PHIL 3something - Logic II

Scott Saunders - 10163541

Winter 2016

Misc. Notation

- The set of positive integers $\{x : x$ is a positive integer $\}$

- The set of positive integers less than3 $\{x : x$ is a positive integer and x is less than 3 $\}$. $= \{1, 2\}$.

- The empty set: $\emptyset$ or Δ

- Member of: $A \subseteq B$ iff $\forall X(x \in A \implies x \in B)$

- Union of A and B: $A \cup B$ iff $\{x : x \in A \vee x \in B\}$

- Intersection of A and B: $A \cap B$ iff $\{x : x \in A \wedge x \in B\}$

- Difference of A and B: $\{x : x \in A \wedge x \notin B\}$

- For any non-empty sets A, B: Cartesian product: A of B: $A \times B$: $\{<x, y> : x \in A \wedge y \in B\}$ (ALL OF THE POSSIBILITIES)

- TOTAL FUNCTION: Every element in the domain is valid

- PARTIAL FUNCTION: Not every element in the domain is valid.

- for any set of sets A:
  - $\mathbb{U}A = \{x : \exists y(y \in A \wedge x \in y)\}$
  - $\mathbb{m}A = \{x : \forall y(y \in A \rightarrow x \in y)\}$

- Relations: R is
  - reflexive : $\forall x Rxx$
  - symmetric : $\forall x \forall y(Rxy \implies Ryx)$
  - transitive : $\forall x \forall y \forall z((Rxy \wedge Ryz) \implies Rxz)$
  - Euclidean : $\forall x \forall y \forall z((Rxy \wedge Rxz) \implies Ryz)$
  - a equivalence relation : it's symmetric,reflexive,transitive.
  - a (partial) function : $\exists x$ and there is at most one y: Rxy : denoted f
  - a (total) function: assigns a value to each number of A : denoted f

- Domain: The set of a functions arguments.

- Range: The set of its values. (Results)

- f is a function from a set A iff the domain of f is included in A

- f is a function to a set B iff its range is included in B.

- f^{-1} is the inverse of the function f from the set A to the set B iff:if for every member $b \in B$, there is exactly one member of $a \in A$ such that $f(a) = b$, then $f^{-1}(b) = a$, otherwise $f^{-1}(b)$ is undefined.

- f is onto B iff B is the range of f (Surjective)
  Alt: (wikipedia) : $\forall y \in Y, \exists x \in X | y = f(x)$

- f is one-to-one iff $\forall x \forall y (f(x) = f(y) \implies x = y)$ (Injective)

- f is a bijection iff f is onto and one-to-one.

- f is a correspondence iff f is total, one-to-one and onto.

- Sets A and B are equinumerous iff there is a correspondence from A to B.

Equinumerous is transitive. Prove: if A is equinumerous with B and B is equinumerous wit C, then A is equinumerous with C. Proof: Suppose A is equinumerous to B, and B is equinumerous to C. Then: There is a total,one-to-one function f from A onto B, and a total one-to-one function g from B to C. Prove equinumerous via h=g(f), such that h(n)=g(f(n))

- h is total: Let a be a member of A. $h(a) = g(f(a))$. Since f is total there is a member of b of B such that $f(a) = b$). since g is total, there is a member of $c \in C$ such that $g(b) = c$. Hence, h is total.

- h is onto C. WLOG Let c be a member of C, as g is onto, $\exists b \in B$ such that $g(b) = c$. As f is onto, then $\exists a \in A$ such that $f(a) = b$. Hence, the composition of $h = f(g)$ is onto C.

- h is one-to-one: Suppose h is not one-to-one.
  Then there $\exists a_1, a_2 \in A$ such that $h(a_1) = h(a_2), a_1 \neq a_2$.
  Giving $g(f(a_1)) = g(f(a_2)), a_1 / not = a_2$
  Since g is one-to-one $g(b_1) = g(b_2)$ iff $b_1 = b_2$.
  So the issue must lie in f. However f is one-to-one $f(a_1) = f(a_2)$ iff $f(a) = f(b)$. Which is a contradiction, giving us that h is one-to-one.

$\square$

A^n : the nth Cartesian product of A with itself.
Suppose that the set of real Numbers $r, r < r < 1$, is enumerable. Then $L_r : r_1, r_2, r_3....$ written in a notation of $0.n_1 n_2 n_3.(n being natural numbers)$
The set of functions form the set of positive integers to positive integers is not enumerable.
The set of total nomadic functions from the set of positive integers, F^1, is not enumerable.
It's a Proof by contradiction.
Turing machines are in the following form: $q_n, S_{1/0}, S_{1/0}/R/L, q_m$ where q_n is our current state, and you see $S_{1/0}$, perform function $S_{1/0}/R/$ and move to state q_m. If there is no operation specified on the current state for a scan, then it halts. (Also Called the Turing Alphabet)

Example with notation: start $\longrightarrow$ (n)   (m)

ex: (These are the same)

$$Q_1 S_1 R Q_1, Q_1 S_0 S_1 Q_2, Q_2 S_1 L Q_2, Q_2 S_0 R Q_3, Q_3 S_1 S_0 Q_3, Q_3 S_0 R Q_4$$

start $\longrightarrow$ (1) 1:R ; 0:1 (2) 1:L ; 0:R (3) 1:0 ; 0:R (4)
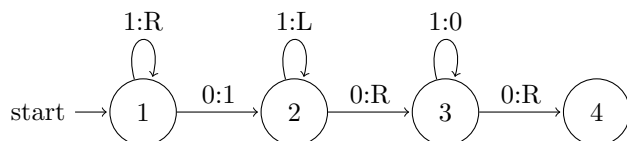
Remark (Turing Machines). • Each Turing machine is a finite set of Turing instructions.

- Each instruction is a 4 letter word of the Turing Alphabet.

- The set of Turing machine is enumerable. (Proof: exercise)