Notes Section 2.2 – Linear Equations

Lesson Objectives

1. The Basics of Linear Equations
2. Steps to solve a linear equation
3. How to deal with fractions
    - Find the Least Common Multiple (lcm) on calculator
4. Classify an equation as either conditional, identity, or contradiction
5. Problem Solving with Equations

A. Linear Equations – The Basics

1. How to Determine if an Equation is Linear

- It has only _____ variable (often x, but it could be a, m, s, etc.).
- The EXPONENT on the variable, wherever it is located, must ALWAYS be _____.

2. Basic terms associated with a linear equation

- _____ – a single number or variable, or numbers and variables mixed together. Terms in a linear equation are separated by the ____ or _____ sign.

- **Examples** of terms:
    - In the equation: $-2(9-7x)-(1-x)=2(x-7)$
    - The terms are: _____

    - Within the parentheses, there are also terms:
        - Within $(9-7x)$, the terms are _____
        - Within $(1-x)$, the terms are _____
        - Within $(x-7)$, the terms are _____

- _____ – the number to the immediate LEFT of a term containing variable.
    - The SIGN of the coefficient _____ the add or the subtract symbol.
    - ADD means the term is _____.
    - SUBTRACT means the term is _____.
    - If a variable has no visible coefficient, then it has an understood value of _____.
    - _____ – a term that has NO variable. It's just a number of some kind.
    - Examples of terms (variable = V, constant = C) and their corresponding coefficients:

term	$-2(9-7x)$	$-(1-x)$	$2(x-7)$	9	$-7x$	1	$-x$	x	-7
type of term	V	V	V	C	V	C	V	V	C
coefficient	-2	-1	2	9	-7	1	-1	1	-7

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- The _____ **Property** is used to "undo" or separate a coefficient next to parentheses.
  - $-2(9-7x)$ becomes $-2\cdot 9+-2\cdot -7x$, simplifying to _____
  - $-(1-x)$ or $-1(1-1x)$ becomes $-1\cdot 1+-1\cdot -1x$, simplifying to _____
  - $2(x-7)$ or $2(1x-7)$ becomes $2\cdot 1x+2\cdot -7$, simplifying to _____
- _____ **terms** – must contain the same type of variable(s), and same exponent(s)
  - Refer back to the original equation: $-2(9-7x)-(1-x)=2(x-7)$
  - After the distributive property: $-18+14x-1+x=2x-14$
- _____ **(Add) Like Terms** – only done on the _____**SIDE** of an equation.
  - NEVER combine like terms "_____" an equation (from opposite sides)!
  - Left side: CONSTANT like terms – 18 and – 1, combine to make – 19.
  - Left side: VARIABLE like terms 14*x* and 1*x*, combine to make 15*x*.
  - Right side: NO like terms.

B. Steps to Solve a Linear Equation

1. **_____ **Like Terms**, if you can.
2. **Undo** _____, using the _____ Property, then ** (see #1).
3. (if necessary) **Clear out** _____ – multiply all terms by the common denominator (also known as the _____, or LCM), then ** (see #1).
4. _____ **go LEFT** – use ADD or SUBTRACT to move variable terms to the LEFT side of the equation, then ** (see #1).
5. _____ **go RIGHT** – use ADD or SUBTRACT to move constant terms to the RIGHT side of the equation, then ** (see #1).
6. _____ – last step is to DIVIDE by the coefficient of your variable and simplify

So, returning to the **EXAMPLE** equation: $-2(9-7x)-(1-x)=2(x-7)$ [2.2.29]

You can't combine like terms yet, so after Distributive Property:

Now you can Combine Like Terms:

Letters go LEFT:

updates to

Numbers go RIGHT:

updates to:

Last step, DIVIDE: updates to simplified: $x=$

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C. How to Deal With Fractions

- **EXAMPLE:** Solve the equation symbolically. [2.2-12]

$$\frac{6x-9}{2}+\frac{3x-2}{5}=\frac{3}{4}$$

A fraction means _____, so first we need use _____ to undo fractions.

You need to multiply by the _____ of all the denominators.

We want the smallest multiple that is common for 2, 5, and 4.

Multiples of 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, **20**, 22, 24, ...

Multiples of 5: 5, 10, 15, **20**, 25, 30, …

Multiples of 4: 4, 8, 12, 16, **20**, 24, 28, …

So _____ is the smallest multiple found in all 3 lists (**Least Common Multiple**).

That process can sometimes take a long time, so here's how it's done on calculator:

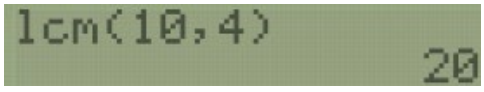
- **Find Least Common Multiple (lcm) on Calculator.**
  - Can only do 2 numbers at a time. If more than 2, "chain" together.
  - No negative numbers. Just ignore the negative temporarily.
  - No variables. Calculator can only do constants.

We need to find the Least Common Multiple (lcm) of 2, 5, and 4.

- **STEP 1:** Press _____, move **right** to **NUM**, select ___**: lcm(**

- **STEP 2:** Enter **first number**, **comma**, **second number**, **close parentheses**, **ENTER**.

- **STEP 3:** If more than 2 numbers, take the answer and do lcm(again with 3rd number, etc.



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Returning to the example problem – here it is written again:

- **EXAMPLE:** Solve the equation symbolically. [2.2-12]

$$\frac{6x-9}{2}+\frac{3x-2}{5}=\frac{3}{4}$$

The least common multiple (lcm) of 2, 5, and 4 is 20, so we need to multiply both sides of the equation by 20. This is called the _____ **Property of Equality**.

$$20\cdot\left(\frac{6x-9}{2}+\frac{3x-2}{5}\right)=20\cdot\left(\frac{3}{4}\right)$$

Use the _____ **Property** next.

Simplify – _____ **out common factors**.

Use the **Distributive Property** again.

Simplify.

Combine like terms.

Numbers go right. (Addition Property of Equality)

Combine like terms.

Divide by the coefficient**.**

Simplify (reduce fraction, if you can, or convert to decimal and round, if needed).

Refer to embedded videos to help you with fractions – you NEED to know how to do these!

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D. Classify an Equation as Conditional, Identity, or Contradiction

	Conditional	Identity	Contradiction
What happens:	Solve "regular" equation, like normal	Variables will drop out, leaving a _____ equation.	Variables will drop out, leaving a _____ equation
Finished equation looks like: (examples)	$x =$ _____	0 = 0 or 7 = 7 (etc.) Both sides are _____.	0 = – 3 or 5 = 14 (etc.) Each side is _____.
Solution (answer) format:	$x = a$ where a is a real number	_____ or $(-\infty, \infty)$	_____

- **EXAMPLE:** Solve the equation symbolically. Classify the equation as a contradiction, an identity, or a conditional equation. [2.2.51]

$$\frac{1-2x}{4} = \frac{5x-2.5}{10}$$

Clear out fractions. The least common multiple of 4 and 10 is _____.

Simplify – **Divide out Common Factors**.

Use the **Distributive Property**.

Simplify

Letters go LEFT.

You have a _____ equation. This is an _____. The solution is _____.

(go on to the next page)

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- **EXAMPLE:** Classify the equation as a contradiction, an identity, or a conditional equation.

$$-12s + 96 + 4(3s - 22) = 0 \qquad [2.2\text{-}18]$$

Use the **Distributive Property** to undo parentheses.

Simplify.

Combine Like Terms.

Simplify.

This is a _____ equation, so this is a _____.
This has _____ **SOLUTION**.

E. Problem Solving with Equations

- **EXAMPLE:** A store is discounting all regularly priced items by 75%. [2.2-29]

(i) Find a function *f* that computes the sale price of an item having a regular price of *x*.

(ii) If an item normally costs $109.45, what is its sale price? Round to the nearest cent.

(solution)

(i) First, we need to identify our variables: $f(x)$ = _____ x = _____

Next, when something is *discounted*, it is _____from the regular price (x).

Discounted 75% means discounted 75% of the regular price = _____

To find a function *f* that computes the sale price of an item having a regular price of *x*:

Sale price = Regular price – the Discount

_____

The function is: _____

(ii) If an item that normally costs $109.45, that means _____

Use the function *f* to find the sale price: $f(x) = x - 0.75x$

Evaluate (plug in) the function for $x = 109.45$: $f(109.45)$ = _____

= _____

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Sources Used:

1. Math is fun website: https://www.mathsisfun.com/definitions/term.html
2. Pearson MyLab Math *College Algebra with Modeling and Visualization, 6th Edition*, Rockswold
3. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website https://archive.codeplex.com/?p=wabbit