# Notes – Section 8.3: Counting

## Lesson Objectives

1. Apply the Fundamental Counting Principle (FCP) for independent events.
2. Consider restrictions/conditions when using FCP.
3. Evaluate permutations or combinations using graphing calculator.
4. Key words associated with permutations or combinations.
5. Solve problems involving permutations or combinations.

- **(Definition)** Two events are __________________ if neither event influences the outcome of the other.

## The Fundamental Counting Principle (FCP)

When there are **m** ways to do one thing and **n** ways to do another, there are _______________ ways of doing **both**.

NOTE: The FCP easily works with more than two events as well.

- **Example:** In how many ways can you answer the questions on an exam that consists of 7 multiple choice questions, each of which has 4 answer choices, followed by 5 true-false questions? [8.3-3]

  For the first 7 multiple choice questions, each having 4 answer choices, that's

  ____ · ____ · ____ · ____ · ____ · ____ · ____ = ______

  and for the last 5 true-false questions (2 choices each), that's

  ____ · ____ · ____ · ____ · ____ = ______

  So, using FCP, there are _____ · _____ = __________________ ways you can do that.

- **Example:** How many automobile license plates can be made involving 2 letters followed by either 3 or 4 digits? [8.3-4]

  We're assuming that letters and digits can be used more than once. We need to do 2 separate calculations, using FCP for each one. Then we will total them.

  - Case 1: ___________, which is ______________________ = ______________
  - Case 2: ___________, which is ______________________ = ______________

  The total is $26^2 \cdot 10^3 + 26^2 \cdot 10^4$ = _________________________ possible license plates that can be made like this.

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## Be careful with __________________ or **CONDITIONS** imposed when using the FCP.

- **(Definition)** When the outcome of one event affects the outcome of another event, they are called ____________________ **events**. This sometimes happens with FCP.

A common situation with dependent events is where ____________ is _____ allowed.

- **Example:** How many automobile license plates can be made involving 3 letters followed by 3 digits, if letters cannot be repeated (used more than once) but digits can be repeated? [8.3-8]

  Since letters cannot repeat, the second letter **depends** on what the first is, and the third letter **depends** on what the first and second letters are. We need to reduce the number of letters available each time by one:

  - License plate format is: _______________________
  - Letters can't repeat, so L L L means ____· ____ · ____
  - Digits can repeat, so DDD means ____ · ____ · ____ = ______
  - Using FCP, there are $26 \cdot 25 \cdot 24 \cdot 10^3$ = ____________________ possible plates.

## Counting Techniques Involving **Dependent** Events (**no repetition**)

- **(Definition)** The ____________ of a natural number is the product of that number and all the natural numbers smaller than it. (NOTE: 0! is defined to equal 1.)

  Simply put, you multiply down, reducing by 1 each time, until you get to 1.

- **Example:** Simplify. 5! [8.3.27]

5! is read as "_______ ____________," and means ___ · ___ · ___ · ___ · ___ = _____

Context problem: How many ways can you arrange 5 different books on a shelf?

Context problem: How many ways can 5 people stand in line (or seated in a row)?

Context problem: How many ways can 5 people compete and finish in a race?

All of those above are solved using the calculation of "Five factorial," 5! = 120.

This can be done on the **calculator** by pressing:

**5, then MATH, (go to PRB), (choose 4: !), ENTER**.


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## Permutation – _____________ matters (the arrangement or sequencing)

A permutation is like a truncated (cut-off) factorial. More on that later. First, let's look at its notation.

- **Notation** (format) used for **Permutation**:

○ ________ is used in MyMathLab and other textbooks. Example: P(6,2)
○ ________ is also found in in textbooks and TI-84 calculator. Example: ${}_6P_2$
○ ___________ is how it looks on the TI-83/82/81 calculator. Example: 6 nPr 2

- Formula for Permutation – but there's an even easier way. (Stay tuned)

$P(n,r) = \frac{n!}{(n-r)!}$ You may see this formula introduced in videos or in the Question Help in MyMathLab, but you can _____________ this formula – there is an easier, faster way on the ________________________.

- What does Permutation mean?

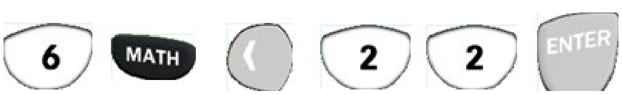
P(6,2) ${}_6P_2$ 6 nPr 2

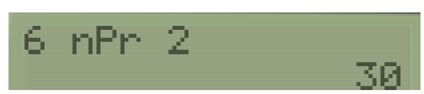
These all mean "_________________ of ______ things taken ______ at a time."

- ○ *n* – is the ____________ number available (the larger number)
- ○ *r* – is the ____________ of the grouping (the smaller number)
- ○ P(6,2) literally means start with 6 and multiply down like a factorial, reducing by one, but stop after 2 positions.

- **Example:** Evaluate the expression. P(6,2) [8.3.31]

Calculator: press **6, then MATH, (go to PRB), (choose 2: nPr), press 2, ENTER**




- **Example:** Context problem for P(6,2)

How many different two-letter codes are there if only the letters A, B, C, D, E, and F can be used and no letter can be used more than once? [8.3.41]

- ○ Is repetition allowed? ______ – the problem states this restriction
- ○ Does order matter? ______ – for example, code AB is different from code BA.
  - ▪ Use ____________________________.
- ○ Total available? _____________
- ○ Size of grouping? _____________

P(6,2) = 6 · 5 = 30 (calculator 6 nPr 2) There are _____ different letter codes.

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## Keywords or Situations for Permutations – order matters

- ___________________________ (Arrange)
- **______________ or ______________________** (note the previous example)
- ___________________ of a club – President, Vice-President, Secretary, Treasurer
- _______________________________ (order or ranking) – First, Second, Third, etc.

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- **Example:** How many ways can a president, vice-president, secretary, and treasurer be chosen from a club with 9 members? [8.3-11]

  - Is repetition allowed? _____ – Assume one person cannot hold 2 different offices
  - Does order matter? _______ – For ex: Amy (Pres), Bill (VP) differs from Bill (Pres), Amy (VP)
    - Use ______________________________
  - Total available? _________ (there are 9 club members)
  - Size of group? _________ (there are 4 officers: Pres, VP, Sec, Treas)

P(9,4) = 9 · 8 · 7 · 6 = 3024 (calculator 9 nPr 4) There are _________ ways for 4 officers.

## Combination – order does NOT matter

In a **combination**, all the ______________________ are removed. More on that later.

### Notation (format) used for Combination:

- _____________ is used in MyMathLab and other textbooks. Example: C(8,3)
- _____________ is also found in in textbooks and TI-84 calculator. Example: ${}_8C_3$
- _________________ is how it looks on the TI-83/82/81 calculator. Example: 8 nCr 3

### Formula for Combination – but there's an even easier way. (Stay tuned)

$C(n,r) = \frac{n!}{r!(n-r)!}$ You may see this formula introduced in videos or in the Question Help in MyMathLab, but you can ______________ this formula – there is an easier, faster way on the ________________________.

Compare the formula $C(n,r) = \frac{n!}{r!(n-r)!}$ with the formula $P(n,r) = \frac{n!}{(n-r)!}$.

How are they different? The _________________________________________________ multiplied to the $(n-r)!$. This extra denominator factor divides out all the duplicates, indicating that order doesn't matter.

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- What does Combination mean?

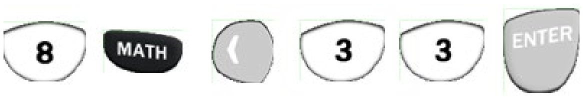
C(8,3) or $_8C_3$ or 8 nCr 3

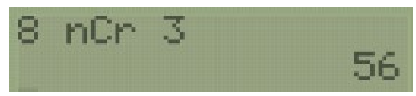
These all mean "______________________ of _____ things taken _____ at a time."

- $n$ – is the total number available (the larger number)
- $r$ – is the size of the grouping (the smaller number)
- this is NOT easily done by hand – please use ______________________!

- **Example:** Evaluate the expression. C(8,3) [8.3.59]

Calculator: press **8, then MATH, (go to PRB), (choose 3: nCr), press 3, ENTER**




What is the **difference between Combination and Permutation**? Why are duplicates removed?

Let's consider an example where both the total available ($n$) and the size of the grouping ($r$) are each 3. Suppose we have three fellas: Al, Bill, and Chuck.

- How can these 3 fellas (Al, Bill, and Chuck) be seated in a row of 3 chairs?

_______ _______ _______ _______ _______ _______

___ total ways – Where they are specifically seated in the row is significant. This is a _______________ because the __________ matters. (calculator 3 nPr 3 or $_3P_3$ equals 6)

Now take these same 3 fellas: Al, Bill, and Chuck and change the problem/situation.

- How many ways can these 3 fellas (Al, Bill, and Chuck) stand together in an elevator?

Order doesn't matter! ABC, ACB, BAC, BCA, CAB, CBA all represent the ___________ 3 fellas in the elevator. So, instead of counting it as 6 separate ways, the five _______________________ are discarded. Instead of 6 ways as a permutation, it's only ______ way as a ________________. (calculator 3 nCr 3 or $_3C_3$ , which equals 1)

There are always **far ________________ combinations** than permutations, assuming you're using the same values for $n$ and r.

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- **Example:** Context problem for C(8,3)

In how many ways can a committee of 3 students be formed from a pool of 8 students? [8.3.68]

- Is repetition allowed? ____ – the same person cannot be duplicated in a group!
- Does order matter? ____ – a committee has no order or special arrangement to it.
    - Use ______________________________
- Total available? _________ (there is a pool of 8 students)
- Size of group? __________ (the size of the committee is 3)

C(8,3) = use calculator (see previous example) = 8 nCr 3 = 56. There are _____ ways.

## Keywords or Situations for Combinations – order does NOT matter

- (look for anything generic, vague, nondescript – such that no particular order, arrangement, sequence is indicated)
- __________________________________________ (note the previous example)
- __________________________of people, including a ________ (see example earlier)
- Chance – ___________ ______________ or __________________________

- **Example:** How many 3 card hands are possible with a 26-card deck? [8.3.72]

- Is repetition allowed? _____ – No duplicates of the same card in a hand
- Does order matter? ______ – how you arrange the cards in your hand doesn't matter; you still have the same three cards.
    - Use ______________________________
- Total available? ___________ (it's a 26-card deck)
- Size of group? __________ (you have a 3-card hand)

C(26,3) = (use calculator) = 26 nCr3. There are __________ possible 3-card hands.

Sources used:

1. Math is Fun website, with content about the Basic Counting Principle, located at https://www.mathsisfun.com/data/basic-counting-principle.html
2. Pearson MyMathLab *College Algebra with Modeling and Visualization, 6th Edition*, Rockswold
3. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website https://archive.codeplex.com/?p=wabbit