Remember:

Statements are represented by the letters: ***p, q, r***
Connectives used in this section are: ***and, or, not***

In this section we will find the truth values, ***T*** or ***F***, for compound statements (two or more statements put together by a connective). First let's use ordinary language and statements to create the rules for finding truth values. Then we will write the rule in symbol form.

## 1. *AND*

Find the truth value, ***T*** or ***F***, of compound statements using the connective, ***and***.

***and*** (∧) is a connective that implies both.

**EXAMPLE:** "I want to go to dinner and a movie." This statement implies doing both activities.

**Compound statement: Last night, we went to dinner and a movie.**

***p*** = went to dinner AND ***q*** = went to a movie

Is ***p*** true? yes Is ***q*** true? yes

Is the compound statement ***p and q*** true? yes

**Compound statement: Last night, we went to dinner and did not go to a movie.**

***p*** = went to dinner AND ***q*** = did not go to movie

Is ***p*** true? yes Is ***q*** true? no

Is the compound statement ***p and q*** true? no

Create a truth value table of all the possible values for compound statements using, ***and***:

Truth Table Rule for ***AND***

| p | q | p ∧ q |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

## 2. *OR*

Find the truth value, ***T*** or ***F***, of compound statements using the connective, ***or***.

***or*** (∨) is a connective that implies one or the other or both.

**EXAMPLE:** "I will help you paint, so I will paint the walls or the ceiling." This statement implies doing one of the activities or maybe both.

**Compound statement: I painted your walls. I painted your ceiling.**

***p*** = I painted walls OR ***q*** = I painted the ceiling

Is ***p*** true? yes Is ***q*** true? yes

Is the compound statement ***p or q*** true? yes

**Compound statement: I painted your walls. I did not paint your ceiling.**

***p*** = I painted walls OR ***q*** = did not paint ceiling

Is ***p*** true? yes Is ***q*** true? no

Is the compound statement ***p or q*** true? yes

Create a truth value table of all the possible values for compound statements using, ***or***:

Truth Table Rule for ***OR***

| p | q | p ∨ q |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

## 3. *NOT*

Find the truth value, ***T*** or ***F***, of statements using ***not***.

***not*** (~) is a negation that implies the opposite, complement.

**EXAMPLE:** The United States has a president.

**Statement: Obama ~~is~~ WAS the President of the United States.**

Is ***p*** true? yes

What is ~***p*** = Obama WAS not president (false)

**Statement: Jimmy Fallon, of The Tonight Show, is the President of the United States.**

Is ***p*** true? no

What is ~***p*** = Jimmy Fallon is not president (true)

Create a truth value table of all the possible values for using, ***not***:

Truth Table Rule for ***NOT***

| p | ~p |
|---|---|
| T | F |
| F | T |

## 4. DeMorgan's Law: The complement (negation) of compound statements.

The complement of *p or q*: $\sim (p \vee q) = \sim p \wedge \sim q$

The compliment of *p and q*: $\sim (p \wedge q) = \sim p \vee \sim q$

**EXAMPLE:** use DeMorgan's law to negate the statements.

Statement: I got an A or I got a B on my test.

Negation: I did not get an A and I did not get a B on my test

Statement: It is summer and there is no snow.

Negation: It is not summer or there is snow

**TRUTH VALUE EXAMPLES:** If ***p* is a false** statement and ***q* is a true** statement, find the truth value for the given compound statements.

1. $\sim p$ = T
2. $p \vee q$ = F or T = T
3. $\sim p \wedge q$ = T and T = T
4. $p \vee \sim q$ = F or F = F

## 5. TRUTH TABLES:

1. Draw a basic table
2. *p* and *q* are the statements and go on the left side of the table.

* You need to know how many basic combinations of TRUE and FALSE will be in your truth table (left side/column). In other words, how many rows are in the truth table? Use the same formula from Chapter 2, when we looked for the number of subsets created from one given set: $2^n$ where n was the number of elements in a set. Now n will be the number of statements.

The formula: $2^n$ How many rows? $2^1=2$ $2^2=4$ $2^3=8$
(p) (pq) (pqr)

3. In many mathematical problems, sometimes you have several steps to get to the final answer. Create a column for each step, the last column being your final answer.
4. We solve math problems using Order of Operations, so you must fill in truth tables in a particular order, too:
   ① Parentheses
   ② Not
   ③ And, Or
5. Refer back the basic truth table rules to follow the pattern and find your answers.

**EXAMPLE:** construct a truth table for the given compound statements. Use this page or your own notebook paper.

1. $\sim (p \wedge q)$ $2^2 = 4$ rows
2. $\sim p \vee \sim q$ $2^2 = 4$ rows
3. $\sim q \wedge (\sim p \vee q)$ $2^2 = 4$ rows
4. $(\sim p \wedge q) \wedge r$ $2^3 = 8$ rows

1) $\overset{②}{\sim}(p \overset{①}{\wedge} q)$

| p | q | $(p \wedge q)$ | $\sim(p \wedge q)$ |
|---|---|---|---|
| T | T | T | F |
| T | F | F | T |
| F | T | F | T |
| F | F | F | T |

↑ final answer

2) $\overset{①}{\sim}p \overset{③}{\vee} \overset{②}{\sim}q$

| p | q | $\sim p$ | $\sim q$ | $\sim p \vee \sim q$ |
|---|---|---|---|---|
| T | T | F | F | F |
| T | F | F | T | T |
| F | T | T | F | T |
| F | F | T | T | T |

↑ Final Answer

3) $\sim q \wedge (\sim p \vee q)$

③ ④ ① ②

| p | q | ~p | (~p ∨ q) | ~q | ~q ∧ (~p ∨ q) |
|---|---|---|---|---|---|
| T | T | F | T | F | F |
| T | F | F | F | T | F |
| F | T | T | T | F | F |
| F | F | T | T | T | T |

↑ Final answer

4) $(\sim p \wedge q) \wedge r$

① ② ③

| p | q | r | ~p | (~p ∧ q) | (~p ∧ q) ∧ r |
|---|---|---|---|---|---|
| T | T | T | F | F | F |
| T | T | F | F | F | F |
| T | F | T | F | F | F |
| T | F | F | F | F | F |
| F | T | T | T | T | T |
| F | T | F | T | T | F |
| F | F | T | T | F | F |
| F | F | F | T | F | F |

↑ Final answer