Notes Section 5.6 – Logarithmic Equations Using Properties

Lesson Objectives

1. Use logarithm properties to solve equations

Expanding/Condensing Logarithm Properties – Reminder for your reference:

- **Product Rule:**

$$\log_a(mn) = \log_a(m) + \log_a(n) \qquad \text{EXPANDING} \qquad \text{(Product to Sum)}$$

or $\quad \log_a(m) + \log_a(n) = \log_a(mn) \qquad$ CONDENSING $\qquad$ (Sum to Product)

- **Quotient Rule:**

$$\log_a\left(\frac{m}{n}\right) = \log_a(m) - \log_a(n) \quad \text{EXPANDING} \quad \text{(Quotient to Difference)}$$

or $\quad \log_a(m) - \log_a(n) = \log_a\left(\frac{m}{n}\right) \quad$ CONDENSING $\quad$ (Difference to Quotient)

- **Power Rule:**

$$\log_a(m^r) = r\log_a(m) \qquad \text{EXPANDING} \qquad \text{(Exponent to Coefficient)}$$

or $\quad r\log_a(m) = \log_a(m^r) \qquad$ CONDENSING $\qquad$ (Coefficient to Exponent)

- **EXAMPLE:** Solve the equation. $\qquad \log_5 2 + \log_5 x = 0 \qquad\qquad$ [*Martin-Gay 9.8.13]

Use **Product Rule** (sum to product) to CONDENSE to a single logarithm. This will also _____ the logarithm.	$\log_5(2) + \log_5(x) = 0$
Do **NOT** divide by the _____ yet! It is trapped inside the logarithm.	$\log_5(\qquad\qquad) = 0$
To undo the logarithm, convert to the _____ form.	$\log_5(2x) = \qquad 0$ A logarithm is an exponent
Simplify. Remember property: $a^0 =$ _____	
Solve the equation. (Divide both sides by 2)	
Simplify.	
Answer:	

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- **EXAMPLE:** Solve the following logarithmic equation.

$$\log_2 x + \log_2 7 = 3 \qquad\qquad \text{[*Angel 13.6.47]}$$

Use _____ **Rule** (_____ to product) to CONDENSE to a single logarithm. This will also **isolate** the logarithm.	$\log_2(x) + \log_2(7) = 3$
Do **NOT** divide by the 7 yet! It is trapped _____ the logarithm.	$\log_2(\qquad) = 3$
To _____ the logarithm, convert to the exponential form.	$\log_2(7x) = \quad 3$ A logarithm is an exponent
Simplify. $2^3 = $ _____	
Solve the equation. (Divide both sides by 7)	
Simplify.	
Answer:	
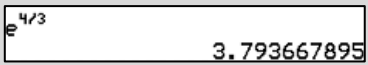
- **EXAMPLE:** Solve the logarithmic equation. $\quad \ln x + \ln x^2 = 4 \qquad$ [5.6.59]
  (Round to the nearest thousandth as needed.)

Use **Product Rule** (sum to product) to CONDENSE to a _____ logarithm. This will also **isolate** the logarithm.	$\ln(x) + \ln(x^2) = 4$
Simplify. $x \cdot x^2 = $ ____ (_____ exponents)	$\ln(\qquad) = 4$
Equation will be EASIER with just x than with x^3. Use _____ **Rule** (exponent to coefficient)	$\ln(\qquad) = 4$
Divide both sides by 3.	
Simplify. Remember that **ln** is same as **log__**	
To undo the logarithm, convert to the exponential form.	A logarithm is an exponent
This is the _____ answer: $x = e^{4/3}$	
Use calculator to get the rounded answer: 2ND LN 4 ÷ 3 ENTER	$e^{4/3}$ 3.793667895 **Rounded Answer:**

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(**RESET** – Here's another way to do the previous problem):

- **EXAMPLE:** Solve the logarithmic equation. $\ln x + \ln x^2 = 4$ [5.6.59]
  (Round to the nearest thousandth as needed.)

Rather than use the Product Rule like before, use the _____ **Rule** (exponent to coefficient) on the second term.	$\ln(x) + \ln(x^2) = 4$
The first term has an understood coefficient ____.	
Combine like terms: **1** of them + **2** of them = _____ of them	$\mathbf{1}\ln(x) + \mathbf{2}\ln(x) = 4$
(From here, the steps are the same as before.) Divide both sides by 3.	
Simplify. Remember that **ln** is same as **log$_e$**	$\dfrac{3\ln(x)}{3} = \dfrac{4}{3}$
To undo the logarithm, convert to the exponential form.	$\log_e(x) = \dfrac{4}{3}$ **A logarithm** is **an exponent**
This is the **exact answer:** $x = e^{4/3}$	$e^{4/3} = x$
Use calculator to get the rounded answer: 2ND LN 4 ÷ 3 ENTER	$e^{4/3}$ 3.793667895 **Rounded Answer:** $x \approx 3.794$

You can use EITHER method when you solve a problem like the previous examples (for Question 7 in the Homework). Be ready to do either the **exact** answer (like $e^{4/3}$) or the rounded answer.

Sources Used:
1. MyLab Math for *Elementary & Intermediate Algebra for College Students*, 5th Edition, Angel, Pearson Education Inc.
2. MyLab Math for *Intermediate Algebra: A Graphing Approach*, 5th Edition, Martin-Gay, Pearson Education Inc.
3. MyLab Math for *College Algebra with Modeling and Visualization*, 6th Edition, Rockswold, Pearson Education Inc.
4. Texas Instruments TI Connect® CE software, https://education.ti.com/en/products/computer-software/ti-connect-ce-sw