Notes Section 3.4 – Quadratic Inequalities

Lesson Objectives

1. Solve Quadratic Inequalities Graphically – when formula *is* or is *not* given.
2. Solve Quadratic Inequalities Symbolically

A. Solve Quadratic Inequalities **Graphically**

$\boldsymbol{f(x) = 0}$ means *y*-coordinate is **zero**

$\boldsymbol{f(x) > 0}$ means *y*-coordinate is **positive**

$\boldsymbol{f(x) < 0}$ means *y*-coordinate is **negative**

$\boldsymbol{f(x) = 0}$ is **ON** the *x*-axis

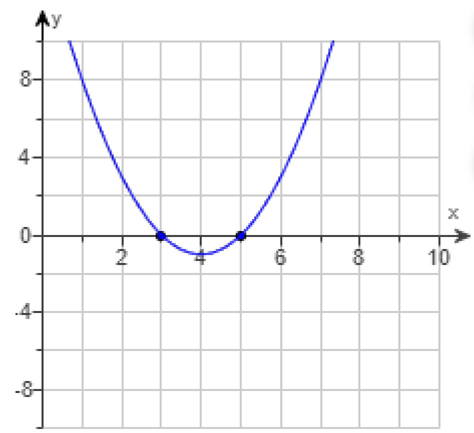
$\boldsymbol{f(x) > 0}$ is **ABOVE** the *x*-axis

$\boldsymbol{f(x) < 0}$ is **BELOW** the *x*-axis

1. Solve Graphically When formula is **NOT** given

- **EXAMPLE:** Given the graph of $f(x)$, solve:
  (a) $f(x) > 0$
  (b) $f(x) < 0$. [3.4.23]

Before we can solve the inequality, we must first solve the **equation** $\boldsymbol{f(x) = 0}$. The solutions are the *x*-intercepts.



The *x*-intercepts are: $(\mathbf{3}, 0)$ and $(\mathbf{5}, 0)$. So, the solution to $f(x) = 0$ based on the given graph is:

$\boldsymbol{x = 3}$ and $\boldsymbol{x = 5}$

These are called **critical points** (CP) for the inequality. These two critical points now divide the domain (*x*) into three (3) distinct parts:

	Left side	Center	Right side
(written as inequality)	$\boldsymbol{x < 3}$	$\boldsymbol{3 < x < 5}$	$\boldsymbol{x > 5}$
(in interval notation)	$(-\infty, \mathbf{3})$	$(\mathbf{3}, \mathbf{5})$	$(\mathbf{5}, \infty)$

NOTE: Make sure you can write these intervals in either inequality or interval notation!

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Notes Section 3.4 – Quadratic Inequalities

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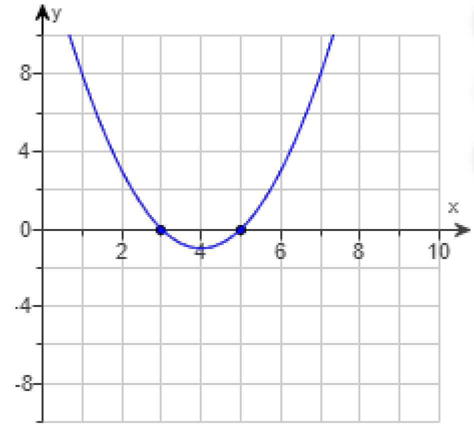
(a) Solve $f(x) > 0$.

This means look **ABOVE** the *x*-axis.

There are **two pieces** of the graph that are **ABOVE** the *x*-axis – the "**tails**" of the graph.

- To the LEFT of $x = 3$, which is $\boldsymbol{x < 3}$
- To the RIGHT of $x = 5$,which is $\boldsymbol{x > 5}$



The correct solution for $f(x) > 0$ is

$\boldsymbol{x < 3 \text{ or } x > 5}$,

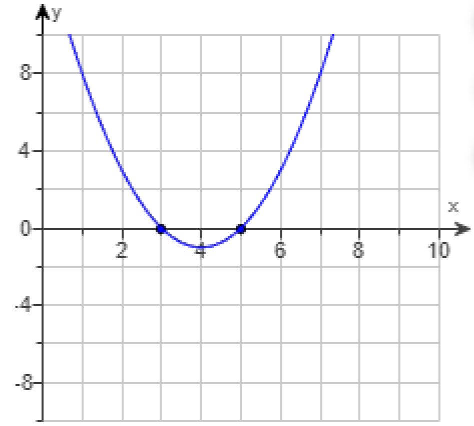
or interval notation $(-\infty, \mathbf{3}) \cup (\mathbf{5}, \infty)$.

(b) Solve $f(x) < 0$.

This means look **BELOW** the *x*-axis.

There is **one piece** of the graph that is **BELOW** the *x*-axis – the "**bowl**" of the graph.

- In BETWEEN $x = 3$ and $x = 5$, which is the interval $\boldsymbol{3 < x < 5}$

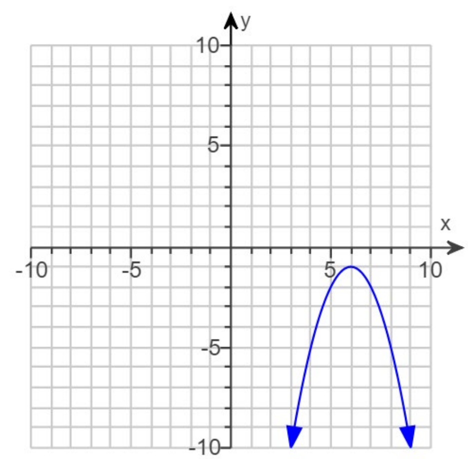


The correct solution for $f(x) > 0$ is $\boldsymbol{3 < x < 5}$, or interval notation $(\mathbf{3}, \mathbf{5})$.

- **EXAMPLE:** The graph of $f(x) = ax^2 + bx + c$ is shown. Solve each inequality. [3.4.27]

(a) $f(x) > 0$

(b) $f(x) < 0$



This time there are **NO** *x*-intercepts. This problem takes a slightly different approach, so **be careful!**

(a) $f(x) > 0$ means **ABOVE** the *x*-axis. The graph of $f(x)$ is **never** above the *x*-axis!

This inequality has **NO SOLUTION** or { } or Ø.

(b) $f(x) < 0$ means **BELOW** the *x*-axis. The graph of $f(x)$ is **always** below the *x*-axis!

The solution to the inequality is

ALL REAL NUMBERS,

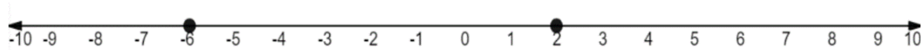
or in interval notation, it is $(-\infty, \infty)$.

Notes Section 3.4 – Quadratic Inequalities

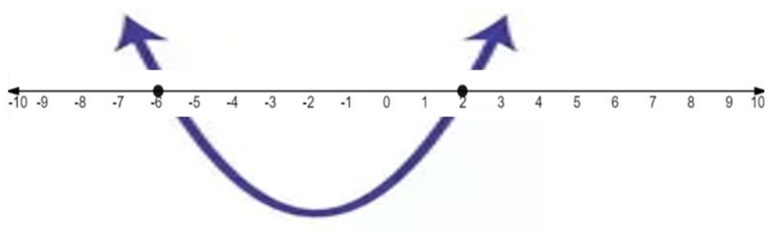
2. Solve Graphically When formula **IS** given

1. Make sure you have **zero** on the right.
2. Find the **solutions** – treat it as if it's an equation.
3. The solutions are the **CRITICAL POINTS** (or *boundary points*).
4. Graph critical points on a **number line** (*x*-axis).
5. **Inspect** leading coefficient (***a***) to see if parabola opens UP or DOWN.
6. **Sketch** parabola passing through number line.
7. **Interpret** inequality symbol as either ABOVE or BELOW *x*-axis.
8. **Write** solution in either inequality or interval notation.

- **EXAMPLE:** Solve the inequality. $x^2 + 4x - 12 \geq 0$ [3.4.39]
  (Type your answer in interval notation. Simplify your answer. Use integers or fractions for any numbers in the expression.)

1. **Zero?** YES $x^2 + 4x - 12 = 0$
2. **Solutions** Factor $\boldsymbol{(x+6)(x-2)} = 0$
   Zero Product Property $\boldsymbol{x+6} = 0$ $\boldsymbol{x-2} = 0$
   Solve each equation
   $x = \boldsymbol{-6}$ $x = \boldsymbol{2}$
3. **Critical Points** $x = -6$ and $x = 2$ (also called ***boundary*** *points*)
4. **Number Line** 

5. **Inspect your "*a*"** $a = \mathbf{1}$, so parabola opens **UP**
6. **Sketch.** Sketch parabola opening UP, passing thru the *x*-intercepts $(-6,0)$ and $(2,0)$.



7. **Interpret.** Inequality is $\geq \mathbf{0}$
   Use bracket or parentheses? **BRACKET**
   Is it ABOVE or BELOW *x*-axis? **ABOVE** *x*-axis
   One or two pieces? **TWO** piece(s), the "**tails**" (use "or" inequality)

8. **Write** solution Inequality: $x \leq -6 \text{ or } x \geq 2$
   Interval Notation: $(-\infty, \mathbf{6}] \cup [\mathbf{2}, \infty)$

Notes Section 3.4 – Quadratic Inequalities

- **EXAMPLE:** Solve the inequality. Write the solution in interval notation. [3.4.7]

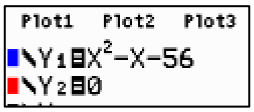
$$x^2 - x - 56 < 0$$

An alternate way to solve by graphing is to use the graphing calculator.
This is how you'll get the *x*-intercepts, or the **critical points** (or *boundary points*).

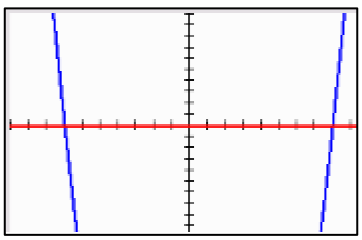
1. Get **zero** on the right, if needed.

2. Press **Y =** button on calculator.

3. Put **LEFT** side into **Y1,**
and put **ZERO** into **Y2**.



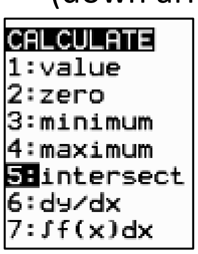
4. **Graph** it (press **ZOOM**, **6**).



- You do NOT need the vertex to solve an inequality.
- Make sure you can see the *x*-intercepts on the screen.
- You may need to Zoom Out. Press ZOOM, 3, ENTER – if needed.

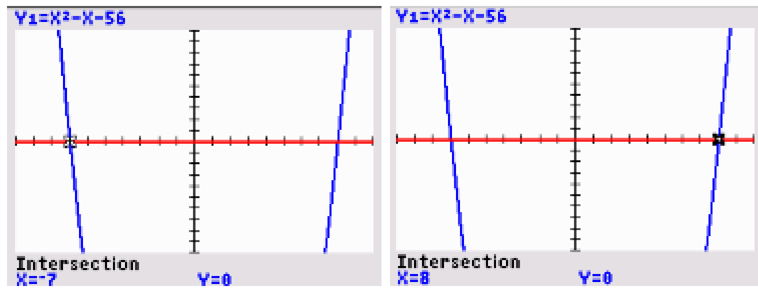
5. To find the *x*-intercepts:

a. Press **2ND**, **TRACE**, **5**: intersect, **↓** (down arrow) to switch to graph Y2.



6. Move cursor to the LEFT *x*-intercept and press **ENTER three** (3) times.

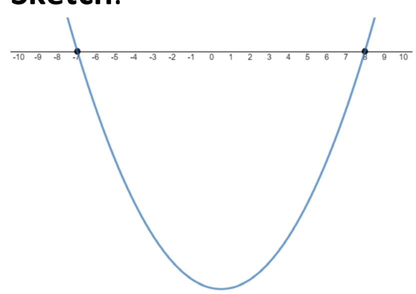
Repeat process to get the RIGHT *x*-intercept.



The *x*-intercepts are the **critical points**, which are $x = -7$ and $x = 8$

7. **Inspect your "*a*"**. The value of $a = 1$, which means opens **UP**.

8. **Sketch**.



9. **Interpret**. Inequality is < 0

Bracket or parentheses? **Parentheses**

Above or Below *x*-axis? **Below** *x*-axis

One or Two pieces? **One** piece(s), the "**bowl**"
(use "in-between" inequality)

10. Write solution

Inequality: $-7 < x < 8$

Interval Notation: $(-7, 8)$

Notes Section 3.4 – Quadratic Inequalities

B. Solve Quadratic Inequalities **Symbolically** (by hand) using **TEST POINTS**

(The first 4 steps are identical to the graphical method at the top of page 3.)

1. Make sure you have **zero** on the right.
2. Find the **solutions** – treat it as if it's an equation.
3. The solutions are the **CRITICAL POINTS** (or *boundary points*).
4. Graph critical points on a **number line** (x-axis).

5. Identify the **intervals** the critical points (or *boundary points*) create.
6. Use a **test point** (TP) from within each interval to test into the inequality.
7. The interval(s) that are **TRUE** are the **solutions**.

- **EXAMPLE:** Solve the inequality. $x^2 - 8x + 15 > 0$ [3.4-10]

1. **Zero?** YES $x^2 - 8x + 15 = 0$

2. **Solutions** Factor: $(\boldsymbol{x} - \mathbf{3})(\boldsymbol{x} - \mathbf{5}) = 0$

   Zero Product Property: $\boldsymbol{x} - \mathbf{3} = 0$ or $\boldsymbol{x} - \mathbf{5} = 0$

   Solve each equation: $x = \mathbf{3}$ or $x = \mathbf{5}$

3. **Critical Points** **(CP)** $x = 3 \text{ or } x = 5$ (also called *boundary points*)

4. **Number Line**

5. **Intervals**

(Inequality)	$\boldsymbol{x} < \mathbf{3}$	$\mathbf{3} < \boldsymbol{x} < \mathbf{5}$	$\boldsymbol{x} > \mathbf{5}$
(Interval Notation)	$(-\infty, \mathbf{3})$	$(\mathbf{3}, \mathbf{5})$	$(\mathbf{5}, \infty)$

NOTE: Your **SOLUTION** will be one or more of these intervals.

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Notes Section 3.4 – Quadratic Inequalities

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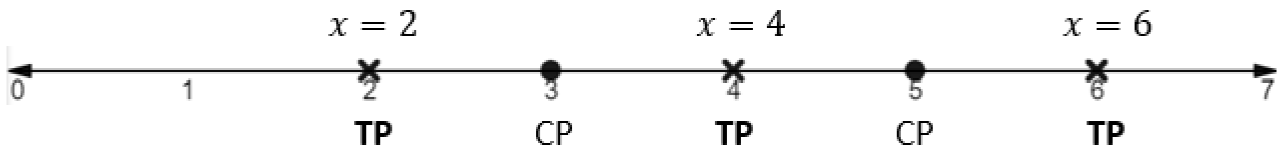
6. **Test Points (TP)**



Using $x^2 - 8x + 15 > 0$

Faster/easier if you use FACTORED form: $\mathbf{(x-3)(x-5)} > 0$

Test $x = 2$	Test $x = 4$	Test $x = 6$
$\mathbf{(2-3)(2-5)} > 0$	$\mathbf{(4-3)(4-5)} > 0$	$\mathbf{(6-3)(6-5)} > 0$
$\mathbf{-1 \cdot -3} > 0$	$\mathbf{1 \cdot -1} > 0$	$\mathbf{3 \cdot 1} > 0$
$\mathbf{3} > 0$	$\mathbf{1} > 0$	$\mathbf{3} > 0$
TRUE	**FALSE**	**TRUE**
All points are TRUE on the interval $x < 3$	All points are FALSE on the interval $3 < x < 5$	All points are TRUE on the interval $x > 5$

7. **TRUE** interval(s) **Solution** to the inequality $x^2 - 8x + 15 > 0$ is:

   (Inequality) $\mathbf{x < 3 \text{ or } x > 5}$

   (Interval Notation) $\mathbf{(-\infty, 3) \cup (5, \infty)}$

Sources Used:

1. MyLab Math for *College Algebra with Modeling and Visualization*, 6th Edition, Rockswold, Pearson Education Inc.
2. Number Line Inequalities (modified) from Desmos, https://www.desmos.com/calculator/evxn1e1njv, © 2019, Desmos, Inc.
3. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website https://archive.codeplex.com/?p=wabbit