Notes Section R.7 – Simplifying Radical Expressions (Square Roots)

Lesson Objectives

1. Simplify a square root – Perfect Square method
2. Simplify a square root – Pairs and Spares method
3. Simplify square roots containing variables

A. Simplify a Square Root – Perfect Square Method

- Review of **Perfect Squares**

A **perfect square** is a number that has two ____________________ factors.

To simplify square roots, it's really helpful if you know at least the first 15 perfect squares:
1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225

- Simplify a Square Root – **Perfect Square Method**

Radicand – the value or amount _____________________ the root
Index – the ________________ of root you are taking

$$\sqrt[\text{index}]{\text{radicand}}$$

With a **square root**, the index of **2** is not written – it is omitted.

A **square root** is considered ________________________ if:
the **radicand contains ______ perfect square factors**.

- **STEP 1.** Inside the square root, divide the radicand into two factors:
  - the _______________________ **perfect square** that divides into the radicand
  - its "___________________" factor that goes with it
- **STEP 2.** Each of those factors gets its _________ square root, multiplied together.
- **STEP 3.** ________________________ the perfect square root into its whole number.
- **STEP 4.** Leave the "buddy" factor _______________________ the square root as the remaining reduced radicand.

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- **EXAMPLE:** Simplify by factoring out the largest perfect square. [R.7.37]

  $\sqrt{192}$

  - **STEP 1.** Inside the square root, divide the radicand into two factors:
    - the **largest perfect square** that divides into the radicand
    - its "buddy" factor that goes with it

    To find the **largest perfect square factor** of 192, you need to:

    - Test the perfect squares by ___________________ 192 by each perfect square
    - No ______________________, no remainder
    - You only need to test perfect squares to about __________-way to 192, or 96

$\frac{192}{4} = 48$	$\frac{192}{9} \approx 21.3$	$\frac{192}{16} = 12$
$\frac{192}{25} = 7.68$	$\frac{192}{36} \approx 5.3$	$\frac{192}{49} \approx 3.9$
$\frac{192}{64} = 3$	$\frac{192}{81} \approx 2.4$	$\frac{192}{100}$ 100 is more than half-way

- **STEP 1.** Rewrite $\sqrt{192}$ as $\sqrt{64 \cdot 3}$

  64 is the largest perfect square factor of 192

  Its "buddy" factor is 3 because $64 \cdot 3 = 192$

- **STEP 2.** Each of those factors gets its own square root, multiplied together.

$$\sqrt{192} = \sqrt{64} \cdot \sqrt{3}$$

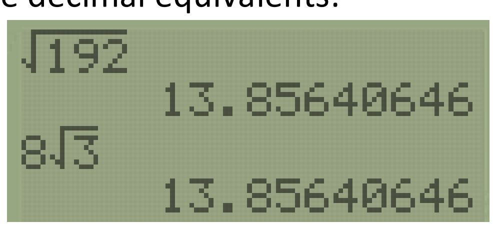
- **STEP 3.** Simplify the perfect square root into its whole number.

$$\sqrt{192} = \_\_\_\_ \cdot \sqrt{3}$$

- **STEP 4.** Leave the "buddy" factor inside the square root as the remaining reduced radicand.

ANSWER: $\sqrt{192} =$ ___________

You can easily verify that $\sqrt{192} = 8\sqrt{3}$ on your calculator.
Just verify the approximate decimal equivalents:



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- **EXAMPLE:** Simplify. $\sqrt{448}$ [*Angel 11.3.11]

448 has several perfect square factors, but we want the largest one.

$\frac{448}{4} = 112$	4 is not the largest perfect square factor because the remaining factor, _____, still divides down by at least the perfect square ____.
$\frac{448}{16} = 28$	16 is not the largest perfect square factor because the remaining factor, _____, still divides down by the perfect square ____.
$\frac{448}{64} = 7$	64 is the largest perfect square factor because the remaining factor, _____, does _________ divide down by any perfect squares.

- **STEP 1.** Rewrite $\sqrt{448}$ as $\sqrt{64 \cdot 7}$

64 is the largest perfect square factor of 448

Its “buddy” factor is 7 because $64 \cdot 7 = 448$

- **STEP 2.** Each of those factors gets its own square root, multiplied together.

$$\sqrt{448} = \sqrt{64} \cdot \sqrt{7}$$

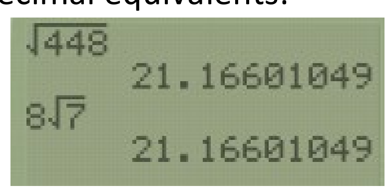
- **STEP 3.** Simplify the perfect square root into its whole number.

$$\sqrt{448} = \_\_\_\_ \cdot \sqrt{7}$$

- **STEP 4.** Leave the “buddy” factor inside the square root as the remaining reduced radicand.

ANSWER: $\sqrt{448} = \_\_\_\_\_\_\_\_\_\_\_$

You can easily verify that $\sqrt{448} = 8\sqrt{7}$ on your calculator.
Just verify the approximate decimal equivalents:



Caution: Don’t be too over-reliant upon the calculator!
For example, $\sqrt{448}$ also equals $4\sqrt{28}$; however, $4\sqrt{28}$ is not simplified because the radicand ______ still divides down by a perfect square, _____.

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B. Simplify a Square Root – "Pairs and Spares" Method

The challenge with the **Perfect Square** method is that sometimes it's ______________ to determine the largest perfect square factor because the radicand is either large or otherwise unfamiliar, or it may not simplify at all.

An alternate, sometimes _________________ and more _________________________ method is called the "Pairs and Spares" method, which utilizes a technique involving a **factor tree**, or the **prime factorization**.

- ### Prime Factorization – make a Factor Tree

_______________ number: a whole number whose **only factors are 1 and itself**.

_____________________ number: a whole number that is **NOT prime**; it is **composed** of **prime factors**. It has additional factors besides 1 and itself.

Note that the number 1 is neither prime nor composite.

Prime factorization: an arrangement of ____________ **factors** whose product is a given number. EVERY whole number (greater than 1) has a UNIQUE prime factorization.

_________________ ____________**:** a systematic way to divide down a whole number into its unique prime factors, or is **prime factorization**.

STEP 1. **"____________________-__________"** the given number into 2 factors.

- If there's more than one way to have 2 factors, then you can simply choose whichever you prefer – it doesn't matter.

STEP 2. If either of the 2 factors is **prime**, then ____________________ it.

STEP 3. If either of the 2 factors is **composite**, then **"branch-off"** of that number into 2 factors as well.

STEP 4. (If needed) Continue the process until your factor tree has ____________________ left but a collection of circled prime numbers.

STEP 5. **Write the prime factorization**:

- List all of the circled numbers together,
- Separated with a multiplication sign in between each factor

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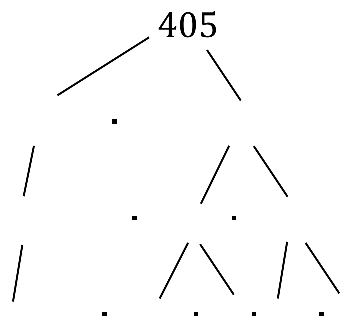
- (EXAMPLE): Use a factor tree to find the prime factorization of 405.



ANSWER: The prime factorization of 405 is:_______________________________________

- Simplify a Square Root – "**Pairs and Spares**" Method

STEP 1. Get the ____________________________ of the radicand using a **factor tree**.

STEP 2. Write the PF as the updated radicand __________________ the square root.

STEP 3A. Circle any _____________ of identical factors; that is, a perfect square.

- Each *pair* of identical factors *inside* the square root simplifies to a _______________ factor ____________________ the square root (to its LEFT).
- Do this for *each* identified pair of identical factors.

STEP 3B. ________________________________ any remaining unpaired factors still in the radicand (*inside* the square root) – these are _______________________.

STEP 4. ______________ **together** either the *outside* factors or the *inside* factors, if needed.

- **EXAMPLE:** Simplify the expression. $\sqrt{405}$ [*Angel 11.3.19]

STEP 1. From the example above, the **prime factorization** of 405 is $\mathbf{3 \cdot 3 \cdot 3 \cdot 3 \cdot 5}$

STEP 2. Update the **radicand**: $\sqrt{405} = \sqrt{3 \cdot 3 \cdot 3 \cdot 3 \cdot 5}$

STEP 3. **Circle pairs**, **Underline spares.** $\sqrt{3 \cdot 3 \cdot 3 \cdot 3 \cdot 5}$

Each pair **simplifies** to a single: $\_\_\_\_ \cdot \_\_\_\_\sqrt{\underline{5}}$

STEP 4. **Multiply *outside*** factors : $\_\_\_\_\_\sqrt{\underline{5}}$

Multiply *inside* factors: (not needed)

ANSWER: $\sqrt{405}$ simplifies to ___________

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C. Simplify a Square Root Containing Variables ("Pairs and Spares" Method)

You can simplify expressions with variables by using the basic definition of an ______________.

For example, you could write out the factors of x^5 as $x \cdot x \cdot x \cdot x \cdot x$ and then circle pairs similar to how you do with constants.

- **EXAMPLE:** Simplify by factoring. Assume that all expressions under radicals represent nonnegative numbers. $\sqrt{x^{17}}$ [*Blitzer 10.3.39]

STEP 1 and 2. Prime factorization, update radicand.

Rewrite x^{17} in the radicand using definition of exponent:

$$\sqrt{x^{17}} = \sqrt{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}$$

STEP 3. Circle pairs, underline spares.

$$\sqrt{x^{17}} = \sqrt{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}$$

Simplify to singles. $\sqrt{x^{17}} =$ ______________________________ $\cdot \sqrt{\underline{x}}$

Multiply outsides **together.** $\sqrt{x^{17}} =$ ____ $\cdot \sqrt{\underline{x}}$

STEP 4. Multiply spares together inside: (not needed)

ANSWER: $\sqrt{x^{17}}$ simplifies to $\boldsymbol{x^8\sqrt{x}}$

Notice when the exponent is very LARGE, this can be rather ________________________.
There's an easier way. Here's the previous problem again, earlier in the problem:

$$\sqrt{x^{17}} = \sqrt{\boxed{x \cdot x} \cdot \boxed{x \cdot x} \cdot \boxed{x \cdot x} \cdot \boxed{x \cdot x} \cdot \boxed{x \cdot x} \cdot \boxed{x \cdot x} \cdot \boxed{x \cdot x} \cdot \boxed{x \cdot x} \cdot \underline{x}}$$

How many **pairs** are there? ______ How many **spares** are there? _____

An __________ exponent can always be written as the previous ______________ exponent and a ________________.

Examples:	$\sqrt{x^{17}} = \sqrt{x^{16}} \cdot \sqrt{x}$ or	$\sqrt{x^{11}} = \sqrt{x^{10}} \cdot \sqrt{x}$ or	$\sqrt{x^{7}} = \sqrt{x^{6}} \cdot \sqrt{x}$
Pairs & Spares:	8 pairs, 1 spare	5 pairs, 1 spare	3 pairs, 1 spare
Sq. Rt. Is Exponent/2:	$16 \div 2 = 8$	$10 \div 2 = 5$	$6 \div 2 = 3$
Simplified	$\sqrt{x^{17}} = x^8\sqrt{x}$	$\sqrt{x^{11}} = x^5\sqrt{x}$	$\sqrt{x^{7}} = x^3\sqrt{x}$

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- Simplify square roots containing both variables and constants

- **EXAMPLE:** Express in simplified form. $\sqrt{180x^6y^{15}}$ [R.7.47]
  Assume that all variables represent positive real numbers.

CONSTANT

STEP 1. Prime Factorization. 180

STEP 2. Update radicand. $\sqrt{180} = \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5}$

STEP 3. Circle pairs, underline spares. $\sqrt{180} = \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot \underline{5}}$

Simplify to singles. $\sqrt{180} = \_\_\_\_\_\_\_\_\_\_\sqrt{\underline{5}}$

STEP 4. Multiply *outside* factors: $\sqrt{180} = \_\_\_\sqrt{\underline{5}}$

Multiply *inside* factors (not needed)

VARIABLES

Write as **separate square roots**: $\sqrt{x^6y^{15}} = \sqrt{x^6} \cdot \sqrt{y^{15}}$

Rewrite **odd** exponents as the previous even and a spare: $= \sqrt{x^6} \cdot \sqrt{y^{14}}\sqrt{\underline{y}}$

Simplify even exponents by dividing by 2: $= \boldsymbol{x^3 \cdot y^7} \cdot \sqrt{\underline{\boldsymbol{y}}}$

MERGE

Merge together the answer portions from the **constants** and the **variables**.

CONSTANTS: ____________________VARIABLES:____________________

MERGED – FINAL ANSWER: $\sqrt{180x^6y^{15}}$ simplifies to ____________________

Sources Used:

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