Notes Section 6.2 –Systems of Inequalities in Two Variables

Lesson Objectives

1. Solve a linear inequality (in one variable) graphically.
2. Solve a linear inequality (in two variables) graphically.
3. Solve a system of 2 linear inequalities graphically.

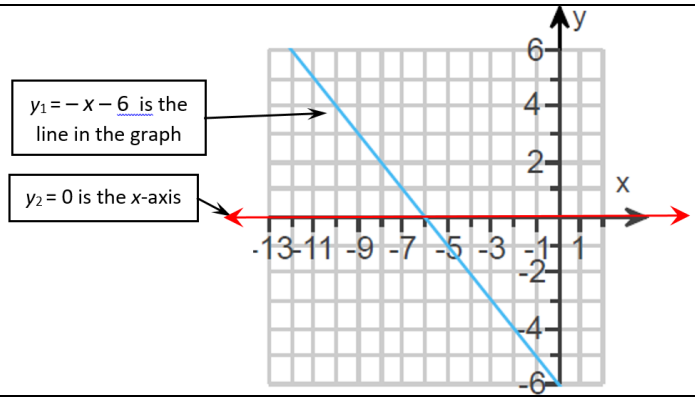
A. Solve a Linear Inequality (in One Variable) Graphically

- **EXAMPLE:** Use the given graph of $y = -x - 6$ to solve each equation and inequality in interval notation. [2.3.55]

  (a) $-x - 6 = 0$ (b) $-x - 6 < 0$ (c) $-x - 6 \geq 0$

(a) The table below describes what's happening graphically in the equation $-x - 6 = 0$

LEFT side of the equation	symbol	RIGHT side of the equation
$-x - 6$	$=$	0
y_1	$=$	y_2
The line you're given	ON	the *x*-axis

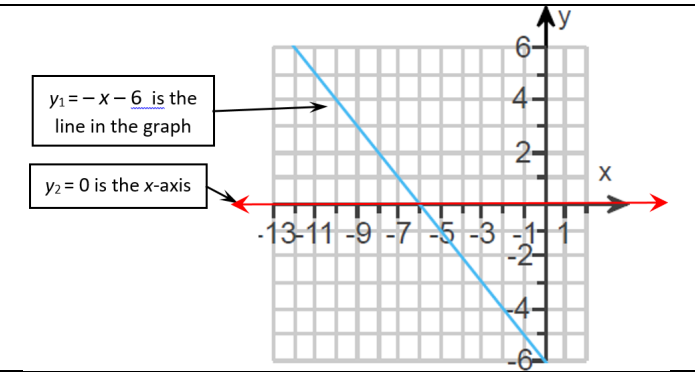


Big Idea: "**Equals** zero" (*something* = 0) means "**ON** the *x*-axis."

WHERE (what value of *x*) is **the graph ON the *x*-axis**? The solution set is $x = -6$.

(b) The table below describes what's happening graphically in the inequality $-x - 6 < 0$

LEFT side of the equation	symbol	RIGHT side of the equation
$-x - 6$	$<$	0
y_1	$<$	y_2
The line you're given	BELOW	the *x*-axis



Big Idea: "**Less** than zero" (*something* < 0) means "**BELOW** the *x*-axis."

WHERE (what values of *x*) is **the graph BELOW the *x*-axis**?

The graph is BELOW the *x*-axis if you go to the **RIGHT** of the intersection point, $x = -6$.

In English: to the RIGHT of $x = -6$ As inequality: $x > -6$ Interval Notation: $(-6, \infty)$

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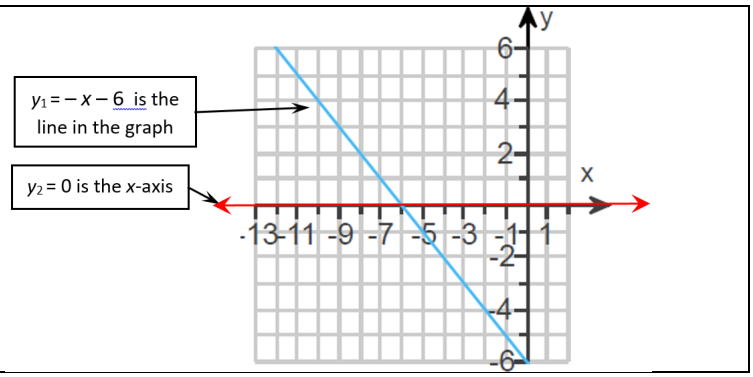
(c) The table below describes what's happening graphically in the inequality $-x - 6 \geq 0$

LEFT side of the equation	symbol	RIGHT side of the equation
$-x - 6$	$\geq$	0
y_1	$\geq$	y_2
The line you're given	on or **ABOVE**	the *x*-axis



Big Idea: "**Greater** than zero" (*something* > 0) means "**ABOVE** the *x*-axis."

WHERE (what values of *x*) is **the graph on or ABOVE the *x*-axis**?
The graph is BELOW the *x*-axis if you go to the **LEFT** of the intersection point, $x = -6$, with the -6 **included**.

In English: to the LEFT of $x = -6$ [included]

As inequality: $x \leq -6$

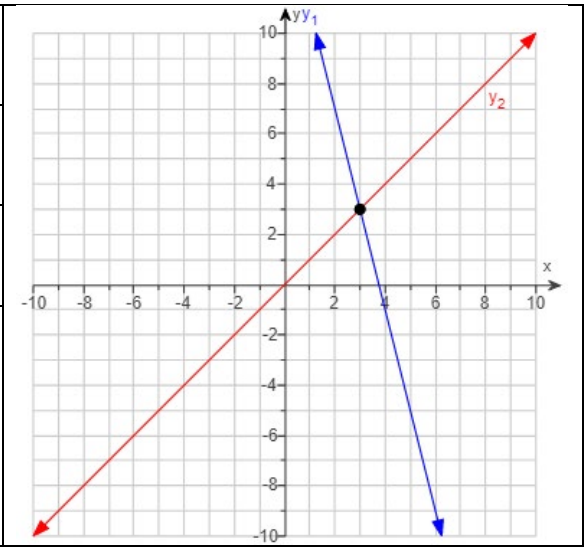
Interval Notation: $(-\infty, -6]$

- **EXAMPLE:** Use the given graphs of y_1 and y_2 to solve each inequality. Write the solution set in interval notation. [2.3.73]

  (a) $y_1 = y_2$ (b) $y_1 > y_2$ (c) $y_1 \leq y_2$

(a) The table below describes what's happening graphically in the equation $y_1 = y_2$

(left side)	(symbol)	(right side)
y_1	=	y_2
blue line	=	red line
BLUE	**ON**	RED



WHERE (what value of *x*) is **BLUE ON RED**? The solution set for $y_1 = y_2$ is $x = 3$.

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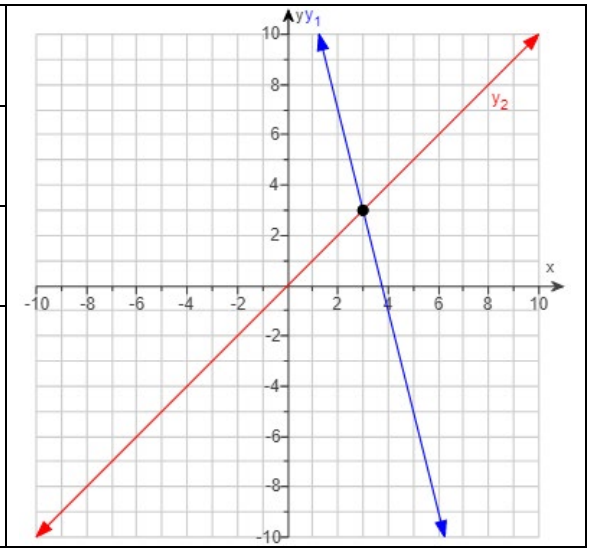
(b) The table below describes what's happening graphically in the equation $\boldsymbol{y_1 > y_2}$

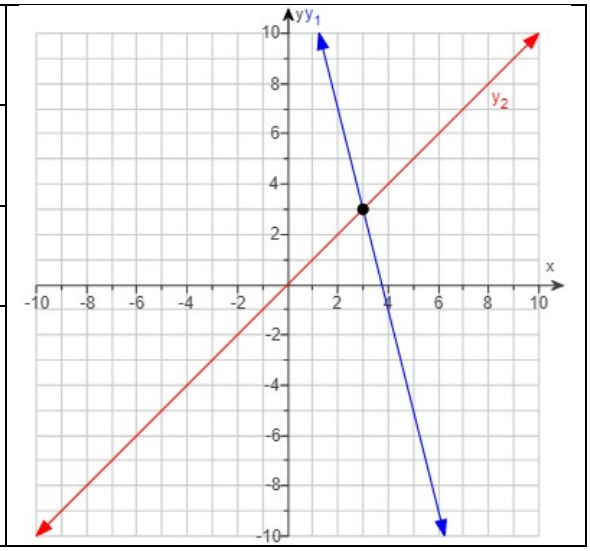
(left side)	(symbol)	(right side)	
y_1	$>$	y_2	
blue line	$>$	red line	
BLUE	**ABOVE**	RED	

WHERE (what values of *x*) is **BLUE ABOVE RED**?

To the **LEFT** of $x = 3$. Inequality: $\boldsymbol{x < 3}$

The solution set (in interval notation) for $y_1 > y_2$ is: $\boldsymbol{(-\infty, 3)}$

(c) The table below describes what's happening graphically in the equation $\boldsymbol{y_1 \leq y_2}$

(left side)	(symbol)	(right side)	
y_1	$\leq$	y_2	
blue line	$\leq$	red line	
BLUE	on or **BELOW**	RED	

WHERE (what values of *x*) is **BLUE on or BELOW RED**?

To the **RIGHT** of $x = 3$ (included). Inequality: $\boldsymbol{x \geq 3}$.

The solution set (in interval notation) for $y_1 \leq y_2$ is: $\boldsymbol{[3, \infty)}$

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B. Solve a Linear Inequality (in Two Variables) Graphically

How to graph a linear inequality in two variables:

Step 1	(If possible) Get your inequality into **SLOPE-INTERCEPT** form. (Be ready to REVERSE the inequality, if needed!)			
Step 2	Graph the **TYPE** of boundary line →	**DASHED** line (without equals)	$>$	$<$
		SOLID line (with equals)	$\geq$	$\leq$
Step 3	Choose **DIRECTION** of shading → →		shade **ABOVE** (greater-than type)	shade **BELOW** (less-than type)
	** exception: for **vertical** lines ($x = a$) →		shade **RIGHT**	shade **LEFT**

- **EXAMPLE:** Graph the inequality $7x + y > 1$
  Use the graphing tool to graph the inequality. [6.2.11]

  - **Step 1.** To graph a linear inequality, you need to convert it to **SLOPE-INTERCEPT** form first.

$$7x + y > 1 \qquad \text{(Subtract } 7x\text{)}$$
$$-7x \qquad -7x$$

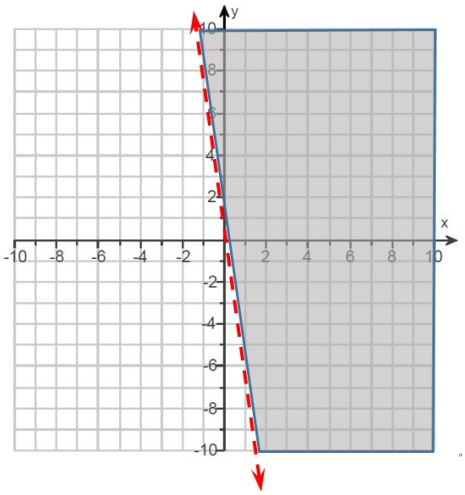
$$y > -7x + 1 \qquad \text{(Simplify)}$$

Graph the boundary line: $y = -7x + 1$

y-intercept: $(0, 1)$ slope: $\frac{rise}{run} = \frac{-7}{+1} = \frac{\text{DOWN 7}}{\text{RIGHT 1}}$

The symbol used: $>$ (**greater**-than)

- **Step 2.** Type of line: **DASHED** (missing equals)
- **Step 3.** Direction to shade: **ABOVE**



The boundary line and shaded area describe the **solution**.

Note: to verify this solution, we can use a **TEST POINT** that is **NOT** on the line. Often the origin **(0,0)** is best to use. If the origin is on the boundary line, then test some other point.

Test it with the inequality: $7x + y > 1$ $7(\mathbf{0}) + \mathbf{0} > 1$ $\mathbf{0 > 1}$ (**FALSE**)

Since testing the origin $(0,0)$ is **FALSE**, that means that the (0,0) region will **NOT** be shaded – the other side will be.

Big Idea! Test point **TRUE** = shade it; Test point **FALSE** = don't shade it

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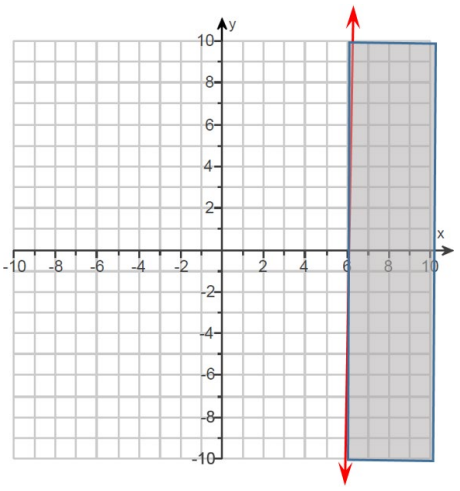
- **EXAMPLE:** Use the graphing tool to graph the given inequality.

$x \geq 6$ [6.2.7]



- **Step 1.** Can't get this into slope-intercept form because this is a **VERTICAL** line.

Graph the boundary line: $x = 6$

Vertical line passing through the ***x***-axis at 6

The symbol used: $\geq$ **greater**-than or equal to

- **Step 2.** Type of line: **SOLID** (it has equals)
- **Step 3.** Direction to shade: **RIGHT**

The boundary line and shaded area describe the **solution.**

Test point $(0,0)$ into $x \geq 6$: $0 \geq 6$ (**FALSE**) So, shaded region will **NOT** contain $(0,0)$.

C. Solve a System of 2 Linear Inequalities Graphically

- **EXAMPLE:** Graph the solution set to the system of inequalities.

Use the graph to identify one solution. Use the graphing tool to graph the system.

$$\begin{cases} 2x + y < 3 \\ x + y < 1 \end{cases}$$ [6.2.23]

- **Step 1.** Convert each to **SLOPE-INTERCEPT** form:

$2x + y < 3$

$-2x \quad -2x$ (subtract 2*x*)

$y < -2x + 3$ (simplify)

$x + y < 1$

$-x \quad -x$ (subtract *x*)

$y < -x + 1$ (simplify)

Graph the boundary line: $y_1 = -2x + 3$

y-intercept: $(0, 3)$

slope: $m = \frac{-2}{+1} = \frac{\text{DOWN 2}}{\text{RIGHT 1}}$

The symbol used: $<$ (**less**-than)

- **Step 2.** Type of line: **DASHED** (no equals)
- **Step 3.** Direction to shade: **BELOW**

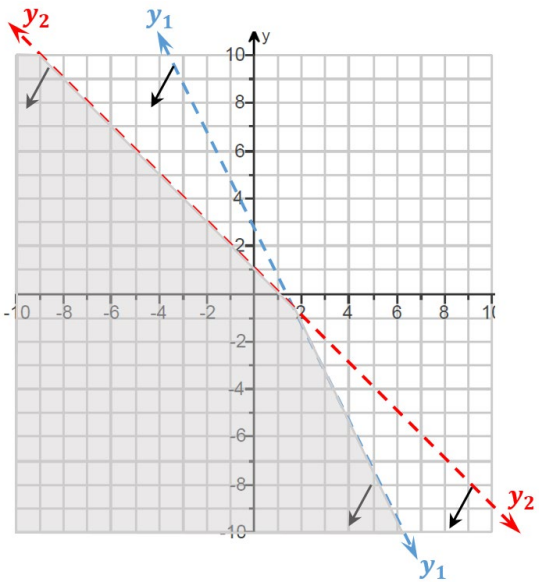
Graph the boundary line: $y_2 = -x + 1$

y-intercept: $(0, 1)$

slope: $m = \frac{-1}{+1} = \frac{\text{DOWN 1}}{\text{RIGHT 1}}$

The symbol used: $<$ (**less**-than)

- **Step 2.** Type of line: **DASHED** (no equals)
- **Step 3.** Direction to shade: **BELOW**



Use small arrows on the **ends** of each line to show the direction of the shading. The region with **TWO** arrows is the solution.

The boundary lines and shaded area describe the **solution.**

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- **EXAMPLE:** Graph the system of inequalities. Which graph is the solution of the system?

$$\begin{cases} x + y \le 1 \\ x - y \le 3 \end{cases}$$ [6.2.17]

- **Step 1.** Convert each to **SLOPE-INTERCEPT** form.

$x + y \le 1$

$-x \quad -x$ (subtract *x*)

$y \le -x + 1$ (simplify)

$y \le -x + 1$

Graph the boundary line:

$y_1 = -x + 1$

y-intercept: $(0, 1)$

slope: $m = \frac{-1}{+1} = \frac{\text{DOWN 1}}{\text{RIGHT 1}}$

The symbol used: $\le$

(**less**-than or equal to)

- **Step 2.** Type of line: **SOLID** (has equals)
- **Step 3.** Direction to shade: **BELOW**

$x - y \le 3$

$-x \quad -x$ (subtract *x*)

$-1y \le -x + 3$ (simplify)

$\frac{-1y}{-1} \ge \frac{-1x}{-1} + \frac{3}{-1}$ (divide by –1)

REVERSE it !!

$y \ge x - 3$

Graph the boundary line:

$y_2 = x - 3$

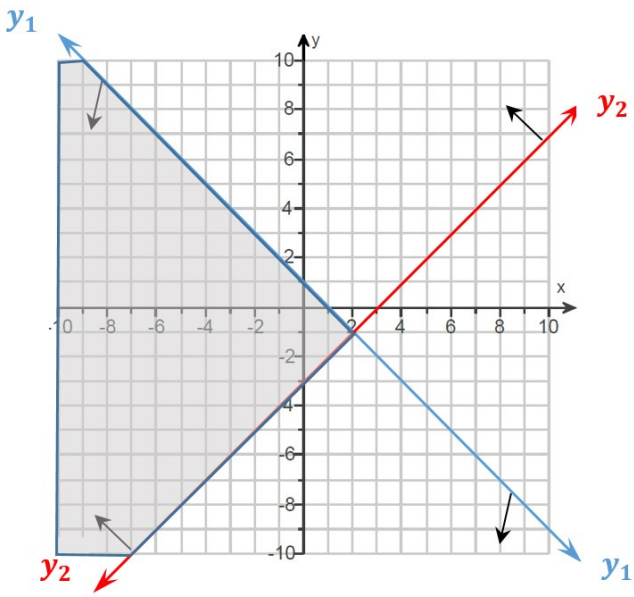
y-intercept: $(0, -3)$

slope: $m = \frac{+1}{+1} = \frac{\text{UP 1}}{\text{RIGHT 1}}$

The symbol used: $\ge$

(**greater**-than or equal to)

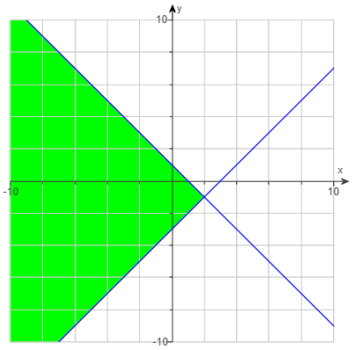
- **Step 2.** Type of line: SOLID (has equals)
- **Step 3.** Direction to shade: **ABOVE**



Use small **arrows** on the ends of each line to show direction of shading.
The region with TWO arrows is the **solution**.

The boundary line and shaded area describe the **solution**. Answer is **A**.

A.



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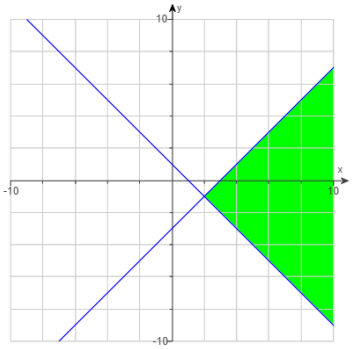

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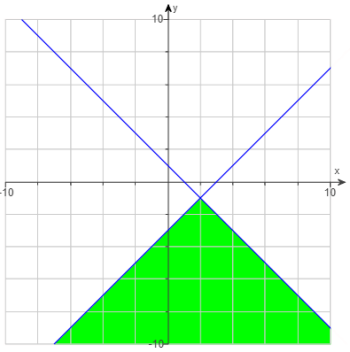

D.