Notes Section 3.2 – Quadratic Equations

Lesson Objectives

1. Zero Product Property
2. Solve by Factoring GCF
3. Solve by Factoring $x^2 + bx + c = 0$ into binomial factors
4. Solve by Square Root Method
5. Solve by the Quadratic Formula
6. Using the Discriminant
7. Solve by Graphing (find *x*-intercepts) on Calculator

A. Zero Product Property

If *a* · *b* = 0, then either or .

- **EXAMPLE:** Solve. $(13s + 8)(5s - 15) = 0$ [*Beecher 3.2.1]

By the Zero Product Property,
Set each factor (parentheses) **equal to zero**: $13s + 8 = 0$ or $5s - 15 = 0$
Solve each equation.

(both are solutions) $\boldsymbol{s} =$ or $\boldsymbol{s} =$

B. Solve by Factoring GCF

- **EXAMPLE:** Solve the quadratic equation. $9x^2 = 54x$ [3.2-1]

NEVER divide by a ________________! Don't do this: $\cancel{\frac{9x^2}{9x} = \frac{54x}{9x}}$ no, No, NO!!! Bad! Stop it! Very illegal!

Set your equation **EQUAL** to **ZERO**! $9x^2 = 54x$

Then, you can **FACTOR out the GCF:**

Now, use the **Zero Product Property**: $= 0$ or $= 0$
Solve each equation:
(both are solutions) $\boldsymbol{x} =$ **or** $\boldsymbol{x} =$

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C. Solve by Factoring $x^2 + bx + c = 0$ into Binomial Factors

- **EXAMPLE:** Solve the equation by factoring. $x^2 = 3x + 40$ [*Blitzer 1.5.3-Setup & Solve]

Set your equation **EQUAL** to **ZERO**! $x^2 = 3x + 40$

Try to factor:

Open 2 sets of parentheses with variable in the first position:

$= \quad (x \qquad)(x \qquad)$

Next, we need 2 integers whose SUM is _____ and whose PRODUCT is _____

To finish factoring, we need 2 numbers:		
Product = -40 (opposite signs)	**Sum = -3** (opposite signs means SUBTRACT)	**Winner?**
$\pm \quad \cdot \mp \quad = -40$	$\pm \quad + (\mp \quad) = \mp$	
$\pm \quad \cdot \mp \quad = -40$	$\pm \quad + (\mp \quad) = \mp$	
$\pm \quad \cdot \mp \quad = -40$	$\pm \quad + (\mp \quad) = \mp$	
$\pm \quad \cdot \mp \quad = -40$	$\pm \quad + (\mp \quad) = \mp$	

$x^2 - 3x - 40$ factors into:

Rewrite the equation in factored form $x^2 - 3x - 40 = 0$

$= 0$

By the Zero Product Property, set each factor (parentheses) equal to zero:

$= 0$ or $= 0$

So, $x =$ or $x =$ (both are solutions)

The solution set is { , }.

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D. Solve by the ________________ _____________ Method

The **Square Root Method** is used when only the **SQUARED** term and the __________________ term are present. That is, the **Square Root Method** is used when your equation is of the form:

$$\boldsymbol{ax^2 - c = 0}.$$

There is no *x* term – only an x^2 term and a constant term.

- **EXAMPLE:** Solve the quadratic equation. Check the answer. $4x^2 - 256 = 0$ [3.2.5]

  Because no "*x*" term, **ISOLATE** the **SQUARED** part: $4x^2 = 256$ (add 256)

  Continue to **ISOLATE** the **SQUARED** part: (divide by 4)

  (take square root)

  (What number could you square to get 64?)

  REALLY IMPORTANT! Don't forget the ___ symbol! $\boldsymbol{x} =$ or $\{ \quad , \quad \}$

- **EXAMPLE:** Solve the following equation. $(x + 21)^2 = 11$ [3.2.29]

  First, _________________ the **SQUARED** part. (DONE!) $(x + 21)^2 = 11$

  Take the ______________ __________ both sides:

  Simplify, if needed. Don't forget the "plus or minus"

  Solve for *x* by subtracting 21:

  $\boldsymbol{x} =$

  (proper format is _______________ part first, followed by the __________________ part)

  Can also be written as: $\{ \qquad\qquad , \qquad\qquad \}$

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E. Solve by the Quadratic Formula

The **Quadratic Formula:** Given $ax^2 + bx + c = 0$ (with $a \neq 0$)

the solutions are: $x =$ ________________ or $x =$ ____________________

Make sure you do the following:

1. Set your equation **EQUAL** to ______________, if needed.
2. Correctly identify the values for *a*, *b*, and *c*.
3. Watch out for negatives! (Use parentheses)

- **EXAMPLE:** Solve the quadratic equation. $x^2 + 6x + 9 = 14$ [3.2-4]

Set your equation **EQUAL** to **ZERO**! $= 0$ (subtracted 14)

You can try to factor first. If it doesn't factor, use the **Quadratic Formula**.

NOTE: You can ALWAYS use Q.F. for ANY quadratic equation, even if other methods do (or don't) work.

Use **Quadratic Formula**: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ with $a =$ $, b =$ $, c =$

Plug in your values: $x =$ ____________________

Simplify inside the square root (no decimals!) $x =$ __________ $=$ ________

Simplify the square root itself:
(pairs and spares, Section R.7)

Now update the solution above: $x =$ __________ $=$ __________

The common denominator is 2. **PULL them APART!** $x = \frac{\qquad}{2} = \frac{\quad}{2} \pm \frac{\quad}{2}$

Reduce each fraction (ignore square root part) $x = \frac{\quad}{2} \pm \frac{\quad}{2} =$ $\pm$

The solutions are: $x =$ $\pm$ or { , }

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Mrs. E! Is there…maybe…an EASIER way to do that last example? Let's revisit it:

- **EXAMPLE:** Solve the quadratic equation. $x^2 + 6x + 9 = 14$ [3.2-4]

There is an interesting opportunity here! Look at just the LEFT side of the equation – do NOT set it equal to zero.

$$x^2 + 6x + 9$$

Let's **factor** that. $x^2 + 6x + 9 =$ $=$

Revisit the equation: $x^2 + 6x + 9 = 14$

Put factored form on the LEFT. $(\qquad)^2 = 14$ Use **square root** property.

Simplify. Don't forget the $\pm$ symbol. Subtract 3.

The solutions are: $\boldsymbol{x} =$ $\pm$ or { , }

F. Using the Discriminant

Recall the **Quadratic Formula:** Given $ax^2 + bx + c = 0$ (with $a \neq 0$)

the solutions are: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The expression inside the square root (the *radicand*), the __________________ is called the __________________________, which can determine the **number of** _________________ to the quadratic equation.

QUADRATIC EQUATIONS AND THE DISCRIMINANT

To determine the number of real solutions to $ax^2 + bx + c = 0$ with $a \neq 0$, evaluate the discriminant $b^2 - 4ac$.

1. If $b^2 - 4ac$ ☐ 0, there are ☐ real solutions.
2. If $b^2 - 4ac$ ☐ 0, there is ☐ real solution.
3. If $b^2 - 4ac$ ☐ 0, there are ☐ real solutions.

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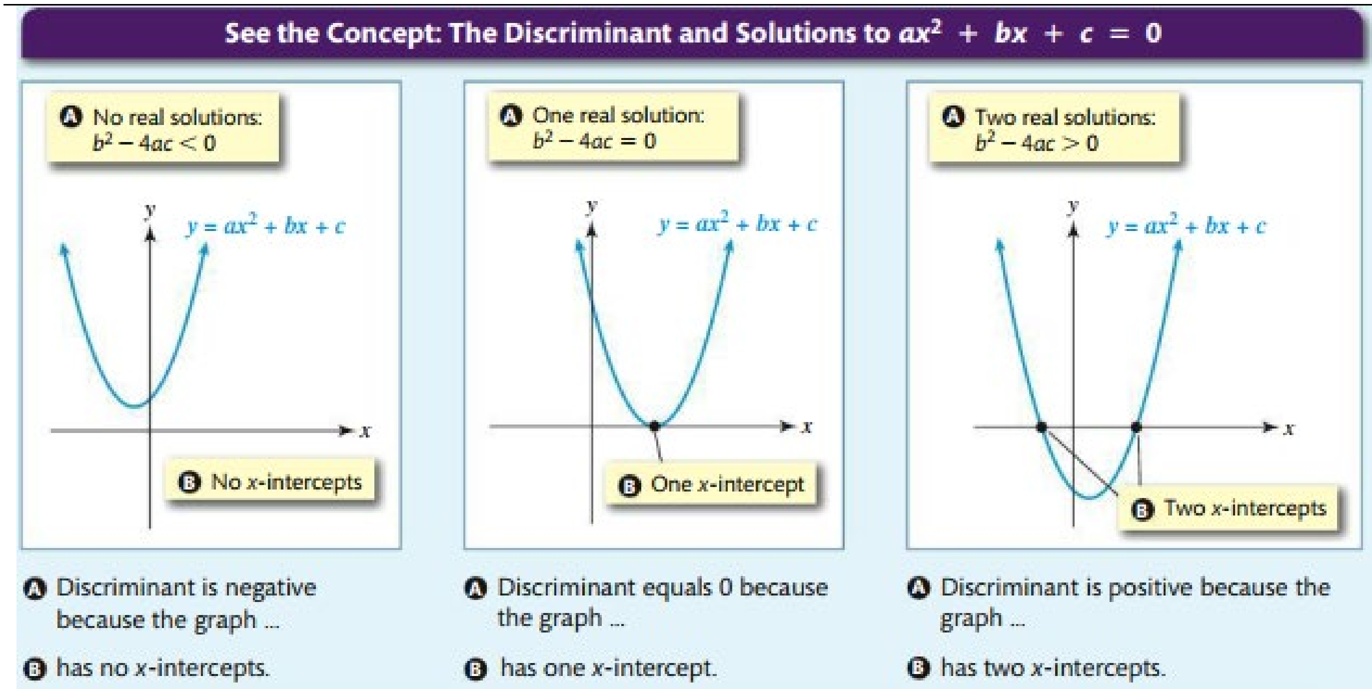


- **EXAMPLE:** Use the discriminant to determine the number of real solutions.

  $w^2 - 2w + 3 = 0$ [3.2-29]

  $a =$ __, $b =$ ___, $c =$ __

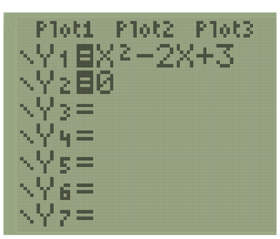
  $b^2 - 4ac =$ ____

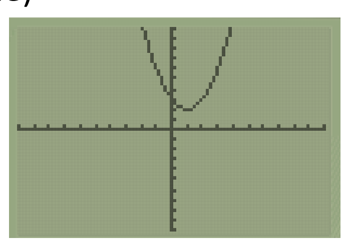
Since the discriminant $b^2 - 4ac$____0 (__________________), the equation will have:

_______ **real solutions**.

Another (easier?) way: **GRAPH** the equation $w^2 - 2w + 3 = 0$ on calculator

(use *x* as your variable)





(Put left side equation in Y1, right side in Y2) (standard window Zoom 6)

Because the parabola does _________ have any *x*-intercepts,
then that also means it has _______ **real solutions**.

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G. Solve by Graphing (finding *x*-intercepts) on Calculator

To solve a quadratic equation by graphing:

1. Set your equation **EQUAL** to __________, if needed.   Go to **Y=** on calculator.
2. Put **left** side of equation into ____ and
   right side (zero) into ___ on calculator (use *x* as your variable).
3. Graph starting with standard window, _________________.
   You may need to Zoom In or Out (ENTER), if needed.
4. Does your graph (parabola) cross or touch ___-axis?
   If YES, go to STEP 5 to find *x*-intercepts.
   If ____, then stop – your equation has _____ **real solutions**.
5. Press _______, ______________, ___ (Intersect).

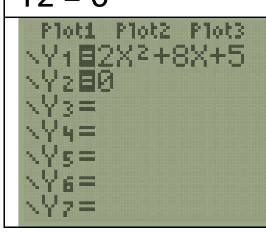
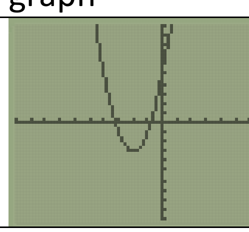
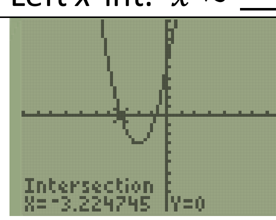
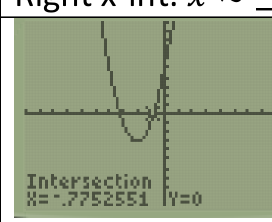
6. Press DOWN Arrow to switch to Y2=0
   and move cursor to where the parabola is touching *x*-axis.
7. Press **ENTER** _____ times.
8. You should see the word INTERSECTION with
   x = some number and *y* = 0. This is an ***x*-intercept**.
9. The **solution** is the ***x*-coordinate** of that *x*-intercept (round the amount accordingly).
10. Repeat STEPS 5 through 9 if there is a second *x*-intercept. It will be the second **solution**.

- **EXAMPLE:** Use a calculator to find the graphical solution to the equation.
  Round to the nearest thousandth.

$$2n^2 = -8n - 5 \qquad [3.2\text{-}16]$$

Set your equation equal to zero (add 8*n* and add 5)

$\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = 0$ Go to Y= on calculator. Use *x* as your variable.

Y1 = $2x^2 + 8x + 5$ Y2 = 0	ZOOM 6 to graph	2nd TRACE 5 Left *x*-int. $x \approx$ _______	2nd TRACE 5 again Right x-int. $x \approx$ _______
			

The solutions to the equation $2n^2 = -8n - 5$ are $\boldsymbol{n} \approx$ _________**or** _________ (rounded to thousandth)

Sources Used:

1. MyLab Math for *College Algebra with Modeling and Visualization*, 6th Edition, Rockswold, Pearson Education Inc.
2. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website https://archive.codeplex.com/?p=wabbit