Notes Section 2.3 – Linear Inequalities

Lesson Objectives

1. Overview of an Inequality Relationship
2. **Remember** the **Reverse Rule** for Inequalities!
3. Basic Linear Inequalities
4. Compound (Three-Part) Linear Inequalities

A. Overview of an Inequality Relationship

Let's examine a simple inequality relationship.

$$-2 < 3$$

That's certainly **TRUE**: negative 2 is less than 3.

- If you multiply both sides by $+\mathbf{5}$, for example:

$$-2(\mathbf{5}) < 3(\mathbf{5}) \quad \text{updates to} \quad -10 < 15$$

So that new inequality relationship $-10 < 15$ Is that still **TRUE**? YES, it is.

Now let's reset back to the original inequality:

$$-2 < 3$$

- Let's examine what happens when we multiply by $-\mathbf{5}$:

$$-2(-\mathbf{5}) < 3(-\mathbf{5}) \quad \text{updates to} \quad 10 < -15$$

Is this inequality relationship $10 < -15$ **TRUE** now? **NO!**

In order to keep the inequality **TRUE**, we need to **REVERSE** the direction of the inequality symbol. This preserves the smaller-bigger or bigger-smaller relationship:

$$10 > -15$$

Is this inequality relationship $10 > -15$ **TRUE** now? **YES!**

That is a *really, really* **BIG IDEA** when solving inequalities!

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B. Remember the Reverse Rule for Inequalities!

Whenever you **MULTIPLY** or **DIVIDE** an inequality by a **NEGATIVE** number, you must **remember** to **REVERSE** the direction of the **inequality symbol**!

This is true for *any* type of inequality, no matter how simple or complex.

You do **NOT** reverse the inequality when:

- Multiplying or dividing by a *positive* number
- Using the *Distributive Property* with a negative number (on one side of equation)
- Adding or subtracting

C. Basic Linear Inequalities

Solving a linear inequality involves the same steps as solving a linear equation (Section 2.2), but you now must **Remember** the **Reverse Rule** and its possibility of being used.

Here is a revised checklist to solve basic linear inequalities:

1. ****Combine Like Terms**, if you can.
2. **Undo Parentheses**, using the Distributive Property, then ** (see #1).
3. (if necessary) **Clear out fractions** – multiply all terms by the common denominator (also known as the Least Common Multiple, or LCM), then ** (see #1).

Remember the **Reverse Rule** – if you **MULTIPLY** by a **NEGATIVE** number, **REVERSE** it!

4. **Letters go LEFT** – use ADD or SUBTRACT to move variable terms to the LEFT side of the equation, then ** (see #1).
5. **Numbers go RIGHT** – use ADD or SUBTRACT to move constant terms to the RIGHT side of the equation, then ** (see #1).
6. **Divide** – last step is to DIVIDE by the coefficient of your variable and simplify.

Remember the **Reverse Rule** – if you **DIVIDE** by a **NEGATIVE** number, **REVERSE** it!

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- **EXAMPLE:** Solve the inequality. Write the solution set in interval notation. [2.3.15]

$$x - 2 \leq 4x + 7$$

We can't combine like terms yet. There are no parentheses nor fractions to deal with.

$$x - 2 \leq 4x + 7$$

Letters go LEFT: $-4x \quad -4x$

Combine like terms: $-3x - 2 \leq 7$

Numbers go RIGHT: $+2 \quad +2$

Combine like terms: $-3x \leq 9$

Divide by coefficient: $\dfrac{-3x}{\mathbf{-3}} \geq \dfrac{9}{\mathbf{-3}}$

Remember the **Reverse Rule!**

Simplify: $x \geq -3$

Is variable on the LEFT? YES

Graph:

Direction of shade? RIGHT

Is endpoint included? YES

Interval Notation: $[-\mathbf{3}, \infty)$ **(solution)**

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- **EXAMPLE:** Solve the inequality symbolically. Express the solution in interval notation.

$$6x - 4 < \frac{-36-x}{-2} \qquad [2.3.21]$$

We can't combine like terms at this point. There are no parentheses to deal with yet. There is one fraction (denominator), which is – 2.

To clear out fractions, we **MULTIPLY** both sides by the – 2. **Remember** the **Reverse Rule**.

$$6x - 4 < \frac{-36-x}{-2}$$

MULTIPLY both sides by – 2 $\quad -2(6x-4) > -2\left(\frac{-36-x}{-2}\right)$

Remember the **Reverse Rule!** $\quad \frac{-2}{-2} = 1$

Simplify – **Divide out common factors**. $\quad -2(6x-4) > -36 - x$

Use the **Distributive Property**. $\quad -12x + 8 > -36 - 1x$

Letters go LEFT. $\quad +1x \qquad\qquad +1x$

Simplify – **Combine like terms**. $\quad -11x + 8 > -36$

Numbers go RIGHT. $\quad -8 \qquad -8$

Simplify – **Combine like terms.** $\quad -11x > -44$

Divide – **Remember** the **Reverse Rule!** $\quad \frac{-11x}{-11} < \frac{-44}{-11}$

Simplify. $\quad x < 4$

Is variable on the left? YES

Graph.

Direction to shade? LEFT

Is endpoint included? NO

Interval Notation: $(-\infty, 4)$

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D. Compound (Three-Part) Linear Inequalities

Recall the basic structure of an "in-between" inequality (or the Three-Part Inequality):

$$\textbf{smaller number} \begin{matrix} < \\ \text{or} \\ \leq \end{matrix} (\boldsymbol{variable}) \begin{matrix} < \\ \text{or} \\ \leq \end{matrix} \textbf{larger number}$$

Here's a summary of how it looks in interval notation. Assume *x* as our variable.
Let *a* represent the smaller number and let *b* represent the larger number.

Inequality	Interval Notation
$a < x < b$	(a, b)
$a < x \leq b$	$(a, b]$
$a \leq x < b$	$[a, b)$
$a \leq x \leq b$	$[a, b]$

When you solve a Three-Part Inequality, focus on the **middle**, where the variable is. Keep your variable in the middle, and UNDO anything attached to it:

- Undo addition or subtraction first
- Undo multiplication with division second – **Remember** the **Reverse Rule!**

- **EXAMPLE:** Solve the inequality symbolically. Express the solution set in interval notation.

$$-3 \leq -8 - 4x \leq 15 \qquad \text{[2.3-13]}$$

We cannot combine like terms yet. There are no parentheses nor fractions to deal with.
Focus on the **middle**, where the variable is.

$$-3 \leq -8 - 4x \leq 15$$

First, undo the subtract 8 with add 8 on all 3 sides: $+8 \quad +8 \qquad +8$

Simplify – **Combine Like Terms**. $5 \leq -4x \leq 23$

Divide by the coefficient.

Remember the **Reverse Rule!** $\dfrac{5}{\mathbf{-4}} \geq \dfrac{-4x}{\mathbf{-4}} \geq \dfrac{23}{\mathbf{-4}}$

Simplify.

Inspect for proper format: $-\frac{5}{4} \geq x \geq -\frac{23}{4}$

Is smaller number on left, larger on right? NO
Are inequality symbols pointing LEFT? NO

We need to "pivot" or reverse the entire inequality: $-\frac{23}{4} \leq x \leq -\frac{5}{4}$

Interval Notation: $\left[-\frac{23}{4}, -\frac{5}{4}\right]$

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- **EXAMPLE:** Solve the inequality symbolically. Express the solution in interval notation.

$$-6 \leq \frac{1-2x}{6} < 12 \qquad [2.3.33]$$

(Simplify your answer. Use integers or decimals for any numbers in the expression.)

We cannot combine like terms yet. There are no parentheses to deal with, but there is a denominator of 6 that is controlling the middle. First, we will clear out fractions.

Multiply all three regions by 6: $\mathbf{6}(-6) \leq \mathbf{6}\left(\frac{1-2x}{6}\right) < \mathbf{6}(12)$

Do we REVERSE the inequalities? NO $\frac{6}{6} = 1$

Simplify – **Divide out common factors**. $-36 \leq 1 - 2x < 72$

Undo positive 1 with subtract 1 all 3 sides. $-1 \quad -1 \qquad -1$

Simplify – **Combine Like Terms**. $-37 \leq -2x < 71$

Divide by the coefficient.

Remember the **Reverse Rule!** $\frac{-37}{-2} \geq \frac{-2x}{-2} > \frac{71}{-2}$

Simplify. (We're using decimals)

Inspect for proper format: $18.5 \geq x > -38.5$

Is smaller number on left, larger on right? NO

Are inequality symbols pointing LEFT? NO

We need to "pivot" or reverse the entire inequality: $-38.5 < x \leq 18.5$

Interval Notation: $\mathbf{(38.5, 18.5]}$

Source Used:

1. Pearson MyLab Math *College Algebra with Modeling and Visualization, 6th Edition*, Rockswold