Notes Section 1.4 – Types of Functions and Their Rates of Change

Lesson Objectives

1. Find the slope of a line that goes through two points.
2. Know the 4 types of slope.
3. Compute the average rate of change between two points.
4. Identify the slope and *y*-intercept of a linear function,
   a. from a formula.
   b. from a graph.
5. Write the slope-intercept formula of a linear function,
   a. given its slope and *y*-intercept.
   b. given the graph of a line.
6. Graph linear functions in slope-intercept form.
   a. Using MyLab Math graphing tool.
   b. Using graphing calculator.

A. Find the SLOPE of a Line between Two Points

Slope Formula: Given two points (x_1, y_1) and (x_2, y_2), the slope of the line between them is:

$$\textbf{Slope} = m =$$

EXAMPLE: Find the slope, if it exists, of the line containing the pair of points $(7, -4)$ and $(7, -10)$.

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} =$$

[1.4.15]

try on calc: `ERR:DIVIDE BY 0`
`1■Quit`
`2:Goto`   slope is _____

B. The 4 Types of SLOPE

- **POSITIVE** slope – _____ (left to right)
- **NEGATIVE** slope – _____ (left to right)

- **ZERO** slope – _____ line
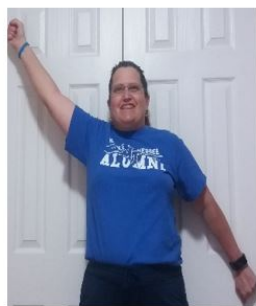

- **UNDEFINED** slope – _____ line

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4 Types of Slope

Positive Slope	Negative Slope
Increasing	Decreasing

Zero Slope	Undefined Slope
Horizontal Line	Vertical Line

The "Slope Cheer"

C. Average Rate of Change (AROC)

- Used for any function, not just a linear function

- Same formula as _____

Average Rate of Change of a function f from x_1 to x_2 is:
$$\text{AROC} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \text{ or } \frac{y_2 - y_1}{x_2 - x_1}$$

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- **EXAMPLE:** Compute the average rate of change from x_1 to x_2. Round your answer to two decimal places when appropriate. Interpret your result graphically. [1.4-47]

$$f(x) = 7x + 3 \qquad x_1 = -4 \text{ and } x_2 = -1$$

To get the *y*-coordinates for the formula, just plug in *x*-values into the function:
- $y_2 = f(x_2) = f(-1) =$
- $y_1 = f(x_1) = f(-4) =$

To calculate AROC from x_1 to x_2, you must subtract in the correct order. Using the formula:

$$AROC = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \text{ or } \frac{y_2 - y_1}{x_2 - x_1} =$$

To interpret this graphically, this means that:

the slope of the line passing through $(-4, f(-4))$ and $(-1, f(-1))$ is _____.

Look back at the original function: $f(x) = 7x + 3$. Notice also where you see a 7 !

- **EXAMPLE:** The following table lists remaining life expectancy, *E*, in years for females of age *x*.

x (yr)	50	60	70	80
E (yr)	33.8	25.2	18.1	13.2

Find the average rate of change of remaining life expectancy between the ages of 50 and 60. (type an integer or decimal). Then, interpret this value. [1.4.103]

Average Rate of Change (AROC) = $\dfrac{y_2 - y_1}{x_2 - x_1} =$

This means that for each 1-year increase between the ages of 50 and 60 years,

the remaining life expectancy _____, on average, by _____ years.

(you don't use a negative sign there because of the context word "decreases.")

D. Identify the SLOPE and *y*-INTERCEPT of a Linear Function

- From a formula

Linear Function: can be written in the form $f(x) =$ _____

is written in **slope-intercept form**, where *m* is the _____, and *b* is the ____-intercept (0,*b*) .

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- **EXAMPLE:** The amount of money, in dollars, raised each year by a band booster club can be estimated by the function $f(x) = 198x - 389{,}053$, where x is the year with $1990 \leq x \leq 2000$.   What is the slope of the graph of f?   [1.4-54]

The formula $f(x) = 198x - 389{,}053$ is given in slope-intercept form, so the slope is _____.

The slope is the coefficient of x, provided you are in slope-intercept form.

- **EXAMPLE:** A linear function f can be written in the form $f(x) = m\,x + b$.

Identify m and b for the given $f(x)$. $f(x) = -2x$ [1.4.3]

This can be rewritten as $f(x) = $ _____ , so $m = $ _____ and $b = $ _____.

- **EXAMPLE:** Write a formula for a linear function f whose graph satisfies the given conditions.

  Slope 24 and passes through the origin.   [1.4.49]

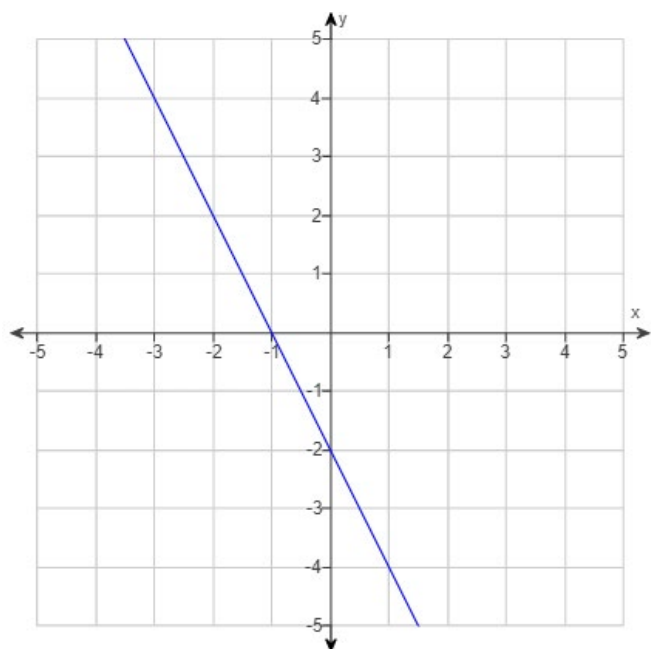The **origin** is the point (____ , ____), so the y-intercept is $b = $ ____, with the given $m = 24$.

So, a formula for a linear function satisfying the given condition is $f(x) = $ _____

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- ● From a graph
- ● **EXAMPLE:** Identify the slope, *y*-intercept, and *x*-intercept.   [1.4-16]

Slope $m = \dfrac{rise}{run} = \dfrac{y_2 - y_1}{x_2 - x_1}$

We're not going to use the coordinate part of the formula; rather, we are going to find rise and run directly in the graph.

Pick any 2 points in the graph that you can easily identify their _____ (coordinates).

Don't "eyeball" it – make sure the graph passes directly through two grid lines that intersect!

Since we need to identify them anyway, Let's use the **x-intercept** (,   ) and the **y-intercept** (,   )

If we go high-to-low,

rise = _____

run = _____.   So the slope is

$$m = \frac{rise}{run} = \frac{\text{DOWN 2}}{\text{RIGHT 1}} =$$

Or, if we go low-to-high,

rise = _____

run = _____.   So the slope is

$$m = \frac{rise}{run} = \frac{\text{UP 2}}{\text{LEFT 1}} =$$

The slope is the same regardless of the direction you calculate the slope.

Notice also that in the graph, the line is going _____, which means the slope will be _____.

This is a way to "sanity check" your calculation for slope.

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E. Write the Slope-Intercept Form of a Linear Function

- Given its slope and y-intercept
- **EXAMPLE:** Write a formula for a linear function f whose graph satisfies the condition slope $-\frac{3}{4}$ and y-intercept $\left(0,\frac{1}{5}\right)$.   [1.4.47]

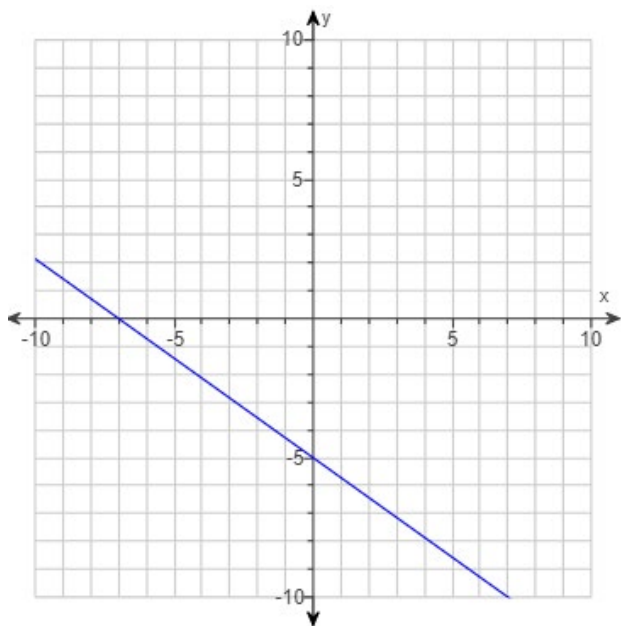A linear function is of the form: $y = mx + b$ or $= mx + b$

The slope m is given, so $m =$ and the y-intercept is given, so $b =$

A formula for a linear function satisfying the given condition is $f(x) =$

- Given the graph of a line
- **EXAMPLE:** Write the equation of the line whose graph is shown.   [1.4-17]

But you actually do them in reverse order when you have a graph.

y-intercept = (0,   )   So **b =** _____

We need a second point to do slope, so use the x-intercept = (_____,0)

Slope , going low-to-high is:

$$m = \frac{rise}{run} =$$

So, equation of the line $y = mx + b$

Is $y =$

Equation of a line always needs:

_____ and _____.

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F. Graph a Linear Function Slope-Intercept Form

- Using *MyLab Math* graphing tool

- **EXAMPLE:** Graph the linear function by hand. Identify the slope and *y*-intercept.

$$g(x) = -\frac{11}{2}x \qquad\qquad [1.4.59]$$

What is the slope of the graph of *g*?   The slope is $m =$

What is the *y*-intercept of *g*?   The *y*-intercept is $(0, \quad)$ so, $b =$ _____.
Use the graphing tool to graph the function.

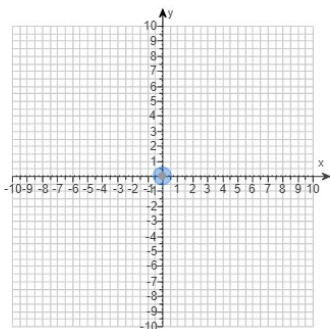1. Press the button **"Click to enlarge graph"**

2. Select the "_____ _____"

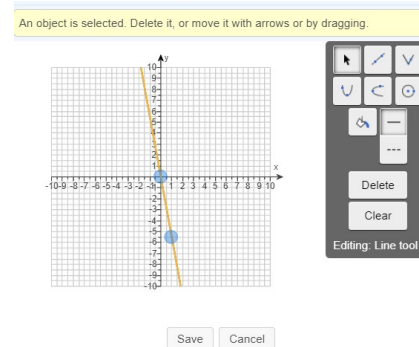3. Click the graph to plot the first point on your line, which is the *y*-intercept (0,0).

4. Click the graph to plot the second point on your line.

Keep your mouse on your *y*-intercept and use the _____ keys to do rise and run for your slope.

- Move _____11 grid lines (you don't need to go to the number 11)
- Move _____ 2 grid lines (you don't need to go to the number 2)
- Press ENTER and line turns _____

5. Press the **SAVE** button, followed by the **CHECK ANSWER** button. Line is _____.
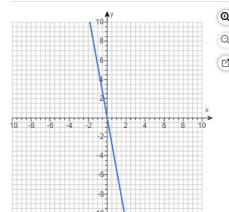
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- ## Using graphing calculator (TI-84/83 Plus)

You can use your graphing calculator to verify that you have the correct graph.

We will use the previous example to graph on the calculator.
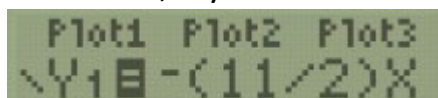
$$g(x) = -\frac{11}{2}x \qquad\qquad [1.4.59]$$

1. Press _____ button at upper left of calculator. **Y=**

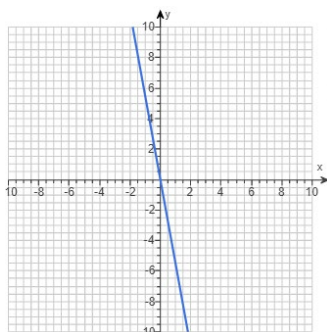   Clear out any functions, if needed, by pressing **CLEAR**. **CLEAR**

2. Enter your function rule next to the **Y₁=** at the top. Use parentheses for fraction, or use the fraction feature on the TI-84, if you want to.

   ```
   Plot1  Plot2  Plot3
   \Y1 = -(11/2)X
   ```
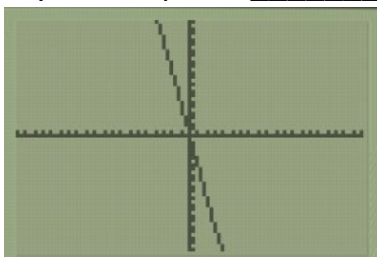
   Make sure Plot1, Plot2, and Plot3 are not highlighted at the top. If they are, move cursor up to it and press ENTER to turn it off.

3. Before you press **GRAPH**, check the graphing area provided either in the problem or the answer. Look at the SCALE – how are the x-axis and y-axis labeled.

   Press _____ **WINDOW** and adjust your x-axis and y-axis values, if needed.

   ```
   WINDOW
    Xmin=-10
    Xmax=10
    Xscl=.5
    Ymin=-10
    Ymax=10
    Yscl=.5
   ```

4. Now you can press _____.

Sources Used:

1. 4 Types of Slope, Mr. Slope Guy:
   http://mrssorensensblog.weebly.com/uploads/5/7/3/6/57368065/7_-_slope.pdf
2. Pearson MyLab Math *College Algebra with Modeling and Visualization, 6th Edition*, Rockswold
3. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website
   https://archive.codeplex.com/?p=wabbit