Notes Section 2.2 – Linear Equations

Lesson Objectives

1. The Basics of Linear Equations
2. Steps to solve a linear equation
3. How to deal with fractions
   - Find the Least Common Multiple (lcm) on calculator
4. Classify an equation as either conditional, identity, or contradiction
5. Problem Solving with Equations

A. Linear Equations – The Basics

1. How to **Determine** if an Equation is **Linear**

- It has only ONE variable (often *x*, but it could be *a*, *m*, *s*, etc.).
- The EXPONENT on the variable, wherever it is located, must ALWAYS be 1.

2. **Basic terms** associated with a linear equation

- **Term** – a single number or variable, or numbers and variables mixed together. Terms in a linear equation are separated by the ADD or SUBTRACT sign.

- **Examples** of terms:
  - In the equation: $-2(9-7x)-(1-x)=2(x-7)$
  - The terms are: $-2(9-7x)$ $-(1-x)$ and $2(x-7)$
  - Within the parentheses, there are also terms:
    - Within $(9-7x)$, the terms are 9 and $-7x$
    - Within $(1-x)$, the terms are 1 and $-x$
    - Within $(x-7)$, the terms are x and -7

- **Coefficient** – the number to the immediate LEFT of a term containing variable.
  - The SIGN of the coefficient includes the add or the subtract symbol.
  - ADD means the term is POSITIVE.
  - SUBTRACT means the term is NEGATIVE.
  - If a variable has no visible coefficient, then it has an understood value of 1.
  - **Constant** – a term that has NO variable. It's just a number of some kind.
  - Examples of terms (variable = V, constant = C) and their corresponding coefficients:

term	$-2(9-7x)$	$-(1-x)$	$2(x-7)$	9	$-7x$	1	$-x$	x	-7
type of term	V	V	V	C	V	C	V	V	C
coefficient	-2	-1	2	9	-7	1	-1	1	-7

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- The **Distributive Property** is used to "undo" or separate a coefficient next to parentheses.
  - $-2(9-7x)$ becomes $-2 \cdot 9 + -2 \cdot -7x$, simplifying to $-18+14x$
  - $-(1-x)$ or $-1(1-1x)$ becomes $-1 \cdot 1 + -1 \cdot -1x$, simplifying to $-1+x$
  - $2(x-7)$ or $2(1x-7)$ becomes $2 \cdot 1x + 2 \cdot -7$, simplifying to $2x-14$
- **Like terms** – must contain the same type of variable(s), and same exponent(s)
  - Refer back to the original equation: $-2(9-7x)-(1-x)=2(x-7)$
  - After the distributive property: $-18+14x-1+x=2x-14$
- **Combine (Add) Like Terms** – only done on the **SAME SIDE** of an equation.
  - NEVER combine like terms "ACROSS" an equation (from opposite sides)!
  - Left side: CONSTANT like terms -18 and -1, combine to make -19.
  - Left side: VARIABLE like terms $14x$ and $1x$, combine to make $15x$.
  - Right side: NO like terms.

B. Steps to Solve a Linear Equation

1. ****Combine Like Terms**, if you can.
2. **Undo Parentheses**, using the Distributive Property, then ** (see #1).
3. (if necessary) **Clear out fractions** – multiply all terms by the common denominator (also known as the Least Common Multiple, or LCM), then ** (see #1).
4. **Letters go LEFT** – use ADD or SUBTRACT to move variable terms to the LEFT side of the equation, then ** (see #1).
5. **Numbers go RIGHT** – use ADD or SUBTRACT to move constant terms to the RIGHT side of the equation, then ** (see #1).
6. **Divide** – last step is to DIVIDE by the coefficient of your variable and simplify

So, returning to the **EXAMPLE** equation: $-2(9-7x)-(1-x)=2(x-7)$ [2.2.29]

You can't combine like terms yet, so after Distributive Property:

$$-18+14x-1+x=2x-14$$

Now you can Combine Like Terms: $-19+15x=2x-14$

Letters go LEFT: $-19+15x-2x=2x-2x-14$

updates to $-19+13x=-14$

Numbers go RIGHT: $-19+19+13x=-14+19$

updates to: $13x=5$

Last step, DIVIDE: $13x=5$ updates to $\frac{13x}{13}=\frac{5}{13}$ simplified: $x=\frac{5}{13}$

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C. How to Deal With Fractions

- **EXAMPLE:** Solve the equation symbolically. [2.2-12]

$$\frac{6x-9}{2}+\frac{3x-2}{5}=\frac{3}{4}$$

A fraction means **DIVISION**, so first we need use **MULTIPLICATION** to undo fractions.

You need to multiply by the **least common multiple** of all the denominators.

We want the smallest multiple that is common for 2, 5, and 4.

Multiples of 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, **20**, 22, 24, ...

Multiples of 5: 5, 10, 15, **20**, 25, 30, …

Multiples of 4: 4, 8, 12, 16, **20**, 24, 28, …

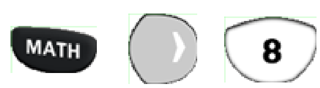
So 20 is the smallest multiple found in all 3 lists (**Least Common Multiple**).

That process can sometimes take a long time, so here's how it's done on calculator:

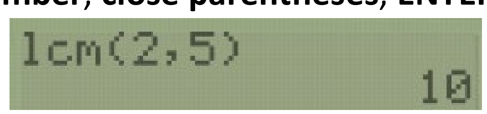
- **Find Least Common Multiple (lcm) on Calculator.**
  - Can only do 2 numbers at a time. If more than 2, "chain" together.
  - No negative numbers. Just ignore the negative temporarily.
  - No variables. Calculator can only do constants.

We need to find the Least Common Multiple (lcm) of 2, 5, and 4.

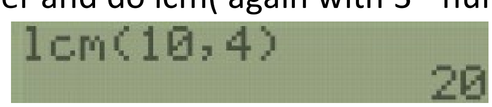
- **STEP 1:** Press **MATH**, move **right** to **NUM**, select **8: lcm(**



- **STEP 2:** Enter **first number**, **comma**, **second number**, **close parentheses**, **ENTER**.



- **STEP 3:** If more than 2 numbers, take the answer and do lcm(again with 3rd number, etc.



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Returning to the example problem – here it is written again:

- **EXAMPLE:** Solve the equation symbolically. [2.2-12]

$$\frac{6x-9}{2}+\frac{3x-2}{5}=\frac{3}{4}$$

The least common multiple (lcm) of 2, 5, and 4 is 20, so we need to multiply both sides of the equation by 20. This is called the **Multiplication Property of Equality**.

$$20\cdot\left(\frac{6x-9}{2}+\frac{3x-2}{5}\right)=20\cdot\left(\frac{3}{4}\right)$$

Use the **Distributive Property** next.

$$20\cdot\left(\frac{6x-9}{2}\right)+20\cdot\left(\frac{3x-2}{5}\right)=20\cdot\left(\frac{3}{4}\right)$$

Simplify – **Divide out common factors**.

$$\frac{20}{2}=10 \qquad \frac{20}{5}=4 \qquad \frac{20}{4}=5$$

$$10\cdot(6x-9)+4\cdot(3x-2)=5\cdot 3$$

Use the **Distributive Property** again.

$$10\cdot 6x+10\cdot(-9)+4\cdot 3x+4\cdot(-2)=5\cdot 3$$

Simplify.

$$60x+(-90)+12x+(-8)=15$$

Combine like terms.

$$60x+12x+(-8)+(-90)=15$$

$$72x-98=15$$

Numbers go right. (Addition Property of Equality)

$$\begin{aligned}72x-98&=15\\ +98\quad&\ \ +98\end{aligned}$$

Combine like terms.

$$72x=113$$

Divide by the coefficient**.**

$$\frac{72}{72}x=\frac{113}{72}$$

Simplify (reduce fraction, if you can, or convert to decimal and round, if needed).

$$x=\frac{113}{72}$$

Refer to embedded videos to help you with fractions – you NEED to know how to do these!

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D. Classify an Equation as Conditional, Identity, or Contradiction

	Conditional	Identity	Contradiction
What happens:	Solve "regular" equation, like normal	Variables will drop out, leaving a **TRUE** equation.	Variables will drop out, leaving a **FALSE** equation
Finished equation looks like: (examples)	x = some number	0 = 0 or 7 = 7 (etc.) Both sides are **IDENTICAL**.	0 = – 3 or 5 = 14 (etc.) Each side is different.
Solution (answer) format:	$x = a$ where a is a real number	**All real numbers** or $(-\infty, \infty)$	**No Solution**

- **EXAMPLE:** Solve the equation symbolically. Classify the equation as a contradiction, an identity, or a conditional equation. [2.2.51]

$$\frac{1-2x}{4} = \frac{5x-2.5}{10}$$

Clear out fractions. The least common multiple of 4 and 10 is 20.

$$20 \cdot \left(\frac{1-2x}{4}\right) = 20 \cdot \left(\frac{5x-2.5}{-10}\right)$$

Simplify – **Divide out Common Factors**.

$$\frac{20}{4} = 5 \qquad \frac{20}{-10} = -2$$

$$5(1-2x) = -2(5x-2.5)$$

Use the **Distributive Property**.

$$5 \cdot 1 + 5 \cdot (-2x) = -2 \cdot 5x + (-2) \cdot (-2.5)$$

Simplify

$$5 - 10x = -10x + 5$$

Letters go LEFT. $\quad +10x \qquad + 10x$

$$5 = 5$$

You have a **TRUE** equation. This is an **IDENTITY**. The solution is **ALL REAL NUMBERS**.

(go on to the next page)

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- **EXAMPLE:** Classify the equation as a contradiction, an identity, or a conditional equation.

$$-12s + 96 + 4(3s - 22) = 0 \qquad [2.2\text{-}18]$$

Use the **Distributive Property** to undo parentheses.

$$-12s + 96 + 4 \cdot 3s + 4 \cdot (-22) = 0$$

Simplify.

$$-12s + 96 + 12s - 88 = 0$$

Combine Like Terms.

$$-12s + 12s + 96 - 88 = 0$$
$$0 + 8 = 0$$

Simplify.

$$8 = 0$$

This is a **FALSE** equation, so this is a **CONTRADICTION**. This has **NO SOLUTION**.

E. Problem Solving with Equations

- **EXAMPLE:** A store is discounting all regularly priced items by 75%. [2.2-29]
  (i) Find a function *f* that computes the sale price of an item having a regular price of *x*.
  (ii) If an item normally costs $109.45, what is its sale price? Round to the nearest cent.

(solution)

(i) First, we need to identify our variables: $f(x)$ = sale price x = regular price

Next, when something is *discounted*, it is *subtracted* from the regular price (x).

Discounted 75% means discounted 75% of the regular price = $0.75(x) = 0.75x$

To find a function *f* that computes the sale price of an item having a regular price of *x*:

Sale price	=	Regular price	–	the Discount
$f(x)$	=	x	–	$0.75x$

The function is: $\boldsymbol{f(x) = x - 0.75x}$

(ii) If an item that normally costs $109.45, that means $x = 109.45$

Use the function *f* to find the sale price: $f(x) = x - 0.75x$

Evaluate (plug in) the function for $x = 109.45$: $f(109.45) = 109.45 - 0.75(109.45)$
$= 27.3625 =$ **$27.36**

Sources Used:

1. Math is fun website: https://www.mathsisfun.com/definitions/term.html
2. Pearson MyLab Math *College Algebra with Modeling and Visualization, 6th Edition*, Rockswold
3. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website https://archive.codeplex.com/?p=wabbit