Notes Section 8.6 – Probability

Lesson Objectives

1. Understand the basic terms, notation, and restrictions of probability.
2. Solve common basic probability problems
3. Use the complement rule for probability
4. Solve compound probability problems using the addition rule (“or”)
5. Solve compound probability problems involving rolling a pair of dice

Basic Probability

Probability measures how **likely** an event is to occur, or the **chance** that it will occur.

Important **Terms** with Probability

- **Outcome** – the result from an experiment
- **Sample Space** – the set of all possible outcomes from an experiment
- **Event** – any subset of the Sample Space

Notation for Probability

P(*E*) is read as “the probability of an event, *E*”

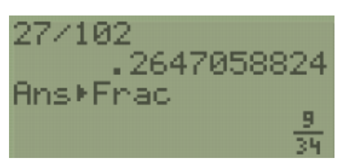
Definition of Probability

$$P(E) = \frac{n(E)}{n(S)} = \frac{number\ of\ times\ the\ event\ E\ occurs}{number\ of\ total\ possible\ outcomes\ (sample\ space, S)}$$

NOTE: since probability is a fraction, always remember to REDUCE or SIMPLIFY the fraction.

- **Example:** A class consists of 27 women and 75 men. If a student is randomly selected, what is the probability that the student is a woman? [*Weiss 4.3-4]
    - Event? Student is a woman, w = 27
    - Total possible? *n* = 27 women + 75 men = 102 students
    - Probability? *P*(woman) = 27 women / 102 students = 27/102 = **9/34**

Note: on the calculator, answers default to decimal. To make fraction, press **MATH, ENTER, ENTER.**



Notes Section 8.6 – Probability

Restrictions of Probability

The probability of some event E, or $P(E)$ is always somewhere between 0 and 1, inclusive (meaning both zero and 1 are included). It can be written as a fraction, a decimal or a percentage. Written in notation:

$$0 \leq P(E) \leq 1 \qquad \text{or} \qquad 0\% \leq P(E) \leq 100\%$$

✓ When $P(E) = 1$, it is called a **certain** event, meaning it always happens.

- Example: Suppose you roll a fair 6-sided die once. What is the probability that you will roll a number less than 7?

  Using notation, *P*(less than 7) = 1 or 100%, because it always happens (certain event)

✓ When $P(E) = 0$, it is called an **impossible** event, meaning it never happens.

- Example: Suppose you roll a fair die once. What is the probability that you will roll a 9?

  Using notation, *P*(9) = 0, because it will never happen (impossible event)

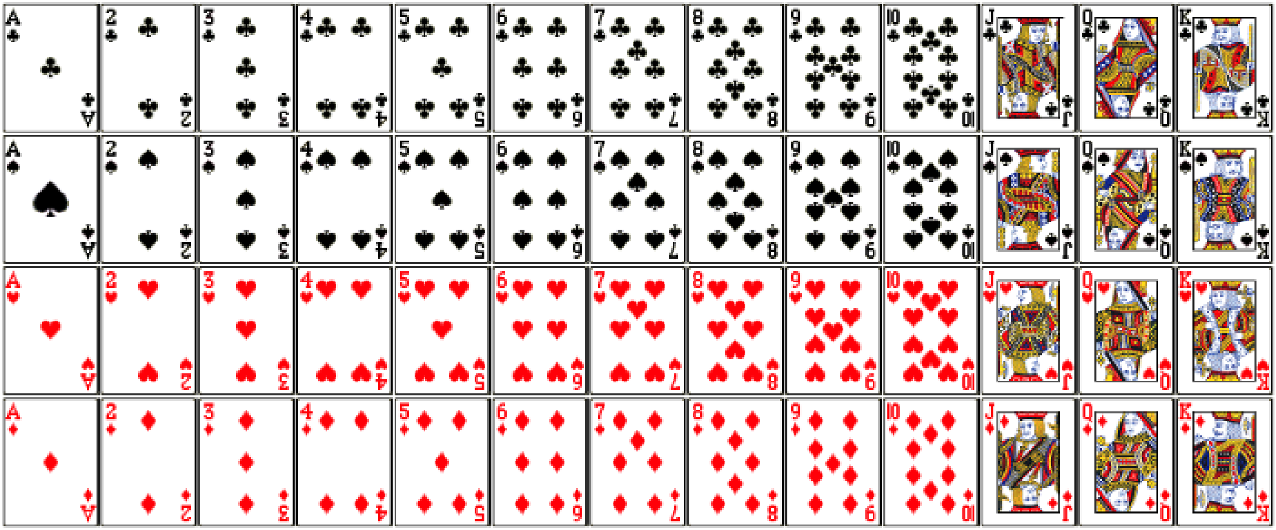
Common Types of Basic Probability Problems

- **Rolling a die** (assume 6-sided die, if not told otherwise)
- **Example:** Give the probability that the roll of a die will show a number less than 3. [8.6-15]
  - Event? Number less than 3, meaning roll a 1 or a 2 (2 ways)
  - Total possible? $n = 6$ (6-sided die)
  - Probability? *P*(less than 3) = 2 ways/6 total = 2/6 = **1/3**

- **Drawing an object (ball, marble, etc.) from a container**
- **Example:** A bag contains 15 balls numbered 1 through 15. What is the probability of selecting a ball that has an even number when one ball is drawn from the bag? [8.6-13]
  - Event? Even number, which are 2, 4, 6, 8, 10, 12, 14 (7 ways)
  - Total possible? $n = 15$ (15 balls in the bag)
  - Probability? *P*(even) = 7 even/15 total = **7/15**

Notes Section 8.6 – Probability

- **Drawing a card**
  - Deck of cards – 52 cards total
  - 4 suits – 2 are black (clubs and spades), and 2 are red (hearts and diamonds)
  - 13 different types – Ace, 2 – 10, face cards (Jack, Queen, King – 12 total face cards)



- **Example:** Suppose a card is drawn from a well-shuffled deck of 52 cards. Determine the following probability. What is the probability of drawing a 9? [8.6.15]
  - Event? Draw a 9 (4 ways – 9 of clubs, 9 of spades, 9 of hearts, 9 of diamonds)
  - Total possible? $n = 52$ (52 cards total)
  - Probability? $P(9)$ = 4 nines/52 total = 4/52 = **1/13**

Probability for the **Complement** of an Event (alternative or opposite)

Suppose you heard on the weather forecast that there's a 70% chance that it will rain. What else does that mean?

It means that there is a 30% chance that it will **NOT** rain. This "alternative" is called the **complement** of the original event.

HOW did we get the 30% for the complement?

By **subtracting** from 100%, which is also equal to **1**.

Definition for the Probability of the **Complement** of an Event

$$P(\text{not } E) = 1 - P(E) \quad \text{or} \quad P(E) = 1 - P(\text{not } E)$$

This is also known as the **complementation rule**. A probability and its complement will always add up to 1.

Notes Section 8.6 – Probability

- **Example:** The probability that Luis will pass his statistics test is 0.94. Use the complementation rule to find the probability that he will fail his statistics test. [*Weiss 4.3-20, Q9]

    - Event? Fail statistics test (fail also means "not pass")
    - Total Possible? not available – does not apply (use **complementation rule** instead)
    - Probability? P(fail) = P(not pass) = 1 – P(pass) = 1 – 0.94 = 0.06
    - So, probability Luis will fail is **0.06**, or 6%.

- **Example:** The distribution of B.A. degrees conferred by a local college is listed below:

Major	Frequency
English	2,073
Mathematics	2,164
Chemistry	318
Physics	856
Liberal Arts	1,358
Business	1,676
Engineering	868

What is the probability that a randomly selected degree is not in Mathematics? [8.6-21]

- Event? not Mathematics = 7149
  (2073 + 318 + 856 + 1358 + 1676 + 868 = 7149 not Math)
- Total possible? n = 9313 (7149 not Math + 2164 Math = 9313 total)
- Probability? P(not Mathematics)
  = 7149 not Math/9313 total = 7149/9313 ≈ **0.768**

Compound Probability – multiple events

Events involving **"OR"** – use the **ADDITION** rule

$P(A \text{ or } B) = P(A) + P(B)$ if A and B are **mutually exclusive** events

Mutually exclusive events (or **disjoint** events) means that there are **no outcomes in common**.

(no overlap; nothing shared).

(Note: in this section, we will only look at disjoint events.)

- **Example:** A card is drawn from a well-shuffled deck of 52 cards. What is the probability of drawing a face card or a 3? [*Weiss 4.3-14]

    - Event? Face card (12 ways) or a 3 (4 ways)
    - Mutually exclusive? YES – a face card cannot also be a 3 at the same time
    - Total possible? *n* = 52 (52 cards total)
    - Probability? *P*(face card or 3) = *P*(face card) + *P*(3) = 12/52 + 4/52 = 16/52 = **4/13**

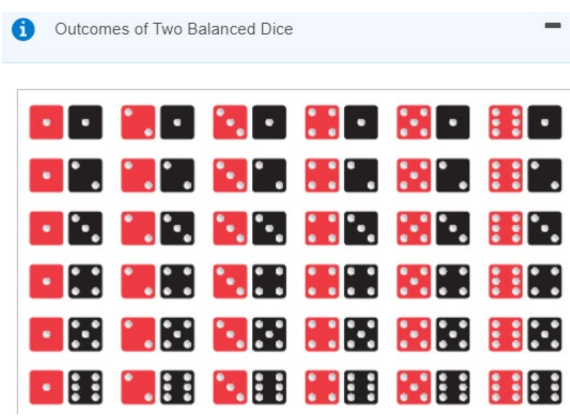
Probability involving a **pair of dice**

Rolling a pair of dice can be thought of as rolling 2 separate dice, one at a time, so this qualifies as a compound event. It's important to keep track of the outcome of each die separately.

Think of the dice being two separate colors (like red and black) or using an ordered pair (1st roll, 2nd roll). This is important to correctly count events. For example:

- (red 3, black 4) is a different from (red 4, black 3) 2 different outcomes
- (red 6, black 6) is the same as (black 6, red 6) Only 1 outcome

Use the graphics below to help you calculate probabilities involving a pair of dice.



If two balanced die are rolled, the possible outcomes

(1, 1) (2, 1) (3, 1) (4, 1) (5, 1) (6, 1)
(1, 2) (2, 2) (3, 2) (4, 2) (5, 2) (6, 2)
(1, 3) (2, 3) (3, 3) (4, 3) (5, 3) (6, 3)
(1, 4) (2, 4) (3, 4) (4, 4) (5, 4) (6, 4)
(1, 5) (2, 5) (3, 5) (4, 5) (5, 5) (6, 5)
(1, 6) (2, 6) (3, 6) (4, 6) (5, 6) (6, 6)

Note: By the Fundamental Counting Principle (FCP), the **total number of outcomes** for a pair of dice is:

6 ways (1st die) · **6 ways** (2nd die) = **36 total ways** (both dice)

So, be sure to use a denominator of 36 for problems about rolling a pair of dice.

Notes Section 8.6 – Probability

- **Example:** If two balanced (or fair) dice are rolled, determine the probability that the sum of the dice is 2 or 7. [*Weiss 4.1-4]

  - Event? Sum of 2 or 7
    - Sum of 2 = (1,1), which is 1 way
    - Sum of 7 = (1,6), (2,5), (3,4), (4,3), (5,2), (6,1), which is 6 ways
  - Total possible? $n = 36$ (36 different ways to roll 2 dice)
  - Probability? P(sum of 2 or 7) = $P(2) + P(7) = 1/36 + 6/36 =$ **7/36**

- **Example:** Two 6-sided dice are rolled. What is the probability that the sum of the numbers will be greater than 10? [*Barnett 8.1-7]
  - Event? Sum greater than 10 (3 ways)
    - Sum of 11 = (5,6), (6,5), which is 2 ways
    - Sum of 12 = (6,6), which is 1 way
  - Total possible? $n = 36$ (36 different ways to roll 2 dice)
  - Probability? P(sum greater than 10) = 3/36 = **1/12**

Sources used:

1. Pearson MyLab Math *College Algebra with Modeling and Visualization, 6th Edition*, Rockswold
2. Website Milefoot.com Mathematics, "Playing Card Frequencies" http://www.milefoot.com/math/discrete/counting/cardfreq.htm
3. Pearson MyLab Math *Finite Mathematics, 12th Edition*, Barnett
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