Notes Section 5.6 – Logarithmic Equations Using Properties

Lesson Objectives

1. Use logarithm properties to solve equations

Expanding/Condensing Logarithm Properties – Reminder for your reference:

- **Product Rule:**

	$\log_a(mn) = \log_a(m) + \log_a(n)$	EXPANDING	(Product to Sum)
or	$\log_a(m) + \log_a(n) = \log_a(mn)$	CONDENSING	(Sum to Product)

- **Quotient Rule:**

	$\log_a\left(\frac{m}{n}\right) = \log_a(m) - \log_a(n)$	EXPANDING	(Quotient to Difference)
or	$\log_a(m) - \log_a(n) = \log_a\left(\frac{m}{n}\right)$	CONDENSING	(Difference to Quotient)

- **Power Rule:**

	$\log_a(m^r) = r\log_a(m)$	EXPANDING	(Exponent to Coefficient)
or	$r\log_a(m) = \log_a(m^r)$	CONDENSING	(Coefficient to Exponent)

- **EXAMPLE:** Solve the equation. $\log_5 2 + \log_5 x = 0$ [*Martin-Gay 9.8.13]

Use **Product Rule** (sum to product) to CONDENSE to single logarithm. This will also **isolate** the logarithm.	$\log_5(2) + \log_5(x) = 0$
Do **NOT** divide by the 2 yet! It is trapped inside the logarithm.	$\log_5(2 \cdot x) = 0$
To undo the logarithm, convert to the exponential form.	$\log_5(2x) = 0$ **A logarithm** is **an exponent**
Simplify. Remember property: $a^0 = 1$	$5^{0} = 2x$
Solve the equation. (Divide both sides by 2)	$1 = 2x$
Simplify.	$\frac{1}{2} = \frac{2x}{2}$
Answer:	$x = \frac{1}{2}$

Notes Section 5.6 – Logarithmic Equations Using Properties

- **EXAMPLE:** Solve the following logarithmic equation.

$$\log_2 x + \log_2 7 = 3$$ [*Angel 13.6.47]

Use **Product Rule** (sum to product) to CONDENSE to single logarithm. This will also **isolate** the logarithm.	$\log_2(x) + \log_2(7) = 3$
Do **NOT** divide by the 7 yet! It is trapped inside the logarithm.	$\log_2(x \cdot 7) = 3$
To undo the logarithm, convert to the exponential form.	$\mathbf{\log_2(7x)} = \mathbf{3}$ **A logarithm is an exponent**
Simplify. $2^3 = 8$	$2^{\mathbf{3}} = 7x$
Solve the equation. (Divide both sides by 7)	$8 = 7x$
Simplify.	$\frac{8}{\mathbf{7}} = \frac{7x}{\mathbf{7}}$
Answer:	$\boldsymbol{x = \frac{8}{7}}$

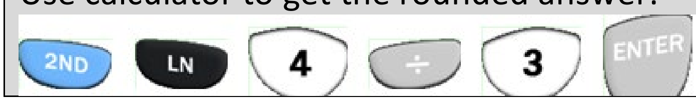
- **EXAMPLE:** Solve the logarithmic equation. $\ln x + \ln x^2 = 4$ [5.6.59]
  (Round to the nearest thousandth as needed.)

Use **Product Rule** (sum to product) to CONDENSE to single logarithm. This will also **isolate** the logarithm.	$\ln(x) + \ln(x^2) = 4$
Simplify. $x \cdot x^2 = x^3$ (add exponents)	$\ln(x \cdot x^2) = 4$
Equation will be EASIER with just x than with x^3. Use **Power Rule** (exponent to coefficient)	$\ln(x^{\mathbf{3}}) = 4$
Divide both sides by 3.	$\mathbf{3}\ln(x) = 4$
Simplify. Remember that **ln** is same as **log** $\mathbf{\log_e}$	$\frac{3\ln(x)}{\mathbf{3}} = \frac{4}{\mathbf{3}}$
To undo the logarithm, convert to the exponential form.	$\boldsymbol{\log_e(x)} = \boldsymbol{\frac{4}{3}}$ **A logarithm** is **an exponent**
This is the **exact answer:** $\boldsymbol{x = e^{4/3}}$	$\boldsymbol{e^{4/3} = x}$
Use calculator to get the rounded answer: 	$e^{4/3}$ 3.793667895 **Rounded Answer:** $\boldsymbol{x \approx 3.794}$

Notes Section 5.6 – Logarithmic Equations Using Properties

(**RESET** – Here's another way to do the previous problem):

- **EXAMPLE:** Solve the logarithmic equation. $\ln x + \ln x^2 = 4$ [5.6.59]
  (Round to the nearest thousandth as needed.)

Rather than use the Product Rule like before, use the **Power Rule** (exponent to coefficient) on the second term.	$\ln(x) + \ln(x^{2}) = 4$
The first term has an understood coefficient **1**.	$1\ln(x) + 2\ln(x) = 4$
Combine like terms: **1** of them + **2** of them = **3** of them	$1\ln(x) + 2\ln(x) = 4$
(From here, the steps are the same as before.) Divide both sides by 3.	$3\ln(x) = 4$
Simplify. Remember that **ln** is same as **log$_e$**	$\dfrac{3\ln(x)}{3} = \dfrac{4}{3}$
To undo the logarithm, convert to the exponential form.	$\log_e(x) = \dfrac{4}{3}$ **A logarithm** is **an exponent**
This is the **exact answer:** $x = e^{4/3}$	$e^{4/3} = x$
Use calculator to get the rounded answer: 	 **Rounded Answer:** $x \approx 3.794$

You can use EITHER method when you solve a problem like the previous examples (for Question 7 in the Homework). Be ready to do either the **exact** answer (like $e^{4/3}$) or the rounded answer.

Sources Used:

1. MyLab Math for *Elementary & Intermediate Algebra for College Students*, 5th Edition, Angel, Pearson Education Inc.
2. MyLab Math for *Intermediate Algebra: A Graphing Approach*, 5th Edition, Martin-Gay, Pearson Education Inc.
3. MyLab Math for *College Algebra with Modeling and Visualization*, 6th Edition, Rockswold, Pearson Education Inc.
4. Texas Instruments TI Connect® CE software, https://education.ti.com/en/products/computer-software/ti-connect-ce-sw