Notes Section 1.3 A – Interval Notation

Lesson Objectives

1. Writing and Graphing Inequalities
2. Interval Notation – Overview
3. Convert among inequality, graph, and interval notation (any variation)
   - **Non-ending** interval (involves positive or negative infinity)
4. Compound Inequalities
   - **Open or Closed** interval (in BETWEEN 2 endpoints – infinity is NOT involved)
   - **Two non-ending intervals** together (2 endpoints going AWAY from each other)

A. Writing & Graphing Inequalities

- Warm-Up

Directions: Compare. Write <, >, or = in the blank.

1) -3 _____ 2

2) 6.5 _____ 6.3

3) $\frac{1}{2}$ _____ $\frac{3}{4}$

4) 0.25 _____ $\frac{1}{4}$

- Graphing Inequalities

To write and graph inequalities, you must connect words with symbols.

Look at the information in the table below.

Symbol	Words	Graph
$<$	"is _____ than"	 Shades to the _____ Endpoint ____CLUDED
$\leq$	"is _____ than or equal to" "at most" "no more than"	 Shades to the _____ Endpoint ____CLUDED
$>$	"is _____ than"	 Shades to the _____ Endpoint ____CLUDED
$\geq$	"is _____ than or equal to" "at least" "no less than"	 Shades to the _____ Endpoint ____CLUDED

Notes Section 1.3 A – Interval Notation

B. Interval Notation - Overview

- Domain and range do not always involve a **discrete** (_____) number of elements.
- **Interval notation** is used for _____ elements (not countable).
- Set-builder notation (_____) can convert to interval notation, and vice-versa.

Interval Notation is describing how the number line is _____ (or "painted"), written as:

- the _____ value first, and
- the _____ value second.

It's identifying the starting and ending points of the _____.
_____ points in _____ are shaded (included).

When using interval notation, the starting and ending points also contain a symbol:

(or) means "not included" or "open"

[or] means "included" or "closed"

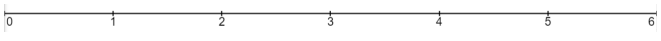
- General format for Interval Notation

The general format for Interval Notation looks like this:

__ or __ _____ number, _____ number __ or __

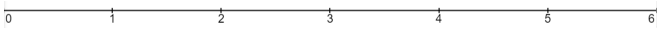
Note that the smaller number could be _____, or the larger number could be _____.
Remember to ALWAYS use _____ with either positive or negative _____.

Non-ending interval: (a, ∞) is interpreted as $x > a$, where a _____ included and infinity is always expressed as being "open" (not included).

Example: $x > 1$ graph: 

Interval Notation: _____

Non-ending interval: $(-\infty, b]$ is interpreted as $x \leq b$, where b _____ included and again, infinity is always expressed as being "open" (not included).

Example: $x \leq 5$ graph: 

Interval Notation: _____

Notes Section 1.3 A – Interval Notation

C. Convert among Inequality, Graph, and Interval Notation (any variation)

- Non-ending interval (involves positive or negative infinity)
- **EXAMPLE:** Write the set $\{x|x < 5\}$ in interval notation. [2.3.1]
  [SOLUTION] The expression there at the end is read as:

  "The set of all values *x*, _____ *x* is _____ than 5."

That expression $\{x|x < 5\}$ is written in _____-_____ **notation**.

For ease of use, you can _____ the braces and the initial "$x|$" part.
Focus just on the INEQUALITY part: $x < 5$

- Ensure the variable is on the _____ of the symbol (it IS) $x < 5$
  - If it isn't, you need to **REVERSE** the inequality (not needed here): $x < 5$
  - By far the most common error students make – forgetting to reverse it.
- $x < 5$ is read as "*x* is _____ than 5"
- First, look at the **graph** of this inequality:

  - What is the SMALLER number? _____
    - Is it INCLUDED or EXCLUDED? _____
    - How do you know? _____
    - Use BRACKET or PARENTHESIS? _____

  - What is the LARGER number? _____
    - Is it INCLUDED or EXCLUDED? _____
    - How do you know? _____
    - Use BRACKET or PARENTHESIS? _____

- Interval notation is always done as: **[** or **(** **smaller number , larger number** **)** or **]**

  ANSWER: Given the set-builder notation, $\{x|x < 5\}$

  In interval notation: _____

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- **EXAMPLE:** Write the interval notation for the graph. [*Beecher JIT.6.8]

[SOLUTION]

- What is the SMALLER number? _____
  - Is it INCLUDED or EXCLUDED? _____
  - How do you know? _____

- What is the LARGER number? _____
  - Is it INCLUDED or EXCLUDED? _____
  - How do you know? _____
  - Use BRACKET or PARENTHESIS? _____

- Interval notation is always done as: **[** or **(** **smaller number , larger number** **)** or **]**

In interval notation: _____

- **EXAMPLE:** Write the interval notation for the graph. [*Beecher JIT.6.10]

[SOLUTION]

- What is the SMALLER number? _____
  - Is it INCLUDED or EXCLUDED? _____
  - How do you know? _____

- What is the LARGER number? ___
  - Is it INCLUDED or EXCLUDED? _____
  - How do you know? _____

- Interval notation is always done as: **[** or **(** **smaller number , larger number** **)** or **]**

In interval notation: _____

(go on to the next page)

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- **EXAMPLE:** Write the inequality in interval notation. $\{x| -3 \leq x\}$ [1.3.5]

The expression there at the end is read as:

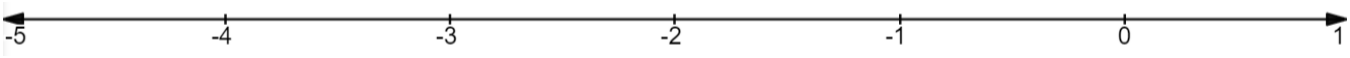
"The set of all values *x*, such that negative 3 is _____ than or equal to *x*."

That expression $\{x| -3 \leq x\}$ is written in **_____-builder notation**.

For ease of use, you can IGNORE the braces and the initial "$x|$" part.

Focus just on the INEQUALITY part: $-3 \leq x$

- Ensure the variable is on the **LEFT** of the symbol (it's _____). $-3 \leq x$
  - If it isn't, you need to _____ the inequality: _____
  - Keep the "_____" of the inequality pointed to the _____ object.
  - By far the **most common error students make – _____ to reverse it**.
- $x \geq -3$ is read as "*x* is _____ than or equal to negative 3"
- First, look at the **graph** of this inequality:



  - What is the SMALLER number? _____
    - Is it INCLUDED or EXCLUDED? _____
    - How do you know? _____
    - Use BRACKET or PARENTHESIS? _____
  - What is the LARGER number? _____
    - Is it INCLUDED or EXCLUDED? _____
    - How do you know? _____
    - Use BRACKET or PARENTHESIS? _____

- Interval notation is always done as: **[** or **(** **smaller number , larger number** **)** or **]**

ANSWER: Given the set-builder notation, $\{x| -3 \leq x\}$

In interval notation: _____

Notes Section 1.3 A – Interval Notation

D. Compound Inequalities

- Open or Closed interval (in _____ two endpoints – infinity is NOT involved)

- **EXAMPLE:** Write the interval notation for the set $\{x| -10 < x < 10$ [*Beecher JIT.6.6]

[SOLUTION]

That expression $\{x| -10 < x < 10\}$ is written in **set-builder notation**.

For ease of use, you can IGNORE the braces and the initial "$x|$" part.
Focus just on the INEQUALITY part: $-10 < x < 10$

This is one type of _____ **INEQUALITY**, because it involves more than one endpoint. Notice that the variable is **IN** _____ the two endpoints.

There is a common structure with this "in-between" inequality:

- The _____ number is always on the _____
- The _____ number is always on the _____
- (This mimics how they truly are on the number line as well.)
- _____ symbols are pointing _____ (_____-than type)
  - The symbols can use any combination of _____ or _____.

Returning to the inequality: $-10 < x < 10$

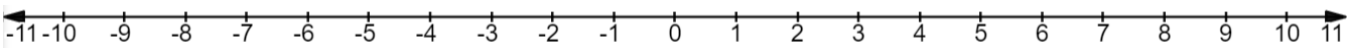
Pull these apart into 2 separate inequalities: _____ and _____

Let's graph these TOGETHER on the _____ number line.

$-10 < x$ _____ ***before*** you graph it: $-10 < x$ reverses to _____

You need to graph TOGETHER these 2 inequalities: _____ and _____

– 10 (___cluded) shading _____ and +10 (___cluded) shading _____



ANSWER: The set $\{x| -10 < x < 10$ in interval notation is: _____

Notes Section 1.3 A – Interval Notation

You can graph compound inequalities MUCH _____ than the previous example, by streamlining the process. Let's look at another example:

- **EXAMPLE:** Write the inequality in interval notation. $-3 < x \leq 2$ [1.3.3]

[SOLUTION]

The smaller number – 3 has the symbol < (– 3 ___cluded = _____)

The larger number 2 has the symbol ≤ (2 ___cluded = _____)

ANSWER: The inequality $-3 < x \leq 2$ in interval notation is _____

- **EXAMPLE:** Write the inequality in interval notation. [1.3.9]

[SOLUTION]

This time the inequality is not given – only the graph is given. This is even easier!

- Smaller number = – 7 _____ (____cluded)
- Larger number = 4 _____ (____cluded)

ANSWER: That inequality as graphed in interval notation is _____

- Two non-ending intervals together (2 endpoints going _____ from each other)

The other type of COMPOUND INEQUALITY is where the shading starts at 2 endpoints and they go _____ from each other. A big giveaway is that you'll see the word "_____" or the "_____" symbol, _____.

- **EXAMPLE:** Express the set in interval notation. [1.3.7]

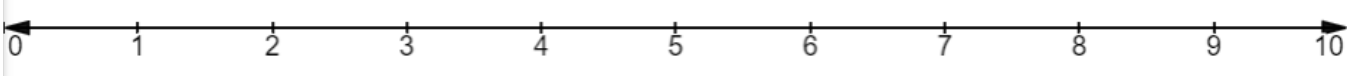
$\{x | x < 4 \text{ or } x \geq 7\}$

[SOLUTION]

There are 2 separate inequalities separated by **"or"**: _____ _____

Make sure the _____ endpoint is written on the _____, the _____ on the _____.

Graph these 2 inequalities TOGETHER on the _____ number line:



Left piece

Smaller number: ____ (_____)

Larger number: ____ (_____)

Interval notation: _____

Right piece

Smaller number: ___ (_____)

Larger number: ____ (_____)

Interval notation: _____

MERGE these two pieces TOGETHER, remove the word "or" and use "_____" symbol **(**U**)**:

ANSWER: The interval notation for $\{x | x < 4 \text{ or } x \geq 7\}$ is _____

Notes Section 1.3 A – Interval Notation

Sources used:

1. Desmos online graphing calculator, located at www.desmos.com/calculator
2. Pearson *MyLab Math*: *College Algebra with Integrated Review*, *5th Edition*, Beecher
3. Pearson *MyLab Math*: *College Algebra with Modeling and Visualization, 6th Edition*, Rockswold