1. Converse, Inverse, and Contrapositive

In mathematics, there are many rules, properties, and theorems that are stated using ***if…then***. These are conditional statements, $p \to q$, if p then q

If ***p*** and ***q*** are interchanged, negated, or both then a new conditional statement is formed.

1. Look at the following statement:

   Conditional: If I go, then you stay.

   p is I go

   q is you stay

2. Let's interchange ***p*** and ***q***, in other words, switch their position in the statement:

   Converse: If you stay then I go

3. Look back at #1, the original conditional statement. Now let's write the ***p*** and ***q*** as a negation:

   Inverse: If I do not go, then you do not stay

4. Look back at #2, the converse statement. Now let's write the ***p*** and ***q*** as a negation:

   Contrapositive: If you do not stay then I do not go

Conditional……opposite……Inverse (the negative)
Converse………opposite……Contrapositive (the negative)

EXAMPLE: Write the three related statements given the conditional statement:

Conditional: If there is smoke then there is fire.

Converse: If there is fire then there is smoke

Inverse: If there is no smoke then there is no fire

Contrapositive: If there is no fire then there is no smoke

Here is a chart that shows all four conditional and related statements in symbol form.

Related Conditional Statements		
Conditional Statement	$p \to q$	(If p, then q.)
Converse	$q \to p$	(If q, then p.)
Inverse	$\sim p \to \sim q$	(If not p, then not q.)
Contrapositive	$\sim q \to \sim p$	(If not q, then not p.)

EXAMPLE: Write in symbol form, the three related statements given the conditional statement. Make sure to simplify two negatives, just like in regular math.

Conditional: $\sim p \to q$

Converse: $q \to \sim p$

Inverse: $p \to \sim q$

Contrapositive: $\sim q \to p$

EXAMPLE: Translate the conditional statement into symbols.
The translation might be the converse, inverse, or contrapositive.

~~A triangle is equilateral only if it has three sides of equal length.~~

p = A triangle is equilateral
q = a triangle has three sides of equal length

1. A triangle is equilateral if it has three sides of equal length. $q \to p$
2. If a triangle is not equilateral then it does not have three sides of equal length. $\sim p \to \sim q$

2. Biconditional statements

A compound statement using the words **"if and only if"** (abbreviated as *iff*) is called a biconditional statement, $p \leftrightarrow q$, *p if and only if q* *p iff q*

Biconditional means there are two conditions: $p \rightarrow q$ and $q \rightarrow p$

Therefore in symbol form: $(p \rightarrow q) \wedge (q \rightarrow p) = p \leftrightarrow q$

Construct the truth table for "biconditional"

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	F	F	F
F	T	T	T	F	F
F	F	T	T	T	T

Notice that a biconditional statement is true when both **p** and **q** statements are the SAme.

EXAMPLE: Determine if each biconditional statement is TRUE or FALSE.

1. $\overset{T}{6 + 8 = 14}$ if and only if $\overset{T}{11 + 5 = 16}$ True, both same
2. $\overset{F}{5 + 2 = 10}$ if and only if $\overset{T}{17 + 19 = 36}$ False
3. $\overset{F}{6 = 5}$ if and only if $\overset{F}{12 \neq 12}$ True both same
4. Apple makes Ipods (T) if and only if Burger King sells Big Macs (F) False

3. Consistent and Contrary

If two statements are **both true** about the same object, the statements are consistent

EXAMPLE: "the apple is red" and "it weighs 12 ounces"

If two statements **cannot** be true about the same object, the statements are contrary

EXAMPLE: "the apple is red" and "the apple is green"

EXAMPLE: decide if each pair of statements are consistent or contrary

contrary Elvis is dead. Elvis is alive!

contrary George Bush was a republican. George Bush was a Democrat.

consistent That animal has four legs. That animal is a tiger.

contrary Pi is an irrational number. Pi is a whole number.

consistent This number is a whole number. This number is an integer.