PRA3021 Assignment 5

Tom Kores Lesjak

October 2024

1 Task 1

1.1 Question 1

Question: Consider $f(t) = 0.8 \sin\left(2\pi \cdot \left(3t - \frac{1}{6}\right)\right) + 0.3 \sin(2\pi \cdot 5t)$. Sample the signal f up to $T = 3$ with a sampling rate $v_s = 50\text{Hz}$.

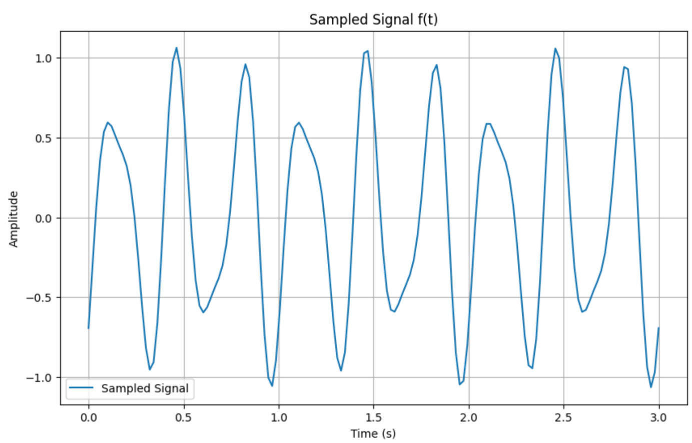


Figure 1: Enter Caption

1.2 Question 2

Question: Use the `dft` function to compute the frequency spectrum of the following functions for the given period at 16Hz and 50Hz. Plot the four functions on the previous slide (at both frequencies) and explain (in words) the connection between the function and the obtained frequency spectrum:

- $f(t) = \sin(10\pi t) + \sin(16\pi t); \quad T = 1$
- $f(t) = \sin(10\pi t) \cdot (2 + \sin(4\pi t)); \quad T = 1$
- $f(t) = (t + 1) \mod 2 - 1; \quad T = 2$
- $f(t) = (t + 1) \mod 2; \quad T = 2$

Plots for 16Hz:

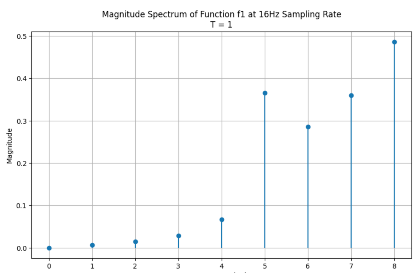


Figure 2: Function 1 at 16Hz: $f(t) = \sin(10\pi t) + \sin(16\pi t)$

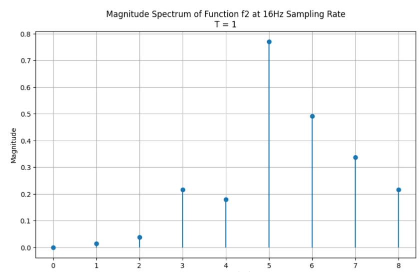


Figure 3: Function 2 at 16Hz: $f(t) = \sin(10\pi t) \cdot (2 + \sin(4\pi t))$

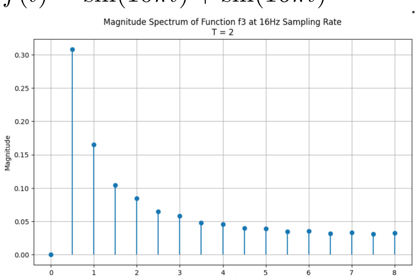


Figure 4: Function 3 at 16Hz: $f(t) = (t + 1) \mod 2 - 1$

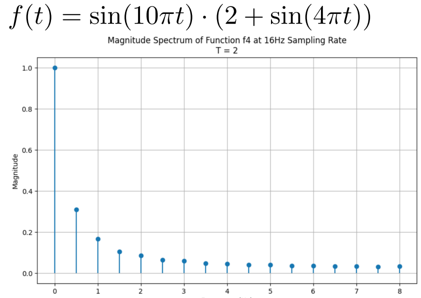


Figure 5: Function 4 at 16Hz: $f(t) = (t + 1) \mod 2$

Plots for 50Hz:

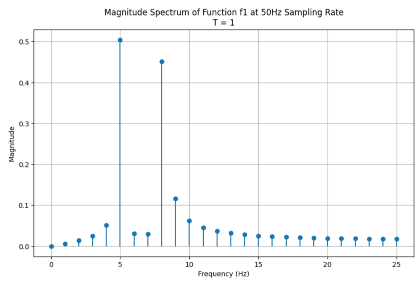


Figure 6: Function 1 at 50Hz: $f(t) = \sin(10\pi t) + \sin(16\pi t)$

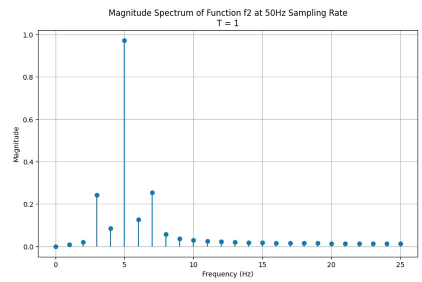


Figure 7: Function 2 at 50Hz: $f(t) = \sin(10\pi t) \cdot (2 + \sin(4\pi t))$

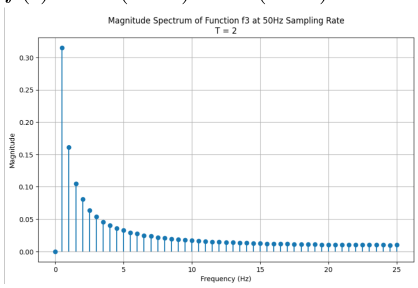


Figure 8: Function 3 at 50Hz: $f(t) = (t + 1) \mod 2 - 1$

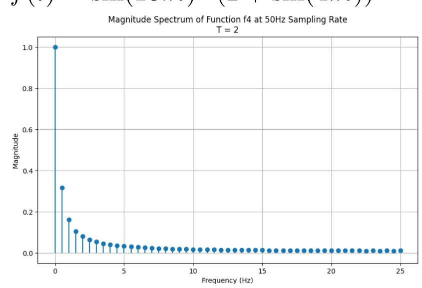


Figure 9: Function 4 at 50Hz: $f(t) = (t + 1) \mod 2$

Each function represents a periodic or quasi-periodic signal. The DFT analyzes these signals over the given time period T by converting them into a sum of sinusoidal components at different frequencies. The frequency spectrum reveals which frequencies contribute to the original signal and how strong each of those contributions is, represented by the magnitude of the Fourier coefficients.In the case of purely sinusoidal functions, the frequency spectrum will show distinct peaks at the frequencies corresponding to those sine waves. The magnitude of these peaks corresponds to the amplitude of the sine waves. For more complex functions, such as those involving division or modulus operations, the DFT reveals contributions from multiple frequencies. These are often harmonics of a base frequency.

According to Nyquist's theorem, the sampling rate must be at least twice the highest frequency present in the signal to avoid aliasing. By analyzing the same function at two different sampling rates (16 Hz and 50 Hz), we can observe how the DFT captures finer or coarser details in the frequency domain. At a higher sampling rate, the frequency spectrum is more detailed and can capture higher frequency components that may be missed at a lower sampling rate. Conversely, at a lower sampling rate, only the main frequencies are captured, and higher frequencies (above 8 Hz in this case) cannot be resolved accurately. For the function $f_1(t) = \sin(10\pi t) + \sin(16\pi t)$ with $T = 1$, we observe two distinct peaks at 5 Hz and 8 Hz in the frequency spectrum, corresponding to the two sine waves in the signal. For $f_2(t) = \frac{\sin(10\pi t)}{2+\sin(4\pi t)}$, the base frequency of 5 Hz remains prominent, but additional frequencies appear due to the modulation effect caused by the denominator. In the case of $f_3(t) = (t+1) \mod 2-1$, with $T = 2$, the spectrum shows multiple harmonics due to the sharp transitions in the signal. Finally, for $f_4(t) = (t+1) \mod 2$, we see a set of harmonics in the frequency spectrum due to the discontinuities.

1.3 Question 3

Question: Plot the reconstruction of the following function obtained from DFT in the previous task:

$$f(t) = \sin(10\pi t) + \sin(16\pi t), \quad T = 1$$

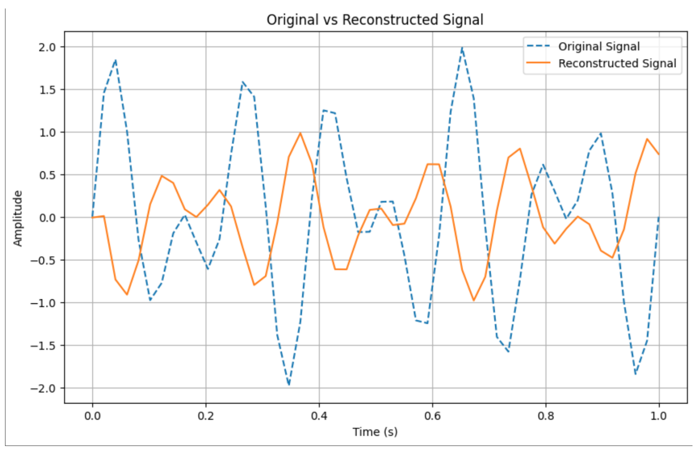


Figure 10: Reconstruction of original function from idft

2 Task 2

2.1 Question 1

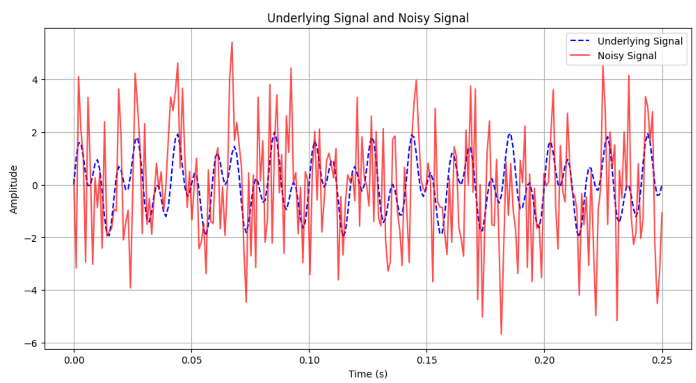
Question: Plot the underlying and noisy signal against time on the same plot. Use different colours for the two signals.

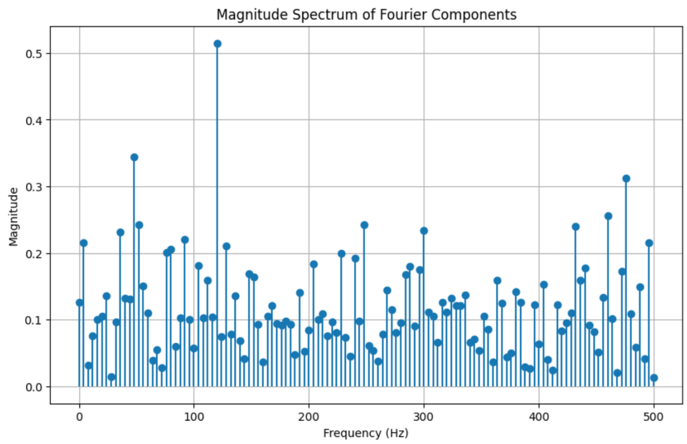
Use DFT to obtain the Fourier coefficients for the noisy signal. Plot the magnitude spectrum obtained.

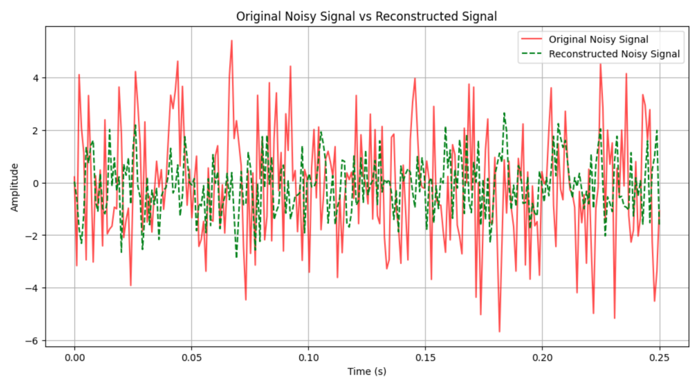
Use IDFT to reconstruct the noisy signal. Plot the original noisy signal and reconstructed noisy signal on the same plot.

Answer: Are the two signals the same? Why or why not?

The two signals are not exactly the same. The finite precision of the DFT plays a role. The DFT operates on a finite number of samples, and when we reconstruct the signal using the IDFT, there is some limitation because the Fourier series is truncated, especially for the high-frequency components. This results in a slight loss of detail, particularly in the high-frequency parts of the signal. Also, the added noise introduces high-frequency components into the signal. While the DFT captures these frequencies, reconstructing the signal using IDFT can smooth out some of these random fluctuations due to the limited number of Fourier coefficients used in the reconstruction.







2.2 Question 2

Question: Perform filtering on the power spectral density of the decomposed noisy signal such that only the two highest frequencies are preserved. In other words, set all other frequencies to zero.

Plot the magnitude spectrum of the noisy and filtered signal on the same plot.

Reconstruct the filtered signal using IDFT and plot the filtered signal and underlying signal on the same plot.

Answer: Are the two signals the same? Why or why not? The two signals are not the same. This is because the filtered signal only retains the two highest frequencies from the noisy signal, whereas the underlying signal consists of the original sinusoidal components without any noise. In the filtering step, we set all other frequencies, including noise and lower-frequency components, to zero, preserving only the two highest frequency components from the noisy signal. This alters the structure of the signal, causing the filtered signal to differ from the underlying signal.

Second, the underlying signal contains both high- and low-frequency components, but the filtering process removes frequencies other than the two highest. As a result, some of the information in the underlying signal is lost during filtering.

In conclusion, the filtered signal is an approximation based on the two dominant frequencies, but it does not capture the full detail of the original underlying signal.