Notes Section 3.5 – Transformations of Graphs

Lesson Objectives

1. Parent Functions
2. Vertical and Horizontal Translations (shifts)

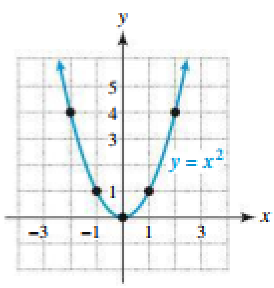
A. Parent Functions

1. **Quadratic (or Square) Function:**

$f(x) = x^2$ or $y = x^2$

Square Function: $f(x) = x^2$

x	-2	-1	0	1	2
$y = x^2$	4	1	0	1	4



$D = (-\infty, \infty)$

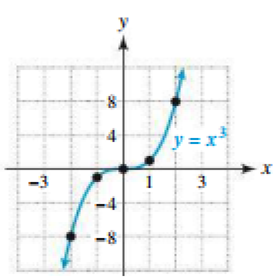
$R = [0, \infty)$

2. **Cubic Function:**

$f(x) = x^3$ or $y = x^3$

Cube Function: $f(x) = x^3$

x	-2	-1	0	1	2
$y = x^3$	-8	-1	0	1	8



$D = (-\infty, \infty)$

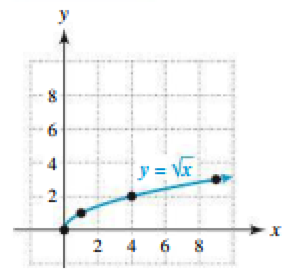
$R = (-\infty, \infty)$

3. **Square Root Function:**

$f(x) = \sqrt{x}$ or $y = \sqrt{x}$

Square Root Function: $f(x) = \sqrt{x}$

x	0	1	4	9
$y = \sqrt{x}$	0	1	2	3



$D = [0, \infty)$

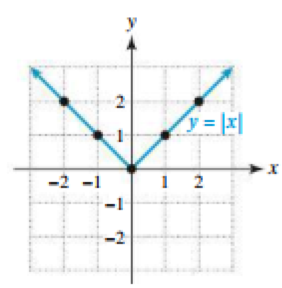
$R = [0, \infty)$

4. **Absolute Value Function:**

$f(x) = |x|$ or $y = |x|$

Absolute Value Function: $f(x) = |x|$

x	-2	-1	0	1	2
$y = \|x\|$	2	1	0	1	2



$D = (-\infty, \infty)$

$R = [0, \infty)$

B. Vertical and Horizontal Shifts

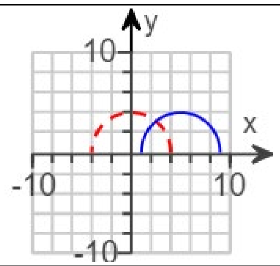
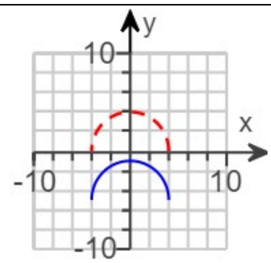
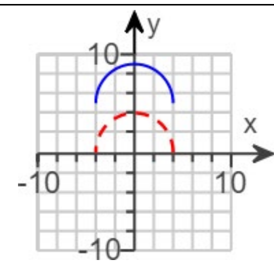
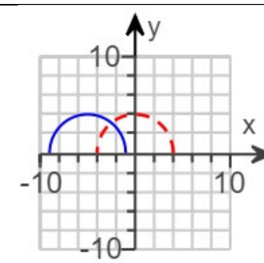
Let f be a function, and let c be a positive number.

To Graph	Shift the Graph of $y = f(x)$ by c Units	(NOTES below)
$y = f(x) + c$	upward	Adding or subtracting a number OUTSIDE parentheses with y causes a VERTICAL SHIFT in the SAME direction as that number. **OUTSIDE – *y* is "do what you see."** (↓ , ↑)
$y = f(x) - c$	downward	
$y = f(x - c)$	right	Adding or subtracting a number INSIDE parentheses with x causes a HORIZONTAL SHIFT in the **OPPOSITE** direction of that number. **INSIDE – *x* goes OPPOSITE!** (← →)
$y = f(x + c)$	left	

Notes Section 3.5 – Transformations of Graphs

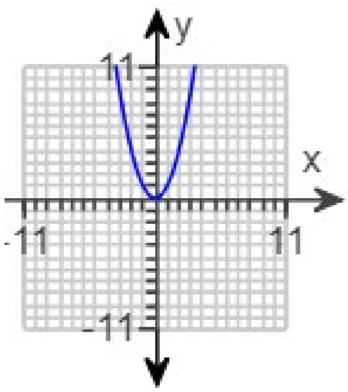
- **EXAMPLE:** The graph of $y = f(x)$ is shown with dashed (red) lines. Graph $y = f(x) + 5$.
  Choose the correct graph in solid (blue). [3.5.31]

$y = f(x) + 5$ **OUTSIDE – *y* is "do what you see."** + 5 means **UP 5** Graph **C**.

A.	B.	C.	D.
			
RIGHT 5, or *f*(*x* – 5) INCORRECT	DOWN 5, or *f*(*x*) – 5 INCORRECT	**UP 5, or *f*(*x*) + 5** **CORRECT**	LEFT 5, or *f*(*x* + 5) INCORRECT

- **EXAMPLE:** Determine which graph indicates the shift in the indicated equation. [3.5-6]

$y = f(x - 3) - 5$

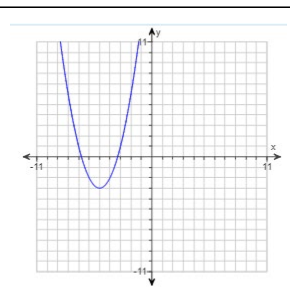
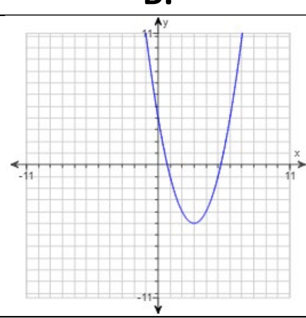
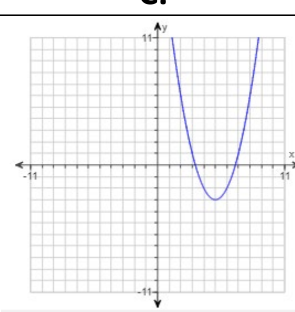
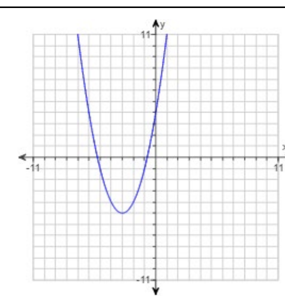


The graph to the left is the given graph of *y* = *f*(*x*).
To graph *y* = *f*(*x* – 3) – 5

<u>**INSIDE** parentheses</u>:
***x* goes OPPOSITE!**
I see – 3 with *x*, so the shift is to the **RIGHT**, not the left.

<u>**OUTSIDE** parentheses</u>:
***y* is "do what you see"**
I see – 5 outside parentheses, so the shift is also **DOWN** 5.

So, together, the shift for $y = f(x - 3) - 5$ is
RIGHT 3, DOWN 5. Correct answer is **B**.

A.	B.	C.	D.
			
Left 5, Down 3	**Right 3, Down 5**	Right 5, Down 3	Left 3, Down 5
INCORRECT	**CORRECT**	INCORRECT	INCORRECT

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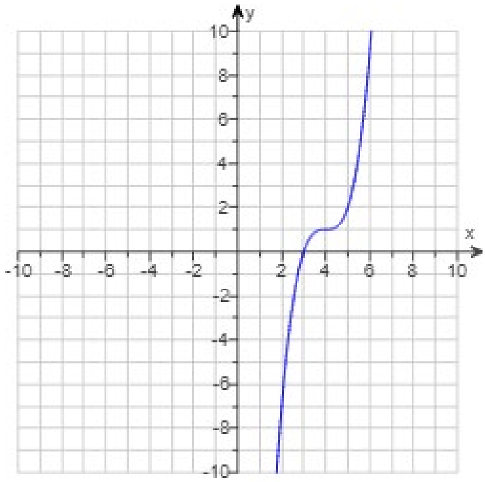
Notes Section 3.5 – Transformations of Graphs

- **EXAMPLE:** The graph is a translation of one of the basic functions $y = x^2, y = x^3, y = \sqrt{x}, y = |x|$. Find the equation that defines the function. [3.5.1] (Type an expression using *x* as the variable. Do not simplify).



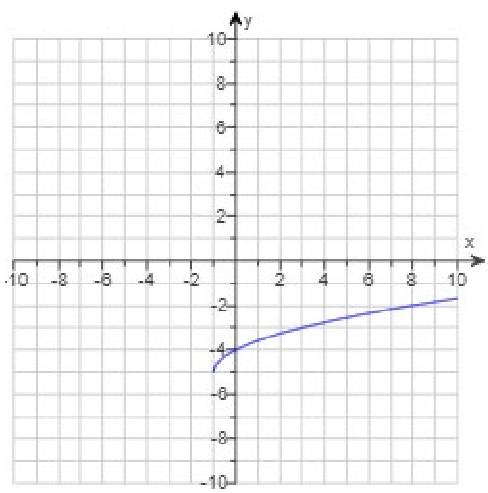
The graph to the left is a translation of $y = x^3$.
The INFLECTION point for $y = x^3$ is normally at the origin.
In this graph, though, it has moved: **RIGHT 4, UP 1**

RIGHT 4 is a change in ***x***, so be sure to **SWITCH** the value.
(remember: **INSIDE – *x* goes OPPOSITE**!)
So, RIGHT 4 is written as $(x - 4)$.

UP 1 is a change in ***y***, so that is written as **+ 1**.
(remember: **OUTSIDE – *y* is "do what you see."**)

When $y = x^3$ that goes RIGHT 4, UP 1,
the equation is *y* = $\boldsymbol{(x-4)^3 + 1}$

- **EXAMPLE:** The graph is a translation of one of the basic functions $y = x^2, y = x^3, y = \sqrt{x}, y = |x|$. Find the equation that defines the function. [3.5.5] (Type an expression using *x* as the variable. Do not simplify).



The graph to the left is a translation of $y = \sqrt{x}$.
The starting point for $y = \sqrt{x}$ is normally at the origin.
In this graph, though, it has moved: **LEFT 1, DOWN 5**.

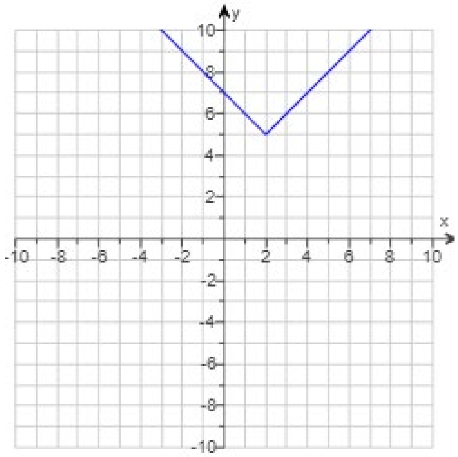
LEFT 1 is a change in ***x***, so be sure to SWITCH the value.
(remember: **INSIDE – *x* goes OPPOSITE**!)
So, LEFT 1 is written as $(x + 1)$.

DOWN 5 is a change in *y*, so that is written as **– 5**.
(remember: **OUTSIDE – *y* is "do what you see."**)

When $y = \sqrt{x}$ goes LEFT 1, DOWN 5,
The equation is *y* = $\boldsymbol{\sqrt{x+1} - 5}$.

Notes Section 3.5 – Transformations of Graphs

- **EXAMPLE:** The graph is a translation of one of the basic functions $y = x^2, y = x^3, y = \sqrt{x}, y = |x|$. Find the equation that defines the function. [3.5.7] (Type an expression using *x* as the variable. Do not simplify).



The graph to the left is a translation of $y = |x|$.
The vertex for $y = |x|$ is normally at the origin.
In this graph, though, it has moved: **RIGHT 2, UP 5**.

RIGHT 2 is a change in ***x***, so be sure to SWITCH the value.
(remember: **INSIDE – *x* goes OPPOSITE**!)
So, RIGHT 2 is written as $(x - 2)$.

UP 5 is a change in ***y***, so that would be written as **+ 5**.
(remember: **OUTSIDE – *y* is "do what you see."**)

When $y = |x|$ that goes RIGHT 2, UP 5,
the equation is *y* = $|x - 2| + 5$.

- **EXAMPLE:** Find the equation that shifts the graph of *f* by the indicated amounts.
  $f(x) = x^4$ right 8 units, up 7 units [3.5-1]

Right 8 units is a change in ***x***, so be sure to SWITCH the value.

So, RIGHT 8 is written as $(x - 8)$. (remember: **INSIDE – *x* goes OPPOSITE**!)

Up 7 units is a change in ***y***, so UP 7 is written with **+ 7** at the end.
(remember: **OUTSIDE – *y* is "do what you see."**)

A.	$y = -(x-8)^4 + 7$	INCORRECT. The negative sign in front of parentheses inverts the graph upside-down, which is a reflection over the *x*-axis.
B.	$y = -(x-8)^4 + 56$	INCORRECT. Similar to answer A., and + 56 also incorrect.
C.	$\mathbf{y = (x-8)^4 + 7}$	**CORRECT.**
D.	$y = (x+8)^4 - 7$	INCORRECT. This graph goes LEFT 8, DOWN 7

Notes Section 3.5 – Transformations of Graphs

- **EXAMPLE:** Use transformations to explain how the graph of f can be found using the graph of $y = x^2$.

$$f(x) = (x-3)^2 + 2 \qquad [3.5.53]$$

I see **– 3** with x, so the shift is to the **RIGHT**, not the left.
(remember: **INSIDE – *x* goes OPPOSITE**!)

So $(x-3)$ means it moves **RIGHT 3**.

I see **+ 2** outside parentheses, so the shift is also **UP 2**.
(remember: **OUTSIDE – *y* is "do what you see."**)

So, together, the shift for $f(x) = (x-3)^2 + 2$ from $y = x^2$ is **RIGHT 3, UP 2**.

- **EXAMPLE:** Use transformations of the graphs of $y = x^2$ or $y = |x|$ to sketch a graph of f by hand.

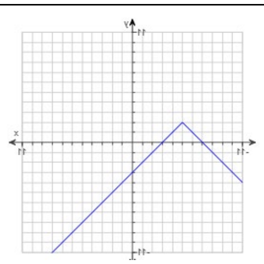
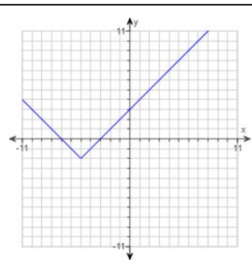
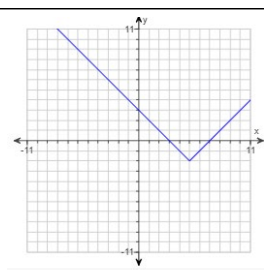
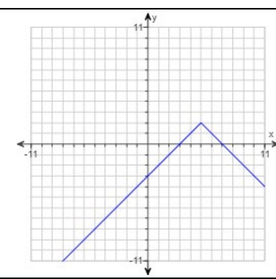
$$f(x) = |x-5| - 2 \qquad [3.5\text{-}11]$$

I see **– 5** with ***x***, so the shift is to the **RIGHT**, not the **left**.
(remember: **INSIDE – *x* goes OPPOSITE**!)

So $|x-5|$ means it moves **RIGHT 5**.

I see **– 2** outside parentheses, so the shift is also **DOWN 2**.
(remember: **OUTSIDE – *y* is "do what you see."**)

So, together, the shift for $f(x) = |x-5| - 2$ from $y = |x|$ is **RIGHT 5, DOWN 2**.

A.	**B.**	**C.**	**D.**
			
Right 5, Up 2, Inverted	Left 5, Down 2	**Right 5, Down 2**	Right 5, Up 2, Inverted
INCORRECT	INCORRECT	**CORRECT**	INCORRECT

Notes Section 3.5 – Transformations of Graphs

- **EXAMPLE:** Find the equation that shifts the graph of *f* by the desired amounts. Graph *f* and the shifted graph in the same *xy*-plane. [3.5.15]

  $f(x) = x^2 - 2x + 2$ right 5 units, upward 3 units

Right 5 units is a change in ***x***, so be sure to switch the value.

RIGHT 5 is written as: $(x - 5)$. (remember: **INSIDE – *x* goes OPPOSITE**!)

Use $(x - 5)$ everywhere you see an ***x*** in the function.

$x^2 \quad - 2x \quad + 2$ changes to

$(x - 5)^2 - 2(x - 5) + 2$

Upward 3 units is a change in ***y***, so jut include a **+ 3** at the end.
(remember: **OUTSIDE – *y* is "do what you see."**)

$(x - 5)^2 - 2(x - 5) + 2 \; + 3$ Combine like terms on the end

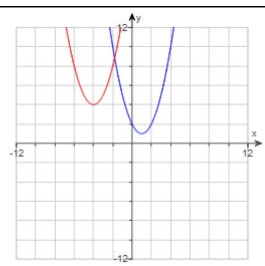
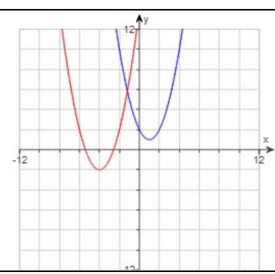
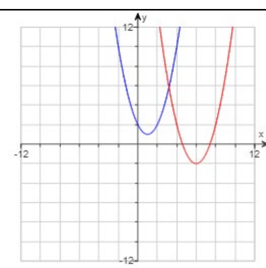
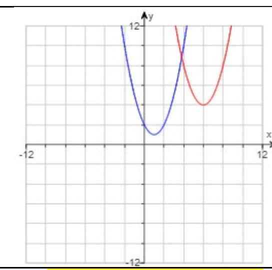
right 5 **right 5** **up 3** **2 + 3 = 5**

$(x - 5)^2 - 2(x - 5) + 5$

Updated: $y = (x - 5)^2 - 2(x - 5) + 5$

(Make sure BOTH sets of parentheses have the SAME value!)

Notice that in all four graphs, one of the graphs is always in the same place, with vertex at about (1,1). Remember that the overall shift is right 5 units and upward 3 units, or more simply: **RIGHT and UP**.

A.	B.	C.	D.
			
LEFT and UP	LEFT and DOWN	RIGHT and DOWN	RIGHT and UP
INCORRECT	INCORRECT	INCORRECT	CORRECT

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Notes Section 3.5 – Transformations of Graphs

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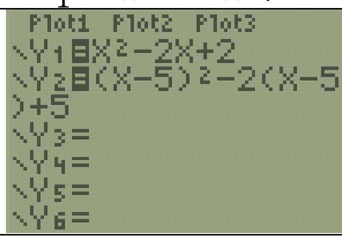
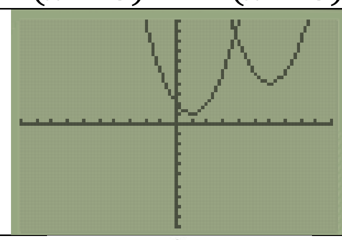
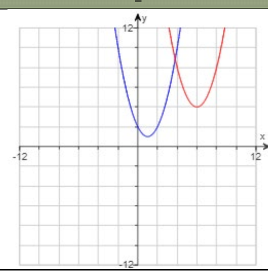
- **EXAMPLE:** Find the equation that shifts the graph of *f* by the desired amounts.

Graph *f* and the shifted graph in the same *xy*-plane. [3.5.15]

$$f(x) = x^2 - 2x + 2 \qquad \text{right 5 units, upward 3 units}$$

NOTE: You can also verify the graph on your graphing calculator:

Original function	Modified function: right 5, upward 3
$f(x) = x^2 - 2x + 2$	$y = (x-5)^2 - 2(x-5) + 5$
$Y_1 = x^2 - 2x + 2$	$Y_2 = (x-5)^2 - 2(x-5) + 5$
	
The graph on the calculator matches the answer we got on the previous page:	

Sources Used:

1. MyLab Math for *College Algebra with Modeling and Visualization*, 6th Edition, Rockswold, Pearson Education Inc.
2. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website https://archive.codeplex.com/?p=wabbit