AMAT 491 - Assignment4

Scott Saunders - 10163541

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1. (15 marks) Answer the following questions briefly

   a) Does every closed 2D manifold necessarily divide the space into an inside and an outside? Justify your answer.
   No - For example the mobious strip. It only has a single side, making it rather hard to distinguish between inside/outside. Especially as at a particular segment or if cut in half, it appears to have two sides.

   b) Show that general cones and general cylinders are special cases of Coons surfaces.
   As we are aware a generalized cone is a subset of a ruled surface, and as is generalized cylinder, if we show that a coon can represent a given ruled surface, we have shown it can make generalized cones/cylinders.

   A ruled surface is parameterized as follows: $Q(u, v) = (1 - v)P_0(u) + vP_1(u)$.
   Where as a Coons Patch can be expressed as: $Q(u, v) =$
   $[Q_0(v)(1 - u) + Q_1(v)u]$ (the first ruled surface)
   $+[P_0(u)(1 - v) + P_1(u)v]$ (the second ruled surface)
   $-[(1 - u)(1 - v)P_{00} + u(1 - v)P_{01} + (1 - u)vP_{10} + uvP_{11}]$ (The bi-linear patch)

   If we define $P_0(u) = (1 - u)P_{00} + uP_{01}$
   If we define $P_1(u) = (1 - u)P_{10} + uP_{11}$
   we get:
   $Q_o(v)(1u) + Q_1(v)(u)$
   as the remaining terms cancel out:

   $$\begin{aligned}
   [P_0(u)(1 - v) + P_1(u)v] = \\
   &= (1 - v)((1 - u)P_{00} + uP_{01}) + v((1 - u)P_{10} + uP_{11}) \\
   &= [(1 - u)(1 - v)P_{00} + u(1 - v)P_{01} + (1 - u)vP_{10} + uvP_{11}]
   \end{aligned}$$

   Which is what is being subtracted as the last term/bilinear patch, canceling it out.

   c) Find a set of subdivision filters that guarantees C 4 continuity. You may ignore boundary conditions.
   (1/32)(1,6,15,20,15,6,1)
   Generated via pascals triangle and the properties of general B-spline.

The pascal triangle $/(2^n)$, where n is the layer down in pascal triangle (with a slight offset), is just a pattern recognized to match the B-spline's filters, as discussed in class and in the notes. And as these are the filters of B-splines, we need an order 7 B-spline, which is the row chosen here.

2. (20 marks) A closed mesh (i.e. no boundary vertices) is saved in a half-edge data structure. For a given face f , write, in pseudocode, an algorithm that

   a) finds all faces adjacent to f

```
let f be our current face,
e = f.edge;
adj<faces>;
do
  adj.push_back( e.pair.face );
  e=e.next;
while(e != f.getEdge);

return adj;
```

   This simply iterates of each edge of the face, and appends it's face to the returned value.

   b) all vertices that are on f or connected to f by an edge.

```
let f be our current face,
e = f.edge;
verts<vertices>;

do
  verts.push_back(e.source);
  e2 = e;
  do
    verts.push_back(e2.source);
    e2 = e2.next.pair
  while ( e2 != e || e2 != e.next.pair)
  e=e.next;
while(e != f.getEdge);

return verts;
```

   Outer loop, iterates over all all vertices that are on f, and adds them.
   Inner loop, iterates over all edges of a given vertex, and adds their vertices (except the one we're currently one, and it's next one, as that'll get added in the next iteration of the outer loop)

3. (20 marks) Consider the following local subdivision matrix for a new subdivision scheme (a modification..

   a) Compare the smoothness of this scheme with the cubic B-spline subdivision scheme.
   You can tell how smooth it is based on when the Eigen values start repeating:
   Where quadratic is $1/2, 1/4, 1/4$
   and cubic is $1, 1/2, 1/4, 1/8, 1/8$
   Hence, this provided filter is less-smooth than cubic, and smoother than quadratic.

   b) Is the resulting curve "coordinate free"? Justify your answer.
   Yes, as all the weights of the filter are sum-to-one in any given row, which is a sufficient condition for convex-hull property.

4. (15 marks) Consider a mesh with e edges, v vertices, and f faces. Compute the new number of edges (e 0), vertices (v 0), and faces (f 0) after one level of subdivision in the cases of the Loop and Doo-Sabin methods. Show your calculations. Assume the mesh doesn't have any boundaries.

   - Doo-Sabin method:
     From the original paper:
     "In this process, 3 types of new faces can be formed:
     a) Type F: A 5-sided face will give a new smaller 5-sided face within itself and bears a similar shape, this type of new face is termed Type F (formed by face).
     b) Type V: A vertex common to 4 faces, i.e. a corner where 3 faces joined together having three common boundaries, will produce a 3-sided face, this is Termed type V, (formed by vertex).
     c) Type E: On each common boundary of two adjacent faces, a 4-sided face will be formed, this is termed Type E (formed by edge).

     The new polyhedron will consist of these 3 types of new faces. A n-sided face will provide a basis for a smaller n-sided F type new face, it will remain n-sided as the subdivision carries on and will gradually converge to the centroid and diminish to an acceptable size. A common edge will always produce a 4-sided new face, and m-spoked vertex will produce a m-sided V type face, which will, in turn, become the basis of a smaller m-sided F type face in the next subdivision process."

     Taking this together, you have:
     $f' = f + e + v$

     (Again from the paper, as I have read it and cannot un-read it)
     Each e will give a 4-sided polygon with 4 new vertices, however any two adjacent edges on a face will generate only one new vertex on that face, therefore the number of new vertices formed will be:
     $v' = 4e/2 = 2e$
     Note: This can also be verified by the expansion/shrinking method of thinking about it: As each edge is shrunk/extended, each end of each old-edge, now creates a vertex, independent of any other edge. This gives $v' = 2e$.

     Finally, we use Euler's formula for polyhedron: $F + V - E = 2$ to get the number of edges:
     $e' = f' + v' - 2$
     $e' = f + e + v + 2e + -2$
     $e' = f + v + 3e - 2$

     In summary:
     $f' = f + e + v$
     $v' = 2e$
     $e' = f + v + 3e - 2$

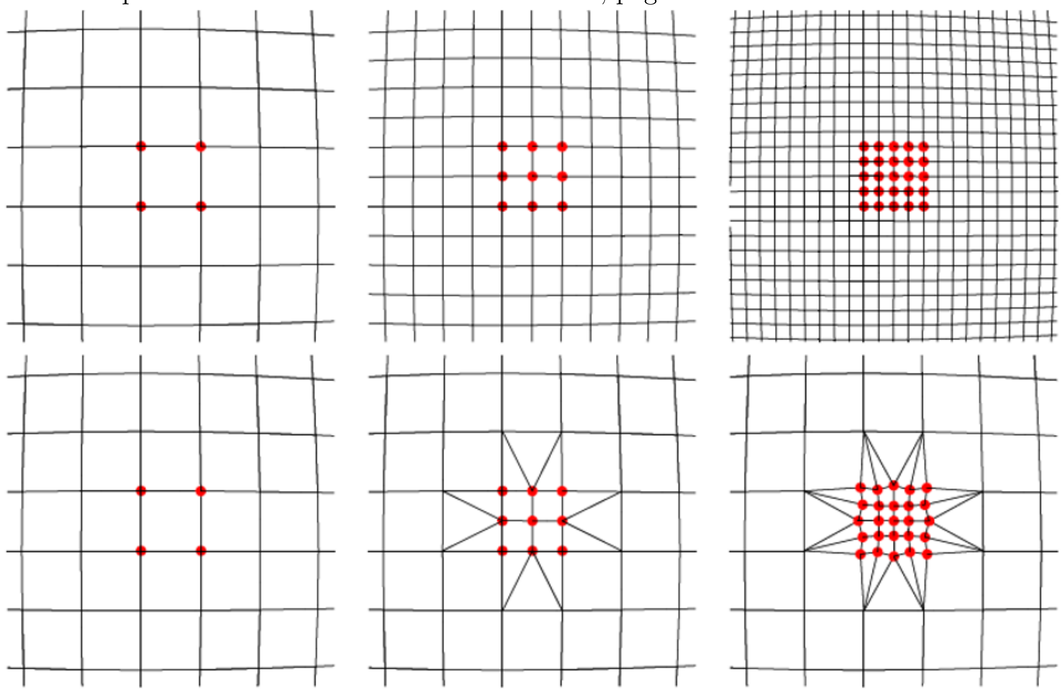
- Loop method:
  Each edge is doubled, and each face produces 3 new edges giving: $f' = 2e + 3 * f$
  Each vertex is kept, while each edge is cut in half with a new vertex giving: $v' = v + e$;
  Each face produces 3 new ones, and persists through giving: $f' = f * 4$

5. (10 marks) Show that Loop subdivision has the strong convex hull property.

   To show this, we need only to show that all weights on all vertices are non-negative.
   For each edge weighting:
   $e^{i+1} = \frac{3}{8}v_1^i + \frac{3}{8}v_2^i + \frac{1}{8}v_3^i + \frac{1}{8}v_4^i$
   Clearly, each component is non-negative. (and is sum to 1)
   For each vertex weighting:
   $v^{i+1} = (1 - n\alpha)v^i + \alpha \sum v_j^i$
   Where $\alpha = \frac{1}{n}(\frac{5}{8} - (\frac{3}{8} + \frac{1}{4}\cos\frac{2\pi}{n})^2)$
   Here, cos will return [-1,1], and hence, the largest value of the inner term is 1/2, squared is 1/4.
   Giving $\alpha = \frac{1}{n}(\frac{5}{8} - \frac{1}{4})$ or $\alpha = \frac{1}{n}\frac{1}{4}$.
   Which, is always positive, as n is always positive, and at max size: $\frac{1}{4n}$
   So $v^{i+1} = (1 - n\alpha)v^i + \alpha \sum v_j^i$ has the coefficients of $1 - \frac{1}{4}$, $\frac{1}{4n}$
   And hence, it's coefficients are always positive.
   Hence, it has convex hull property.

6. (20 marks) For Catmull-Clark subdivision, we would like to subdivide only a particular area of the mesh using an adaptive subdivision method.

   a) Extend the simple (naive) T-junction insertion method of Adaptive Loop subdivision to create an Adaptive Catmull-Clark subdivision. Draw a figure to support your description.
   Note: the picture is taken from the CPSC589 notes, page 126.

   To make a local subdivision via Catmull-Clark, a simple method is to perform Catmull-Clark, as normal in the local area, but then modify the immediately surrounding barrier to fill-in the gaps of the new edge-vertexes (gaps exist if it is not perfectly flat).
   To do this, we have to add two edges, (instead of 1 for loop), connecting the new mid-edge point, to the two closest vertices outside the subdivided area.
   (Note: This is only true for the first subdivision, after this they're triangles, and use the same T-junctions from loop for smoothing to the un-divided area)

   b) Describe the disadvantages of this simple adaptive subdivision.
   It creates a lot of 'long' triangles, getting worse as it repeatedly subdivides.
   It also ends up creating a high difference-of-detail on some vertices. Ex: on the far-right of the picture, the surrounding vertices, some edges go to vertices with valence 4, while some goto vertices with valence 7. This also implies that these are no longer regular, possibly creating extra-ordinary vertices.