1. Probability Overview

The study of probability is concerned with random phenomena.
Even though we cannot be certain, or guaranteed, a given result will happen, we can often calculate the likelihood or probability of it occurring.

Experiment: any observation or measurement of random phenomena.

Outcomes: possible results

Sample Space (S): set of all possible outcomes (total)

Event (E): subset of the sample space (S) - what you want to happen

Probability can be written in one of three formats: fraction, decimal, percent.

Since probability is calculating likelihood, it always falls somewhere between:

Probability of 0 or 0 % (impossible event, will never happen)
and
Probability of 1 or 100 % (certain event, will always happen)

Probability can NEVER be negative and it can NEVER be greater than 1.

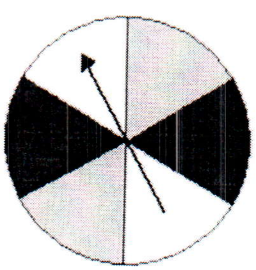
2. Basic Probability (simple fraction)

If all outcomes in a Sample (S) are equally likely to happen and the Event (E) is an event we want to see happen, then we can calculate the mathematical (theoretical) probability.

Mathematical or Theoretical probability of an event is:

$$P(E) = \frac{number\ of\ possible\ event\ outcomes}{total\ number\ of\ possible\ outcomes\ in\ sample\ space} = \frac{n(E)}{n(S)}$$

EXAMPLE: Give the probability that the spinner shown would land on the color grey.



$$P(\text{grey}) = \frac{\text{number of grey sectors}}{\text{total number of sectors } (S)} = \frac{2}{6}$$ reduce

$$= \boxed{\frac{1}{3}}$$

EXAMPLE: A bag contains 8 red marbles, 4 blue marbles, and 1 green marble. What is the probability that a randomly selected marble is not blue?

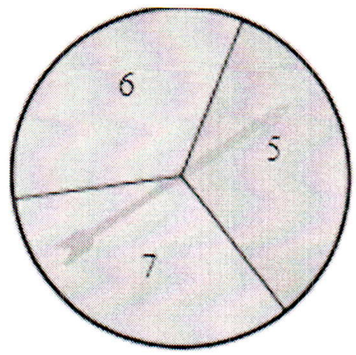
$$P(\text{even}) = \frac{\text{number of } \textbf{NOT blue} \text{ marbles}}{\text{total number of marbles } (S)} = \frac{\overset{R}{8}+\overset{G}{1}}{\underset{R}{8}+\underset{B}{4}+\underset{G}{1}} = \boxed{\frac{9}{13}}$$

EXAMPLE: Two fair 6-sided dice are rolled What is the probability that the sum of the two numbers is 8?

$$P(\text{sum } 8) = \frac{\text{number of ways to get sum 8}}{\text{total number of outcomes from rolling 2 dice } (S)} = \boxed{\frac{5}{36}}$$

sum 8 = (2,6), (3,5), (4,4), (5,3), (6,2) = 5 ways

EXAMPLE: The spinner to the right is spun twice in succession to determine a two-digit number. The first spin gives the first digit and the second spin gives the second digit. Give each of the following. (Write as a simplified fraction.)



(a) the sample space (S)
(b) the probability of an odd number
(c) the probability of a number with no repeated digits
(d) the probability of a number greater than 60
(e) the probability of a composite number

(a) the sample space (S) is all possible 2-digit numbers created by 2 spins on the spinner:
{ 55, 56, 57, 65, 66, 67, 75, 76, 77 = 9 ways }

(b) $P(\text{odd}) = \dfrac{\text{amount of 2-digit ODD numbers in } S}{\text{total outcomes in } S} = \dfrac{6}{9} = \boxed{\dfrac{2}{3}}$

(c) $P(\text{no repeated digits}) = \dfrac{\text{amount of 2-digit numbers with no repeated digits in } S}{\text{total outcomes in } S} = \dfrac{6}{9} = \boxed{\dfrac{2}{3}}$

(d) $P(\text{number greater than 60}) = \dfrac{\text{amount of 2-digit numbers greater than 60 in } S}{\text{total outcomes in } S} = \dfrac{6}{9} = \boxed{\dfrac{2}{3}}$

(e) NOTE: A prime number is only divisible by 1 and itself.
A composite number is a number that is not prime; that is, divides by numbers other than 1 and itself.

$$P(\text{composite number}) = \frac{\text{amount of 2-digit } \textbf{\textit{composite}} \text{ numbers in } S}{\text{total outcomes in } S} = \boxed{\frac{8}{9}}$$

only 67 is prime
all others are composite

Law of Large numbers:

If an experiment is repeated more and more times, the proportion of favorable events (events we want to see happen) will tend to come closer and closer to the actual mathematical (theoretical) probability of that event.

An experiment must be performed a very large number of times for the experimental (empirical) probability to come closer to the mathematical (theoretical) probability.

If an Event (E) occurs when an **experiment** is performed, then we can calculate the experimental (or empirical) probability after collecting the results and counting.

Experimental or Empirical probability of an event:

$$P(E) \approx \frac{number\ of\ times\ an\ event\ happened}{total\ number\ of\ times\ the\ experiments\ was\ performed} = \frac{n(E)}{n(S)}$$

EXAMPLE: Of the last 60 people who went to the cash register at a department store, 12 had blond hair, 19 had black hair, 22 had brown hair, and 7 had red hair. Determine the empirical probability that the next person to come to the cash register has blond hair.

$$\text{Probability(blond)} = \text{P(E)} = \frac{\text{number of times blond hair}}{\text{total number of customers}} = \frac{12}{60} = \boxed{\frac{1}{5}} \text{ or } \boxed{0.2}$$

EXAMPLE: A pair of dice is rolled 50 times and the sum of the dots on the faces is noted. Compute the empirical probability that the sum rolled is greater than 9.

Outcome (sum of dots)	2	3	4	5	6	7	8	9	10	11	12
Frequency	15	1	4	4	0	4	12	4	4	2	0

$$\text{Probability(sum greater than 9)} = \text{P(E)} = \frac{\text{number of times sum greater than 9}}{\text{total number dice rolls}} = \frac{10}{50} = \boxed{\frac{1}{5}}$$

or $\boxed{0.2}$

EXAMPLE: A school has 820 male students and 903 female students. If a student from that school is selected at random, what is the probability that the student will be female?

$$P(\text{female}) = \frac{\text{number of females}}{\text{total number of students } (S)} = \frac{903}{820+903} = \boxed{\frac{903}{1723}} \text{ or approx } \boxed{0.524}$$

(rounded)

EXAMPLE: The table shows the number of college students who prefer a given pizza topping.

Toppings	freshman	sophomore	junior	senior
cheese	13	11	26	27
meat	19	27	11	13
veggie	11	13	19	27

(a) Determine the empirical probability that a student prefers veggie toppings. (Round 3 places)

Total amount of veggie lovers = $11 + 13 + 19 + 27 = 70$

Total amount of students (sample space S) = add all values 217

$P(\text{veggie}) = \dfrac{\text{number of veggie lovers}}{\text{total number of students } (S)} = \dfrac{70}{217} = \boxed{\dfrac{10}{31}}$ or approx $\boxed{0.323}$ rounded

EXAMPLE: The table shows the number of college students who prefer a given pizza topping.

Toppings	freshman	sophomore	junior	senior
cheese	13	11	26	27
meat	19	27	11	13
veggie	11	13	19	27

(b) Determine the empirical probability that a student prefers meat toppings. (Round 3 places)

Total amount of meat lovers = $19 + 27 + 11 + 13 = 70$

Total amount of students (sample space S) = add all values 217

$P(\text{meat}) = \dfrac{\text{number of meat lovers}}{\text{total number of students } (S)} = \dfrac{70}{217} = \boxed{\dfrac{10}{31}}$ or approx $\boxed{0.323}$ rounded

EXAMPLE: The table shows the number of college students who prefer a given pizza topping.

Toppings	freshman	sophomore	junior	senior
cheese	13	11	26	27
meat	19	27	11	13
veggie	11	13	19	27

(c) Determine the empirical probability that a freshman prefers cheese toppings. (Round 3 places)

Total amount of freshman cheese lovers = 13

Total amount of freshmen (sample space S) = $13 + 19 + 11 = 43$

$P(\text{meat}) = \dfrac{\text{number of freshman cheese lovers}}{\text{total number of freshmen } (S)} = \boxed{\dfrac{13}{43}}$ or approx $\boxed{0.302}$ rounded

3. Odds

While probability compares favorable outcomes to the total outcomes,

- **Odds compares favorable outcomes to unfavorable outcomes** (or vice-versa).

Odds are commonly quoted in horse racing, lotteries, and most gambling situations.

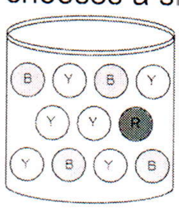
- Odds in favor of an event $E = \dfrac{\text{number of favorable outcomes }(E)}{\text{number of unfavorable outcomes (not }E)} = \dfrac{n(E)}{n(E')}$

  or alternatively, odds in favor of $E = \dfrac{\text{probability of } E \text{ happening}}{\text{probability of } E \textbf{ not} \text{ happening}} = \dfrac{P(E)}{P(E')}$

- Odds against an event $E = \dfrac{\text{number of unfavorable outcomes (not }E)}{\text{number of favorable outcomes (not }E)} = \dfrac{n(E')}{n(E)}$

  or alternatively, odds against $E = \dfrac{\text{probability of } E \textbf{ not} \text{ happening (unfavorable outcomes)}}{\text{probability of } E \text{ happening (favorable outcomes)}} = \dfrac{P(E')}{P(E)}$

EXAMPLE: The given jar contains yellow (Y), blue (B) and red (R) balls. Anne randomly chooses a single ball from the can shown. Find the odds against the event yellow (Y).

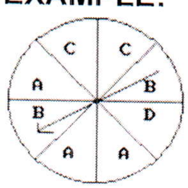


total = 11 balls

Odds against yellow (Y) = $\dfrac{\text{number of balls } \textbf{NOT} \text{ yellow}}{\text{number of balls that } \textbf{ARE} \text{ yellow}} = \dfrac{5}{6}$

also written as 5 to 6

EXAMPLE: What are the odds in favor of spinning an A on this spinner?



Odds in favor of A = $\dfrac{\text{number of sectors that are } \mathbf{A}}{\text{number of sectors that are } \textbf{not A}} = \dfrac{3}{5}$

total Sectors = 8

also written as 3 : 5

EXAMPLE: If it has been determined that the probability of an earthquake occurring on a certain day in a certain area is 0.05, what are the odds against an earthquake.

Complement principle: A probability and its complement MUST always add up to 1.

Given: P(earthquake) = 0.05, so P(**not** earthquake) = 1 − 0.05 = 0.95

Odds **against** earthquake = $\dfrac{P(\textbf{not} \text{ earthquake})}{P(\text{earthquake})} = \dfrac{0.95}{0.05} = \dfrac{19}{1}$

also written as 19 to 1