CPSC 589 - Assignment 2

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1. (10 marks) For the case of a planar B-spline curve, does symmetry of the control polygon with respect to the y-axis imply the same symmetry for the curve?

   No.

   Assuming uniform knot sequences, this is. $P_0 = (-1,-1), P_1 = (0,0), P_2 = (1,-1)$, is symmetric, as is
   $//P_0 = (-1,1), P_1 = (1,1), P_2 = (-1,1), P_3 = (1,1)$

   However, with a different knot sequence: 0,0,1 This is no longer symmetric.

2. (10 marks) Derive the formula of third order B-spline with uniform, integer knot sequence from deBoor's recursive formula.

$$N_{i,r}(u) = \frac{u - u_i}{u_{i+r-1} - u_i} N_{i,r-1}(u) + \frac{u_{i+r} - u}{u_{i+r} - u_{i+1}} N_{i+1,r-1}(u)$$

and

$$N_{i,1}(u) = \{1 if u_i \leq u \leq U_{i+1} 0 otherwise$$

Thus:

$$\begin{aligned}
N_{i,3}(u) &= \frac{u - u_i}{u_{i+2} - u_i} N_{i,2}(u) + \frac{u_{i+3} - u}{u_{i+3} - u_{i+1}} N_{i+1,2}(u) \\
&= \frac{u - u_i}{u_{i+2} - u_i} (\frac{u - u_i}{u_{i+1} - u_i} N_{i,1}(u) + \frac{u_{i+2} - u}{u_{i+2} - u_{i+1}} N_{i+1,1}) \\
&+ \frac{u_{i+3} - u}{u_{i+3} - u_{i+1}} (\frac{u - u_i}{u_{i+1} - u_i} N_{i,1}(u) + \frac{u_{i+2} - u}{u_{i+2} - u_{i+1}} N_{i+1,1}(u)) \\
&= \frac{u - u_i}{u_{i+2} - u_i} \frac{u - u_i}{u_{i+1} - u_i} && (\text{ if } (u_i \leq u \leq u_{i+1})\ ) \\
&+ \frac{u - u_i}{u_{i+2} - u_i} \frac{u_{i+2} - u}{u_{i+2} - u_{i+1}} && (\text{ if } (\ u_{i+1} \leq u \leq u_{i+2}\ )) \\
&+ \frac{u_{i+3} - u}{u_{i+3} - u_{i+1}} \frac{u - u_i}{u_{i+1} - u_i} && (\text{ if } (\ u_{i+2} \leq u \leq u_{i+3})) \\
&+ \frac{u_{i+3} - u}{u_{i+3} - u_{i+1}} \frac{u_{i+2} - u}{u_{i+2} - u_{i+1}} && (\text{ if } (u_{i+3} \leq u \leq u_{i+4}))
\end{aligned}$$

Note: I have made a mistake in the above, where on every branch on the right-side, is wronly offset.

3. 3 (10 marks) Show that B-splines of order 3 on 0,0,0,1,1,1 are second degree Berenstein polynomials.

The second degree berenstein polynomial:

$$u^2P_2 + (1-u)*uP_1 + (1-u)^2P_0$$

So for $i = 0$:

$$N_{i=0,3}(u) = \frac{u-0}{0-0}\frac{u-0}{0-0} \qquad + \frac{u-0}{0-0}\frac{0-u}{0-0} + \frac{1-u}{1-0}\frac{u-0}{0-0} \qquad + \frac{1-u}{1-0}\frac{0-u}{0-0}$$

So for $i = 1$:

$$\begin{aligned}
N_{i=1,3}(u) = {} & \frac{u-u_i}{u_{i+2}-u_i}\frac{u-u_i}{u_{i+1}-u_i} && (\text{ if } (u_i \le u \le u_{i+1})\ ) \\
& + \frac{u-u_i}{u_{i+2}-u_i}\frac{u_{i+2}-u}{u_{i+2}-u_{i+1}} && (\text{ if } (\ u_{i+1} \le u \le u_{i+2}\ )) \\
& + \frac{u_{i+3}-u}{u_{i+3}-u_{i+1}}\frac{u-u_i}{u_{i+1}-u_i} && (\text{ if } (\ u_{i+2} \le u \le u_{i+3})) \\
& + \frac{u_{i+3}-u}{u_{i+3}-u_{i+1}}\frac{u_{i+2}-u}{u_{i+2}-u_{i+1}} \qquad = \frac{u-0}{1-0}\frac{u-0}{0-0} && (\text{ if } (u_{i+3} \le u \le u_{i+4})) \\
& + \frac{u-0}{1-0}\frac{1-u}{1-0} \\
& + \frac{1-u}{1-0}\frac{u-0}{0-0} \\
& = u*(1-u) + (1-u)(1-u)
\end{aligned}$$

So for $i = 2$:

$$\begin{aligned}
N_{i=2,3}(u) = {} & \frac{u-u_2}{u_4-u_2}\frac{u-u_2}{u_3-u_2} \\
& + \frac{u-u_2}{u_4-u_2}\frac{u_4-u}{u_4-u_3} \\
& + \frac{u_5-u}{u_3-u_3}\frac{u-u_2}{u_3-u_2} \\
& + \frac{u_5-u}{u_3-u_3}\frac{u_4-u}{u_4-u_3} \\
& = \frac{u-0}{1-0}\frac{u-0}{1-0} \\
& + \frac{u-0}{1-0}\frac{1-u}{1-1} \\
& + \frac{1-u}{1-1}\frac{u-0}{1-0} \\
& + \frac{1-u}{1-1}\frac{1-u}{1-1} \qquad = u*u + (1-u)*u + 0 + 0
\end{aligned}$$

Note: Due to the prior question mistakes, i=0, and i=1 is wrong.

4. (10 marks) Consider the following B-spline surface:

   a) How many non-zero terms does $S(u, v)$ have?
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   b) Which terms are the non-zero values? use the δ notation.
   We use u_v, where v are these values:

$$\begin{matrix} \delta_{0,0} & \delta_{1,0} & \delta_{2,0} & \delta_{3,0} \\ \delta_{0,1} & \delta_{1,1} & \delta_{2,1} & \delta_{3,1} \\ \delta_{0,2} & \delta_{1,2} & \delta_{2,2} & \delta_{3,2} \end{matrix}$$

5. (10 marks) The first quadratic B-spline basis function defining on positive integer numbers (knot sequence) is given as:

   a) Show that $N_{0,3}(u)$ is really a third order spline function (verify the necessary properties only at u = 0).

      At $N_{0,3}$, we must show that it is continious, and hence a spline function:
      At $u = 0$: $C^0 : 0 = u^2/2 \rightarrow 0 = 0$ $C^1 : 0 = u^2/2 \rightarrow 0 = 0$ $C^2 : 0 = u \rightarrow 0 = 0$ $C^3 : 0 = 0 \rightarrow 0 = 0$
      Hence, it has c^3 continuity at $u = 0$, which is all that was requested.

   b) Determine $N_{5,3}(u)$ (try to use $N + 0, 3(u)$). Show your work.
      As the above equation is for the uniform-knot sequence, and there isn't any reason to assume it is not uniform, we can $N_{i,3}(u) = N_{0,3}(u - i)$. Which gives us:
      $N_{5,3}(u) = N_{0,3}(u - 5)$