Notes Section 5.5 – Properties of Logarithms

Lesson Objectives

1. Change of Base Formula
2. Evaluating logarithms using TI-84 Plus series calculators
3. Expanding or Condensing (Combining) Logarithm Properties for:
    a. Multiplication
    b. Division
    c. Powers (Exponents)
    d. Various mixtures of these

Remember from when we introduced logarithms in the previous section:

A logarithm is ____________________.

A. The Change of Base Formula

Also, in the previous section, we discussed the two special types of logarithms.
These are the only logarithms that have their own buttons on the calculator:

1. **Common** logarithm – base is **10**, but not explicitly written. It is understood to be 10.
   If you see a logarithm written **without** a base, then the base is **10**.
    - Examples: $\log x$ means $\log_{10}(x)$ $\log\frac{1}{100}$ means $\log_{10}\left(\frac{1}{100}\right)$
    - Calculator button is **LOG** 

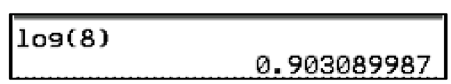
      (to the left of the **7** button)
    - This calculator button is **ONLY** for base **10**, the common logarithm!

2. **Natural** logarithm – base is *e*, but the logarithm is written as **"*ln*"** not "log$_e$".
    - Examples: $\ln x$ means $\log_e(x)$ $\ln e^7$ means $\log_e(e^7)$
    - Calculator button is **LN** 

      (to the left of the **4** button)
    - This calculator button is **ONLY** for base ***e***, the natural logarithm!

Consider the following logarithm: $\log_2(8)$ We know this equals 3, because $2^3 = 8$.

Sometimes students assume that the LOG button on the calculator works for any logarithm.

We know $\log_2(8) =$ ___, but $\log(8) \approx$ ___ on calculator. 

They're different values because they're different ______________.

If we want to determine a logarithm with a base other than 10 or *e* using calculator, we need another means to do it.

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- Change of Base Formula

Let $x, a \neq 1, b \neq 1$ be positive real numbers. Then, $\log_b(x) =$

(Alliteration: Remember that the **b-b-b**ase goes on the **b-b-b**ottom.)

Technically, you can use the Change of Base formula to convert to ANY base $(\boldsymbol{a})$, but for rounding purposes, base 10 (_____) or base *e* (____) is the way to go.

- **EXAMPLE:** Find the logarithm using the change of base formula. [5.5.79]

$$4\log_3(20)$$

(Simplify your answer. Do not round until the final answer. Then round to the nearest thousandth as needed.)

First of all, the 4 is like a coefficient, so it will just be multiplied onto the logarithm. Using the change of base formula:

$$4\log_3(20) = 4\cdot\frac{\qquad}{\qquad} \text{ or } 4\cdot\frac{\qquad}{\qquad} \approx \underline{\qquad\qquad}$$

```
4*log(20)/log(3)
             10.90733211
```

```
4*ln(20)/ln(3)
             10.90733211
```

Note that for your calculator, you can use either common or natural logarithm. Be careful with your parentheses when using change of base formula. It's easy to make a mistake with it. Here are 2 common mistakes:

```
4log(20/log(3)
             6.489604927
4ln(20/ln(3)
             11.60673778
```

```
4log(20)/ln(3)
             4.736994148
4ln(20)/log(3)
             25.11506032
```

If you don't _________ parentheses with the 20, you will get an **INCORRECT** answer.

If you ______________ logarithms when you divide, you will also get **INCORRECT** answer.

If you are using a **TI-83 Plus** calculator (or a TI-84 Plus calculator with older software), the **change-of-base formula** __________ be used to evaluate logarithms that are not base 10 or base *e*.

If you use a **TI-84 Plus** (includes color screen models, too), there is an easier, faster way to calculate logarithms that are not base 10 or base *e*.

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B. Evaluating logarithms using TI-84 Plus series calculator

Let's re-examine the previous example, this time using the TI-84 Plus CE calculator (it works for TI-84 Plus calculator, too, as long as it has updated software):

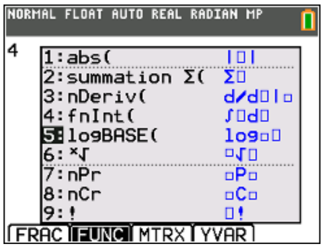
- **EXAMPLE:** Find the logarithm using the change of base formula. [5.5.79]

$$4\log_3(20)$$

(Simplify your answer. Do not round until the final answer. Then round to the nearest thousandth as needed.)

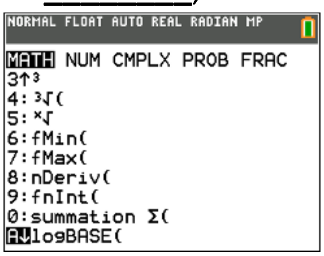
1. Press ________________________

Choose number __________________



Or

Press ________, scroll to _____________



2. Enter the base of **3** and the value of **20** in parentheses, then press **ENTER**

3. Round answer accordingly.

$4\log_3(20) \approx$ _____________

C. Expanding/Condensing Logarithm Properties

- **Product Rule:**

$\log_a(mn) =$ _________________ ____________ (_________________)

or _________________ $= \log_a(mn)$ ____________ (_________________)

- **Quotient Rule:**

$\log_a\left(\frac{m}{n}\right) =$ _________________ EXPANDING (_______________________)

or _________________ $= \log_a\left(\frac{m}{n}\right)$ CONDENSING (_______________________)

- **Power Rule:**

$\log_a(m^r) =$ _________ EXPANDING (_______________________)

or ________ $= \log_a(m^r)$ CONDENSING (_______________________)

Notes Section 5.5 – Properties of Logarithms

- EXPANDING Logarithms

- **EXAMPLE:** Expand the expression. If possible, write your answer without exponents.

  $\log_4(64k^4x)$ (Simplify your answer.) [5.5.17]

The value of the logarithm includes a __________: $\log_4(64k^4x) = \log_4(64 \cdot \boldsymbol{k^4} \cdot x)$

Use the **Product Rule** to ________, so you use __________ and keep the _______ base:

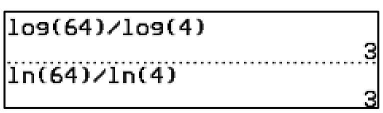
$\log_4(64 \cdot k^4 \cdot x) =$ ____

If a logarithm has NO variables, try to ________________:

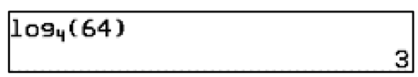
$\log_4(64)$ means ______________ $= 64$. It's ___.

(NOTE: If you are unsure if it simplifies, try it on ___________________with either **Change of Base** formula or **logBASE** feature. If you get a "nice, pretty" rational number, like **3**, then go ahead and ____________ to that value. If you get a "messy" decimal that doesn't convert to a fraction, then it does NOT simplify.

$$\log_4(64) = \frac{\log(64)}{\log(4)} \text{ or } \frac{\ln(64)}{\ln(3)} = \mathbf{3}$$



Using **Change of Base Formula**



Using the **logBASE** command

Continuing on:

$$\log_4(64 \cdot k^4 \cdot x) = \log_4(64) + \log_4(k^4) + \log_4(x)$$

$$= \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

Next, use the **Power Rule** (exponent to coefficient) to simplify 2nd term:

$$\log_4(k^4) = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

Now update:

$\log_4(64k^4x) =$ ______________________________ **Answer**

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- **EXAMPLE:** Expand the expression. If possible, write your answer without exponents.

$$\log_3\left(\frac{9m^5}{n^5}\right) \qquad [5.5.19]$$

(Simplify your answer. Use integers or fractions in the expression.)

This logarithm has a mixture of multiplication, division, and exponents.

First, use the ______________ **Rule** (SUBTRACTION) to EXPAND the fraction:

$$\log_3\left(\frac{\mathbf{9m^5}}{\mathbf{n^5}}\right) = \_\_\_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_\_\_\_\_$$

Next, use the ______________ **Rule** (ADDITION) to EXPAND $\log_3(9m^5)$

$$\log_3(\mathbf{9 \cdot m^5}) = \_\_\_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_\_\_$$

Update the entire logarithm:

$$\log_3\left(\frac{9m^5}{n^5}\right) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Now, we use the ______________ **Rule** (exponent to coefficient)

for both $\mathbf{\log_3(m^5)}$ and $\mathbf{\log_3(n^5)}$

Update the entire logarithm:

$$\log_3\left(\frac{9m^5}{n^5}\right) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Finally, we **simplify** (if possible) any logarithms with no variables:

$$\log_3(9) \text{ means } \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = 9. \text{ It's } \_\_\_.$$

```
log(9)/log(3)
                    2
ln(9)/ln(3)
                    2
```

Using **Change of Base Formula**

```
log₃(9)
                    2
```

Using **logBASE** command

NOTE: In general, if you get a decimal, do _________ convert to a rounded number, unless directed to do so. If it is not an _________ number, leave it alone as a logarithm.

Update your entire logarithm:

$$\log_3\left(\frac{9m^5}{n^5}\right) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \qquad \textbf{Answer}$$

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Notes Section 5.5 – Properties of Logarithms

- **CONDENSING** (or COMBINING) **Logarithms**

- **EXAMPLE:** Write the following expression as a logarithm of a single expression

  $\log 27 + \log \frac{1}{9}$ (Simplify your answer. Type an exact answer.) [5.5.37]

Write as a **single expression** means to ______________ (or COMBINE) logarithms.

$\log(27) + \log\left(\frac{1}{9}\right)$ Use _______________ **Rule** (sum to product)

$\log(27) + \log\left(\frac{1}{9}\right) = \log(\underline{\qquad\quad})$ Simplify $27 \cdot \frac{1}{9} = \underline{\quad}$

Update the entire expression: $= \underline{\qquad\quad}$ **Answer**

```
log(3)
                0.4771212547
```

not exact – _______________!

Notice that $\mathbf{log}(\mathbf{3})$ does not simplify into a "nice, pretty" number – it's ___________.
Therefore, $\mathbf{log}\ (\mathbf{3})$ is already an _________ answer, as required in the instructions.

- **EXAMPLE:** Write the expression as a logarithm of a single expression. [5.5.39]

  $\log 7 + \log 30 - \log 6$ (Simplify your answer.)

By the order of operations, add and subtract go in order, left to right.

Start with the first two terms: $\log(7) + \log(30)$

Addition of logarithms means use ____________ **Rule** (sum to product):

$\log(7) + \log(30) = \log(\underline{\qquad\quad}) = \log\ (\underline{\qquad})$

Update the entire expression:

$\log(7) + \log(30) - \log(6) = \underline{\qquad\qquad\qquad\qquad\qquad}$

Subtraction of logarithms means use ___________ **Rule** (difference to quotient):

$\log(210) - \log(6) = \log(\underline{\quad}) = \log(\underline{\quad})$

Update the entire expression:

$\log(7) + \log(30) - \log(6) \qquad = \underline{\qquad\qquad}$ **Answer**

```
log(35)
                 1.544068044
```

not __________ – leave it be!

Notice that $\mathbf{log}(\mathbf{35})$ does not simplify into a "nice, pretty" number – it's irrational.
The instructions ______ indicate to _______ the answer, so leave it as an exact answer.

Notes Section 5.5 – Properties of Logarithms

- **EXAMPLE:** Use properties of logarithms to condense the logarithmic expression below. Write the expression as a single logarithm whose coefficient is 1. Where possible, evaluate logarithmic expressions. [5.5.51]

$$5\ln x + 3\ln y - 2\ln z$$

All three terms have ____________, so use the _____ **Rule** (coefficient to exponent):

$$5\ln(x\ ) + 3\ln(y\ ) - 2\ln(z\ ) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Next, Addition of logarithms means use ___________ **Rule** (sum to product):

$$\ln(x^5) + \ln(y^3) = \ln(\_\_\_\_\_\_\_) = \mathbf{ln}(\_\_\_\_\_\_)$$

Update the entire expression:

$$5\ln(x) + 3\ln(y) - 2\ln(z) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Subtraction of logarithms means use ____________ **Rule** (difference to quotient):

$$\ln(x^5y^3) - \ln(z^2) = \ln\left(\frac{}{}\right)$$

Update the entire expression:

$$5\ln(x) + 3\ln(y) - 2\ln(z) \quad = \_\_\_\_\_\_\_\_\_\_\_$$ **Answer**

Sources Used:

1. MyLab Math for *College Algebra with Modeling and Visualization*, 6th Edition, Rockswold, Pearson Education Inc.
2. Texas Instruments TI Connect® CE software, https://education.ti.com/en/products/computer-software/ti-connect-ce-sw