Notes Section 5.1 – Combining Functions

Lesson Objectives

1. Perform Arithmetic Operations (+, –, ×, ÷) on Two Functions
   a. Symbolically (with formula)
   b. Numerically (with table)
   c. Graphically
2. Perform a Composition of Two Functions
   a. Symbolically (with formula)
   b. Numerically (with table)
   c. Graphically

A. Perform **Arithmetic Operations** (+, – , ×, ÷) on Two Functions

1. **Symbolically** (by hand)

Properties

If *f*(*x*) and *g*(*x*) both exist, the sum, difference, product, and quotient are defined as:

Sum of Functions: $(f+g)(x) =$ ____________________

Difference of Functions: $(f-g)(x) =$ ____________________

Product of Functions: $(fg)(x) =$ ____________________

Quotient of Functions $\left(\frac{f}{g}\right)(x) =$ ______________, with $g(x) \neq 0$

(go on to the next page)

Notes Section 5.1 – Combining Functions

- **EXAMPLE:** Let $f(x) = 3x + 2$ and $g(x) = \frac{1}{x}$.
  Evaluate each expression symbolically. [5.1.9]

(a) $(f + g)(4)$

(b) $(f - g)\left(\frac{1}{3}\right)$

(c) $(fg)(2)$

(d) $\left(\frac{f}{g}\right)(0)$

(a) $(f + g)(4) = \_\_\_\_\_ + \_\_\_\_\_$ $\qquad f(x) = 3x + 2$ and $g(x) = \frac{1}{x}$

$= 3(\_\_) + 2 + \frac{1}{\_\_} \qquad = \_\_\_ + \frac{1}{4} \qquad = \_\_\_\_\_$

(b) $(f - g)\left(\frac{1}{3}\right) = \_\_\_\_\_ - \_\_\_\_\_$ $\qquad f(x) = 3x + 2$ and $g(x) = \frac{1}{x}$

$= 3(\_\_) + 2 \qquad - \qquad \frac{1}{\_\_} \qquad = \_\_ - \_\_ \quad = \_\_$

(c) $(fg)(2) = \_\_\_\_\_ \cdot \_\_\_\_\_$ $\qquad f(x) = 3x + 2$ and $g(x) = \frac{1}{x}$

$= 3(\_\_) + 2 \cdot \frac{1}{\_\_} \qquad = \_\_ \cdot \frac{1}{2} \qquad = \_\_$

(d) $\frac{f}{g}(0) = \frac{f(0)}{g(0)} \qquad = \frac{3(\_\_)+2}{\frac{1}{\_\_}}$ $\qquad$ but, $\frac{1}{0}$ is $\_\_\_\_\_\_\_\_\_\_$

$f(x) = 3x + 2$ and $g(x) = \frac{1}{x}$

(go on to the next page)

Notes Section 5.1 – Combining Functions

2. Numerically (with table)

- **EXAMPLE:** Use the given table to complete the table below. [5.1.47]

Given table:

x	$f(x)$	$g(x)$
-2	0	8
0	6	0
2	7	-4
4	14	7

Complete the table. (Simplify your answers. Type N if the answer is undefined.)

x	$(f+g)(x)$	$(f-g)(x)$	$(fg)(x)$	$\left(\frac{f}{g}\right)(x)$
-2				
0				
2				
4				

(go on to the next page)

Notes Section 5.1 – Combining Functions

3. Graphically

- **EXAMPLE:** Use the graph to the right to evaluate the following functions. [5.1.39]

(a) $(f+g)(0)$

$= f(0) + g(0)$

(get the *y*-coordinates at *x* = 0)

$=$ ____ $+$ ____ $=$ ____

(b) $(f-g)(-1)$

$= f(-1) - g(-1)$

(get the *y*-coordinates at *x* = – 1)

$=$ ____ $-$ ____ $=$ ____

(c) $(fg)(1)$

$= f(1) \cdot g(1)$

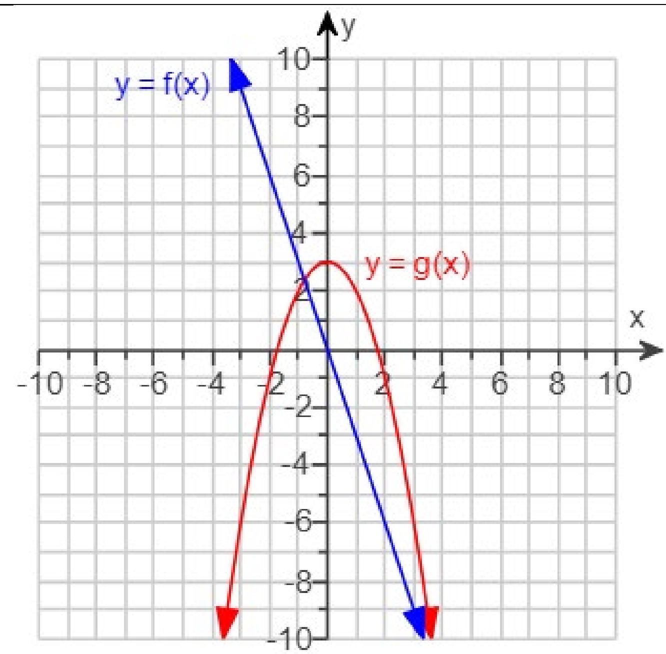
(get the *y*-coordinates at *x* = 1)

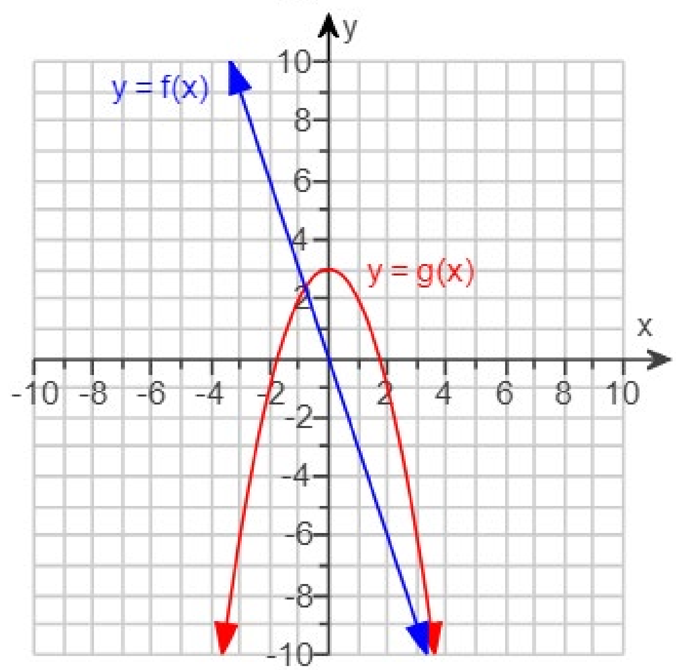
$=$ ____ $\cdot$ ____ $=$ ____

(d) $\left(\frac{f}{g}\right)(2) = \frac{f(2)}{g(2)}$

$= \frac{}{} =$ ____

(get the *y*-coordinates at *x* = 2)





(go on to the next page)

Notes Section 5.1 – Combining Functions

B. Perform a **Composition** of Two Functions

Definition **Function Composition** is defined as follows:

$(f \circ g)(x) =$ ______________ "________________________."

The output of the second function is the input into the first function

1. **Symbolically** (by hand)

- **EXAMPLE:** Find $(g \circ f)(5)$ when $f(x) = -3x - 2$ and $g(x) = -5x^2 - 2x - 9.$ [5.1-26]

Always start with the ______________________ function, and use the given input value.

$(g \circ f)(5)$

$f(5) = -3(\_\_\_) - 2 = \_\_\_\_\_\_\_$

Take that **OUTPUT** (answer) from 2^{ND} function and **INPUT** into **FIRST** function:

$$g(\_\_\_\_\_) = -5(\_\_\_\_\_)^2 - 2(\_\_\_\_\_) - 9$$
$$= -5(\_\_\_\_\_)^2 \quad \_\_\_\_\_\_\_ \quad - 9$$
$$= \_\_\_\_\_\_\_\_\_ \quad \_\_\_\_\_\_\_\_\_ \quad = \_\_\_\_\_\_\_\_\_\_\_$$

2. **Numerically** (with table)

- **EXAMPLE:** Use the tables to evaluate the expressions. [5.1.89]

x	1	2	5	7
$f(x)$	5	7	1	2

x	1	2	5	7
$g(x)$	2	5	7	8

Find $(g \circ f)(2)$ You do these similar to the symbolic (formula) way.

Always start with the ______________ function, and use the given input value.	Take that **OUTPUT** (answer) from 2^{ND} function and **INPUT** into ________ function:
$(g \circ f)(2)$ $f(2) = \_\_$	$g(7) = \_\_\_\_$ So, $(g \circ f)(2) = \_\_\_\_$

Notes Section 5.1 – Combining Functions

(continued from previous page – same problem)

- **EXAMPLE:** Use the tables to evaluate the expressions. [5.1.89]

x	1	2	5	7
$f(x)$	5	7	1	2

x	1	2	5	7
$g(x)$	2	5	7	8

Find $(f \circ g)(5)$ You do these similar to the symbolic (formula) way.

Always start with the **SECOND** function, and use the given input value.	Take that **OUTPUT** (answer) from 2ND function and **INPUT** into **FIRST** function:
$(f \circ g)(5)$ $g(5) =$ ___	$f(7) =$ ___ So, $(f \circ g)(5) =$ ___

(continuation of same problem)

- **EXAMPLE:** Use the tables to evaluate the expressions. [5.1.89]

x	1	2	5	7
$f(x)$	5	7	1	2

x	1	2	5	7
$g(x)$	2	5	7	8

Find $(g \circ g)(7)$ You do these similar to the symbolic (formula) way.

Always start with the **SECOND** function, and use the given input value.	Take that **OUTPUT** (answer) from 2ND function and **INPUT** into **FIRST** function:
$(g \circ g)(7)$ $g(7) =$ ___	$g(8)$ is ________ in the table. There's no $x = 8$ in the $g(x)$ table. So, $(g \circ g)(7) =$ ________________

(go on to the next page)

Notes Section 5.1 – Combining Functions

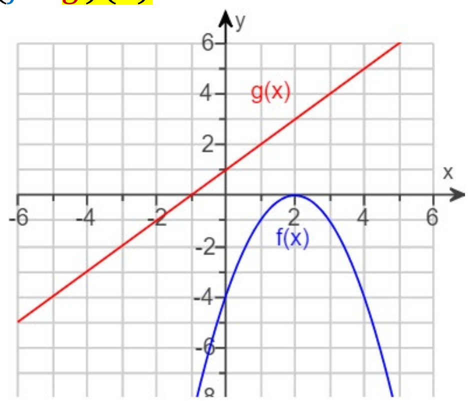
3. Graphically

- **EXAMPLE:** Use the graph to evaluate the following expressions. [5.1.87]

(a) $(f \circ g)(1)$
(b) $(g \circ f)(1)$
(c) $(g \circ g)(0)$

SOLUTION

(a) $(f \circ g)(1)$



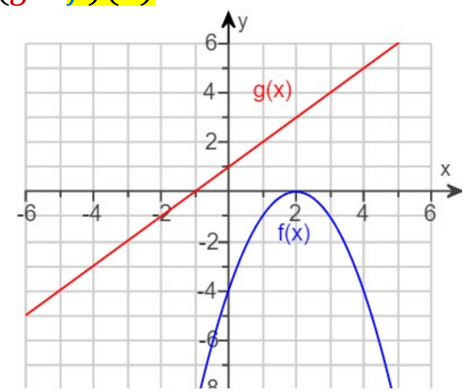
$g(1)$ means using the $g(x)$ graph, find the *y*-coordinate when $x = 1$.

$g(1) =$ ___

$f($___$)$ means using the $f(x)$ graph, find the *y*-coordinate when $x =$ ___. $f($___$) =$ ___

So, $(f \circ g)(1) =$ ___

(b) $(g \circ f)(1)$



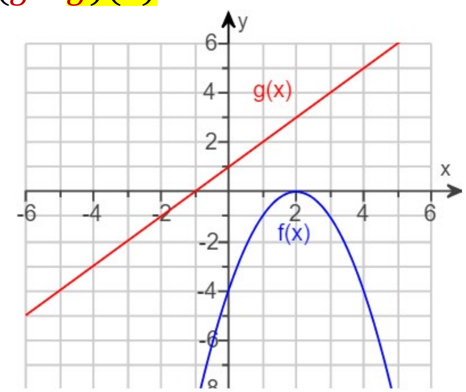
$f(1)$ means using the $f(x)$ graph, find the *y*-coordinate when $x = 1$.

$f(1) =$ ___

$g($___$)$ means using the $g(x)$ graph, find the *y*-coordinate when $x =$ ___. $g($___$) =$ ___

So, $(g \circ f)(1) =$ ___

(c) $(g \circ g)(0)$



$g(0)$ means using the $g(x)$ graph, find the *y*-coordinate when $x = 0$.

$g(0) =$ ___

$g($___$)$ means using the $g(x)$ graph, find the *y*-coordinate when $x =$ ___. $g($___$) =$ ___

So, $(g \circ g)(0) =$ ___

Source Used: MyLab Math for *College Algebra with Modeling and Visualization*, 6th Edition, Rockswold, Pearson Education Inc.