Counting _to find the number of objects of a certain type_

This section involves the listing of possible results. It is important to use a systematic approach so that no possibilities are missed.

1. **One-Part Counting** – counting one item or one way at a time.

EXAMPLE: simple list
A club has 5 members Alan, Bill, Cathy, David, and Evelyn

2. **Two-Part Counting** – counting two items, consider using a table

EXAMPLE:
Using the same club members as above: the set of club members = {A, B, C, D, E}
How many ways can a president and a secretary be elected? ___ _20_ ___

This time we can make a chart instead of a simple list. A chart is an organized method of showing all the different ways something occurs so not to miss any possibility.

For this example, a person elected cannot be both president and secretary. The first person is president and the second person is secretary.

SECRETARY

	A	**B**	**C**	**D**	**E**
A	X	AB	AC	AD	AE
B	BA	X	BC	BD	BE
C	CA	CB	X	CD	CE
D	DA	DB	DC	X	DE
E	EA	EB	EC	ED	X

(left margin label: PRESIDENT)

EXAMPLE: Determine the number of different possible results when two dice are rolled. Let's say one dice is red and the other dice is white so we can tell each dice apart from one another.

Since we are still looking at counting two objects, two dice, we can construct a table of the possible results. Fill in the table with the results:

		WHITE dice				
		1	**2**	**3**	**4**	**5**	**6**
RED dice	**1**	1,1	1,2	1,3	1,4	1,5	1,6
	2	2,1	2,2	2,3	2,4	2,5	2,6
	3	3,1	3,2	3,3	3,4	3,5	3,6
	4	4,1	4,2	4,3	4,4	4,5	4,6
	5	5,1	5,2	5,3	5,4	5,5	5,6
	6	6,1	6,2	6,3	6,4	6,5	6,6

How many possible ways are there to roll two dice? __36__

How many possible ways can you roll two dice where their sum is 7? __6__

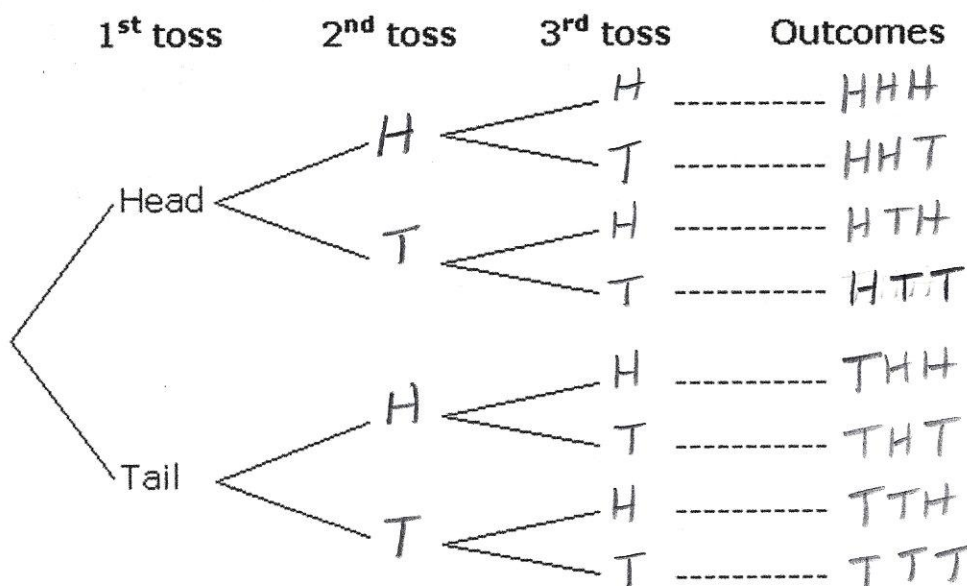How many ways can you roll two dice where their sum is an odd number? __18__

How many ways can you roll two dice with a sum greater than 8? __10__

Keep this table: it will be used in homework for chapters 10, 11, 12.

3. **Multiple-Part counting** – counting with multiple sets.

Sometimes a list or a chart is not useful when counting multiple parts. A tree diagram might be more useful.

EXAMPLE: How many different results can come from tossing 3 coins? ___8___

1st toss	2nd toss	3rd toss	Outcomes

```
                              H ---------- HHH
                    H <
                              T ---------- HHT
          Head <
                              H ---------- HTH
                    T <
                              T ---------- HTT

                              H ---------- THH
                    H <
                              T ---------- THT
          Tail <
                              H ---------- TTH
                    T <
                              T ---------- TTT
```

When counting multiple possible results/outcomes, there are two ways the information can be sorted: **with replacement** and **without replacement**.

1. **With Replacement:** Use each item multiple times. All items can be used repeatedly

2. **Without Replacement:** Use each item only one time. Once used, items cannot be repeated

Both of these will be used again in chapter 11 in probability.

EXAMPLE: How many 3-digit numbers can be created using only the numbers 4, 5, 6?

a) Repeated digits not allowed   b) Repeated digits allowed

digit one	digit two	digit three	outcomes (results)

5 → 6 ------ 456

4

6 → 5 ------ 465

4 → 6 ------ 546

5

6 → 4 ------ 564

4 → 5 ------ 645

6

5 → 4 ------ 654

6

digit one	digit two	digit three	outcomes (results)

4 —— 444
4 5 —— 445
6 —— 446

4 —— 454
4 → 5 5 —— 455
6 —— 456

4 —— 464
6 5 —— 465
6 —— 466

4 —— 544
4 5 —— 545
6 —— 546

4 —— 554
5 → 5 5 —— 555
6 —— 556

4 —— 564
6 5 —— 565
6 —— 566

4 —— 644
4 5 —— 645
6 —— 646

4 —— 654
6 → 5 5 —— 655
6 —— 656

4 —— 664
6 5 —— 665
6 —— 666

27

4. **Counting shapes** – another way to systematically count is to count shapes by finding a starting point, looking at different sizes, and label & move around the figure.

EXAMPLE: Count the number of triangles (any size) in the figure: ___22___
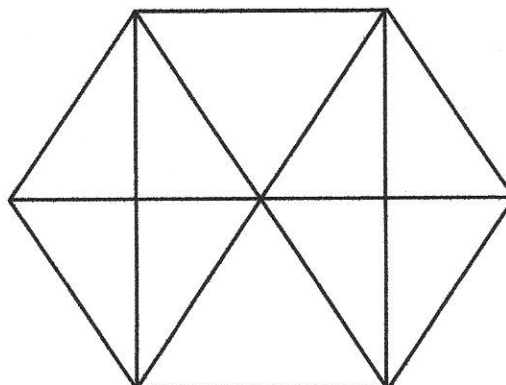
8 small
6 medium
4 long tall
4 large right
_____
22 total