Notes – Section 8.3: Counting

Lesson Objectives

1. Apply the Fundamental Counting Principle (FCP) for independent events.
2. Consider restrictions/conditions when using FCP.
3. Evaluate permutations or combinations using graphing calculator.
4. Key words associated with permutations or combinations.
5. Solve problems involving permutations or combinations.

- **(Definition)** Two events are **independent** if neither event influences the outcome of the other.

The Fundamental Counting Principle (FCP)

When there are **m** ways to do one thing and **n** ways to do another, there are **m × n** ways of doing **both**.

NOTE: The FCP easily works with more than two events as well.

- **Example:** In how many ways can you answer the questions on an exam that consists of 7 multiple choice questions, each of which has 4 answer choices, followed by 5 true-false questions? [8.3-3]

  For the first 7 multiple choice questions, each having 4 answer choices, that's

  $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^7$

  and for the last 5 true-false questions (2 choices each), that's

  $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$

  So, using FCP, there are $4^7 \cdot 2^5$ = **524,288** ways you can do that.

- **Example:** How many automobile license plates can be made involving 2 letters followed by either 3 or 4 digits? [8.3-4]

  We're assuming that letters and digits can be used more than once. We need to do 2 separate calculations, using FCP for each one. Then we will total them.

  - Case 1: L L D D D, which is $26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 26^2 \cdot 10^3$
  - Case 2: L L D D D D, which is $26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 26^2 \cdot 10^4$

  The total is $26^2 \cdot 10^3 + 26^2 \cdot 10^4$ = **7,436,000** possible license plates that can be made like this.

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Be careful with **RESTRICTIONS** or **CONDITIONS** imposed when using the FCP.

- **(Definition)** When the outcome of one event affects the outcome of another event, they are called **dependent events**. This sometimes happens with FCP.

  A common situation with dependent events is where **repetition** is **not** allowed.

- **Example:** How many automobile license plates can be made involving 3 letters followed by 3 digits, if letters cannot be repeated (used more than once) but digits can be repeated? [8.3-8]

  Since letters cannot repeat, the second letter **depends** on what the first is, and the third letter **depends** on what the first and second letters are. We need to reduce the number of letters available each time by one:

  - License plate format is: **L L L D D D**
  - Letters can't repeat, so L L L means **26 · 25 · 24**
  - Digits can repeat, so DDD means **10 · 10 · 10** = $\mathbf{10^3}$
  - Using FCP, there are 26 · 25 · 24 · 10^3 = **15,600,000** possible license plates.

Counting Techniques Involving **Dependent** Events (**no repetition**)

- **(Definition)** The **factorial** of a natural number is the product of that number and all the natural numbers smaller than it. (NOTE: 0! is defined to equal 1.)
  Simply put, you multiply down, reducing by 1 each time, until you get to 1.

- **Example:** Simplify. 5! [8.3.27]

  5! is read as "**Five factorial**," and means **5 · 4 · 3 · 2 · 1** = **120**

Context problem: How many ways can you arrange 5 different books on a shelf?

Context problem: How many ways can 5 people stand in line (or seated in a row)?

Context problem: How many ways can 5 people compete and finish in a race?

All of those above are solved using the calculation of "Five factorial," 5! = 120.

This can be done on the calculator by pressing:

5, then MATH, (go to PRB), (choose 4: !), ENTER.

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Permutation – order (arrangement or sequencing) matters

A permutation is like a truncated (cut-off) factorial. More on that later. First, let's look at its notation.

- **Notation (format) used for Permutation:**

  - P(*n*,*r*) is used in MyMathLab and other textbooks. Example: P(6,2)
  - $_{n}P_{r}$ is also found in in textbooks and TI-84 calculator. Example: $_{6}P_{2}$
  - *n* nPr *r* is how it looks on the TI-83/82/81 calculator. Example: 6 nPr 2

- **Formula for Permutation – but there's an even easier way. (Stay tuned)**

$P(n,r) = \frac{n!}{(n-r)!}$ You may see this formula introduced in videos or in the Question Help in MyMathLab, but you can IGNORE this formula – there is an easier, faster way on the calculator.

- **What does Permutation mean?**

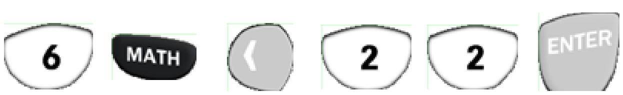
P(6,2) $_{6}P_{2}$ 6 nPr 2

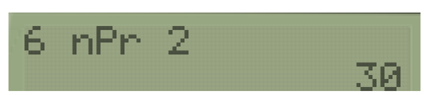
These all mean "permutation of six things taken 2 at a time."

  - *n* – is the total number available
  - *r* – is the size of the grouping
  - P(6,2) literally means start with 6 and multiply down like a factorial, reducing by one, but stop after 2 positions.

- **Example:** Evaluate the expression. P(6,2) [8.3.31]

Calculator: press **6, then MATH, (go to PRB), (choose 2: nPr), press 2, ENTER**





- **Example:** Context problem for P(6,2)

How many different two-letter codes are there if only the letters A, B, C, D, E, and F can be used and no letter can be used more than once? [8.3.41]

  - Is repetition allowed? NO – the problem states this restriction
  - Does order matter? YES – code AB is different from code BA.
    - Use **permutation** P(*n*,*r*).
  - Total available? ***n* = 6**
  - Size of grouping? ***r* = 2**

P(6,2) = 6 · 5 = 30 (calculator 6 nPr 2) There are **30** different letter codes.

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Key Words or Situations for Permutations – order matters

- **Arrangement** (Arrange)
- **Codes or Passwords** (note the previous example)
- **Officers** of a club – President, Vice-President, Secretary, Treasurer
- **Race/Competition** (order or ranking) – First, Second, Third, etc.

- **Example:** How many ways can a president, vice-president, secretary, and treasurer be chosen from a club with 9 members? [8.3-11]

- Is repetition allowed? NO – Assume one person cannot hold 2 different offices
- Does order matter? YES – Amy (Pres), Bill (VP) is different from Bill (Pres), Amy (VP)
  - Use **Permutation** P(*n*,*r*)
- Total available? ***n* = 9** (there are 9 club members)
- Size of group? ***r* = 4** (there are 4 officers: Pres, VP, Sec, Treas)

P(9,4) = 9 · 8 · 7 · 6 = 3024 (calculator 9 nPr 4) There are **3024** ways for 4 officers.

Combination – order does NOT matter

In a **combination**, all the duplicates are removed. More on that later.

- Notation (format) used for Combination:

- C(*n*,*r*) is used in MyMathLab and other textbooks. Example: C(8,3)
- ${}_nC_r$ is also found in in textbooks and TI-84 calculator. Example: ${}_8C_3$
- *n* nCr *r* is how it looks on the TI-83/82/81 calculator. Example: 8 nCr 3

- Formula for Combination – but there's an even easier way. (Stay tuned)

$C(n,r) = \frac{n!}{r!(n-r)!}$ You may see this formula introduced in videos or in the Question Help in MyMathLab, but you can IGNORE this formula – there is an easier, faster way on the calculator.

Compare the formula $C(n,r) = \frac{n!}{r!(n-r)!}$ with the formula $P(n,r) = \frac{n!}{(n-r)!}$.

How are they different? The denominator has an extra *r*! multiplied to the (*n* – *r*)!.

This extra denominator factor divides out all the duplicates, indicating that order doesn't matter.

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- What does Combination mean?

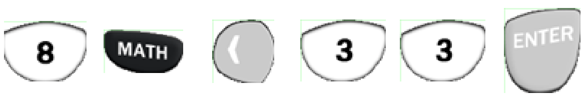
C(8,3) or ${}_8C_3$ or 8 nCr 3

These all mean "combination of eight things taken 3 at a time."

- n – is the total number available
- r – is the size of the grouping
- this is not easily done by hand – please use calculator!

- **Example:** Evaluate the expression. C(8,3) [8.3.59]

Calculator: press **8, then MATH, (go to PRB), (choose 3: nCr), press 3, ENTER**



What is the **difference between Combination and Permutation**? Why are duplicates removed?

Let's consider an example where both the total available (n) and the size of the grouping (r) are each 3. Suppose we have three fellas: Al, Bill, and Chuck.

- How can these 3 fellas (Al, Bill, and Chuck) be seated in a row of 3 chairs?

A B C   A C B   B A C   B C A   C A B   C B A

6 total ways – Where they are specifically seated in the row is significant. This is a permutation because the order matters. (calculator 3 nPr 3 or ${}_3P_3$, which equals 6)

Now take these same 3 fellas: Al, Bill, and Chuck and change the problem/situation.

- How many ways can these 3 fellas (Al, Bill, and Chuck) stand together in an elevator?

Order doesn't matter! ABC, ACB, BAC, BCA, CAB, CBA all represent the same 3 fellas in the elevator. So, instead of counting it as 6 separate ways, the five duplicates are discarded. Instead of 6 ways as a permutation, it's only **one** way as a combination. (calculator 3 nCr 3 or ${}_3C_3$, which equals 1)

There are always **far fewer combinations** than permutations, assuming you're using the same values for n and r.

- **Example:** Context problem for C(8,3)

In how many ways can a committee of 3 students be formed from a pool of 8 students? [8.3.68]

- Is repetition allowed? NO – the same person cannot be duplicated in a group!
- Does order matter? NO – a committee has no order or special arrangement to it.
  - Use **Combination** C(*n*,*r*)
- Total available? n = 8 (there is a pool of 8 students)
- Size of group? r = 3 (the size of the committee is 3)

C(8,3) = use calculator (see previous example) = 8 nCr 3 = 56. There are **56** ways.

Keywords or Situations for Combinations – order does NOT matter

- (look for anything generic, vague, nondescript – such that no particular order, arrangement, sequence is indicated)
- **Collection or Group/grouping** (note the previous example)
- **Committee** or **team** of people, including a **jury** (note example earlier)
- Chance – **Card Hands** or **Lotteries**

- **Example:** How many 3 card hands are possible with a 26-card deck? [8.3.72]

- Is repetition allowed? NO – No duplicates of the same card in a hand
- Does order matter? NO – how you arrange the cards in your hand doesn't matter; you still have the same three cards.
  - Use **Combination** C(*n*,*r*)
- Total available? ***n* = 26** (it's a 26-card deck)
- Size of group? ***r* = 3** (you have a 3-card hand)

C(26,3) = (use calculator) = 26 nCr3 = 2600. There are **2600** possible 3-card hands.

Sources used:

1. Math is Fun website, with content about the Basic Counting Principle, located at https://www.mathsisfun.com/data/basic-counting-principle.html
2. Pearson MyMathLab *College Algebra with Modeling and Visualization, 6th Edition*, Rockswold
3. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website https://archive.codeplex.com/?p=wabbit