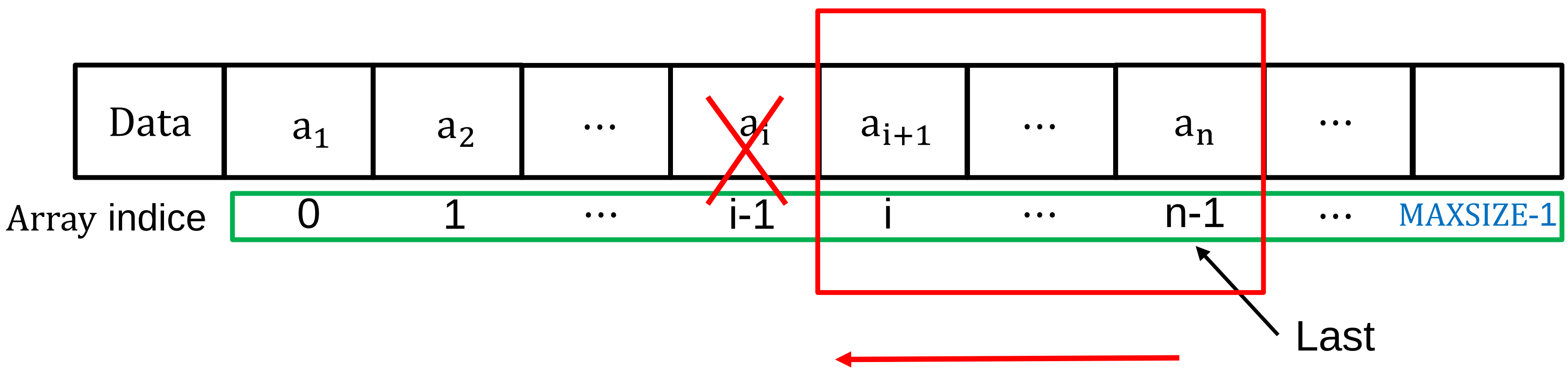


List based on array

- **Insert element**



- **Delete element**



Outline

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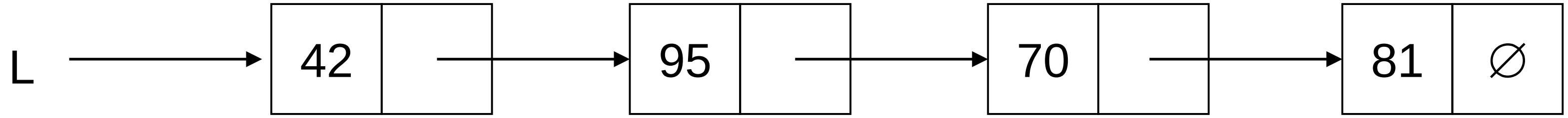
- Application

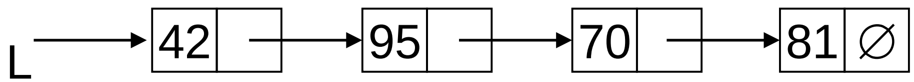
Definition

A linked list is a data structure where each object is stored in a *node*

As well as storing data, the node must also contains a reference/pointer to the node containing the next item of data



Node Class



The node must store data and a pointer:

```
class Node {
    private:
        int element;
        Node *next_node;
    public:
        Node( int = 0, Node * = nullptr );

        int retrieve() const;
        Node *next() const;
};
```

Node Constructor

The constructor assigns the two member variables based on the arguments

```
Node::Node( int e, Node *n ):
element( e ),
next_node( n ) {
    // empty constructor
}
```

The default values are given in the class definition:

```
Node( int = 0, Node * = nullptr );
```

Accessors

The two member functions are accessors which simply return the **`element`** and the **`next_node`** member variables, respectively

```
int Node::retrieve() const {
    return element;
}

Node *Node::next() const {
    return next_node;
}
```

Linked List Class

Because each node in a linked lists refers to the next, the linked list class need only link to the first node in the list

The linked list class requires member variable: a pointer to a node

```
class List {
    private:
        Node *list_head;
    // ...
};
```

Structure

Let us look at the internal representation of a linked list

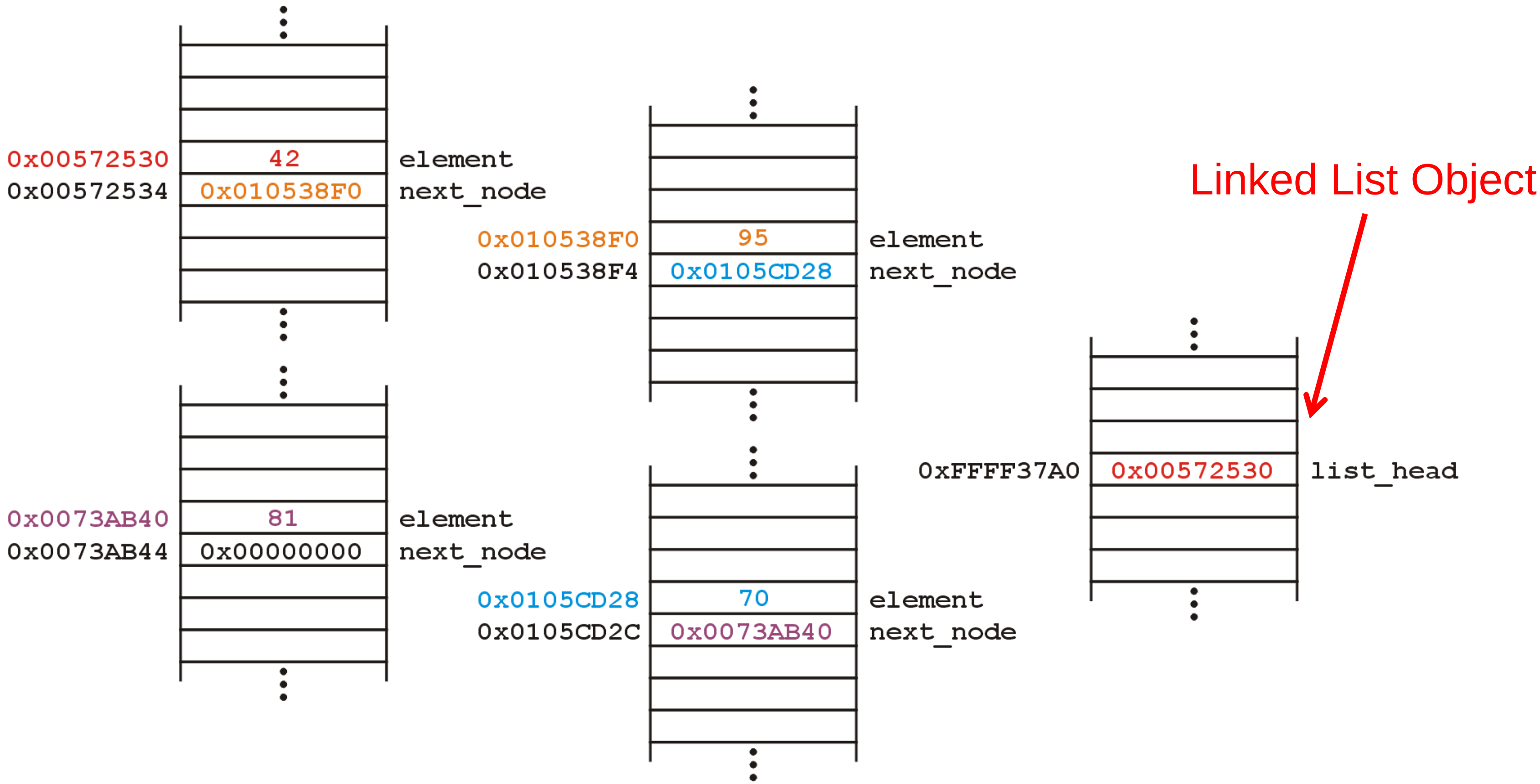
Suppose we want a linked list to store the values

`42   95   70   81`

in this order

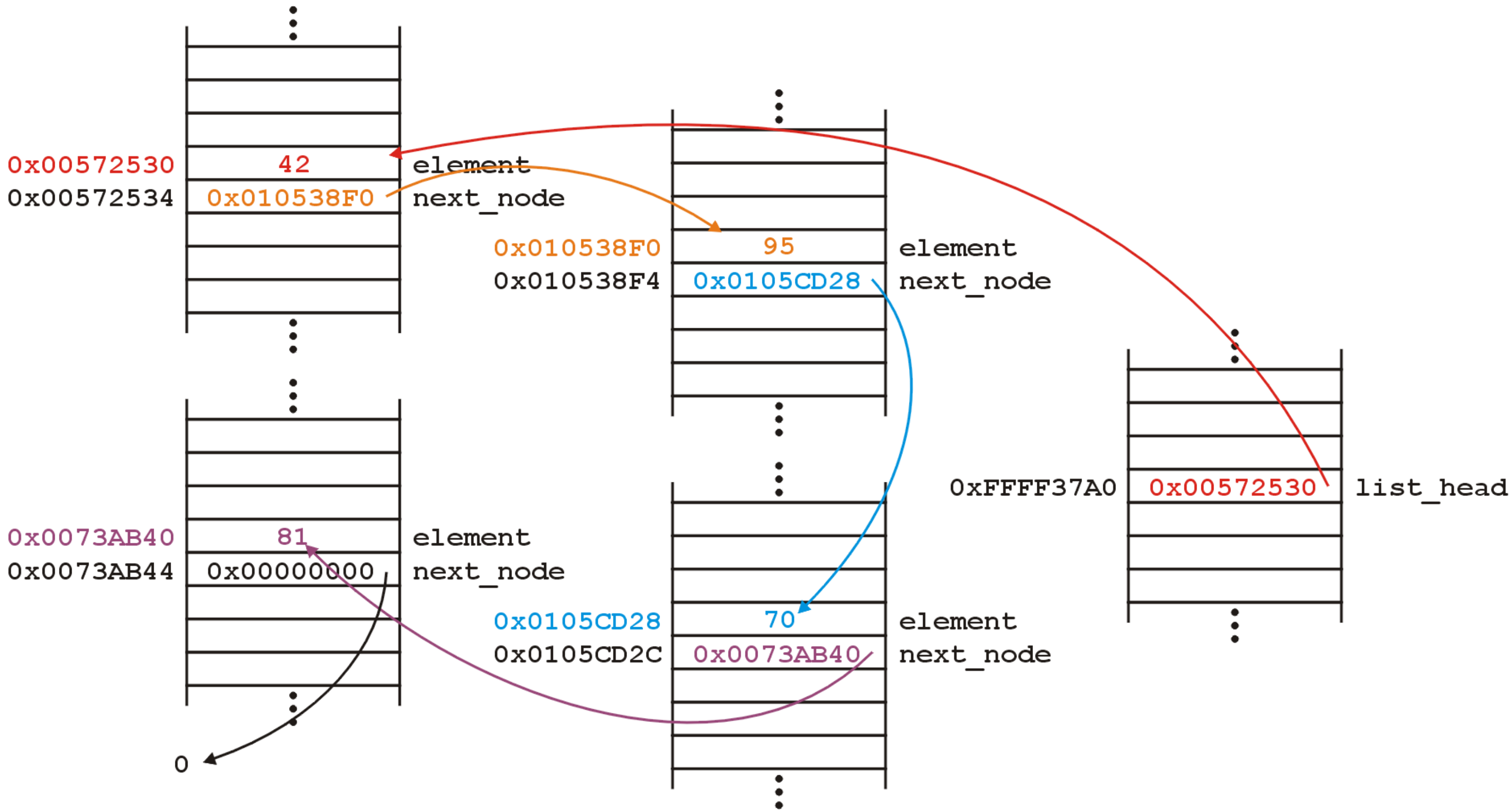
Structure

A linked list uses linked allocation, and therefore each node may appear anywhere in memory:



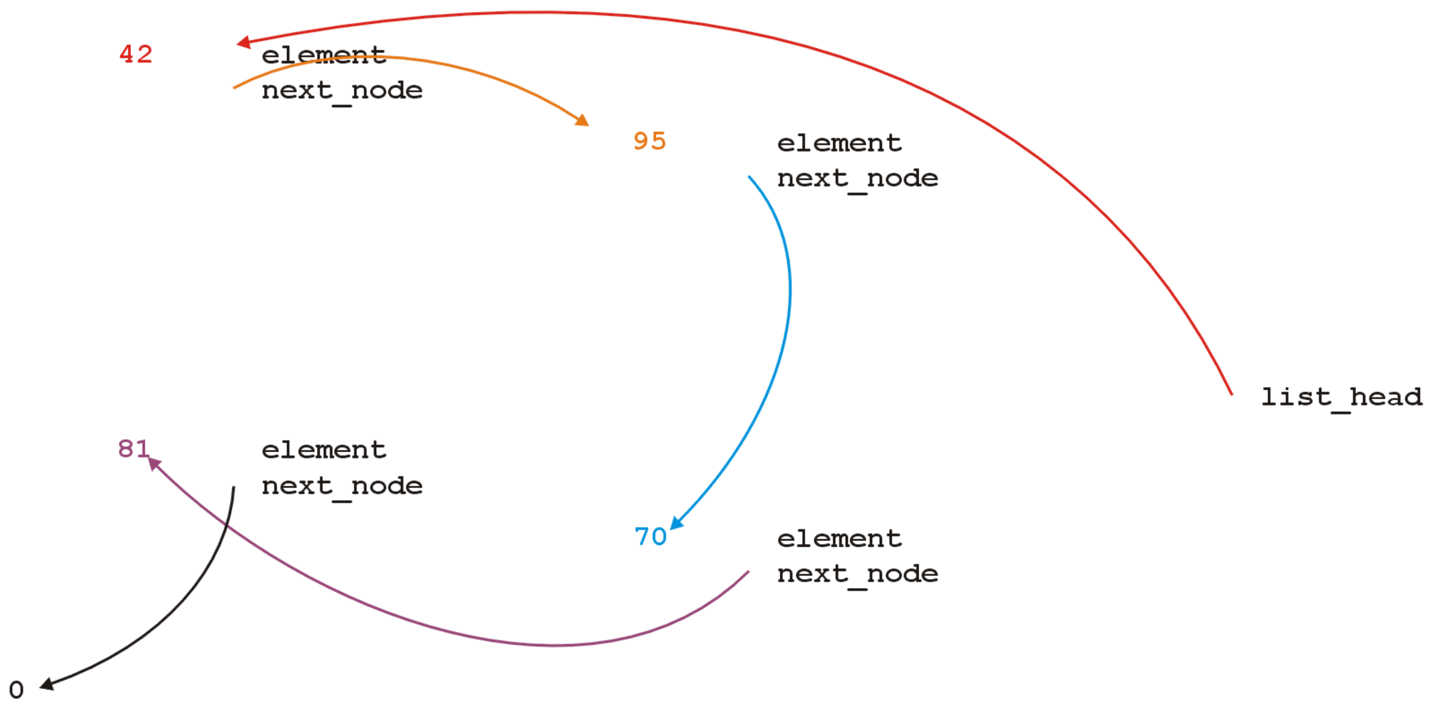
Structure

The **next_node** pointers store the addresses of the next node in the list



Structure

Because the addresses are arbitrary, we can remove that information:



Structure

We will clean up the representation as follows:

We do not specify the addresses because they are arbitrary and:

- The contents of the circle is the element
- The `next_node` pointer is represented by an arrow

Operations

First, we want to create a linked list

We also want to be able to:

- insert into,
- access, and
- erase from

the elements stored in the linked list

Operations

We can do them with the following operations:

- Adding, retrieving, or removing the value at the front of the linked list

```
void push_front( int );
int front() const;
void pop_front();
```

- We may also want to access the head of the linked list

```
Node *head() const;
```

Operations

All these operations relate to the first node of the linked list

We may want to perform operations on an arbitrary node of the linked list, for example:

- Find the number of instances of an integer in the list:

```
int count( int ) const;
```

- Remove all instances of an integer from the list:

```
int erase( int );
```

Linked Lists

Additionally, we may wish to check the state:

- Is the linked list empty?

```
bool empty() const;
```

- How many objects are in the list?

```
int size() const;
```

The list is empty when the `list_head` pointer is set to `nullptr`

Linked Lists

Consider this simple (but incomplete) linked list class:

```
class List {
    private:
        Node *list_head;

    public:
        List();

        // Accessors
        bool empty() const;
        int size() const;
        int front() const;
        Node *head() const;
        int count( int ) const;

        // Mutators
        void push_front( int );
        int pop_front();
        int erase( int );
};
```

The Constructor

In the constructor, we assign `list_head` the value `nullptr`

```
List::List():list_head( nullptr ) {
    // empty constructor
}
```

We will always ensure that when a linked list is empty, the list head is assigned `nullptr`

bool empty() const

Starting with the easier member functions:

```
bool List::empty() const {
    if ( list_head == nullptr ) {
        return true;
    } else {
        return false;
    }
}
```

Better yet:

```
bool List::empty() const {
    return ( list_head == nullptr );
}
```

`Node *head() const`

The member function `Node *head() const` is easy enough to implement:

```
Node *List::head() const {
    return list_head;
}
```

This will always work: if the list is empty, it will return `nullptr`

`int front() const`

To get the first element in the linked list, we must access the node to which the `list_head` is pointing

Because we have a pointer, we must use the `->` operator to call the member function:

```
int List::front() const {
    return head()->retrieve();
}
```

`int front() const`

What if the list is empty?

If we tried to access a member function of a pointer set to `nullptr`, we would access restricted memory and the OS would terminate the running program

`int front() const`

Instead, we can use an exception handling mechanism where we thrown an exception

We define a class

```
class underflow {
    // emtpy
};
```

and then we *throw* an instance of this class:

```
throw underflow();
```

int front() const

Thus, the full function is

```
int List::front() const {
    if ( empty() ) {
        throw underflow();
    }

    return head()->retrieve();
}
```

int front() const

Why is `emtpy()` better than

```
int List::front() const {
    if ( list_head == nullptr ) {
        throw underflow();
    }

    return list_head->element;
}
```

Two benefits:

- More readable
- If the implementation changes we do nothing

void push_front(int)

Next, let us add an element to the list

If it is empty, we start with:

and, if we try to add 81, we should end up with:

`void push_front( int )`

We must:

- create a new node which:
  - stores the value **81**, and
  - is pointing to **0**
- assign its address to `list_head`

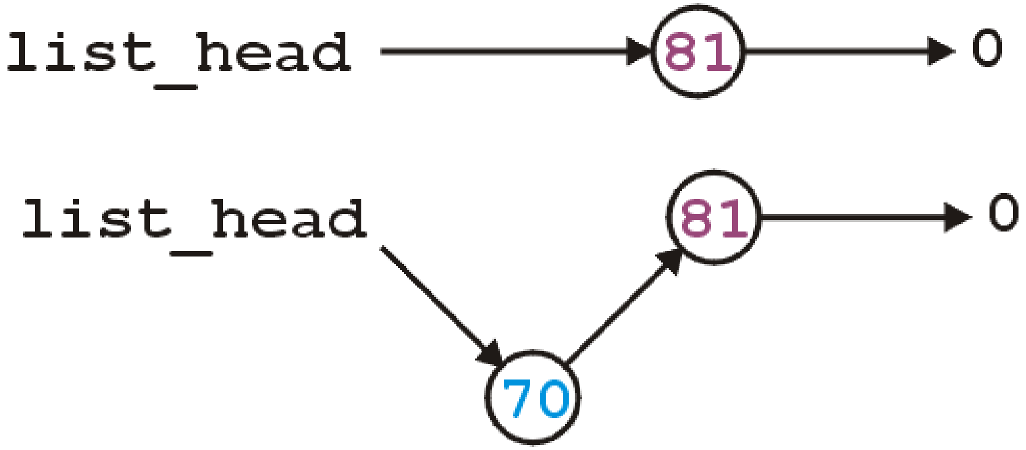
We can do this as follows:

```
list_head = new Node( 81, nullptr );
```

`void push_front( int )`

Suppose however, we already have a non-empty list

Adding **`70`**, we want:



`void push_front( int )`

To achieve this, we must we must create a new node which:

- stores the value `70`, and
- is pointing to the current list head

– we must then assign its address to `list_head`

We can do this as follows:

```
list_head = new Node( 70, list_head );
```

void push_front(int)

Thus, our implementation could be:

```
void List::push_front( int n ) {
    if ( empty() ) {
        list_head = new Node( n, nullptr );
    } else {
        list_head = new Node( n, head() );
    }
}
```

void push_front(int)

We could, however, note that when the list is empty, `list_head == 0`, thus we could shorten this to:

```
void List::push_front( int n ) {
    list_head = new Node( n, list_head );
}
```

`void push_front( int )`

Are we allowed to do this?

```
void List::push_front( int n ) {
    list_head = new Node( n, head() );
}
```

Yes: the right-hand side of an assignment is evaluated first

– The original value of `list_head` is accessed first before the function call is made

void push_front(int)

Question: does this work?

```
void List::push_front( int n ) {
    Node new_node( n, head() );
    list_head = &new_node;
}
```

Why or why not? What happens to new_node?

How does this differ from

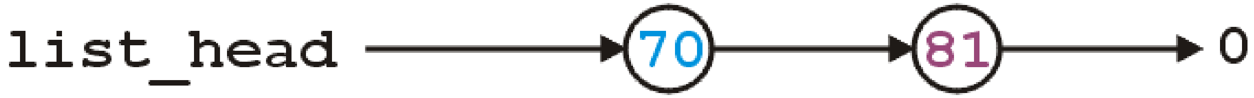
```
void List::push_front( int n ) {
    Node *new_node = new Node( n, head() );
    list_head = new_node;
}
```

`int pop_front()`

Erasing from the front of a linked list is even easier:

– We assign the list head to the next pointer of the first node

Graphically, given:



we want:

`int pop_front()`

Easy enough:

```
int List::pop_front() {
    int e = front();
    list_head = head()->next();
    return e;
}
```

Unfortunately, we have some problems:

- The list may be empty
- We still have the memory allocated for the node containing **`70`**

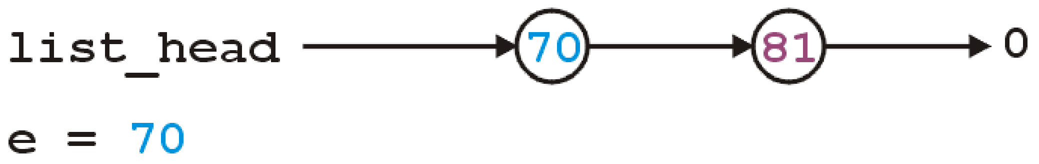
int pop_front()

Does this work?

```
int List::pop_front() {
    if ( empty() ) {
        throw underflow();
    }

    int e = front();
    delete head();
    list_head = head()->next();
    return e;
}
```

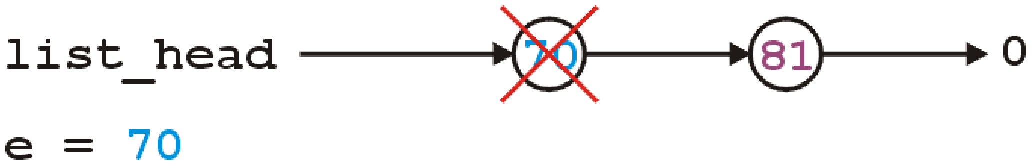
int pop_front()

```
int List::pop_front() {
    if ( empty() ) {
        throw underflow();
    }

    int e = front();

    delete head();

    list_head = head()->next();

    return e;
}
```



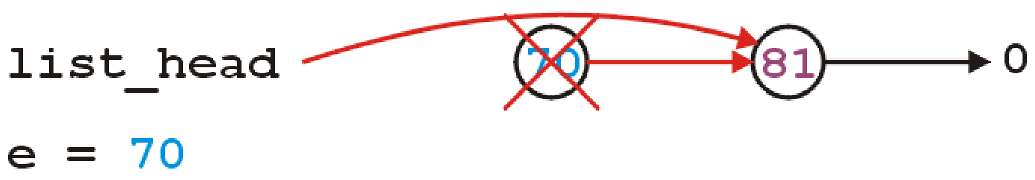
int pop_front()

```
int List::pop_front() {
    if ( empty() ) {
        throw underflow();
    }

    int e = front();

    delete head();

    list_head = head()->next();

    return e;
}
```



int pop_front()

```
int List::pop_front() {
    if ( empty() ) {
        throw underflow();
    }

    int e = front();

    delete head();

    list_head = head()->next();

    return e;
}
```



Any problem with the above code?

int pop_front()

The correct implementation assigns a temporary pointer to point to the node being deleted:

```
int List::pop_front() {
    if ( empty() ) {
        throw underflow();
    }

    int e = front();
    Node *ptr = list_head;
    list_head = list_head->next();
    delete ptr;
    return e;
}
```

Stepping through a Linked List

The next step is to look at member functions which potentially require us to step through the entire list:

```
int size() const;
int count( int ) const;
int erase( int );
```

The second counts the number of instances of an integer, and the last removes the nodes containing that integer

Stepping through a Linked List

The process of stepping through a linked list can be thought of as being analogous to a for-loop:

- We initialize a temporary pointer with the list head
- We continue iterating until the pointer equals `nullptr`
- With each step, we set the pointer to point to the next object

Stepping through a Linked List

Thus, we have:

```
for ( Node *ptr = head(); ptr != nullptr; ptr = ptr->next() ) {
    // do something
    // use ptr->fn() to call member functions
    // use ptr->var to assign/access member variables
}
```

Stepping through a Linked List

Analogously:

```
for ( Node *ptr = head(); ptr != nullptr; ptr = ptr->next() )
for ( int i = 0;          i != N;         ++i               )
```

`int count( int ) const`

To implement `int count(int) const`, we simply check if the argument matches the element with each step

- Each time we find a match, we increment the count
- When the loop is finished, we return the count
- The size function is simplification of count

int count(int) const

The implementation:

```
int List::count( int n ) const {
    int node_count = 0;

    for ( Node *ptr = list(); ptr != nullptr; ptr = ptr->next() ) {
        if ( ptr->retrieve() == n ) {
            ++node_count;
        }
    }

    return node_count;
}
```

`int erase( int )`

To remove an arbitrary element, *i.e.*, to implement `int erase( int )`, we must update the previous node

For example, given

if we delete **70**, we want to end up with

Accessing Private Member Variables

Notice that the `erase` function must modify the member variables of the node prior to the node being removed

Thus, it must have access to the member variable `next_node`

We could supply the member function

```
void set_next( Node * );
```

however, this would be globally accessible

Possible solutions:

- Friends
- Nested classes
- Inner classes (Java/C#)

C++ Friends

In C++, you explicitly break encapsulation by declaring the class List to be a *friend* of the class Node:

```
class Node {
    Node *next() const;
    // ... declaration ...
    friend class List;
};
```

Now, inside `erase` (a member function of `List`), you can modify all the member variables of any instance of the `Node` class

C++ Friends

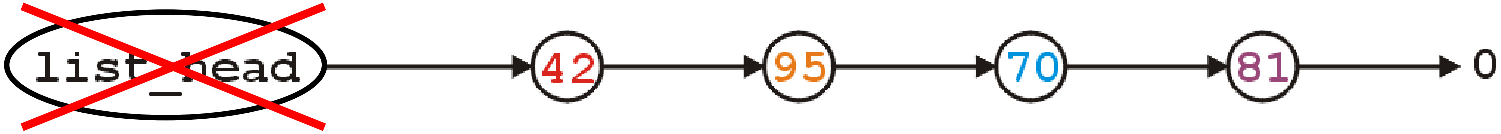
For example, the erase member function could be implemented using the following code:

```
int List::erase( int n ) {
    int node_count = 0;
    // ...

    for ( Node *ptr = head(); ptr != nullptr; ptr = ptr->next() ) {
        // ...

        if ( some condition ) {
            ptr->next_node = ptr->next()->next();
            // ...
            ++node_count;
        }
    }

    return node_count;
}
```

Destructor

We dynamically allocated memory each time we added a new **`int`** into this list

Suppose we delete a list before we remove everything from it
- This would leave the memory allocated with no reference to it



Destructor

Thus, we need a destructor:

```
class List {
    private:
        Node *list_head;
    public:
        List();
        ~List();
        // ...etc...
};
```

Destructor

The destructor has to delete any memory which had been allocated but has not yet been deallocated

This is straight-forward enough:

```
while ( !empty() ) {
    pop_front();
}
```

Making Copies

Is this sufficient for a linked list class?

Initially, it may appear yes, but we now have to look at how C++ copies objects during:

- Passing by value (making a copy), and
- Assignment

Pass by Value

Recall that when you pass an integer to a function, a copy is made, so any changes to that parameter does not affect the original:

```
#include <iostream>

 void increment( int n ) {
     ++n;
 }

 int main() {
     int counter = 0;

     increment( counter );

     std::cout << counter << std::endl;  // counter is still 0
 }
```

Pass by Reference

If you want to change the value, you can pass by reference:

```
#include <iostream>

 void increment( int &n ) {
     ++n;
 }

 int main() {
     int counter = 0;

     increment( counter );

     std::cout << counter << std::endl;  // counter is now 1
 }
```

Pass by Pointer (C)

In C, you would pass the address of the object to change it:

```
#include <stdio.h>

 void increment( int *pn ) {
     ++(*pn);
 }

 int main() {
     int counter = 0;

     increment( &counter );

     printf( "%d", counter );            // counter is now 1
 }
```

Modifying Arguments

Pass by reference could be used to modify a list

```
void reverse( List &list ) {
    List tmp;

    while ( !list.empty() ) {
        tmp.push_front( ls.pop_front() );
    }

    // All the member variables of 'list' and 'tmp' are swapped
    std::swap( list, tmp );

    // The memory for 'tmp' will be cleaned up
}
```

Modifying Arguments

If you wanted to prevent the argument from being modified, you could declare it `const`:

```
double average( List const &ls, int min, int max ) {
    double sum = 0, count = 0;

    for ( Node *ptr = head(); ptr != nullptr; ptr = ptr->next() ) {
        sum += ptr->retrieve();
        ++count;
    }

    return sum/count;
}
```

Note: this reveals a weakness in our model—we will discuss iterators later…

Modifying Arguments

You want to pass a copy of a linked list to a function—where the function may modify the copy, but the original list shall be unchanged.

```
void func( List ls ) {
    // The compiler creates a new instance and copies the values
    // The function does something with 'ls'
    // The compiler ensures the destructor is called on 'ls'
}
```

With the *default copy constructor*, all the member variables are simply copied over into the new instance of the class

Modifying Arguments

```
void send_copy( List ls ) {
    // The compiler creates a new instance and copies the values
    // The function does something with 'ls'
    // The compiler ensures the destructor is called on 'ls'
}

int main() {
    List prim;

    for ( int i = 2; i <= 4; ++i ) {
        prim.push_front( i*i );
    }

    send_copy( prim );

    std::cout << prim.empty() << std::endl;

    return 0;
}
```

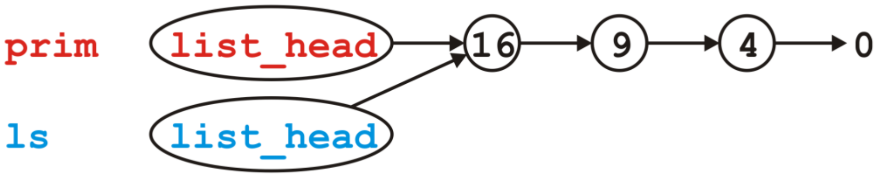
First, the list prim is created and three elements are pushed onto it

Modifying Arguments

```
void send_copy( List ls ) {
    // The compiler creates a new instance and copies the values
    // The function does something with 'ls'
    // The compiler ensures the destructor is called on 'ls'
}

int main() {
    List prim;

    for ( int i = 2; i <= 4; ++i ) {
        prim.push_front( i*i );
    }

    send_copy( prim );

    std::cout << prim.empty() << std::endl;

    return 0;
}
```

Next, we call send_copy and assigns a copy of `prim` to `ls`. The default is to copy member variables:

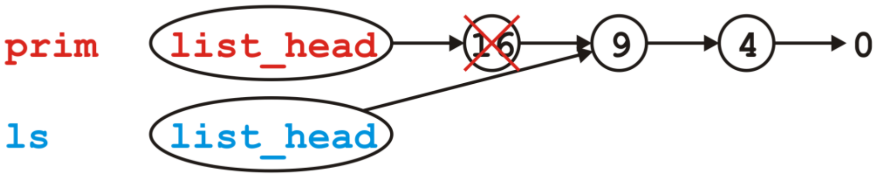
```
ls.list_head = prim.list_head
```



Modifying Arguments

```
void send_copy( List ls ) {
    // The compiler creates a new instance and copies the values
    // The function does something with 'ls'
    // The compiler ensures the destructor is called on 'ls'
}

int main() {
    List prim;

    for ( int i = 2; i <= 4; ++i ) {
        prim.push_front( i*i );
    }

    send_copy( prim );

    std::cout << prim.empty() << std::endl;

    return 0;
}
```

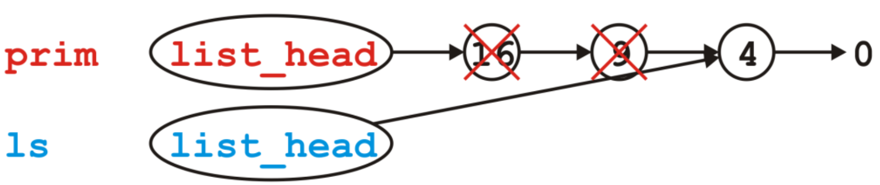
When send_copy returns, the destructor is called on `ls`



Modifying Arguments

```
void send_copy( List ls ) {
    // The compiler creates a new instance and copies the values
    // The function does something with 'ls'
    // The compiler ensures the destructor is called on 'ls'
}

int main() {
    List prim;

    for ( int i = 2; i <= 4; ++i ) {
        prim.push_front( i*i );
    }

    send_copy( prim );

    std::cout << prim.empty() << std::endl;

    return 0;
}
```

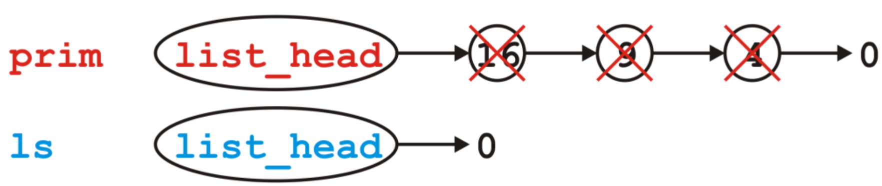
When send_copy returns, the destructor is called on ls



Modifying Arguments

```
void send_copy( List ls ) {
    // The compiler creates a new instance and copies the values
    // The function does something with 'ls'
    // The compiler ensures the destructor is called on 'ls'
}

int main() {
    List prim;

    for ( int i = 2; i <= 4; ++i ) {
        prim.push_front( i*i );
    }

    send_copy( prim );

    std::cout << prim.empty() << std::endl;

    return 0;
}
```

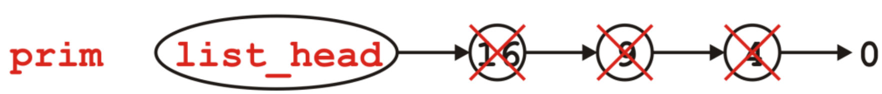
When `send_copy` returns, the destructor is called on `ls`



Modifying Arguments

```
void send_copy( List ls ) {
    // The compiler creates a new instance and copies the values
    // The function does something with 'ls'
    // The compiler ensures the destructor is called on 'ls'
}

int main() {
    List prim;

    for ( int i = 2; i <= 4; ++i ) {
        prim.push_front( i*i );
    }

    send_copy( prim );

    std::cout << prim.empty() << std::endl;

    return 0;
}
```

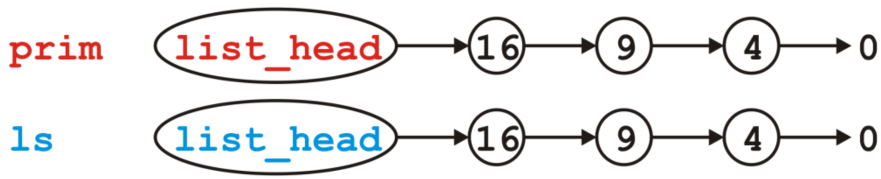
Back in `main()`, `prim.list_head` still stores the address of the Node containing 16, memory that has since been returned to the OS



Modifying Arguments

What do we really want?

- We really want a copy of the linked list
- If this copy is modified, it leaves the original unchanged

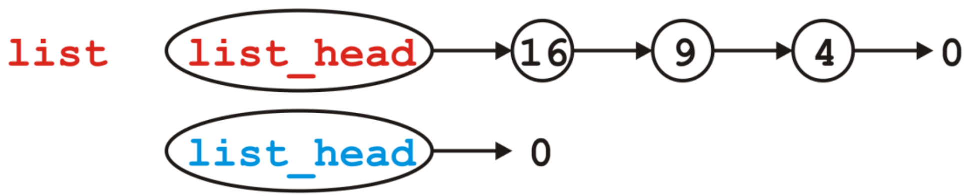


Copy Constructor

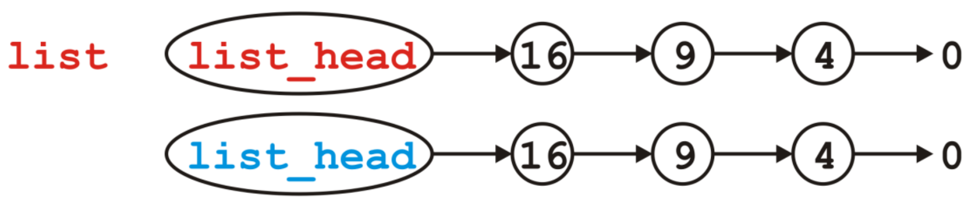
You can modify how copies are made by defining a *copy constructor*

```
List::List( List const &list ):list_head( nullptr ) {
    // Make a copy of list
}
```

We now want to go from



to



Copy Constructor

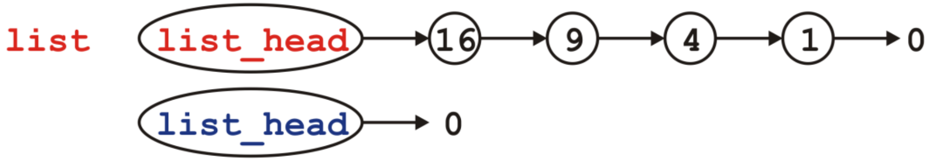
First, make life simple: if `list` is empty, we are finished, so return

```
List::List( List const &list ):list_head( nullptr ) {
    if ( list.empty() ) {
        return;
    }









}
```

Copy Constructor

Otherwise, the list being copied is not empty…

```
List::List( List const &list ):list_head( nullptr ) {
    if ( list.empty() ) {
        return;
    }
```

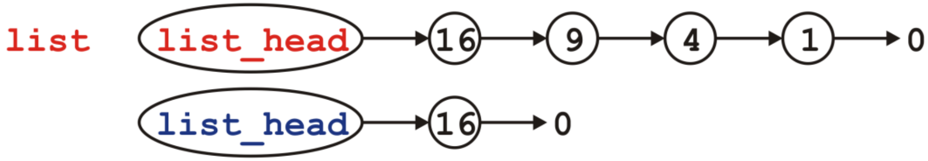



```
}
```

Copy Constructor

Copy the first node—we no longer modifying `list_head`

```
List::List( List const &list ):list_head( nullptr ) {
    if ( list.empty() ) {
        return;
    }

    push_front( list.front() );
```

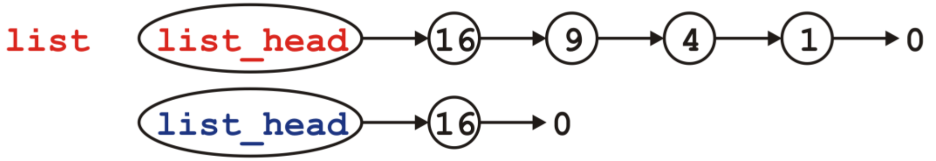



```
}
```

Copy Constructor

We will need to loop through the list… How about a for loop?

```
List::List( List const &list ):list_head( nullptr ) {
    if ( list.empty() ) {
        return;
    }

    push_front( list.front() );
```

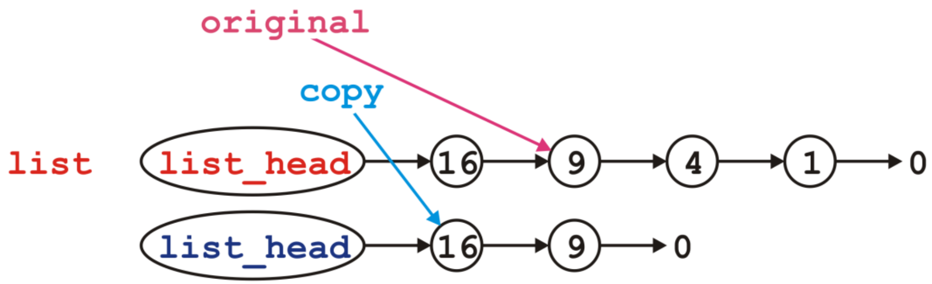



```
}
```

Copy Constructor

We modify the next pointer of the node pointed to by copy

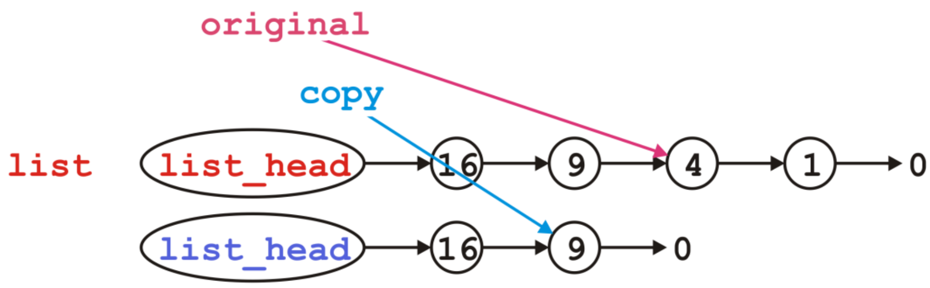
```
List::List( List const &list ):list_head( nullptr ) {
    if ( list.empty() ) {
        return;
    }

    push_front( list.front() );

    for (
        Node *original = list.head()->next(), *copy = head();
        original != nullptr;
        original = original->next(), copy = copy->next()
    ) {
        copy->next_node = new Node( original->retrieve(), nullptr );
    }
}
```



Copy Constructor

Then we move each pointer forward:

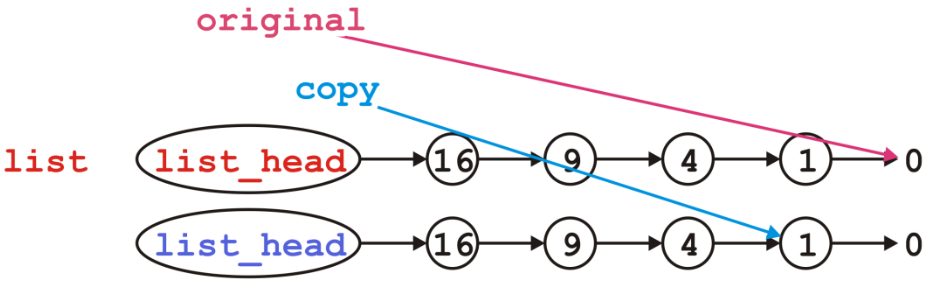
```
List::List( List const &list ):list_head( nullptr ) {
    if ( list.empty() ) {
        return;
    }

    push_front( list.front() );

    for (
        Node *original = list.head()->next(), *copy = head();
        original != nullptr;
        original = original->next(), copy = copy->next()
    ) {
        copy->next_node = new Node( original->retrieve(), nullptr );
    }
}
```



Copy Constructor

We'd continue copying until we reach the end

```
List::List( List const &list ):list_head( nullptr ) {
    if ( list.empty() ) {
        return;
    }

    push_front( list.front() );

    for (
        Node *original = list.head()->next(), *copy = head();
        original != nullptr;
        original = original->next(), copy = copy->next()
    ) {
        copy->next_node = new Node( original->retrieve(), nullptr );
    }
}
```



Assignment

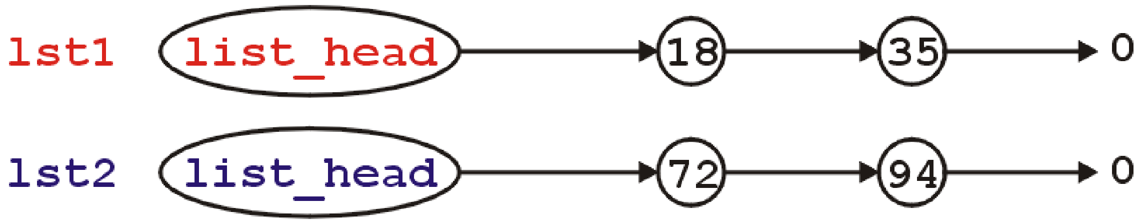
What about assignment?

– Suppose you have linked lists:

```
List lst1, lst2;

lst1.push_front( 35 );
lst1.push_front( 18 );
lst2.push_front( 94 );
lst2.push_front( 72 );
```

Assignment

This is the current state:



Consider an assignment:

```
lst2 = lst1;
```

What do we want? What do we actually do?

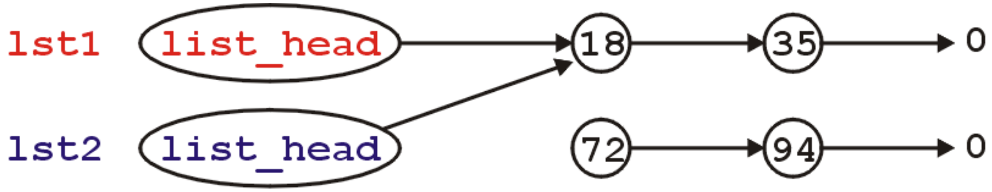
Assignment

The default behavior: the member variables of this class are copied over

It is equivalent to writing:

```
lst2.list_head = lst1.list_head;
```

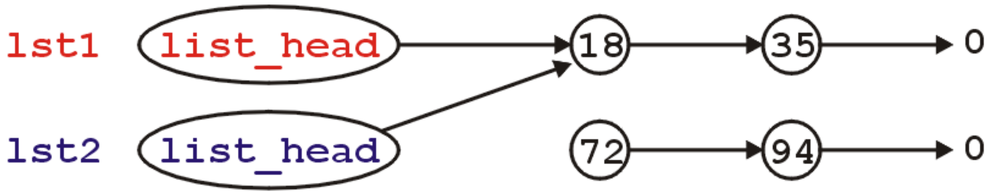
Graphically:



Assignment

What's wrong with this picture?

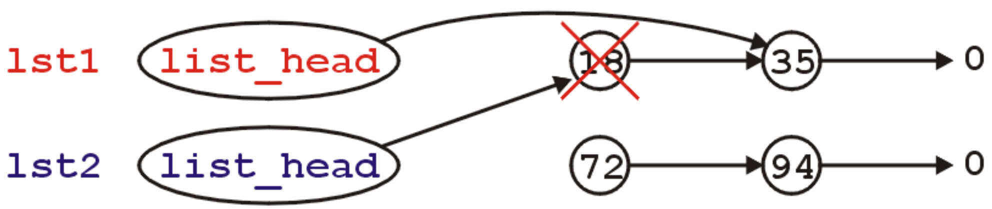
– We no longer have links to either of the nodes storing 72 or 94 (memory leak)



– Also, suppose we call the member function

```
lst1.pop_front();
```

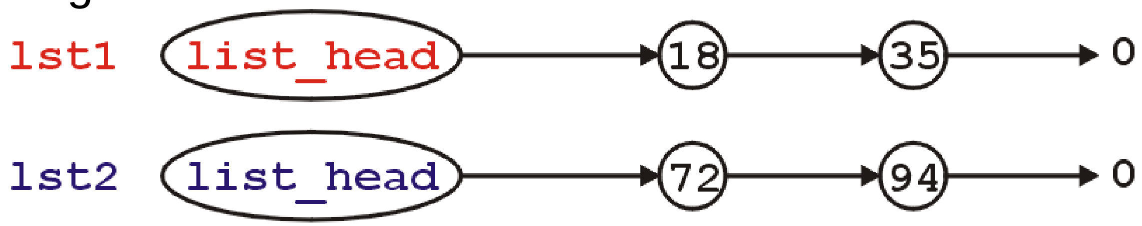
– `lst2` is now invalid



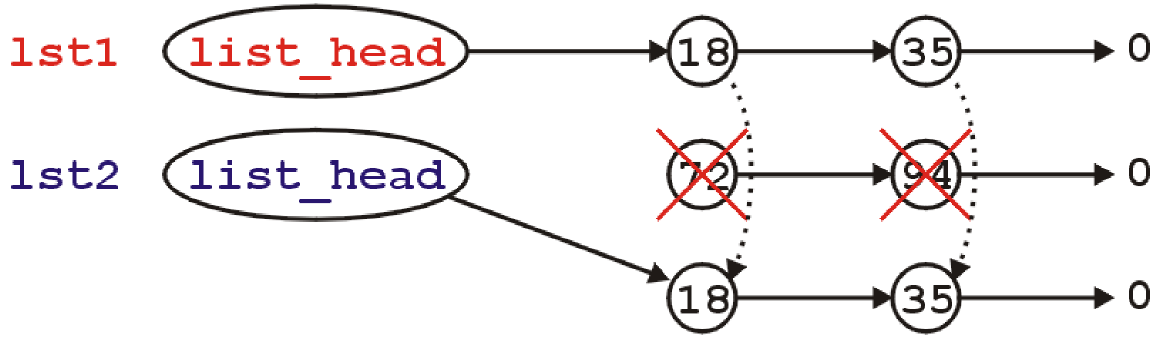
Assignment

Like making copies, we must have a reasonable means of assigning

– Starting with



– We need to erase the content of `lst2` and copy over the nodes in `lst1`



Assignment

First, to overload the assignment operator, we must overload the function named `operator =`

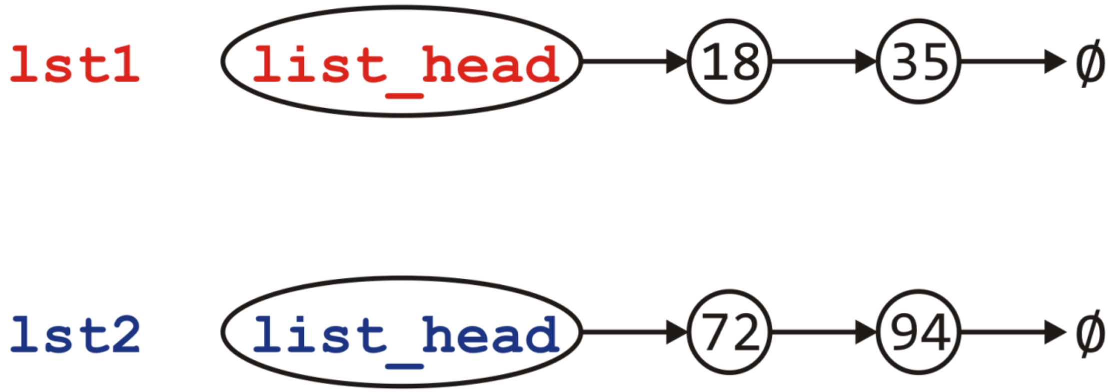
– This is a how you indicate to the compiler that you are overloading the assignment (`=`) operator

The signature is:

```
List &operator = ( List );
```

Assignment

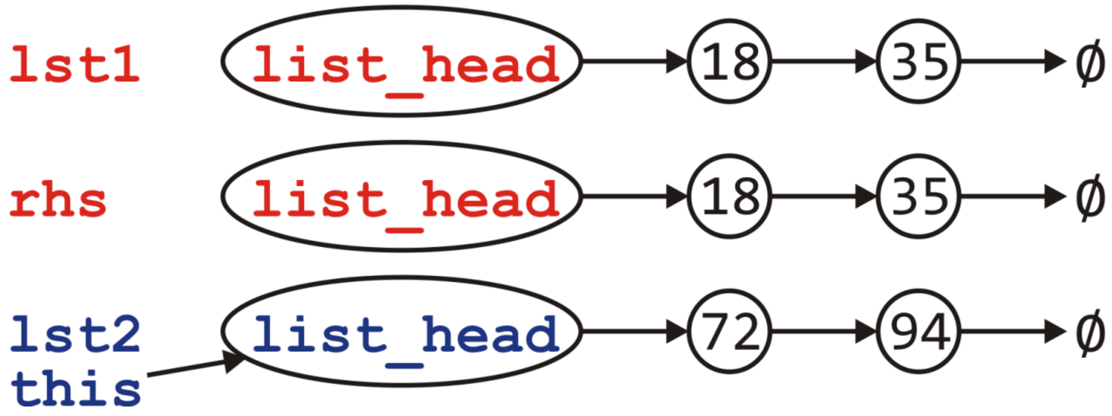
Visually, we are doing the following:



Assignment

Visually, we are doing the following:

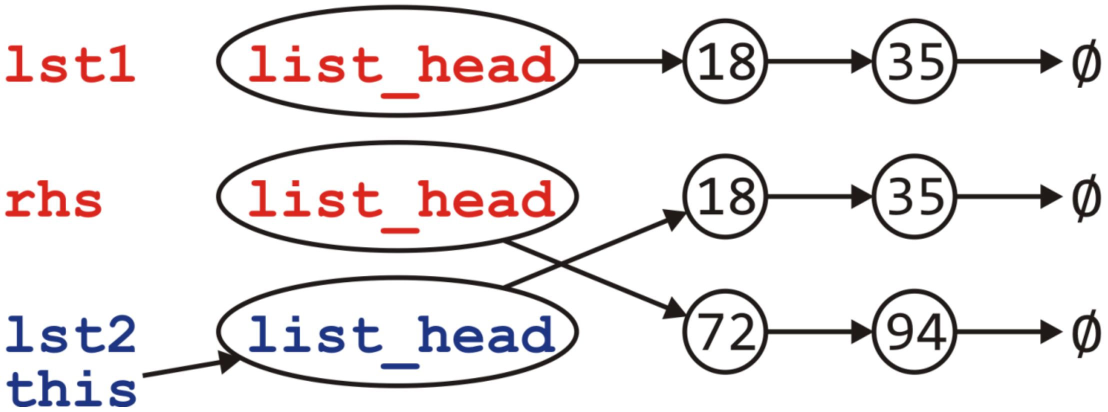
– Call the copy constructor to create `rhs`



Assignment

Visually, we are doing the following:

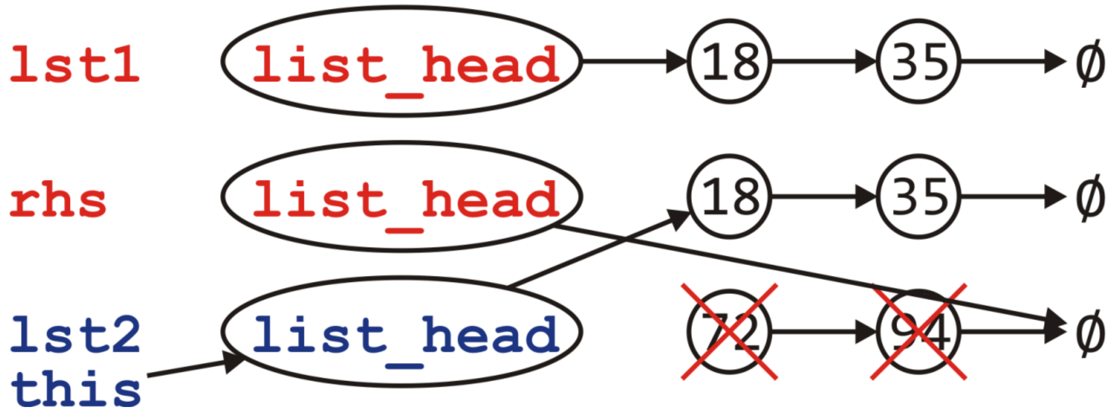
- Call the copy constructor to create `rhs`
- Swapping the member variables of `*this` and `rhs`



Assignment

Visually, we are doing the following:

– Call the copy constructor to create `rhs`
– Swapping the member variables of `*this` and `rhs`
– The destructor is called on `rhs`



Assignment

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- Call the copy constructor to create `rhs`
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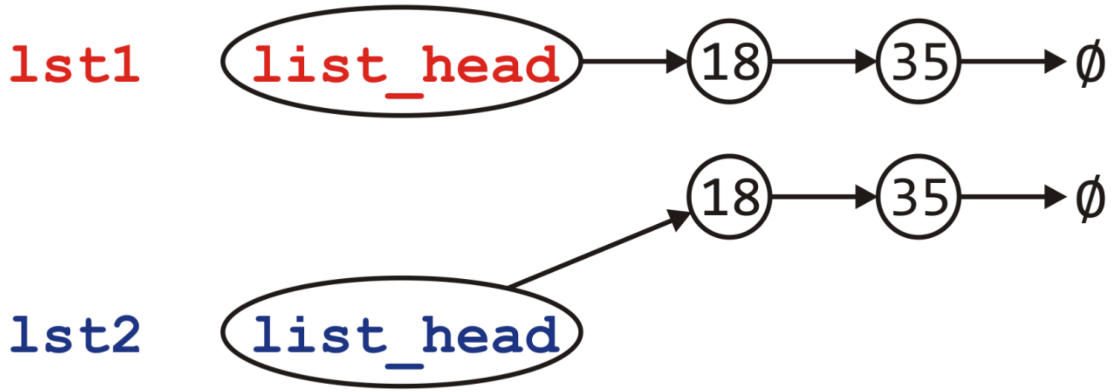
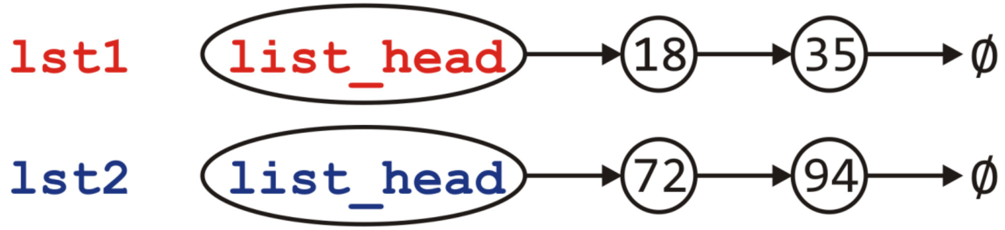


Assignment

Can we do better?

Consider the calls to `new` and `delete`

- Each of these is very expensive…
- Would it not be better to reuse the nodes if possible?



Assignment

Can we do better?

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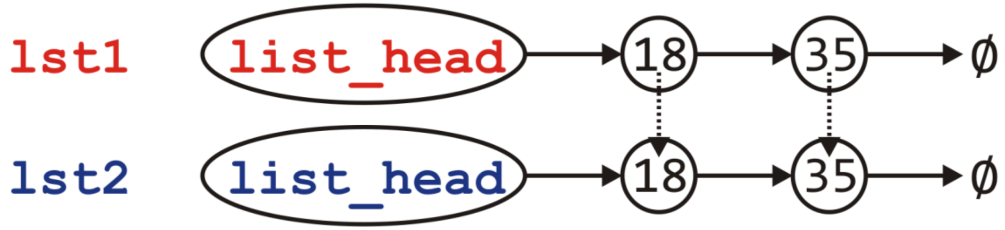


- No calls to `new` or `delete`

Assignment

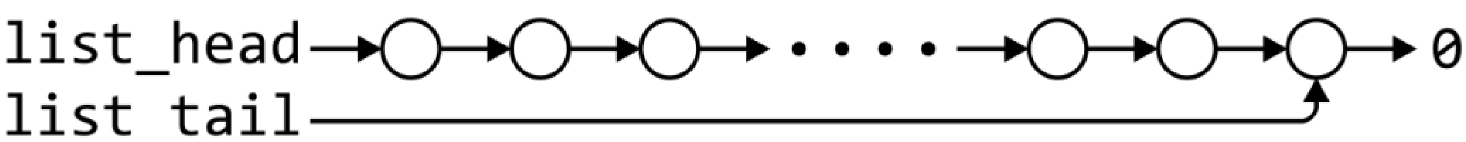
What is the plan?

- If the right-hand side is empty, it’s straight-forward:
  - Just empty this list
- Otherwise, step through the right-hand side list and for each node there
  - If there is a corresponding node in this, copy over the value, else
  - There is no corresponding node; create a new node and append it
- If there are any nodes remaining in this, delete them

Linked list

	Front/1^{st} node	k^{th} node	Back/n^{th} node
Find	$\Theta(1)$	$O(n)$	$\Theta(1)$
Insert Before	$\Theta(1)$	$O(n)$	$\Theta(n)$
Insert After	$\Theta(1)$	$\Theta(1)^*$	$\Theta(1)$
Replace	$\Theta(1)$	$\Theta(1)^*$	$\Theta(1)$
Erase	$\Theta(1)$	$O(n)$	$\Theta(n)$
Next	$\Theta(1)$	$\Theta(1)^*$	n/a
Previous	n/a	$O(n)$	$\Theta(n)$

* These assume we have already accessed the k^{th} entry—an $O(n)$ operation



Assume we have a tail pointer

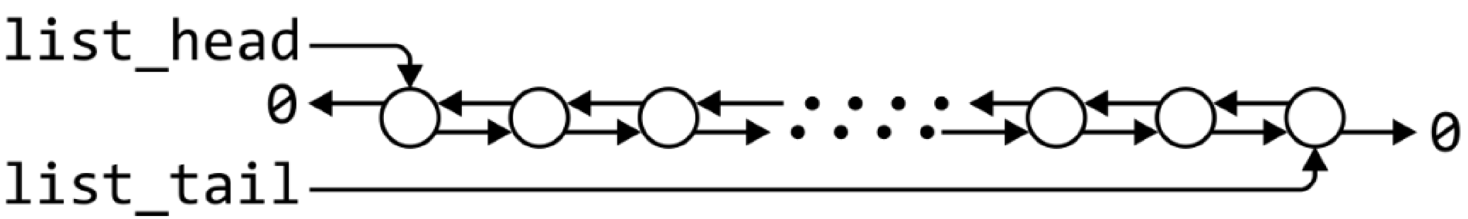
Outline

- List ADT
- Linked list
- Doubly linked list
- Node-based storage with arrays
- Application

Doubly linked lists

	Front/1st node	k^{th} node	Back/n^{th} node
Find	$\Theta(1)$	$O(n)$	$\Theta(1)$
Insert Before	$\Theta(1)$	$\Theta(1)^*$	$\Theta(1)$
Insert After	$\Theta(1)$	$\Theta(1)^*$	$\Theta(1)$
Replace	$\Theta(1)$	$\Theta(1)^*$	$\Theta(1)$
Erase	$\Theta(1)$	$\Theta(1)^*$	$\Theta(1)$
Next	$\Theta(1)$	$\Theta(1)^*$	n/a
Previous	n/a	$\Theta(1)^*$	$\Theta(1)$

* These assume we have already accessed the k^{th} entry—an $O(n)$ operation



Memory usage versus run times

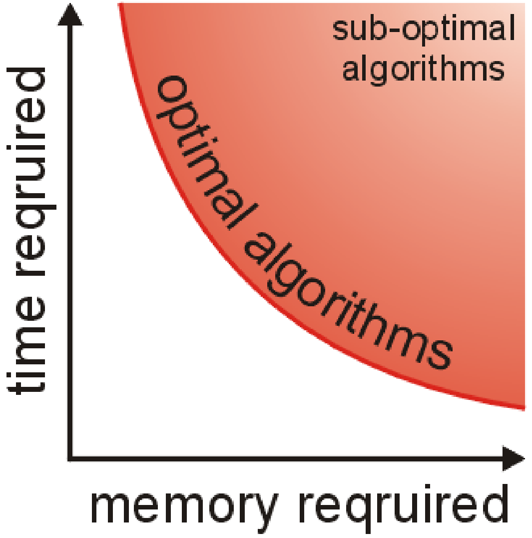
Using a doubly linked list requires $\Theta(n)$ additional memory, but it speeds up many operations

Memory usage versus run times

In general, there is an interesting relationship between memory and time efficiency

For a data structure/algorithm:

- Improving the run time usually requires more memory
- Reducing the required memory usually requires more run time



Memory usage versus run times

Warning: programmers often mistake this to suggest that given any solution to a problem, any solution which may be faster must require more memory

This guideline not true in general: there may be different data structures and/or algorithms which are both faster and require less memory

– This requires thought and research

Outline

- List ADT
- Linked list
- Doubly linked list
- Node-based storage with arrays
- Application

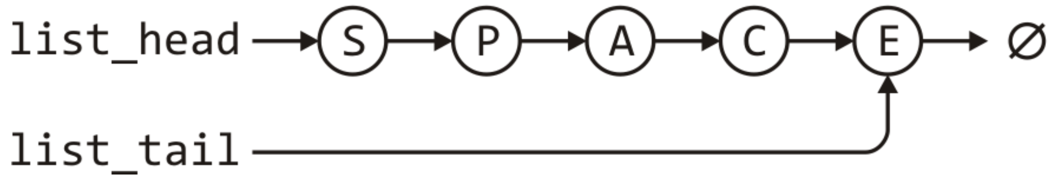
The issue

A significant issue with linked lists: node-based data structures require $\Theta(n)$ calls to `new`

– Each `new` operation requires a call to the operating system requesting a memory allocation

Using an array?

Suppose we store this linked list in an array?




```
list_head = 5;
list_tail = 2;
```

0	1	2	3	4	5	6	7
A		E	P		S	C	
6		-1	0		3	2	

Using an array?

Rather than using, `-1`, use a constant assigned that value

- This makes reading your code easier

```
list_head = 5;
list_tail = 2;
```

0	1	2	3	4	5	6	7
A		E	P		S	C	
6		NULLPTR	0		3	2	

A solution

To achieve this, we must create an array of objects that:

- Store the value
- Store the array index where the next entry is stored

```
template <typename Type>
class Single_node {
    private:
        Type element;
        int next_node;
    public:
        Type retrieve() const;
        int next() const;
};
```

A solution

Now, memory allocation is done once in the constructor:

```
template <typename Type>
class Single_list {
    private:
        int list_capacity;
        int list_head;
        int list_tail;
        int list_size;
        Single_node<Type> *node_pool;

        static const int NULLPTR;
    public:
        Single_list( int = 16 );
        // member functions
};

const int Single_list::NULLPTR = -1;
```

```
template <typename Type>
Single_list<Type>::Single_list( int n ):
list_capacity( n ),
list_head( NULLPTR ),
list_tail( NULLPTR ),
list_size( 0 ),
node_pool( new Single_node<Type>[n] ) {
    // Empty constructor
}
```

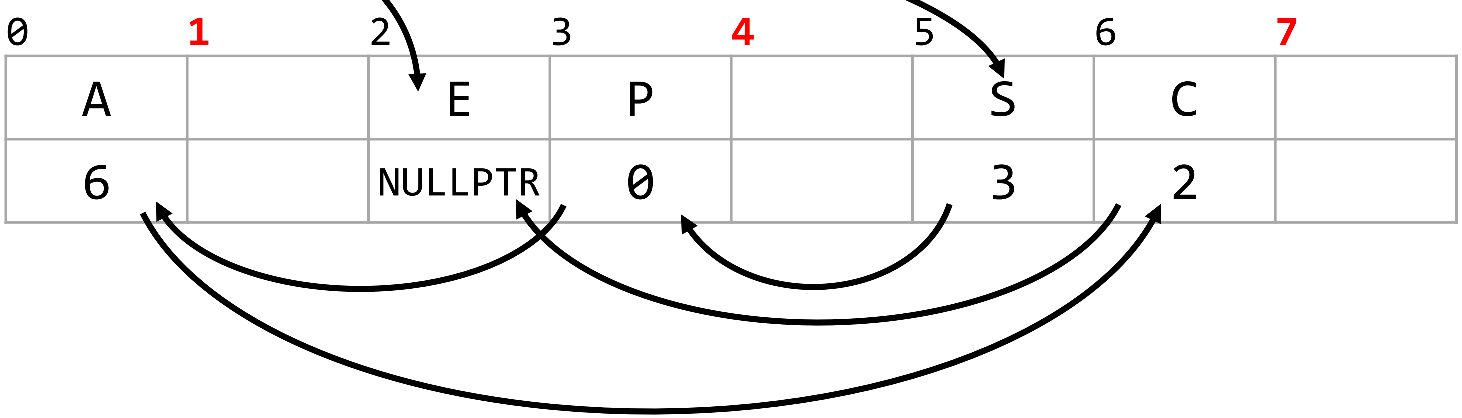
A solution

Problem: when inserting a new element…

how do you know which cell to use?

– Solution: keep a container (a stack) of the indices of unused nodes

```
list_head = 5;
list_tail = 2;
```




```
stack_size = 3;
```

0	1	2	3	4	5	6	7
7	1	4					

A solution

The stack would be initialized with all the entries

```
list_head = NULLPTR;
list_tail = NULLPTR;
```

0	1	2	3	4	5	6	7

```
stack_size = 8;
```

0	1	2	3	4	5	6	7
0	1	2	3	4	5	6	7

A solution

When pushing onto the list, the entry at the top of the stack is used

```
list_head = NULLPTR;
list_tail = NULLPTR;
```

0	1	2	3	4	5	6	7

```
stack_size = 8;
```

0	1	2	3	4	5	6	7
0	1	2	3	4	5	6	7

A solution

Now, `push_front( 'O' )` would result in

```
list_head = 7;
list_tail = 7;
```

0	1	2	3	4	5	6	7
							O
							NULLPTR

```
stack_size = 7;
```

0	1	2	3	4	5	6	7
0	1	2	3	4	5	6	7

A solution

Suppose we call `push_front( 'N' )`

```
list_head = 7;
list_tail = 7;
```

0	1	2	3	4	5	6	7
							O
							NULLPTR

```
stack_size = 7;
```

0	1	2	3	4	5	6	7
0	1	2	3	4	5	6	7

A solution

Suppose we call `push_front( 'N' )`

– The next node is at index 6

```
list_head = 6;
list_tail = 7;
```

0	1	2	3	4	5	6	7
						N	O
						7	NULLPTR

```
stack_size = 6;
```

0	1	2	3	4	5	6	7
0	1	2	3	4	5	6	7

A solution

Suppose we call `push_back( 'R' )`

```
list_head = 6;
list_tail = 7;
```

0	1	2	3	4	5	6	7
						N	O
						7	NULLPTR

```
stack_size = 6;
```

0	1	2	3	4	5	6	7
0	1	2	3	4	5	6	7

A solution

Suppose we call `push_back( 'R' )`

– The next node is at index 5

```
list_head = 6;
list_tail = 5;
```

0	1	2	3	4	5	6	7
					R	N	O
					NULLPTR	7	5

```
stack_size = 5;
```

0	1	2	3	4	5	6	7
0	1	2	3	4	5	6	7

A solution

Finally, suppose we call `pop_front()`

```
list_head = 6;
list_tail = 5;
```

0	1	2	3	4	5	6	7
					R	N	O
					NULLPTR	7	5

```
stack_size = 5;
```

0	1	2	3	4	5	6	7
0	1	2	3	4	5	6	7

A solution

Finally, suppose we call `pop_front()`

– The popped node is placed back into the stack

```
list_head = 7;
list_tail = 5;
```

0	1	2	3	4	5	6	7
					R	N	O
					NULLPTR	7	5

```
stack_size = 6;
```

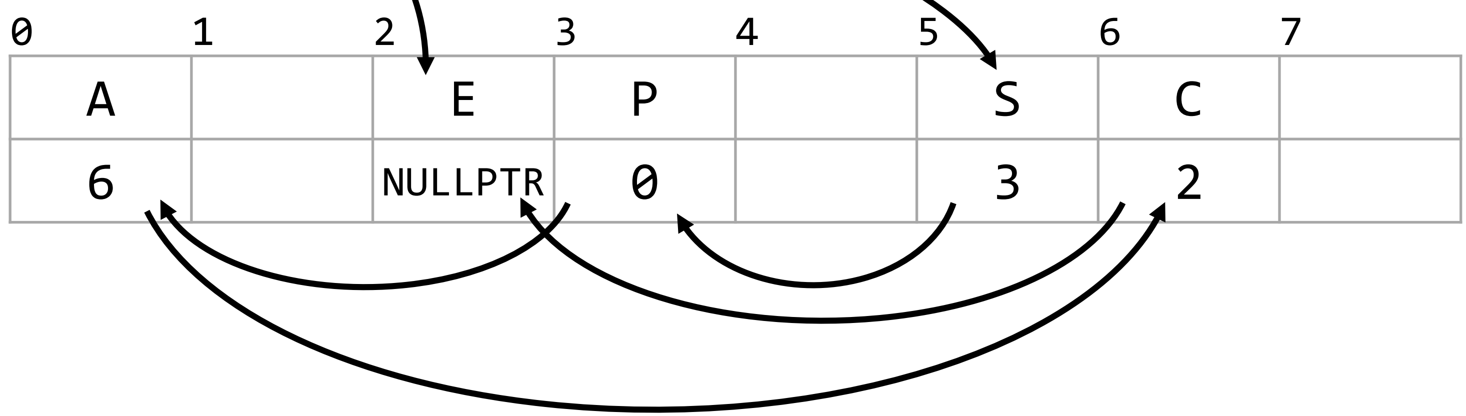
0	1	2	3	4	5	6	7
0	1	2	3	4	6	6	7

A better solution

Problem:

- Our solution requires $\Theta(N)$ additional memory
- In our initial example, the unused nodes are 1, 4 and 7
- How about using these to define a second stack-as-linked-list?

```
list_head = 5;
list_tail = 2;
```



A better solution

Problem:

- Our solution requires $\Theta(N)$ additional memory
- In our initial example, the unused nodes are 1, 4 and 7
- How about using these to define a second stack-as-linked-list?

```
list_head = 5;
list_tail = 2;
stack_top = 1;
```

0	1	2	3	4	5	6	7
A		E	P		S	C	
6	4	NULLPTR	0	7	3	2	NULLPTR

- We only need a head pointer for the stack-as-linked-list

A better solution

Suppose we call `pop_front()`

```
list_head = 5;
list_tail = 2;
stack_top = 1;
```

0	1	2	3	4	5	6	7
A		E	P		S	C	
6	4	NULLPTR	0	7	3	2	NULLPTR

A better solution

Suppose we call `pop_front()`

– The extra node is placed onto the stack

```
list_head = 3;
list_tail = 2;
stack_top = 5;
```

0	1	2	3	4	5	6	7
A		E	P		S	C	
6	4	NULLPTR	0	7	1	2	NULLPTR

A better solution

Suppose we now call `push_back( 'D' )`

```
list_head = 3;
list_tail = 2;
stack_top = 5;
```

0	1	2	3	4	5	6	7
A		E	P		S	C	
6	4	NULLPTR	0	7	1	2	NULLPTR

A better solution

Suppose we now call `push_back( 'D' )`

- We pop the node off of the top of the stack

```
list_head = 3;
list_tail = 5;
stack_top = 1;
```

0	1	2	3	4	5	6	7
A		E	P		D	C	
6	4	5	0	7	NULLPTR	2	NULLPTR

A better solution

Suppose we finally call `pop_front()` again

```
list_head = 3;
list_tail = 5;
stack_top = 1;
```

0	1	2	3	4	5	6	7
A		E	P		D	C	
6	4	5	0	7	NULLPTR	2	NULLPTR

A better solution

Suppose we finally call `pop_front()` again

– The node containing `'P'` is pushed back onto the stack

```
list_head = 0;
list_tail = 5;
stack_top = 3;
```

0	1	2	3	4	5	6	7
A		E	P		D	C	
6	4	5	1	7	NULLPTR	2	NULLPTR

A better solution

In this case, our data structure would be initialized to:

```
list_head = NULLPTR;
list_tail = NULLPTR;
stack_top = 0;
```

0	1	2	3	4	5	6	7
1	2	3	4	5	6	7	NULLPTR

A solution

Our class would look something like:

```
template <typename Type>
class Single_list {
    private:
        int list_head;
        int list_tail;
        int list_size;
        int list_capacity;
        Single_node<Type> *node_pool;
        int stack_top;

        static const int NULL;
    public:
        Single_list( int = 16 );
        // member functions
};

const int Single_list::NULLPTR = -1;
```

```
template <typename Type>
Single_list<Type>::Single_list( int n ):
list_head( NULLPTR ),
list_tail( NULLPTR ),
list_size( 0 ),
list_capacity( n ),
node_pool( new Single_node<Type>[n] ),
stack_top( 0 ) {
    for ( int i = 1; i < n; ++i ) {
        node_pool[i - 1].next = i;
    }

    node_pool[n - 1] = NULLPTR;
}
```

Analysis

This solution:

- Requires only three more member variable than our linked list class
- It still requires $\mathrm{O}(N)$ additional memory over an array
- All the run-times are identical to that of a linked list
- Only one call to `new`, as opposed to $\Theta(n)$
- There is a potential for up to $\mathrm{O}(N)$ wasted memory

Question: What happens if we run out of memory?

Reallocation of memory

Suppose we start with a capacity N but after a while, all the entries have been allocated

– We can double the size of the array and copy the entries over

```
list_head = 6;
list_tail = 4;
list_size = 8;
list_capacity = 8;
stack_top = NULLPTR;
```

0	1	2	3	4	5	6	7
C	R	U	T	R	U	S	T
7	2	0	1	NULLPTR	4	3	5

Reallocation of memory

Suppose we start with a capacity N but after a while, all the entries have been allocated

- We can double the size of the array and copy the entries over
- Only the stack needs to be updated and the old array deleted

```
list_head = 6;
list_tail = 4;
list_size = 8;
list_capacity = 16;
stack_top = 8;
```

0	1	2	3	4	5	6	7
C	R	U	T	R	U	S	T
7	2	0	1	NULLPTR	4	3	5

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
C	R	U	T	R	U	S	T								
7	2	0	1	NULLPTR	4	3	5	9	10	11	12	13	14	15	NULLPTR

Reallocation of memory

Now `push_back( 'E' )` would use the next location

```
list_head = 6;
list_tail = 4;
list_size = 8;
list_capacity = 16;
stack_top = 8;
```

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
C	R	U	T	R	U	S	T								
7	2	0	1	NULLPTR	4	3	5	9	10	11	12	13	14	15	NULLPTR

Reallocation of memory

Now `push_back( 'E' )` would use the next location

```
list_head = 6;
list_tail = 8;
list_size = 9;
list_capacity = 16;
stack_top = 9;
```

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
C	R	U	T	R	U	S	T	E							
7	2	0	1	8	4	3	5	NULLPTR	10	11	12	13	14	15	NULLPTR

Reallocation of memory

If at some point, we decide it is desirable to reduce the memory allocated, it might be easier to just insert the entries into a newer and smaller table

```
list_head = 4;
list_tail = 5;
list_size = 4;
list_capacity = 16;
stack_top = 7;
```

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		T		D	A					A					
14	9	5	0	10	NULLPTR	8	12	NULLPTR	3	2	13	1	15	11	6

Reallocation of memory

If at some point, we decide it is desirable to reduce the memory allocated, it might be easier to just insert the entries into a newer and smaller table

```
list_head = 4;
list_tail = 5;
list_size = 4;
list_capacity = 16;
stack_top = 7;
```

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		T		D	A					A					
14	9	5	0	10	NULLPTR	8	12	NULLPTR	3	2	13	1	15	11	6

0	1	2	3	4	5	6	7
D	A	T	A				
1	2	3	NULLPTR	5	6	7	NULLPTR

Reallocation of memory

If at some point, we decide it is desirable to reduce the memory allocated, it might be easier to just insert the entries into a newer, and smaller table

- Now, delete the old array and update the member variables

```
list_head = 0;
list_tail = 3;
list_size = 4;
list_capacity = 8;
stack_top = 4;
```

0	1	2	3	4	5	6	7
D	A	T	A				
1	2	3	NULLPTR	5	6	7	NULLPTR

Outline

- List ADT
- Linked list
- Doubly linked list
- Node-based storage with arrays
- Application

Polynomial

- Possible linked list implementation
  - A_i is the coefficient of the x^{i-1} term

$5 + 2x + 3x^2$	**(5 2 3)**
$7 + 8x$	**(7 8)**
$3 + x^2$	**(3 0 2)**

Problem?

$$4 + 3x^{2001}$$

(4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
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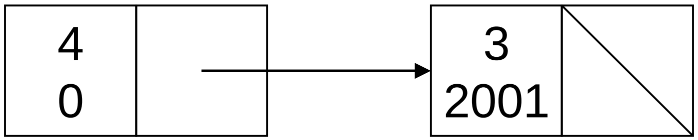
Sparse Vector Data Structure:

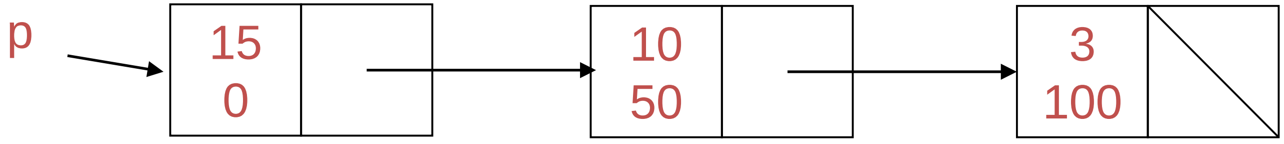
$4 + 3x^{2001}$

`(<4 0>  <2001 3>)`

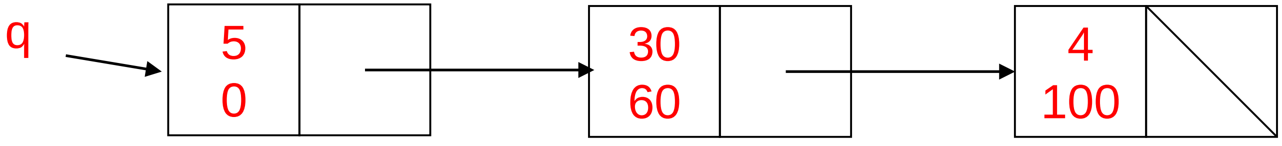


Addition of Two Polynomials?

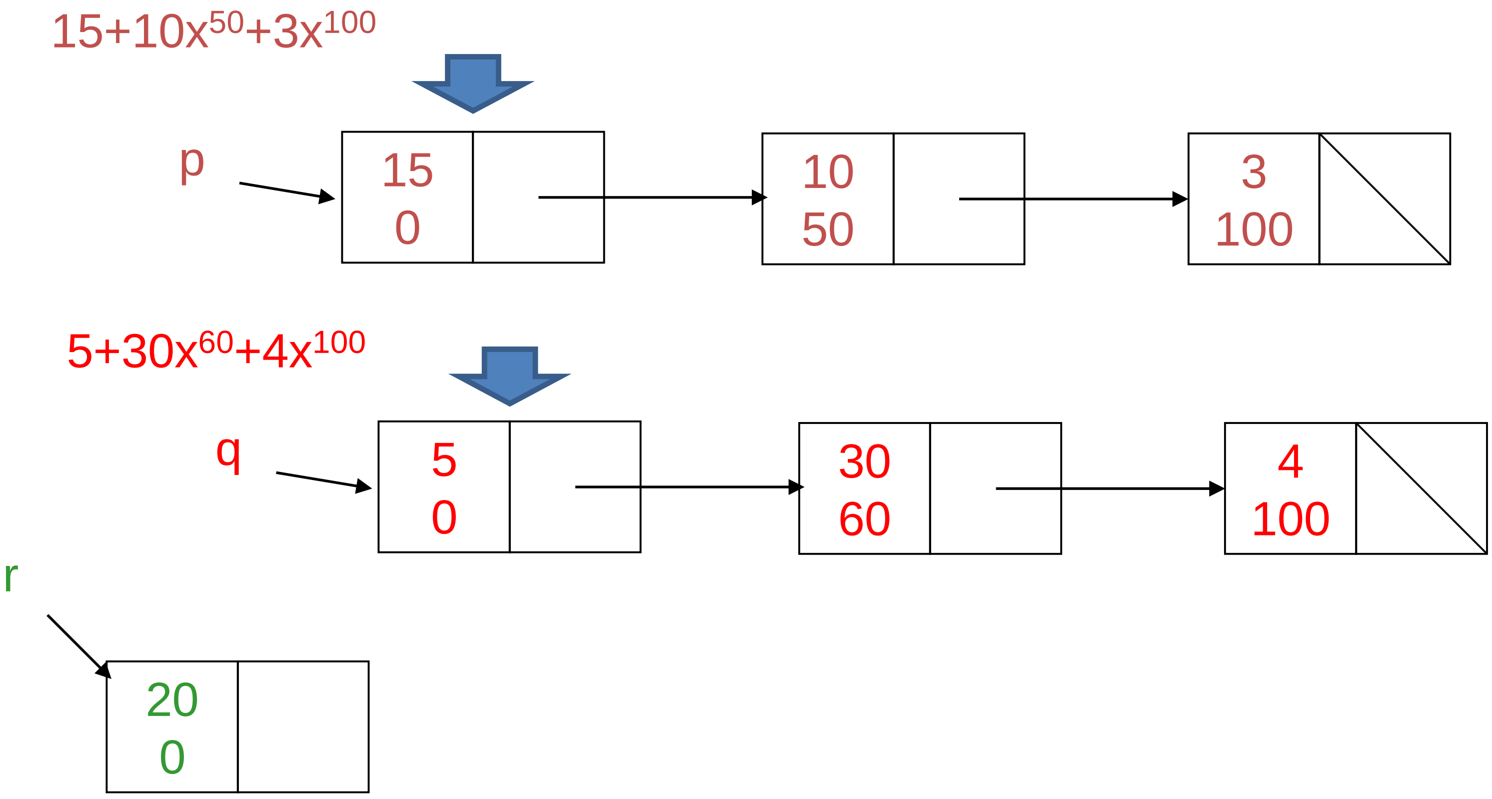
$15+10x^{50}+3x^{100}$



$5+30x^{60}+4x^{100}$



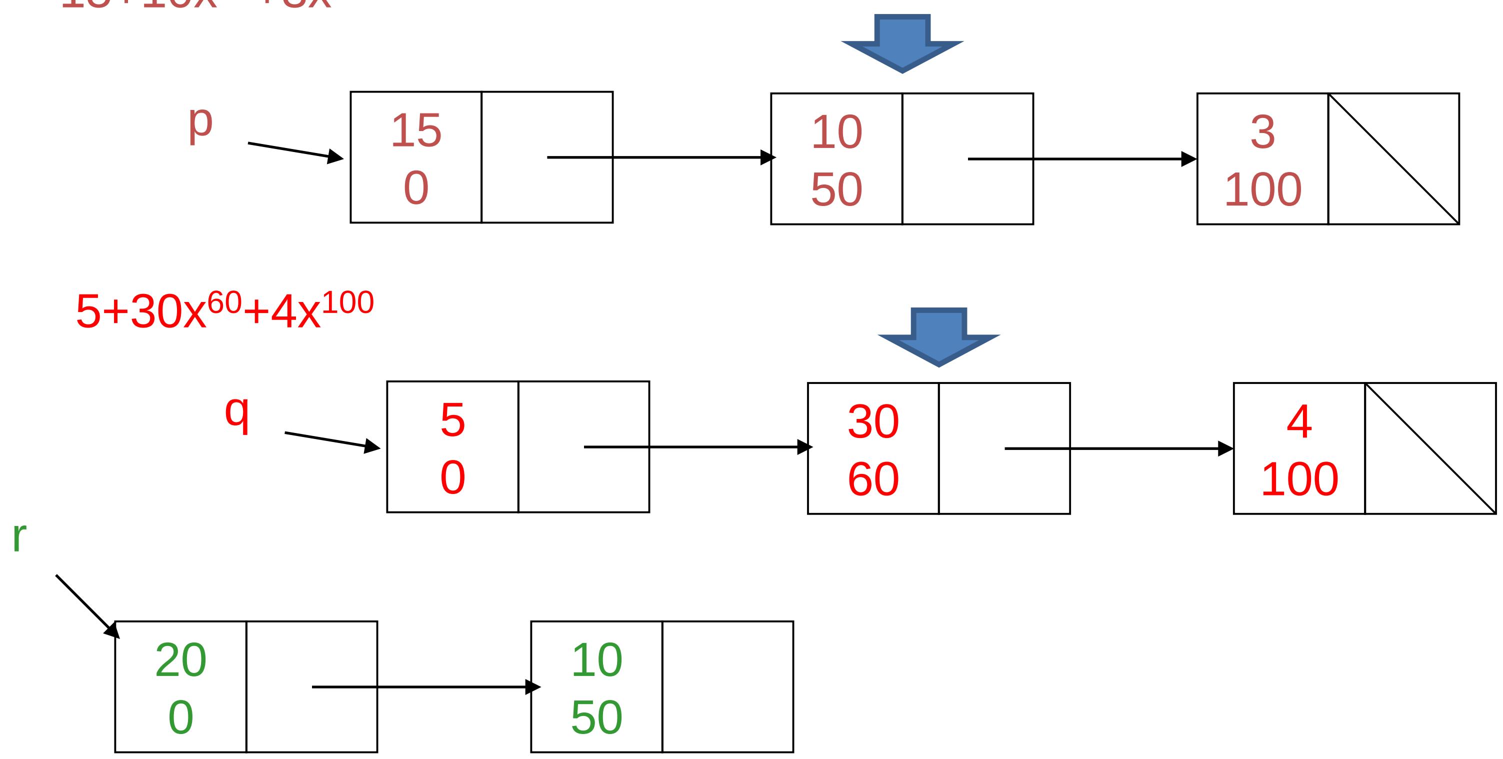
Addition of Two Polynomials



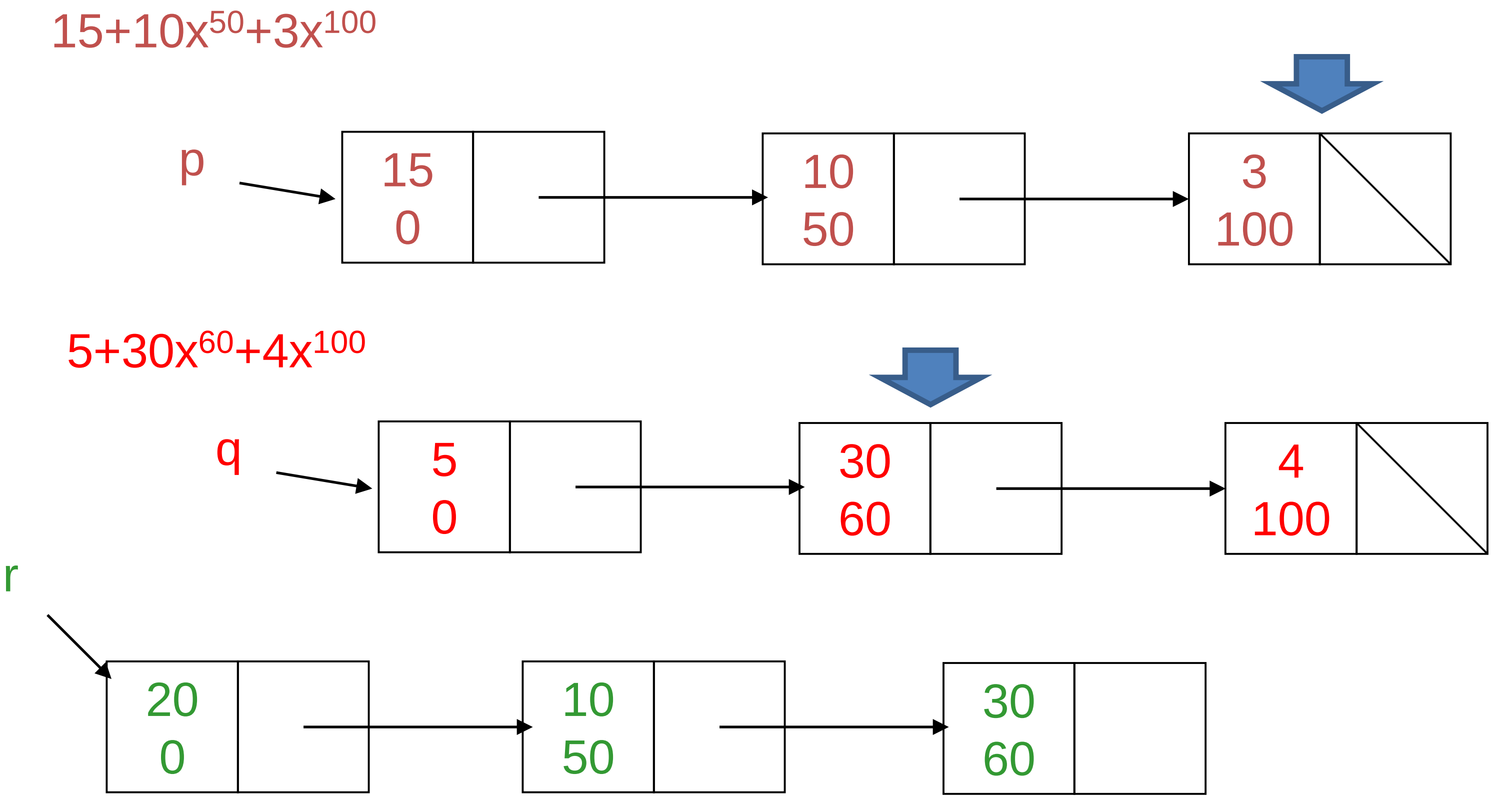
Addition of Two Polynomials

$15+10x^{50}+3x^{100}$

$5+30x^{60}+4x^{100}$



Addition of Two Polynomials

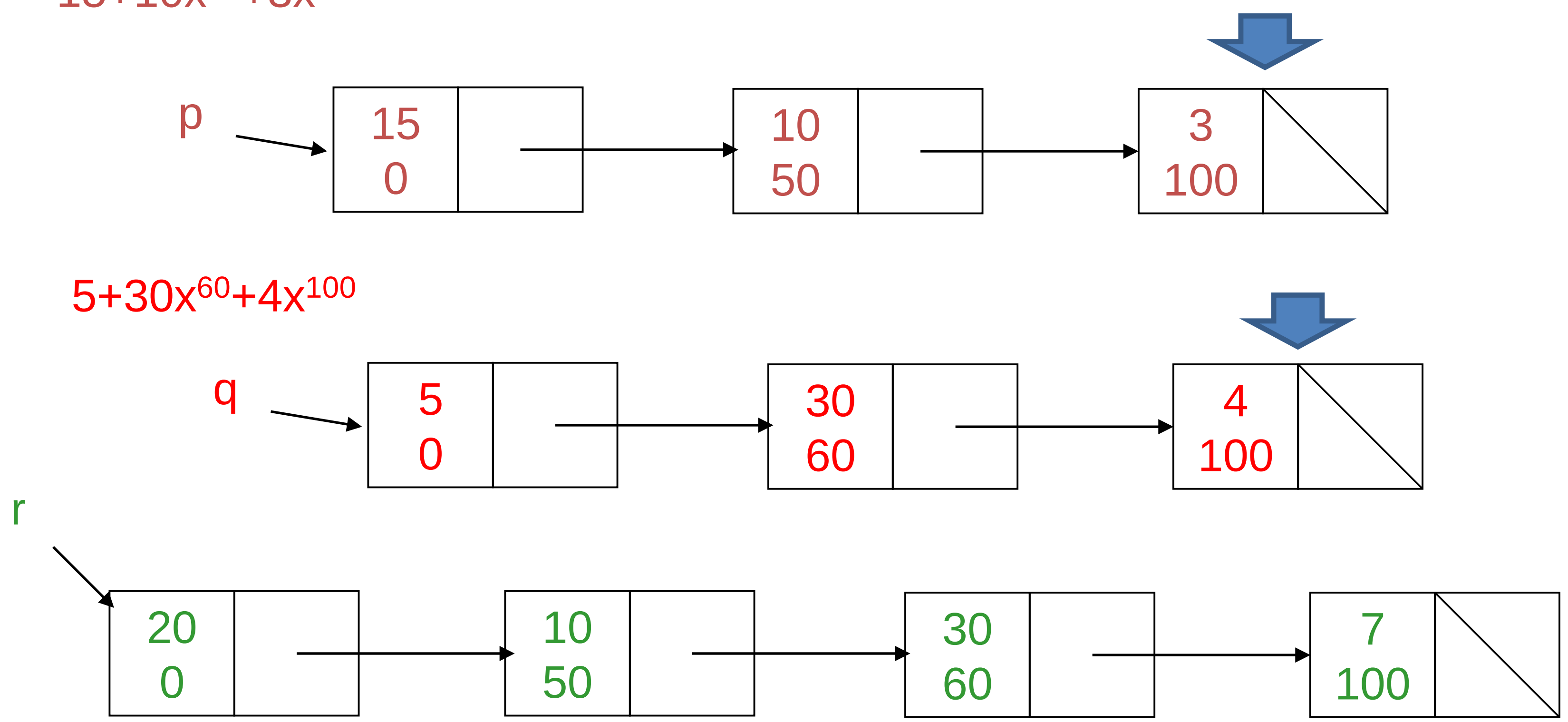


Addition of Two Polynomials

- One pass down each list: $\Theta(n+m)$

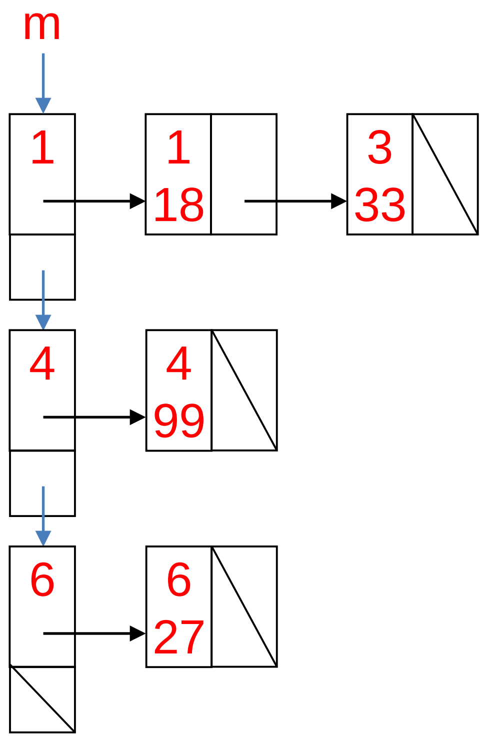
$15+10x^{50}+3x^{100}$

$5+30x^{60}+4x^{100}$



Sparse Matrices

18	0	33	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	99	0	0
0	0	0	0	0	0
0	0	0	0	0	27



Summary

- List ADT
  - A sequence of elements (special case: string)
  - Array
- Linked list
  - Accessors and mutators
  - Stepping through a linked list
  - Copy and assignment operator
- Doubly linked list
  - Memory usage versus run times
- Node-based storage with arrays
  - No longer need to call new for each new node
- Application
  - Polynomial, sparse matrix