1. Permutations

Permutation is a way of counting Arrangements. Arrangements can be items like, how winning prizes are given, who gives a speech first or second, and even voting. Because items are arranged in a particular order then repetitions are not allowed. Once an item is used, it cannot be used again.

For example: At a high school, the yearbook staff is made up of 10 students. How many ways can the staff elect an editor, a secretary, and a treasurer?

Using the Fundamental Counting Theorem (arrangement): $10 \cdot 9 \cdot 8 = 720$

Using Permutations: $P(10,3) = \frac{10!}{(10-3)!} = \frac{10!}{7!} = 720$

When arrangements become more complicated then use permutations:

Permutations Formula:

The number of permutations, or arrangements, of **n** total things (total to choose from) taken **r** at a time (number in group) can be calculated by the following formula:

$$P(n,r) = {}_nP_r = \frac{n!}{(n-r)!}$$

Permutations can be done in your calculator: MATH →→→ PRB (probability), Permutation is number **2: ${}_nP_r$**

EXAMPLE: Evaluate the permutations in your calculator

(1) $P(7, 3) = {}_7P_3 =$ 210

(2) $P(100, 2) = {}_{100}P_2 =$ 9,900

in calculator:
type "n" number
select ${}_nP_r$
type "r" number
enter

2. Combinations

Combination is not an arrangement or order but rather a subset. It is usually a smaller group made from the larger group. Combination is a general group and order is not important and once again, repetitions are not allowed. Once an item is used, it cannot be used again.

For example: How many ways can a club of 25-members choose a committee of three members?

Using Combinations: $C(25,3) = \frac{25!}{3!(25-3)!} = \frac{25!}{3!22!} = 2{,}300$

Combinations Formula:

The number of combinations, or subsets, of **n** total things (total to choose from) taken **r** at a time (number in group) can be calculated by the following formula:

$$C(n,r) = {}_nC_r = \frac{n!}{r!(n-r)!}$$

Divide by $r!$ to take away duplicates of the same subsets.

Combinations can be done in your calculator: MATH →→→ PRB (probability), Combination is number **3: $_nC_r$**

EXAMPLE: Evaluate the combinations in your calculator:

(1) $C(10, 2) = {}_{10}C_2 = 45$

(2) $C(9, 4) = {}_9C_4 = 126$

in calculator:
type "n" number
select $_nC_r$
type "r" number
enter

Guidelines on Which Method to Use

The following table summarizes the similarities and differences between permutations and combinations and appropriate formulas for calculating their values.

Permutations	Combinations
Number of ways of selecting r items out of n items	
Repetitions are not allowed.	
Order is important.	Order is not important.
Arrangements of n items taken r at a time	Subsets of n items taken r at a time
${}_nP_r = \dfrac{n!}{(n-r)!}$	${}_nC_r = \dfrac{n!}{r!(n-r)!}$
Clue words: arrangement, schedule, order	Clue words: set, group, sample, selection

Decide if the examples are permutations or combinations and answer the question.

EXAMPLE: How many ways can first, second, and third place finishers occur in a race with 23 runners competing?

Permutation $P(23,3) = {}_{23}P_3 = 10,626$

EXAMPLE: How many 5-member committees can be formed from 100 US Senators?

combination

$C(100,5) = {}_{100}C_5 = 75,287,520$

EXAMPLE: How many different ways can a lottery select 5 numbers from the numbers 1-39?

combination $C(39,5) = {}_{39}C_5 = 575,757$

EXAMPLE: How many ways are there to draw a 5-card hand from a 52-card deck?

Combination

$$C(52,5) = {}_{52}C_5 = 2,598,960$$

EXAMPLE: A speech class has 8 students. How many ways can the class of students give a speech?

Permutation

$$P(8,8) = {}_8P_8 = 40,320$$

Same as $8!$

EXAMPLE: How many ways can fifteen people be divided into five groups containing one, two, three, four, and five respectively in each group?

Combination and fundamental counting principle

$${}_{15}C_1 \cdot {}_{14}C_2 \cdot {}_{12}C_3 \cdot {}_9C_4 \cdot {}_5C_5 = 37,837,800$$

EXAMPLE: Using a standard 52-card deck, how many possible 5-card hands would consist of the following cards? Combination and fundamental counting principle

(a) Four clubs and one non-club:

$${}_{13}C_4 \cdot 39 = 27,885$$

(b) Two red cards, two clubs and a spade:

$${}_{26}C_2 \cdot {}_{13}C_2 \cdot 13 = 329,550$$