I. The z-score

Measures of Central Tendency (mean, median, mode) give us a number in which a data set is centered. Measures of Dispersion (range, standard deviation) tells us how much the data set is spread out from that centered point.

This section is about Measures of Position which looks at individual items within a data set and those measures are z-score, percentile, quartile.

Chebyshev's theorem states that 89% of items in any set, lie within 3 standard deviations of the mean. In other words, 89% of the items will have a z-score between -3 and +3. Actually 99% of items are within 3 standard deviations so a z-score less than -3 or greater than +3 is a rare occurrence.

The z-score

If x is a data item in a sample with mean $\bar{x}$ and standard deviation s, then the **z-score** of x is calculated as follows.

$$z = \frac{x - \bar{x}}{s}$$

EXAMPLE: Ann made an 86 on her history midterm and Kay made a 78 on hers. Which student is doing better (ranked higher) in their history classes?

	Ann	Kay
Class mean	73	69
Class standard deviation	8	5

Ann

$$z = \frac{86-73}{8} = \frac{13}{8} = 1.625$$

Kay

$$z = \frac{78-69}{5} = \frac{9}{5} = 1.8$$

Kay is ranked higher in her class.

EXAMPLE: Using the chart of populations of some countries, compute the z-score for Japan's population.

First, use calculator to computer mean and standard deviation.
Second, find z-score using formula.

$\bar{x} = 191.8 \quad s = 405$

$z = \frac{127-191.8}{405} = \frac{-64.8}{405} = \boxed{-.16}$

Country	Population (millions)
Canada	33
China	1339
Mexico	110
Japan	127
Germany	82
United Kingdom	61
Saudi Arabia	28
Venezuela	26
South Korea	48
France	64

$n = 10$

II. Percentiles

If you have ever taken a standardized test, such as t-cap, ACT, SAT, MCAT, etc, then your raw score is converted to a percentile. If you scored at the 83rd percentile on the SAT, then that means you did better than 83% of the people who took the test. This does **not** mean you scored an 83% on your test.

Remember a percent is always multiplied.

Percentile = percent times the number of data items = the position of the item
Percentile is rounded up to the next number to begin the percentile

To locate the position of the item, all items must be ranked from least to greatest

EXAMPLE: Below is a list of the number of customers served by a restaurant for 40 days. Find the 35th percentile and the 86th percentile.

46	51	52	55	56	56	58	59	59	59
61	61	62	62	63	63	64	64	64	65
66	66	66	67	67	67	68	68	69	69
70	70	71	71	72	75	79	79	83	88

35th percentile:

$.35(40) = 14$

use 15^{th} item for the 35^{th} percentile

$= 63$

86th percentile:

$.86(40) = 34.4$

use 35^{th} item for the 86^{th} percentile

$= 72$

III. Quartiles

Quartiles of a data set are three values that divide the data set into four "equal-sized" parts.

Quartiles use the median values which means the data set must be in numerical order.

Finding Quartiles

For any set of data (ranked in order from least to greatest):

The **second quartile, Q_2,** is just the median, the middle item when the number of items is odd, or the mean of the two middle items when the number of items is even.

The **first quartile, Q_1,** is the median of all items below Q_2.

The **third quartile, Q_3,** is the median of all items above Q_2.

EXAMPLE: Below is a list of the number of customers served by a restaurant for 40 days. Find the three quartiles for the data set (median means middle number).

46	51	52	55	56	56	58	59	59	59
61	61	62	62	63	63	64	64	64	65
66	66	66	67	67	67	68	68	69	69
70	70	71	71	72	75	79	79	83	88

Second quartile, Q_2:

$$\frac{65+66}{2} = 65.5$$

First quartile, Q_1:

$$\frac{59+61}{2} = 60$$

Third quartile, Q_3:

$$\frac{69+70}{2} = 69.5$$