Section 11.2 – Probability of Events Involving "**NOT**" and "**OR**" math 1010

This table shows the connections that are the basis for the probability rules in this chapter.

	Set Theory	**Logic**	**Arithmetic**
Section 11.2 Operation Connective Symbol	Compliment (′)	Not (~)	Subtraction (–)
Section 11.2 Operation Connective Symbol	Union (∪)	Or (**V**)	Addition (**+**)
Section 11.3 Operation Connective Symbol	Intersection (∩)	And (**Λ**)	Multiplication (•)

1. Events Involving "NOT"

The probability of an event ***not*** happening involves the *compliment* and *subtraction* of probabilities.

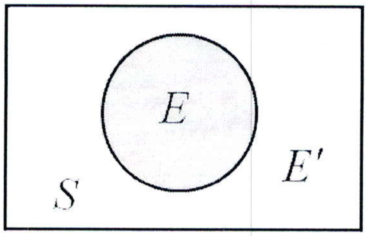
Probability of a Compliment – "NOT"

The probability that an event (E) will ***not*** occur is equal to one minus the probability that an event will occur, (the opposite). Look at the Venn diagram of probability.

$P(\text{not } E) = 1 - P(E)$

Because, $P(E) + P(E') = 1$

Basically, it involves subtraction



EXAMPLE: The probability that it will rain today is 75% or 0.75. What is the probability that it will not rain today?

$P(\text{not rain}) = 1 - P(\text{rain})$
$= 1 - 0.75 = \boxed{0.25 \text{ or } 25\%}$

EXAMPLE: The distribution of degrees conferred at a local college is listed in the table to the right. What is the probability that a randomly selected student's degree is not in liberal arts?

Business	1676
Chemistry	318
Engineering	868
English	2073
Liberal Arts	1358
Mathematics	2164
Physics	856

$P(\text{not Liberal Arts}) = 1 - P(\text{Liberal Arts})$

$P(\text{Liberal Arts}) = \dfrac{1358}{9313} = \underset{\text{rounded}}{0.146}$

* Total = 9313

$P(\text{not liberal Arts}) = 1 - \dfrac{1358}{9313} = \underset{\text{rounded}}{\boxed{0.854}}$

— Alternate Way —

$P(\text{Not Liberal Arts}) = \dfrac{\text{not Liberal Arts}}{\text{Total}} = \dfrac{9313 - 1358}{9313} = \dfrac{7955}{9313} = \underset{\text{rounded}}{\boxed{0.854}}$

EXAMPLE: A single card is drawn from a standard 52-card deck. Answer the following questions:

(a) • What is the probability that your one card is not an ace? 4 Aces; $52-4=48$ not Aces

$P(\text{not Ace}) = 1 - P(\text{Ace}) = 1 - \frac{4}{52} = \frac{48}{52} = \boxed{\frac{12}{13}}$ Alternate way: $P(\text{not Ace}) = \frac{48 \text{ not Ace}}{52 \text{ total}} = \boxed{\frac{12}{13}}$

• What are the odds in favor of not drawing an ace?

$\frac{P(\text{Not ace})}{\underbrace{1-P(\text{Not Ace})}_{\text{complement}}} = \frac{\frac{12}{13}}{1-\frac{12}{13}} = \frac{\frac{12}{13}}{\frac{1}{13}} = \frac{12}{1}$ or $\boxed{12 \text{ to } 1}$

(b) • What is the probability that your one card is not a diamond? 13 Diamonds; $52-13=39$ Not diamonds

$P(\text{not diamond}) = 1 - P(\text{diamond}) = 1 - \frac{13}{52} = \frac{39}{52} = \boxed{\frac{3}{4}}$ Alternate way: $P(\text{not diamond}) = \frac{39 \text{ not diamonds}}{52 \text{ total}} = \boxed{\frac{3}{4}}$

• What are the odds in favor of not drawing a diamond?

$\frac{P(\text{not diamond})}{\underbrace{1-P(\text{not diamond})}_{\text{complement}}} = \frac{\frac{3}{4}}{1-\frac{3}{4}} = \frac{\frac{3}{4}}{\frac{1}{4}} = \frac{3}{1}$ or $\boxed{3 \text{ to } 1}$

(c) • What is the probability that your one card will not be red? 26 Red; $52-26=26$ Not Red

$P(\text{Not Red}) = 1 - P(\text{Red}) = 1 - \frac{26}{52} = \frac{26}{52} = \boxed{\frac{1}{2}}$ Alternate way: $P(\text{Not Red}) = \frac{26 \text{ Not Red}}{52 \text{ total}} = \boxed{\frac{1}{2}}$

• What are the odds in favor of not drawing a red card?

$\frac{P(\text{Not Red})}{\underbrace{1-P(\text{Not Red})}_{\text{complement}}} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = \frac{1}{1}$ or $\boxed{1 \text{ to } 1}$

2. Events Involving "OR"

The probability of one event **OR** another event happening involves the *union* and *addition* of probabilities. Remember **OR** involves one or the other or both.

If two events have nothing in common (disjoint), then the two events, event *A* and event *B*, are said to be Mutually exclusive events. This means they have NO *outcomes in common.*

Probability for *A* "or" *B* uses the addition rule:

(1) If A and B are mutually exclusive (have nothing in common) then add their probabilities:

$$P(A \text{ or } B) = P(A) + P(B)$$

(2) If A and B are two events with something in common then add their probabilities and subtract what they have in common:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Duplicate numbers (numbers being repeated) are subtracted. You CANNOT count the same thing twice.

EXAMPLE: For the experiment of drawing a single card from a standard 52-card deck, find the **(a)** probability of the event below, and **(b)** the odds in favor of the given event.

Ace or Jack

- How many Aces? 4
- How many Jacks? 4
- Mutually exclusive? (anything in common) YES, mutually exclusive (nothing in common)
- Total number of cards in the deck 52
- **(a)** Calculate the probability of drawing an Ace or a Jack:

$$P(\text{Ace or Jack}) = P(\text{Ace}) + P(\text{Jack}) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \boxed{\frac{2}{13}}$$

- **(b)** Calculate the odds in favor of drawing an Ace or a Jack:

$$\text{odds in favor of drawing Ace or Jack} = \frac{P(\text{drawing Ace oR Jack})}{P(\text{not drawing Ace or Jack})} = \frac{\frac{2}{13}}{1-\frac{2}{13}} = \frac{\frac{2}{13}}{\frac{11}{13}} = \frac{2}{11} \text{ or } \boxed{2 \text{ to } 11}$$

EXAMPLE: If a single, six-sided die is rolled, what is the probability of rolling a number greater than 4 or less than 3?

- How many ways to roll greater than 4? roll 5, roll 6 = 2ways
- How many ways to roll less than 3? roll 2, roll 1 = 2ways
- Mutually exclusive? (anything in common) YES, mutually exclusive (nothing in common)
- Total number of outcomes 6
- Calculate the probability of rolling a number greater than 4 or less than 3:

$$P(\text{greater than 4 or less than 3}) = P(\text{greater than 4}) + P(\text{less than 3}) = \frac{2}{6} + \frac{2}{6} = \frac{4}{6} = \boxed{\frac{2}{3}}$$

EXAMPLE: For the experiment of rolling an ordinary pair of dice, find the probability that the sum will be less than 4 or greater than 7.

- How many ways sum less than 4? sum 3: (1,2), (2,1) ; sum 2: (1,1) Total = 3ways
- How many ways sum greater than 7?

Sum 8 (5)	Sum 9 (4)	Sum 10 (3)	Sum 11 (2)	Sum 12 (1)	Total
(2,6),(3,5),(4,4),(5,3),(6,2)	(3,6),(4,5),(5,4),(6,3)	(4,6),(5,5),(6,4)	(5,6),(6,5)	(6,6)	15 ways

- Mutually exclusive? (anything in common) YES, mutually exclusive
- Total number of outcomes rolling 2 dice 36
- Calculate the probability of rolling a sum less than 4 or greater than 7:

$$P(\text{sum less than 4 or greater than 7}) = P(\text{sum Less than 4}) + P(\text{sum greater than 7})$$
$$= \frac{3}{36} + \frac{15}{36}$$
$$= \frac{18}{36} = \boxed{\frac{1}{2}}$$

EXAMPLE: A lottery game has balls numbered 0 through 13. If a ball is selected at random, what is the probability of selecting an even numbered ball or a 3?

- How many even numbered balls? 0, 2, 4, 6, 8, 10, 12 = 7 ways
- How many balls numbered "3"? one = 1 way
- Mutually exclusive? (anything in common) NO
- Total number of lottery balls 0 through 13 = 14 total
- Calculate the probability of selecting even numbered ball or a 3:

$$P(\text{even or } 3) = P(\text{even}) + P(3)$$
$$= \frac{7}{14} + \frac{1}{14} = \frac{8}{14} = \boxed{\frac{4}{7}}$$

EXAMPLE: For the experiment of drawing a single card from a standard 52-card deck, find the **(a)** probability of the event below, and **(b)** the odds in favor of the given event.

spade or face card

- How many spades? 13
- How many face cards? 12
- Mutually exclusive? (anything in common) NO – J, Q, K spades = 3 ways (subtract)
- Total number of cards in the deck 52
- **(a)** Calculate the probability of drawing a spade or a face card:

$$P(\text{spade or face card}) = P(\text{spade}) + P(\text{facecard}) - P(\text{spade and facecard})$$
$$= \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{22}{52} = \boxed{\frac{11}{26}}$$

- **(b)** Calculate the odds in favor of drawing a spade or a face card:

$$\text{odds in favor of spade or face card} = \frac{P(\text{spade or face card})}{P(\text{not spade or face card})} = \frac{\frac{11}{26}}{1-\frac{11}{26}} = \frac{\frac{11}{26}}{\frac{15}{26}} = \frac{11}{15} \text{ or } \boxed{11 \text{ to } 15}$$

EXAMPLE: For the experiment of rolling an ordinary pair of dice, find the probability that the sum will be odd or a multiple of 3?

- How many specific ways can the sum be odd?

Sum 3 (2)	Sum 5 (4)	Sum 7 (6)	Sum 9 (4)	Sum 11 (2)	Total
(1,2), (2,1)	(1,4), (2,3) (3,2), (4,1)	(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)	(3,6), (4,5), (5,4), (6,3)	(5,6) (6,5)	18

- How many specific ways can the sum be a multiple of 3?

Sum 3 (2)	Sum 6 (5)	Sum 9 (4)	Sum 12 (1)	Total
(1,2), (2,1)	(1,5), (2,4), (3,3), (4,2), (5,1)	(3,6), (4,5), (5,4), (6,3)	(6,6)	12

- Mutually exclusive? (anything in common) NO – sum 3, sum 9 = 6 ways (subtract)
- Total number of outcomes rolling a pair of dice? 36
- Calculate the probability that the sum will be odd or a multiple of 3:

$$P(\text{sum odd or multiple of } 3) = P(\text{odd}) + P(\text{mult. }3) - P(\text{odd and mult. }3)$$
$$= \frac{18}{36} + \frac{12}{36} - \frac{6}{36} = \frac{24}{36} = \boxed{\frac{2}{3}}$$

EXAMPLE: For the experiment of drawing a single card from a standard 52-card deck, find the **(a)** probability of the event below, and **(b)** the odds in favor of the given event.

Neither a heart nor a king or queen

- How many hearts? 13
- How many kings or queens? 4K + 4Q = 8
- Mutually exclusive? (anything in common) NO - King♡, Queen♡ = 2 ways ↑ subtract
- Total number of cards in the deck 52
- **(a)** Calculate the probability of drawing neither a heart nor a king or queen:

$$P(\text{heart or K or Q}) = P(\text{heart}) + P(\text{K or Q}) - P(\text{heart and K or Q})$$

$$= \frac{13}{52} + \frac{8}{52} - \frac{2}{52} = \frac{19}{52}$$

$$P(\text{neither heart nor K or Q}) = 1 - P(\text{heart or K or Q})$$

*using complements rule for "NOT"

$$= 1 - \frac{19}{52} = \boxed{\frac{33}{52}}$$

- **(b)** Calculate the odds in favor of drawing neither a heart nor a king or queen:

odds in favor of drawing neither heart nor K or Q

$$= \frac{P(\text{neither heart nor K or Q})}{P(\text{heart or K or Q})}$$

prob. it occurs

prob. it does not occur

$$= \frac{\frac{33}{52}}{\frac{19}{52}} = \frac{33}{19} \text{ or } \boxed{33 \text{ to } 19}$$