Notes Section 5.4 – Logarithmic Functions and Their Representations

Lesson Objectives

1. The Basics of a Logarithm
2. The Two "Special" Types of Logarithms
3. Basic Properties of Logarithms
4. Simplify (or evaluate) logarithms
5. Convert Between Exponential and Logarithmic (and vice-versa)
6. Solve Basic Logarithmic and Exponential Equations

A. The Basics of a Logarithm

Suppose we have the exponential equation: $2^x = 8$

We know the answer is $x = 3$. $(2^3 = 8)$

Because our variable is in the exponent, we _____ just use _____ to solve this equation:

NO, NO, NO!! Don't do this!! → → $\frac{2^x}{2} = \frac{8}{2}$

If you did, you would get $x = 4$. (incorrect) Division is used to undo multiplication, right?

But, we don't have multiplication; we have exponential.

BIG IDEA! To undo an exponential function, we need to use its inverse – the _____.

A logarithm is _____.

$log =$ _____

Since a logarithm is an exponent, then it must necessarily have a _____.

Logarithmic form (definition):

_____ "**Logarithm** base *b* of ***x*** equals *y*"

$\log_{\text{base}}(\text{value}) = \text{exponent}$

Rewrite (or convert) to **exponential form**:

$\log_b x = y$ means the same thing as _____

$\text{base}^{\text{exponent}} = \text{value}$

BIG IDEA! _____ and _____ are interchangeable in meaning.

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B. The Two "Special" Types of Logarithms

1. _____ logarithm – base is ___, but not explicitly written. It is understood to be 10.
If you see a logarithm written _____ a base, then the base is _____.

- Examples: $\log x$ means $\log_{10}(x)$ $\log \frac{1}{100}$ means $\log_{10}\left(\frac{1}{100}\right)$
- Calculator button is **LOG** 

(to the left of the **7** button)
- This calculator button is **ONLY** for base **10**, the common logarithm!

2. _____ logarithm – base is ___, but the logarithm is written as "___" not "$\log_e$".

- Examples: $\ln x$ means $\log_e(x)$ $\ln e^7$ means $\log_e(e^7)$
- Calculator button is **LN** 

(to the left of the **4** button)
- This calculator button is **ONLY** for base ***e***, the natural logarithm!

C. Basic Properties of Logarithms

Recall **BIG IDEA!**: $\log_b x = y \iff b^y = x$ (these are interchangeable in meaning)

Here are some **Basic Logarithm Properties** to remember:

1. $\log_b 1 =$ ___ because $b^0 = 1$ (Any base with _____ power is 1)

2. $\log_b b =$ ___ because $b^1 = b$ (Any base to the power of ___ is the base itself)

3. $\log_b b^x =$ ___ because $b^x = b^x$ (_____ base *b* will undo _____ base b)
~~$\log_b b^x$~~ = ___ (Logarithm base b will undo "big" base b)

4. $b^{\log_b x} =$ ___ because $\log_b x = \log_b x$ (_____ base b will undo ____ base b)
~~$b^{\log_b x}$~~ = ___ ("Big" base b will undo logarithm base b)

- **EXAMPLE:** Simplify the expression. $\log_5 1$ [5.4-18]

$\log_5 1$ means _____ Property 1 Answer: $\log_5 1 =$ ___

- **EXAMPLE:** Evaluate the logarithm. $\ln 1$ [5.4-15]
This is a natural logarithm (ln) – it has base *e*.

$\ln 1$ or $\log_e 1$ means _____ Property 1 Answer: $\ln 1 = \log_e 1 =$ ___

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- **EXAMPLE:** Evaluate the logarithm. $\ln(e)$ [5.4-14]
  This is a natural logarithm (ln) – it has base *e*.

  $\ln e$ or $\log_e e$ means _____ Property 2 Answer: $\ln(e) = \log_e(e) =$ ___

- **EXAMPLE:** Simplify the expression, if possible. $\log 10^{7.4}$ [5.4.1]
  Notice that the base of the logarithm is not written – it is a common logarithm, base 10.

  $\log 10^{7.4} = \log_{10}(10^{7.4})$ (Logarithm base 10 will undo exponential base 10)

  $\log_{10}(10^{7.4})$ Property 3 Answer: $\log 10^{7.4} =$ ___

- **EXAMPLE:** Simplify the expression, if possible. $\ln e^6$ [5.4.29]
  This is a natural logarithm (ln) – it has base *e*.

  $\ln e^6 = \log_e e^6$ (Logarithm base *e* will undo exponential base *e*)

  $\log_e e^6$ Property 3 Answer: $\ln e^6 =$ ___

- **EXAMPLE:** Find the indicated value of the logarithmic function.
  $\log_7(7)^{4x}$ [5.4.23]

  $\log_7(7)^{4x}$ (Logarithm base 7 will undo exponential base 7)

  $\log_7(7)^{4x}$ Property 3 Answer: $\log_7(7)^{4x} =$ ___

- **EXAMPLE:** Simplify. $4^{\log_4(5)}$ [5.4.25]

  $4^{\log_4(5)}$ (Exponential base 4 will undo logarithm base 4)

  $4^{\log_4(5)}$ Property 4 Answer: $4^{\log_4(5)} =$ ___

D. Simplify (or Evaluate) Logarithms

- **EXAMPLE:** Find the logarithm $\log_5 \frac{1}{625}$ [5.4.51]
  Put "= *y*" on the end of the expression: $\log_5 \frac{1}{625} = y$
  Chant: "A logarithm is an exponent."

  $\log_5 \frac{1}{625}$ means: _____ or $5^y = \frac{1}{625}$

  Since the value $\frac{1}{625}$ is a fraction, the exponent must be _____.

  625 is a power of 5, since $5 \cdot 5 \cdot 5 \cdot 5 = 625$, or $5^4 = 625$. So $\log_5 \frac{1}{625} =$ ___

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E. Convert between Exponential and Logarithmic (and vice-versa)

- **EXAMPLE:** Write in exponential form. $\log_{10} \frac{1}{1000000} = -6$ [*Lial 10.3.19]

Chant: "A logarithm is an exponent."

$\log_{10} \frac{1}{1000000} = -6$

What is the base? ____ What is the exponent? _____ put them together: _____

What is the "value"? _____ In exponential form: _____

- **EXAMPLE:** Write in exponential form. $\log_{15} 1 = 0$ [*Lial 10.3-11]

Chant: "A logarithm is an exponent."

$\log_{15} 1 = 0$

base = ___, exponent = ___, value = ___ In exponential form: _____

- **EXAMPLE:** Write in logarithmic form. $7^3 = 343$ [*Lial 10.3-1]

$7^3 = 343$

base = ___, exponent = ___, value = _____

Chant: "A logarithm is an exponent."

Setup: $\log_{(\text{base})}(\text{value}) = \text{exponent}$

In logarithmic form: _____

- **EXAMPLE:** Write in logarithmic form. $10^{-5} = 0.00001$ [*Lial 10.3-4]

$10^{-5} = 0.00001$

base = ___, exponent = ____, value = _____

Chant: "A logarithm is an exponent."

Setup: $\log_{(\text{base})}(\text{value}) = \text{exponent}$

In logarithmic form: _____ or _____

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F. Solve Basic Logarithmic and Exponential Equations

- Solve Basic Logarithmic Equations – ISOLATE and convert to EXPONENTIAL

- **EXAMPLE:** Solve the equation. $9\log(3x) = 27$ [5.4.95]

First, _____ the logarithm. Divide by 9. $\frac{9\log(3x)}{} = \frac{27}{}$ (log has base ___)

Simplify. Do ____ divide by 3 in parentheses yet! _____
(It is stuck inside the logarithm.)

Chant: "A logarithm is an exponent."

_____ the $\log_{10}(3x) = 3$ to exponential form with base 10. $10^3 = 3x$

Divide both sides by 3 and simplify. $\frac{1000}{} = \frac{3x}{}$

Leave answer as a _____ – do not round. ANSWER: $x =$ _____

- **EXAMPLE:** Solve the equation symbolically for the unknown. [5.4-30]

$3\ln(4x) = 21$

First, _____ the logarithm. Divide by 3. $\frac{3\ln(4x)}{} = \frac{21}{}$ (ln means base ___)

Simplify. Do _____ divide by 4 yet! _____
(It is stuck inside the logarithm.)

Chant: "A logarithm is an exponent."

_____ the $\log_e(4x) = 7$ to exponential form with base *e*. _____

Divide both sides by 4 and simplify. $\frac{e^7}{} = \frac{4x}{}$

Leave as exact answer with *e* – do not _____. ANSWER: $x = \frac{e^7}{4}$ or _____

Answer is **NOT**: $x =$ _____ (the divide 4 is _____ part of the exponent!)

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- **EXAMPLE:** Solve the equation. $4 - 2\log_3 x = 2$ [5.4.10]

First, **ISOLATE** the logarithm. Subtract 4 both sides. $4 - 2\log_3 x = 2$
(do _____ do 4 – 2 = 2 at beginning!)

Combine like terms and simplify. _____

Divide both sides by – 2 _____ = ___

Simplify. _____

Chant: "A logarithm is an exponent."

_____ the $\log_3 x = 1$ to exponential form with base 3. _____

Simplify. ANSWER: _____

- Solve Basic Exponential Equations – ISOLATE and convert to LOGARITHM

- **EXAMPLE:** Solve the equation. Use the change of base formula as appropriate.

$3(10^{2x}) = 17$ [5.4.73]

(Type an integer or decimal rounded to the nearest hundredth as needed.)

First, _____ the exponential. Divide by 3. $3(10^{2x}) = 17$

Simplify. Do NOT round $\frac{17}{3}$. Leave as _____ to the end! $\frac{3(10^{2x})}{} = \frac{17}{}$

Update the equation. _____

Do _____ divide by the 2 yet. It's stuck in the exponential.

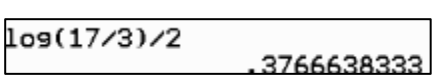
$$10^{2x} = \frac{17}{3}$$

Chant: "A logarithm is an exponent."

_____ the exponential $10^{2x} = \frac{17}{3}$ to a logarithm base 10. _____

Divide both sides by 2 and simplify. $\frac{\log\left(\frac{17}{3}\right)}{} = \frac{2x}{}$

Simplify. Use calculator to round to hundredth. $x = \frac{\log\left(\frac{17}{3}\right)}{2} \approx$ _____



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- **EXAMPLE:** Solve the equation. $5e^x + 5 = 8$ [5.4.81]
  (Round to 4 decimal places as needed. Use a comma to separate answers as needed.)

  _____ the exponential. First, subtract 5. $5e^x + 5 = 8$

  Combine like terms and simplify.

  Divide both sides by 5 and simplify. $\frac{5e^x}{\_\_\_\_} = \frac{3}{\_\_\_\_}$

  Chant: "A logarithm is an exponent."

  _____ the exponential $e^x = \frac{3}{5}$ to a logarithm base *e*. _____

  A logarithm base *e* is same as natural logarithm (____) $\ln\left(\frac{3}{5}\right) = x$

  Use calculator to get answer rounded to 4 decimal places.

  ln(3/5)
  -.5108256238

  Answer: _____

- **EXAMPLE:** Solve the equation for *x*. $e^{-x} = 258$ [5.4-26]
  (Type an integer or decimal rounded to the nearest hundredth as needed.)

  Do _____ divide by – 1 yet. (It's stuck in the exponential.) $e^{-1x} = 258$

  No need to ISOLATE the exponential – it's already there!

  $e^{-1x} = 258$

  Chant: "A logarithm is an exponent."

  _____ the exponential $e^{-1x} = 258$ to a logarithm base *e*. _____

  A logarithm base *e* is same as natural logarithm (___) _____

  Divide by the – 1 both sides and simplify. $\frac{\ln(258)}{\_\_\_\_} = \frac{-1x}{\_\_\_\_}$

  INCORRECT: $x =$ _____

  ln(-258)
  Error

  ERROR: NONREAL ANSWERS
  1:Quit
  2:Goto

  That would be _____.

  In general, you ***cannot*** take logarithm of zero or negative. Only $\log_b(\_\_\_\_\_)$ works!

  $x = -\ln(258)$ is "_____" answer

  Use calculator to round

  -ln(258)
  -5.552959585

  Answer: _____

Sources Used:

1. MyLab Math for *Algebra for College Students*, 8th Edition, Lial, Pearson Education Inc.
2. MyLab Math for *College Algebra with Modeling and Visualization*, 6th Edition, Rockswold, Pearson Education Inc.