Notes Section 6.5 – Properties and Applications of Matrices

Lesson Objectives

1. Addition or subtraction of matrices
2. Scalar multiplication of a matrix
3. Multiplying matrices

A. Addition or subtraction of matrices

In order to either add or subtract matrices, they must have the **SAME dimension**.

- **EXAMPLE:** Find $A+B$. [6.5.7]

$A=\begin{bmatrix} 7 & -9 \\ 8 & 0 \end{bmatrix}$ $B=\begin{bmatrix} -5 & -9 \\ 9 & -8 \end{bmatrix}$

To find $A+B$, simply add in corresponding positions. $\begin{bmatrix} \mathbf{7+(-5)} & \mathbf{-9+(-9)} \\ \mathbf{8+9} & \mathbf{0+(-8)} \end{bmatrix}$

$$A+B=\begin{bmatrix} \mathbf{2} & \mathbf{-18} \\ \mathbf{17} & \mathbf{-8} \end{bmatrix}$$

- **EXAMPLE:** Perform the matrix operation. [6.5-13]

$[-3 \quad 9 \quad 4]-[4 \quad 3]$ This is not defined – they are not the same dimensions.

B. Scalar multiplication of a matrix

A scalar is like a coefficient (or "multiplier") to a matrix. It works sort of like using the distributive property – multiply all elements in the matrix by that scalar.

- **EXAMPLE:** If possible, use the given matrices A and B to find the following.

(a) $A+B$ **(b)** $3A$ **(c)** $4A-3B$ [6.5.13]

$A=\begin{bmatrix} 2 & -3 & 2 \\ 1 & 4 & 7 \\ -6 & -1 & 4 \end{bmatrix}$ $B=\begin{bmatrix} -4 & 5 & 2 \\ 6 & 3 & 6 \end{bmatrix}$

(a) $\boldsymbol{A+B}$ **is** undefined because they are not the same dimensions.

(b) $3A=3\begin{bmatrix} 2 & -3 & 2 \\ 1 & 4 & 7 \\ -6 & -1 & 4 \end{bmatrix}=\begin{bmatrix} 3\cdot 2 & 3(-3) & 3\cdot 2 \\ 3\cdot 1 & 3\cdot 4 & 3\cdot 7 \\ 3(-6) & 3(-1) & 3\cdot 4 \end{bmatrix}=\begin{bmatrix} \mathbf{6} & \mathbf{-9} & \mathbf{6} \\ \mathbf{3} & \mathbf{12} & \mathbf{21} \\ \mathbf{-18} & \mathbf{-3} & \mathbf{12} \end{bmatrix}$

NOTE: Multiplying by a scalar does NOT affect the dimensions of a matrix.

(c) $4A-3B=4\begin{bmatrix} 2 & -3 & 2 \\ 1 & 4 & 7 \\ -6 & -1 & 4 \end{bmatrix}-3\begin{bmatrix} -4 & 5 & 2 \\ 6 & 3 & 6 \end{bmatrix}$

$\boldsymbol{4A-3B}$ **is** undefined because they are not the same dimensions.

Notes Section 6.5 – Properties and Applications of Matrices

- **EXAMPLE:** Find the following matrices where $A = \begin{bmatrix} 7 & -5 \\ 7 & -8 \\ 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 5 \\ 7 & -9 \\ 6 & 5 \end{bmatrix}$.

  (we will be using graphing calculator)

  a. $A + B$ **b.** $-6A$ **c.** $-9A - 9B$ [6.5.15]

  First, you need to enter your matrices into your calculator

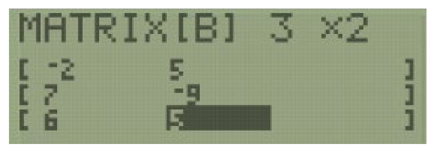
  1. Press **2ND**, $\boldsymbol{x^{-1}}$ (MATRIX), go to EDIT, **ENTER** for matrix [A].

  2. Enter the dimensions of matrix [A]; in this problem, it's 3 rows × 2 columns.

  3. Enter each of the elements of the matrix.
  4. Press **2ND**, **MODE** (QUIT).

  5. Repeat the process for matrix [B].

  Next, to call up a matrix to do a calculation:

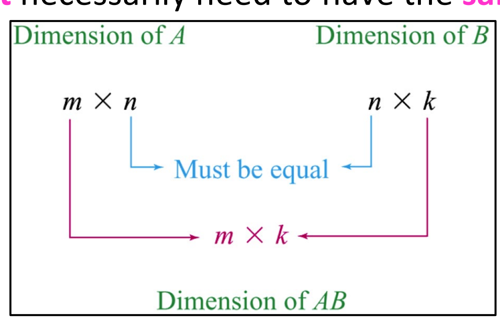
  Press **2ND**, $\boldsymbol{x^{-1}}$ (MATRIX), stay on NAMES, select your matrix, and press **ENTER**.

a. $A + B$	**b.** $-6A$	**c.** $-9A - 9B$
[A]+[B] [[5 0] [14 -17] [9 6]]	-6[A] [[-42 30] [-42 48] [-18 -6]]	-9[A]-9[B] [[-45 0] [-126 153] [-81 -54]]
$A + B = \begin{bmatrix} 5 & 0 \\ 14 & -17 \\ 9 & 6 \end{bmatrix}$	$-6A = \begin{bmatrix} -42 & 30 \\ -42 & 48 \\ -18 & -6 \end{bmatrix}$	$-9A - 9B = \begin{bmatrix} -45 & 0 \\ -126 & 153 \\ -81 & -54 \end{bmatrix}$

C. Multiplying matrices – do NOT do this by hand! Use your CALCULATOR!

Multiplying matrices is different than adding or subtracting matrices in two ways:

- You **don't** multiply corresponding positions like how add or subtract works.
- The two matrices **don't** necessarily need to have the **same** dimensions.



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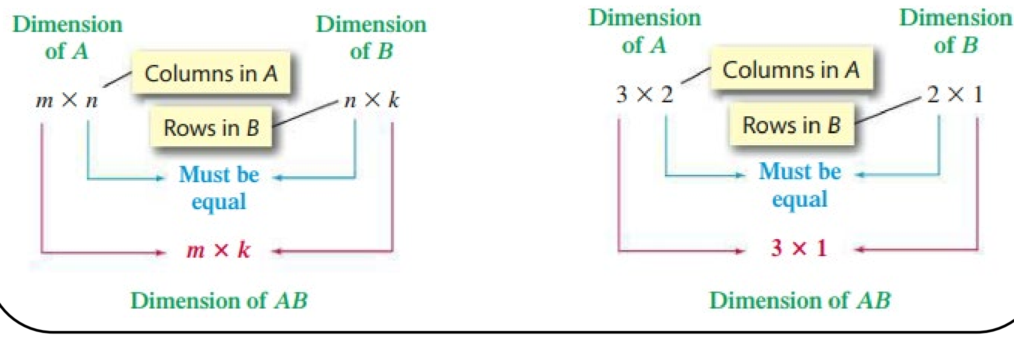
How Matrix Multiplication Works



NOTE: **Do NOT do matrix multiplication by hand…EVER! Use CALCULATOR!!**

- **EXAMPLE:** Find (if possible) **a.** AB and **b.** BA, if [6.5.31]

$$A = \begin{bmatrix} 2 & 4 \\ -4 & 4 \\ 5 & -3 \end{bmatrix}, B = \begin{bmatrix} 5 & -3 & -1 \\ -3 & 0 & -3 \end{bmatrix}$$

a. AB means [A][B] on calculator. **b.** BA means [B][A] on calculator.

First, check **dimensions** to see if multiplication even works.

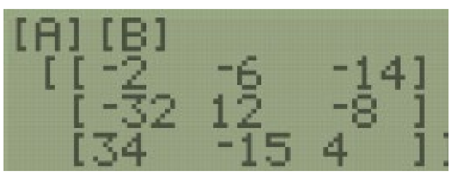
Dimension of [A]	Dimension of [B]	Dimension of [B]	Dimension of [A]
$[3 \times \boxed{2}]$	$[\boxed{2} \times 3]$	$[2 \times \boxed{3}]$	$[\boxed{3} \times 2]$

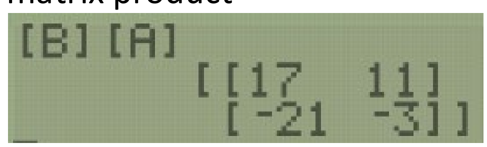
Do inside numbers match? **YES** Do inside numbers match? **YES**

If yes, look at **outside** numbers – that's dimension of product matrix.

Product matrix [A][B] will be $[3 \times 3]$ Product matrix [B][A] will be $[2 \times 2]$

Use Calculator to find the matrix product





$$AB = \begin{bmatrix} -2 & -6 & -14 \\ -32 & 12 & -8 \\ 34 & -15 & 4 \end{bmatrix} \qquad BA = \begin{bmatrix} 17 & 11 \\ -21 & -3 \end{bmatrix}$$

- **EXAMPLE:** Find the product of the following matrices, if possible. [6.5.29]

$$\begin{bmatrix} 7 & -8 & 4 \\ -6 & 0 & 8 \end{bmatrix} \begin{bmatrix} 6 & 2 & -2 \\ -8 & 9 & 7 \end{bmatrix}$$

Dimension of first matrix: $[2 \times 3]$ Dimension of second matrix: $[2 \times 3]$

Do the inside numbers match? **NO** **3** and **2**

Conclusion: **The multiplication is not possible**, even though they are same dimension.

Sources Used:

1. Pearson MyLab Math *College Algebra with Modeling and Visualization, 6th Edition*, Rockswold
2. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website https://archive.codeplex.com/?p=wabbit