

Notes Section 3.4 – Quadratic Inequalities

Lesson Objectives

1. Solve Quadratic Inequalities Graphically – when formula *is* or is *not* given.
2. Solve Quadratic Inequalities Symbolically

A. Solve Quadratic Inequalities Graphically

$f(x) = 0$ means y-coordinate is _____

$f(x) > 0$ means y-coordinate is _____

$f(x) < 0$ means y-coordinate is _____

$f(x) = 0$ is _____ the x-axis

$f(x) > 0$ is _____ x-axis

$f(x) < 0$ is _____ x-axis

1. Solve Graphically When formula is **NOT** given

- **EXAMPLE:** Given the graph of $f(x)$, solve:

(a) $f(x) > 0$

(b) $f(x) < 0$. [3.4.23]

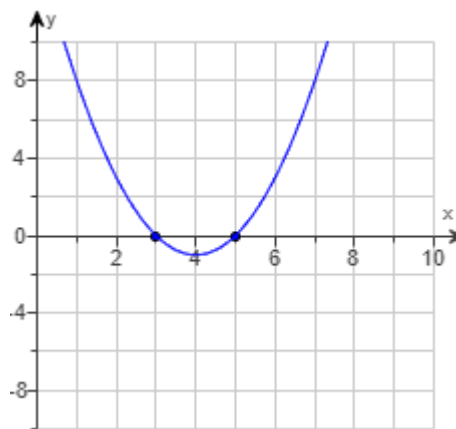
Before we can solve the inequality, we must first solve the **equation** _____

The solutions are the x-intercepts.

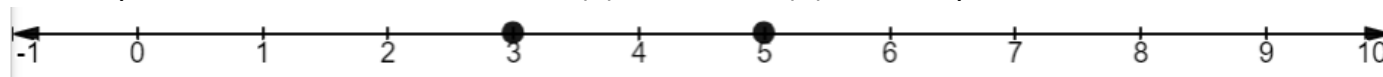
The x-intercepts are: (____, 0) and (____, 0).

So, the solution to $f(x) = 0$ based on the given graph is:

_____ and _____



These are called _____ (CP) for the inequality. These two critical points now divide the domain (x) into three (3) distinct parts:



Left side Center Right side

(written as
inequality)

(in interval
notation)

NOTE: Make sure you can write these intervals in either inequality or interval notation!
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(a) Solve $f(x) > 0$.

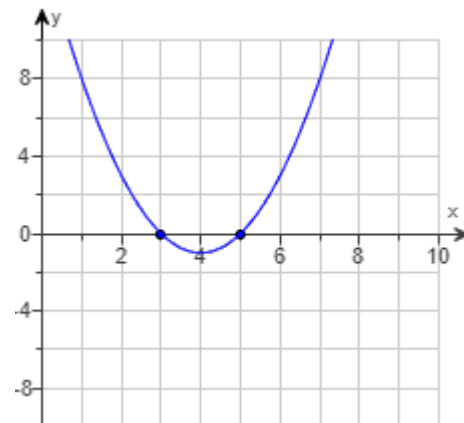
This means look _____ the x-axis.

There are _____ of the graph that are **ABOVE** the x-axis – the “_____” of the graph.

- To the LEFT of $x = 3$, which is _____
- To the RIGHT of $x = 5$, which is _____

The correct solution for $f(x) > 0$ is

or interval notation _____.



(b) Solve $f(x) < 0$.

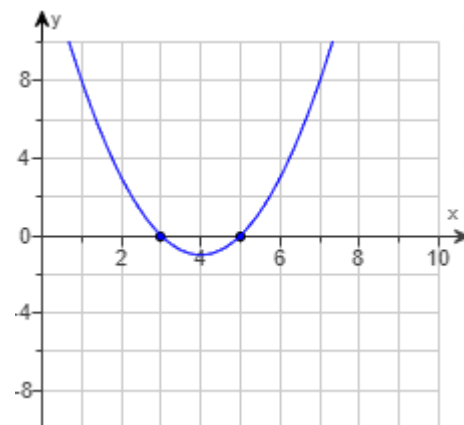
This means look _____ the x-axis.

There is _____ of the graph that is **BELOW** the x-axis – the “_____” of the graph.

- In BETWEEN $x = 3$ and $x = 5$, which is the interval _____

The correct solution for $f(x) < 0$ is

or interval notation _____.



- **EXAMPLE:** The graph of $f(x) = ax^2 + bx + c$ is shown. Solve each inequality. [3.4.27]

(a) $f(x) > 0$

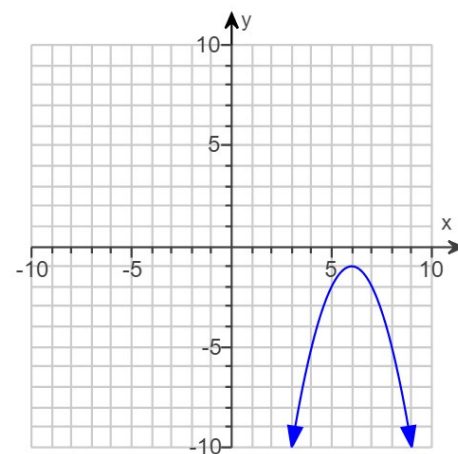
(b) $f(x) < 0$

This time there are **NO** x-intercepts. This problem takes a slightly different approach, so **be careful!**

(a) $f(x) > 0$ means _____ the x-axis. The graph of $f(x)$ is _____ above the x-axis! This inequality has _____ or $\{ \}$ or \emptyset .

(b) $f(x) < 0$ means _____ the x-axis. The graph of $f(x)$ is _____ below the x-axis! The solution to the inequality is

or in interval notation, it is _____.



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2. Solve Graphically When formula IS given


1. Make sure you have _____ on the right.
2. Find the _____ – treat it as if it's an equation.
3. The solutions are the _____ (or *boundary points*).
4. Graph critical points on a _____ (x-axis).
5. _____ leading coefficient (____) to see if parabola opens UP or DOWN.
6. _____ parabola passing through number line.
7. _____ inequality symbol as either ABOVE or BELOW x-axis.
8. _____ solution in either inequality or interval notation.

- **EXAMPLE:** Solve the inequality. $x^2 + 4x - 12 \geq 0$ [3.4.39]

(Type your answer in interval notation. Simplify your answer. Use integers or fractions for any numbers in the expression.)

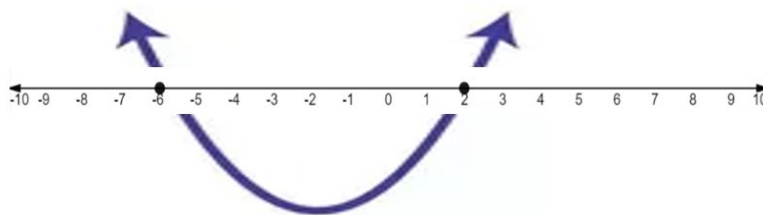
1. **Zero?** YES $x^2 + 4x - 12 = 0$
2. **Solutions** Factor $(\quad)(\quad) = 0$
Zero Product Property $\quad = 0$ $\quad = 0$
Solve each equation

3. **Critical Points** $x = \quad$ $x = \quad$
 $x = -6$ and $x = 2$ (also called _____ points)

4. **Number Line** 

5. **Inspect your "a"** $a = \quad$, so parabola opens _____

6. **Sketch.** Sketch parabola opening UP, passing thru the x-intercepts $(-6,0)$ and $(2,0)$.



7. **Interpret.** Inequality is _____
Use bracket or parentheses? _____
Is it ABOVE or BELOW x-axis? _____ x-axis
One or two pieces? _____ piece(s), the "_____" (use "or" inequality)

8. **Write solution** Inequality: $x \leq -6$ or $x \geq 2$
Interval Notation: _____

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- EXAMPLE:** Solve the inequality. Write the solution in interval notation. [3.4.7]

$$x^2 - x - 56 < 0$$

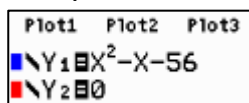
An alternate way to solve by graphing is to use the graphing calculator.

This is how you'll get the x-intercepts, or the **critical points** (or *boundary points*).

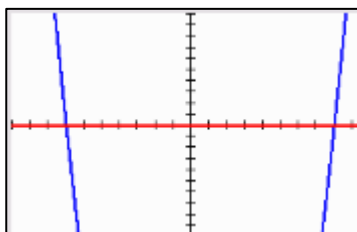
1. Get _____ on the right, if needed.

2. Press _____ button on calculator.

3. Put **LEFT** side into _____,
and put **ZERO** into _____.



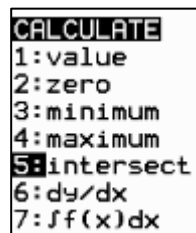
4. **Graph** it (press _____).



- You do NOT need the vertex to solve an inequality.
- Make sure you can see the x-intercepts on the screen.
- You may need to Zoom Out. Press ZOOM, 3, ENTER – if needed.

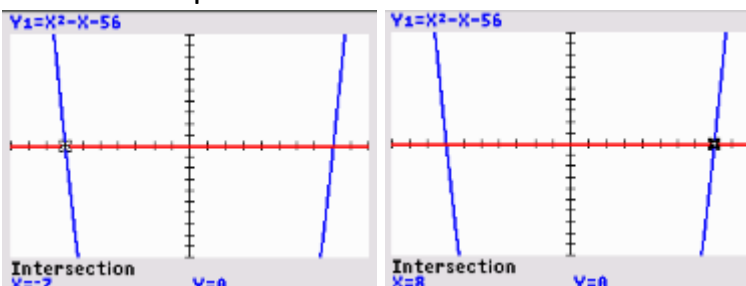
5. To find the x-intercepts:

a. Press _____: intersect, _____
(down arrow) to switch to graph Y2.



6. Move cursor to the **LEFT** x-intercept and press _____ **three (3)** times.

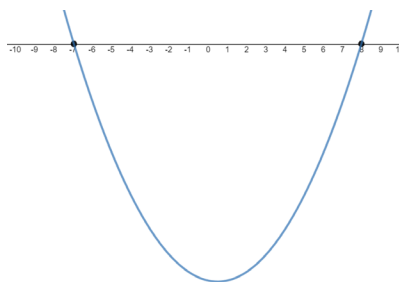
Repeat process to get the **RIGHT** x-intercept.



The x-intercepts are the **critical points**, which are $x = \underline{\hspace{1cm}}$ and $x = \underline{\hspace{1cm}}$

7. **Inspect your "a"**. The value of $a = \underline{\hspace{1cm}}$, which means opens _____.

8. **Sketch.**



9. **Interpret.** Inequality is _____
Bracket or parentheses? _____
Above or Below x-axis? _____ x-axis
One or Two pieces? _____ piece(s), the
"_____" (use "in-between" inequality)

10. **Write solution**

Inequality: _____

Interval Notation: _____

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B. Solve Quadratic Inequalities Symbolically (by hand) using _____

(The first 4 steps are identical to the graphical method at the top of page 3.)

1. Make sure you have **zero** on the right.
2. Find the **solutions** – treat it as if it's an equation.
3. The solutions are the **CRITICAL POINTS** (or *boundary points*).
4. Graph critical points on a **number line** (x-axis).
5. Identify the _____ the critical points (or *boundary points*) create.
6. Use a _____ (TP) from within each interval to test into the inequality.
7. The interval(s) that are _____ are the **solutions**.

- **EXAMPLE:** Solve the inequality. $x^2 - 8x + 15 > 0$ [3.4-10]

1. **Zero?** YES $x^2 - 8x + 15 = 0$

2. **Solutions** Factor: $(\quad)(\quad) = 0$

Zero Product Property: $\quad = 0$ or $\quad = 0$

Solve each equation: $x = \underline{\quad}$ or $x = \underline{\quad}$

3. **Critical Points (CP)** $x = 3$ or $x = 5$ (also called *boundary points*)

4. Number Line



5. Intervals

(Inequality) _____

(Interval Notation) _____

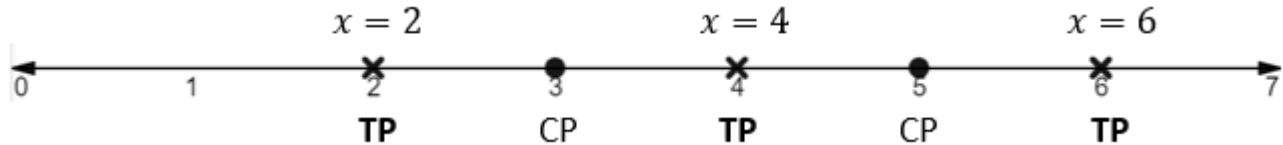
NOTE: Your **SOLUTION** will be one or more of these intervals.

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6. Test Points (TP)



Using $x^2 - 8x + 15 > 0$

Faster/easier if you use FACTORED form: $(\quad)(\quad) > 0$

Test $x = 2$	Test $x = 4$	Test $x = 6$
$(\quad)(\quad) > 0$	$(\quad)(\quad) > 0$	$(\quad)(\quad) > 0$
$\quad > 0$	$\quad > 0$	$\quad > 0$
$\quad > 0$	$\quad > 0$	$\quad > 0$
\quad	\quad	\quad
All points are TRUE on the interval $x < 3$	All points are FALSE on the interval $3 < x < 5$	All points are TRUE on the interval $x > 5$

7. **TRUE** interval(s) **Solution** to the inequality $x^2 - 8x + 15 > 0$ is:

(Inequality)

(Interval Notation) _____

Sources Used:

1. MyLab Math for *College Algebra with Modeling and Visualization*, 6th Edition, Rockswold, Pearson Education Inc.
2. Number Line Inequalities (modified) from Desmos, <https://www.desmos.com/calculator/evxn1e1njv>, © 2019, Desmos, Inc.
3. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website <https://archive.codeplex.com/?p=wabbit>