Notes Section 4.2 – Polynomial Functions and Models

Lesson Objectives

1. Basic Terms with Polynomial Functions
2. Describe the End Behavior of a Polynomial
3. Overview of Polynomials through 5th Degree
4. Find Turning Points using Graphing Calculator
5. Determine Intervals of Increase and/or Decrease

A. Basic Terms with Polynomial Functions

Poly: many **Nomial:** term **Polynomial:** many terms

______________________: the highest-exponent term of a polynomial

Leading Coefficient: the coefficient found with the ________________ exponent (DEGREE).

________________ __________: where the graph of a polynomial changes from increasing to decreasing and vice-versa. Includes local maximum ("hilltop") and local minimum ("valley").

A graph may or may not have turning points.

B. Describe End Behavior of a Polynomial

Polynomials ALWAYS have a domain of all real numbers $(-\infty, \infty)$.

They go on FOREVER, left to right.

The graph of a polynomial is ___________________ (no sharp points) and ___________________________ (no breaks).

_________ ___________________: what happens to a graph when either:

x gets very small ($x \to -\infty$ read as: "*x* approaches negative infinity")

or

x gets very large ($x \to \infty$ read as: "*x* approaches positive infinity")

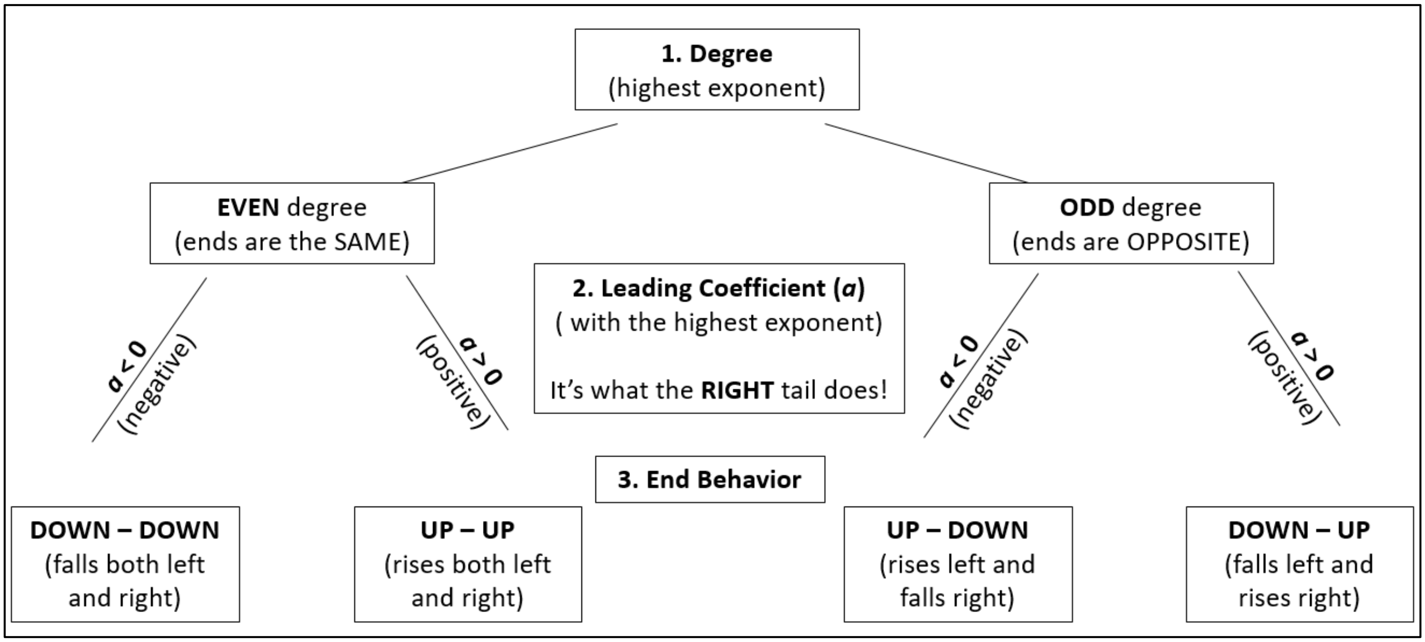
The **end behavior** of polynomials falls into one of 4 categories:

- Left end rises and right end rises (_________ – _________)
- Left end falls and right end falls (_________ – _________)
- Left end rises and right end falls (_________ – _________)
- Left end falls and right end rises (_________ – _________)

Notes Section 4.2 – Polynomial Functions and Models

The term containing the **leading coefficient** tells you how these ends ("tails") of the graph will look. You can perform a **Leading Coefficient (Term) Test** to figure this out.

Decision Chart (diagram) for End Behavior – The Leading Coefficient (Term) Test



- **EXAMPLE:** Complete parts **(a)** and **(b)** for $f(x) = -2x^3 - 2x^4 - 5$. [4.2.39]

  (a) State the degree and leading coefficient of *f*.

  (b) State the end behavior of the graph of *f*.

  A. The graph of *f* falls both to the left and to the right.

  B. The graph of *f* rises to the left and falls to the right.

  C. The graph of *f* falls to the left and rises to the right.

  D. The graph of *f* rises both to the left and to the right.

(a) (Circle the term that contains the degree) $f(x) = -2x^3 - 2x^4 - 5$

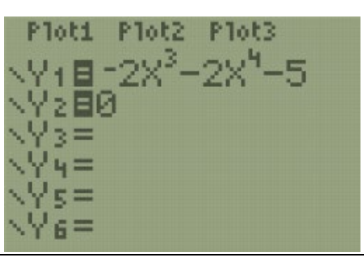
The degree (highest exponent) of *f* is _____ and its leading coefficient is _____.

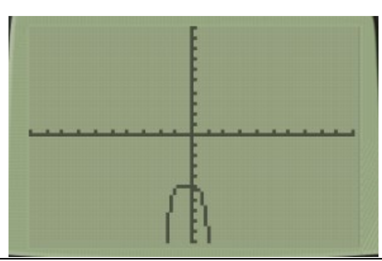
(b) Degree 4 is ____________, which means the __________.

Coefficient – 2 is ________________ (right tail _____________),

so use ___________ – ____________. Correct choice is _____.

When in doubt – **GRAPH IT OUT** !!





Notes Section 4.2 – Polynomial Functions and Models

- **EXAMPLE:** State the end behavior of the graph of f. $f(x) = 2x - \frac{1}{6}x^3$ [4.2-20]
    A. Up on left side, down on right side
    B. Down on both sides
    C. Up on both sides
    D. Down on left side, up on right side

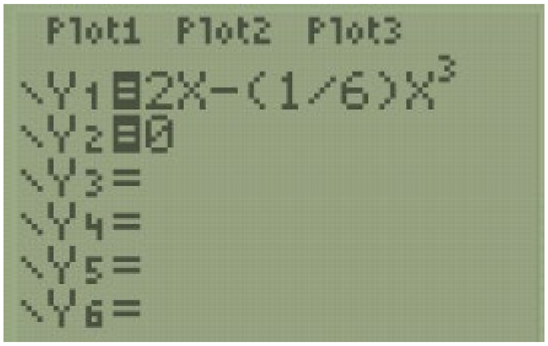
(Circle the term that contains the degree) $f(x) = 2x - \frac{1}{6}x^3$

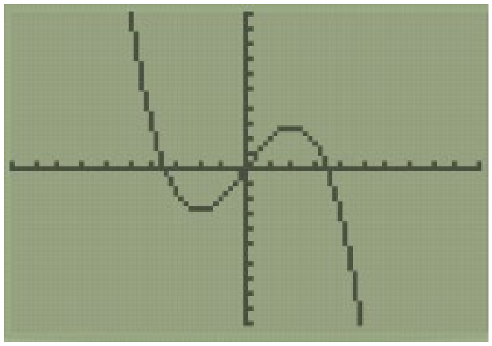
Degree is _____ (odd = _________________)

Leading Coefficient is _______ (negative = right tail ____________)

End Behavior is: ______ – ________ (rises left and falls right) Correct answer: _____

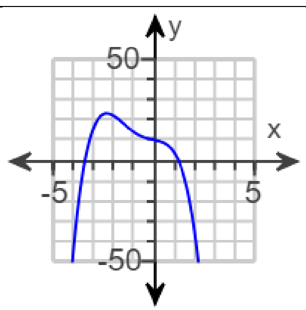
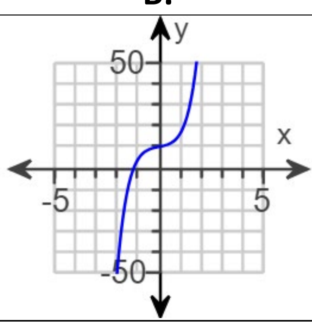
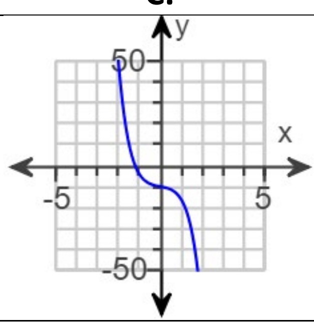
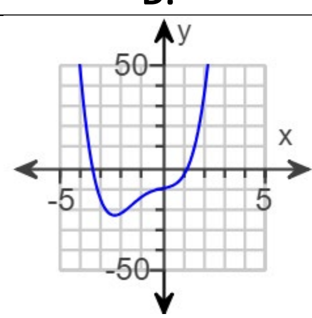
When in doubt – **GRAPH IT OUT** !!





- **EXAMPLE:** Pick which graph satisfies the given conditions. [4.2-38]

Degree 5 with 1 *x*-intercept and a positive leading coefficient.

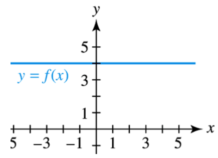
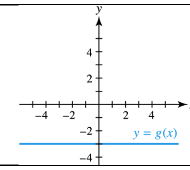
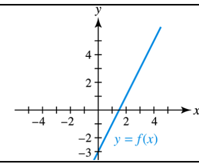
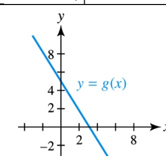
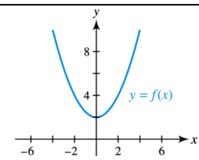
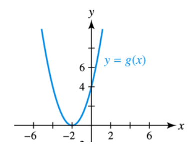
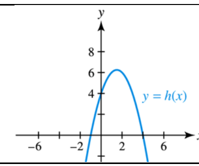
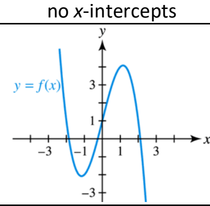
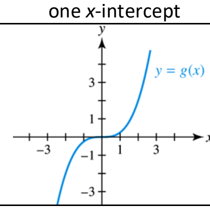
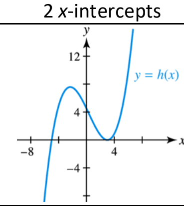
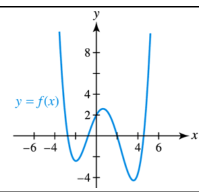
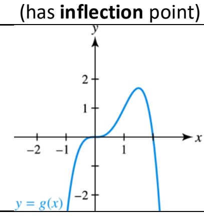
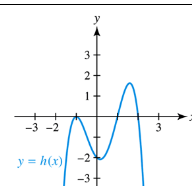
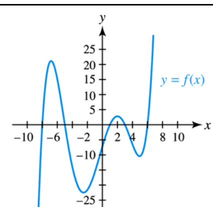
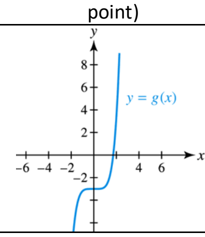
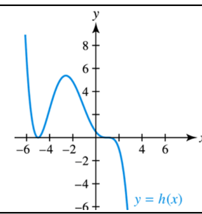
A.	B.	C.	D.
			

Degree 5 (odd = ________________). Answers A and D are incorrect (ends are the _________).

Leading Coefficient positive (right tail ______). Answer C is incorrect (right tail is _________).
Correct answer is _____.

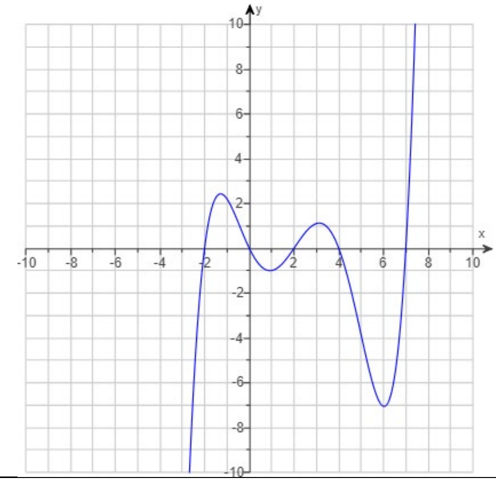
Notes Section 4.2 – Polynomial Functions and Models

C. Overview of Polynomials through 5th Degree

Function Type	Degree	(maximum) *x*-intercepts	(maximum) turning points	**Example Graphs**		
						
				 $a > 0$	 $a < 0$	
				 $a > 0$ no *x*-intercepts	 $a > 0$ one *x*-intercept	 $a < 0$ 2 *x*-intercepts
				 $a < 0$ 3 *x*-intercepts 2 turning points	 $a > 0$ one *x*-intercept no turning points (has **inflection** point)	 $a > 0$ 2 *x*-intercepts 2 turning points
				 $a > 0$ 4 *x*-intercepts 3 turning points	 $a < 0$ 2 *x*-intercepts 1 turning point (also has 1 **inflection** point)	 $a < 0$ 3 *x*-intercepts 3 turning points
				 $a > 0$ 5 *x*-intercepts 4 turning points	 $a > 0$ one *x*-intercept no turning points (has **inflection** point)	 $a < 0$ 2 *x*-intercepts 2 turning points (also has 1 **inflection** point)
7th Degree						
10th Degree						
*n*th Degree						

Notes Section 4.2 – Polynomial Functions and Models

- **EXAMPLE:** Use the graph of the polynomial function shown to the right to complete the following. Let *a* be the leading coefficient of the polynomial *f*(*x*). [4.2.7]



(a) Determine the number of turning points and estimate any *x*-intercepts.

(b) State whether $a > 0$ or $a < 0$.

(c) Determine the minimum degree of *f*.

(a) How many turning points does the graph have? ____

The *x*-intercept(s) is/are: **(,0), (,0), (,0), (,0), (,0)**
(write as ordered pairs, separating separate answers with a comma)

(b) State whether $a > 0$ or $a < 0$. Choose the correct answer below.

A. $a < 0$
B. $a > 0$

Since the **right** tail is _____, the leading coefficient (*a*) is ________________. Answer: ____.

(c) The minimum degree of *f* must be _______ (tails are in _________________ directions).

Since there are 5 *x*-intercepts, degree can't be lower than ____.

Since there are 4 turning points, degree still can't be lower than ____.

Therefore, the minimum degree of *f* is _____.

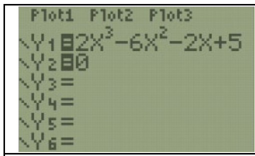
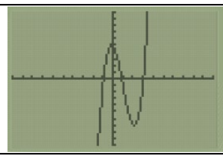
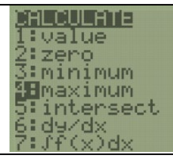
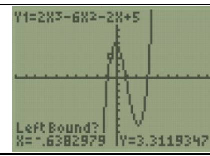
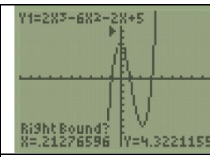
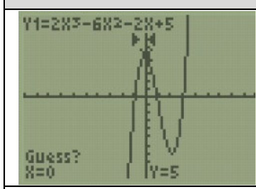
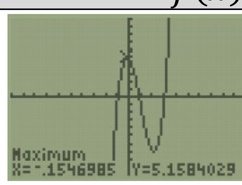
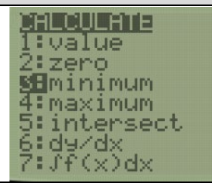
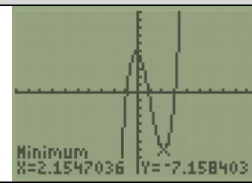
(This means that other, higher ODD-degree functions might look like the given graph. A 7th-degree polynomial, or a 9th-degree polynomial, or a 11th-degree polynomial, etc. could look like that, too.)

Notes Section 4.2 – Polynomial Functions and Models

D. Find **Turning Points** Using Graphing Calculator

- **EXAMPLE:** Approximate the coordinates of each turning point by graphing *f*(*x*) in the standard viewing rectangle.

$$f(x) = 2x^3 - 6x^2 - 2x + 5 \qquad [4.2\text{-}14]$$

				
Press **Y=** button. Put *f*(*x*) into Y1. You can leave zero in Y2.	**ZOOM 6** for standard window	For **Maximum**, use:	Left Bound? Put cursor just left of TP, ENTER.	Right Bound? Put cursor just right of TP, ENTER.
$f(x) = 2x^3 - 6x^2 - 2x + 5$				
				The coordinates of the turning points are:
Guess? Put cursor at approx. TP, ENTER.	Maximum is at:	For **Minimum**, use: (Use same process.)	Minimum is at:	and

(go on to the next page)

Notes Section 4.2 – Polynomial Functions and Models

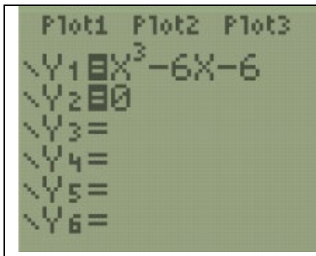
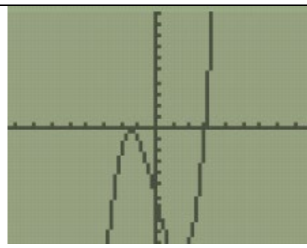
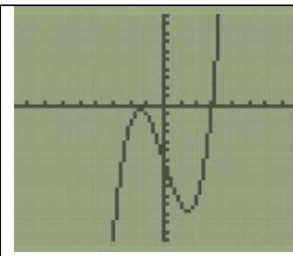
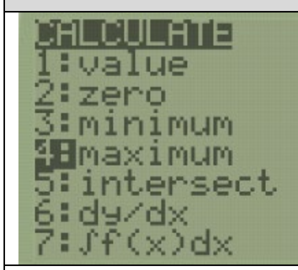
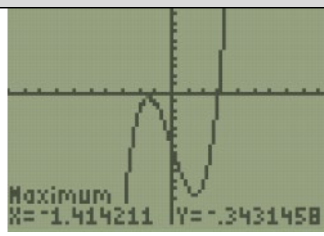
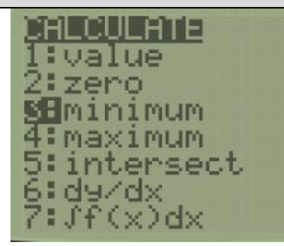
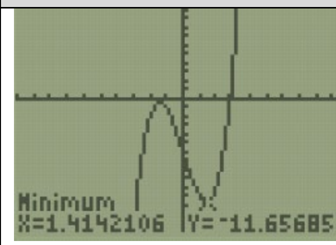
E. Determine Intervals of Increase and/or Decrease

You **MUST** know _______________ ___________ to find intervals of increase and/or decrease!

- **EXAMPLE:** Identify where *f* is increasing or where *f* is decreasing, as indicated.

$f(x) = x^3 - 6x - 6$; decreasing   [1.4-42]

(Use interval notation. Round your answer to two decimal places when appropriate.)

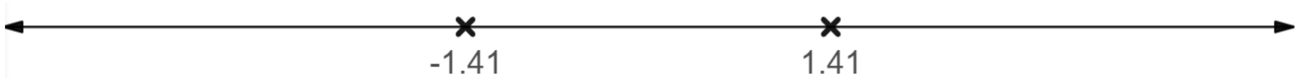
			
Put *f*(*x*) into Y1 in calculator. You can leave zero in Y2.	Start with ZOOM 6 for standard window. Can adjust more if needed.	Need to see **lower** to get the minimum (low point). Press WINDOW.	Set Ymin = – 15 and press GRAPH.
			
For Maximum, use: 2^{ND}, TRACE, 4 (go through process)	Maximum is at: $(-1.41, -0.34)$	For Minimum, use: 2^{ND}, TRACE, 3 (go through process)	Minimum is at: $(1.41, -11.66)$

(go on to the next page)

Notes Section 4.2 – Polynomial Functions and Models

The *x*-coordinates of the two turning points divide the domain (number line) into 3 regions:

With inequalities:



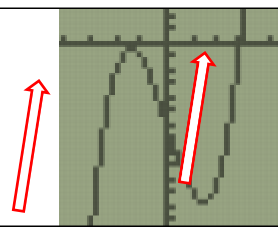
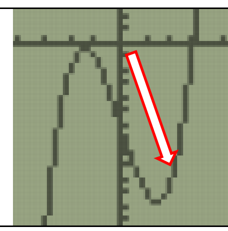
With interval notation:

(here is the problem again, for reference:)

- **EXAMPLE:** Identify where *f* is increasing or where *f* is decreasing, as indicated.

  $f(x) = x^3 - 6x - 6$; decreasing [1.4-42]

  (Use interval notation. Round your answer to two decimal places when appropriate.)

	Increase or Decrease is done from: **_____ to _____,** like moving on a roller coaster. (Use only ____, not *y* !)		
Here's the graph from the calculator again.		Increasing on:	Decreasing on: Final answer

Sources Used:

1. MyLab Math for *College Algebra with Modeling and Visualization*, 6th Edition, Rockswold, Pearson Education Inc.
2. Number Line Inequalities (modified) from Desmos, https://www.desmos.com/calculator/evxn1e1njv, © 2019, Desmos, Inc.
3. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website https://archive.codeplex.com/?p=wabbit