Notes Section 5.1 – Combining Functions

Lesson Objectives

1. Perform Arithmetic Operations (+, –, ×, ÷) on Two Functions
   a. Symbolically (with formula)
   b. Numerically (with table)
   c. Graphically
2. Perform a Composition of Two Functions
   a. Symbolically (with formula)
   b. Numerically (with table)
   c. Graphically

A. Perform Arithmetic Operations (+, – , ×, ÷) on Two Functions

1. Symbolically (by hand)

Properties

If *f*(*x*) and *g*(*x*) both exist, the sum, difference, product, and quotient are defined as:

Sum of Functions: $(f + g)(x) = f(x) + g(x)$

Difference of Functions: $(f - g)(x) = f(x) - g(x)$

Product of Functions: $(fg)(x) = f(x) \cdot g(x)$

Quotient of Functions $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \text{with } g(x) \neq 0$

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- **EXAMPLE:** Let $f(x) = 3x + 2$ and $g(x) = \frac{1}{x}$.

  Evaluate each expression symbolically. [5.1.9]

  (a) $(f + g)(4)$

  (b) $(f - g)\left(\frac{1}{3}\right)$

  (c) $(fg)(2)$

  (d) $\left(\frac{f}{g}\right)(0)$

(a) $(f + g)(4) = f(4) \quad + \quad g(4)$ $\qquad f(x) = 3x + 2$ and $g(x) = \frac{1}{x}$

$= 3(\mathbf{4}) + 2 + \frac{\mathbf{1}}{\mathbf{4}} \qquad = 14 + \frac{1}{4} = \frac{\mathbf{57}}{\mathbf{4}}$

(b) $(f - g)\left(\frac{1}{3}\right) = f\left(\frac{1}{3}\right) - g\left(\frac{1}{3}\right)$ $\qquad f(x) = 3x + 2$ and $g(x) = \frac{1}{x}$

$= 3\left(\frac{\mathbf{1}}{\mathbf{3}}\right) + 2 - \frac{1}{\mathbf{1}/\mathbf{3}} \qquad = 3 - 3 \qquad = \mathbf{0}$

(c) $(fg)(2) = f(2) \quad \cdot \quad g(2)$ $\qquad f(x) = 3x + 2$ and $g(x) = \frac{1}{x}$

$= 3(\mathbf{2}) + 2 \cdot \frac{\mathbf{1}}{\mathbf{2}} \qquad = 8 \cdot \frac{1}{2} \qquad = \mathbf{4}$

(d) $\frac{f}{g}(0) = \frac{f(0)}{g(0)} \qquad = \frac{3(\mathbf{0})+2}{\frac{\mathbf{1}}{\mathbf{0}}}$ $\qquad$ but, $\frac{1}{0}$ is **Undefined**

$f(x) = 3x + 2$ and $g(x) = \frac{1}{x}$

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2. Numerically (with table)

- **EXAMPLE:** Use the given table to complete the table below. [5.1.47]

Given table:

x	$f(x)$	$g(x)$
– 2	0	8
0	6	0
2	7	– 4
4	14	7

Complete the table. (Simplify your answers. Type N if the answer is undefined.)

x	$(f+g)(x)$	$(f-g)(x)$	$(fg)(x)$	$\left(\frac{f}{g}\right)(x)$
– 2	$0+8=\mathbf{8}$	$0-8=\mathbf{-8}$	$0\cdot 8=\mathbf{0}$	$\frac{0}{8}=\mathbf{0}$
0	$6+0=\mathbf{6}$	$6-0=\mathbf{6}$	$6\cdot 0=\mathbf{0}$	$\frac{6}{0}=\mathbf{N}$
2	$7+(-4)=\mathbf{3}$	$7-(-4)=\mathbf{11}$	$7\cdot -4=\mathbf{-28}$	$\frac{7}{-4}=\mathbf{-\frac{7}{4}}$
4	$14+7=\mathbf{21}$	$14-7=\mathbf{7}$	$14\cdot 7=\mathbf{98}$	$\frac{14}{7}=\mathbf{2}$

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3. Graphically

- **EXAMPLE:** Use the graph to the right to evaluate the following functions. [5.1.39]

(a) $(f+g)(0)$

$= f(0) + g(0)$

(get the *y*-coordinates at *x* = 0)

$= 0 + 3 = \mathbf{3}$

(b) $(f-g)(-1)$

$= f(-1) - g(-1)$

(get the *y*-coordinates at *x* = – 1)

$= 3 - 2 = \mathbf{1}$

(c) $(fg)(1)$

$= f(1) \cdot g(1)$

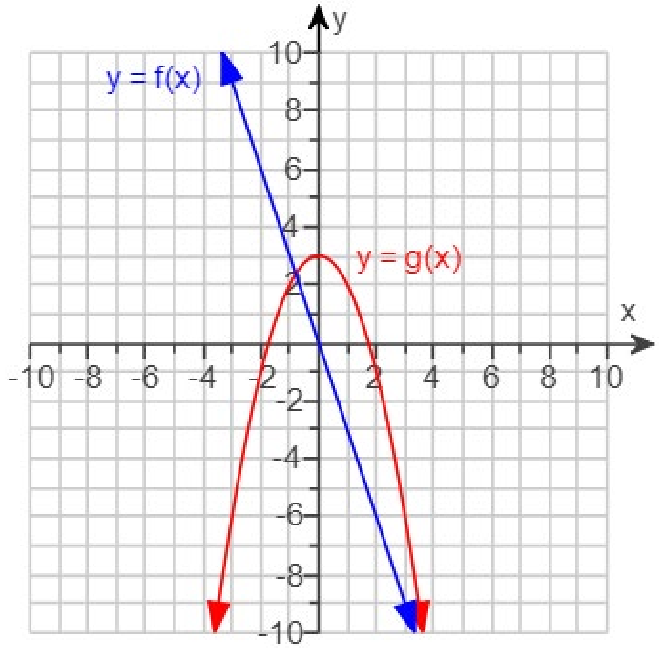
(get the *y*-coordinates at *x* = 1)

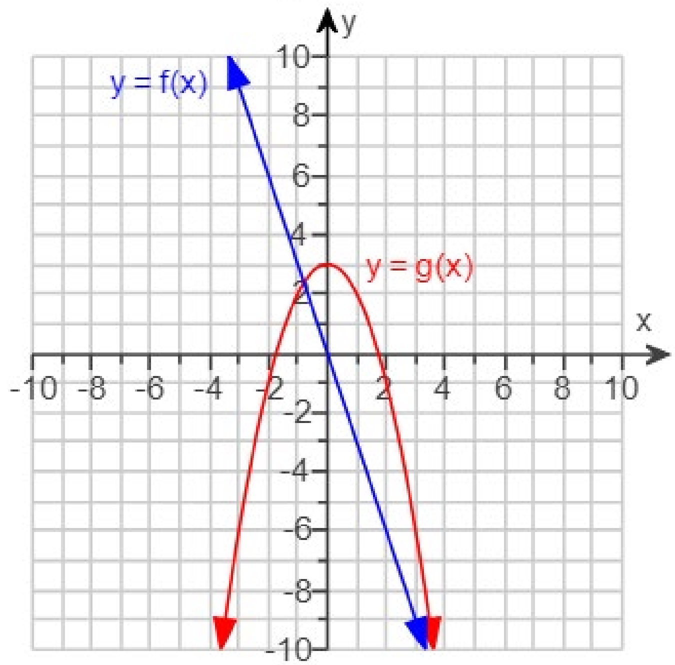
$= -3 \cdot 2 = \mathbf{-6}$

(d) $\left(\frac{f}{g}\right)(2) = \frac{f(2)}{g(2)}$

$= \frac{-6}{-1} = \mathbf{6}$

(get the *y*-coordinates at *x* = 2)





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B. Perform a **Composition** of Two Functions

Definition **Function Composition** is defined as follows:

$(f \circ g)(x) = f(g(x))$ "f of g of x."

The output of the second function is the input into the first function

1. **Symbolically** (by hand)

- **EXAMPLE:** Find $(g \circ f)(5)$ when $f(x) = -3x - 2$
  and $g(x) = -5x^2 - 2x - 9.$ [5.1-26]

Always start with the **SECOND** function, and use the given input value.

$(g \circ f)(5)$

$f(5) = -3(5) - 2 = -17$

Take that **OUTPUT** (answer) from 2[ND] function and **INPUT** into **FIRST** function:

$$g(-17) = -5(-17)^2 - 2(-17) - 9$$
$$= -5(289) + 34 - 9$$
$$= -1445 + 25 = \mathbf{-1420}$$

2. **Numerically** (with table)

- **EXAMPLE:** Use the tables to evaluate the expressions. [5.1.89]

x	1	2	5	7
$f(x)$	5	7	1	2

x	1	2	5	7
$g(x)$	2	5	7	8

Find $(g \circ f)(2)$ You do these similar to the symbolic (formula) way.

Always start with the **SECOND** function, and use the given input value.	Take that **OUTPUT** (answer) from 2[ND] function and **INPUT** into **FIRST** function:
$(g \circ f)(2)$ $f(2) = 7$	$g(7) = 8$ So, $(g \circ f)(2) = \mathbf{8}$

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- **EXAMPLE:** Use the tables to evaluate the expressions. [5.1.89]

x	1	2	5	7
$f(x)$	5	7	1	2

x	1	2	5	7
$g(x)$	2	5	7	8

Find $(f \circ g)(5)$ You do these similar to the symbolic (formula) way.

Always start with the **SECOND** function, and use the given input value.	Take that **OUTPUT** (answer) from 2ND function and **INPUT** into **FIRST** function:
$(f \circ g)(5)$ $g(5) = 7$	$f(7) = 2$ So, $(f \circ g)(5) = \mathbf{2}$

(continuation of same problem)

- **EXAMPLE:** Use the tables to evaluate the expressions. [5.1.89]

x	1	2	5	7
$f(x)$	5	7	1	2

x	1	2	5	7
$g(x)$	2	5	7	8

Find $(g \circ g)(7)$ You do these similar to the symbolic (formula) way.

Always start with the **SECOND** function, and use the given input value.	Take that **OUTPUT** (answer) from 2ND function and **INPUT** into **FIRST** function:
$(g \circ g)(7)$ $g(7) = 8$	$g(8)$ is not in the table. There's no $x = 8$ in the $g(x)$ table. So, $(g \circ g)(7) = \textbf{undefined}$

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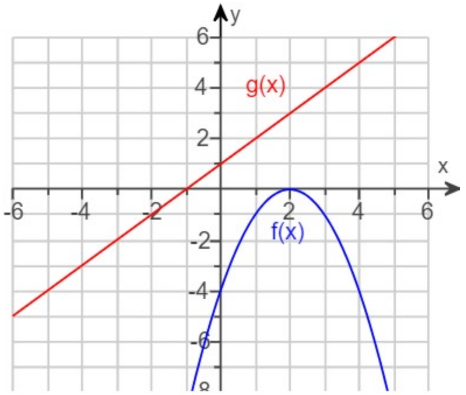
3. Graphically

- **EXAMPLE:** Use the graph to evaluate the following expressions. [5.1.87]

(a) $(f \circ g)(1)$
(b) $(g \circ f)(1)$
(c) $(g \circ g)(0)$

SOLUTION

(a) $(f \circ g)(1)$



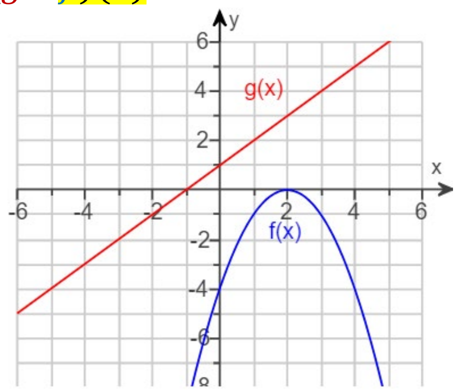
$g(1)$ means using the $g(x)$ graph, find the *y*-coordinate when $x = 1$.

$g(1) = 2$

$f(2)$ means using the $f(x)$ graph, find the *y*-coordinate when $x = 2$. $f(2) = 0$

So, $(f \circ g)(1) = \mathbf{0}$

(b) $(g \circ f)(1)$



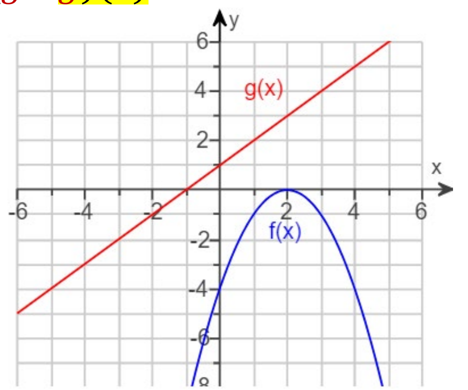
$f(1)$ means using the $f(x)$ graph, find the *y*-coordinate when $x = 1$.

$f(1) = -1$

$g(-1)$ means using the $g(x)$ graph, find the *y*-coordinate when $x = -1$. $g(-1) = 0$

So, $(g \circ f)(1) = \mathbf{0}$

(c) $(g \circ g)(0)$



$g(0)$ means using the $g(x)$ graph, find the *y*-coordinate when $x = 0$.

$g(0) = 1$

$g(1)$ means using the $g(x)$ graph, find the *y*-coordinate when $x = 1$. $g(1) = 2$

So, $(g \circ g)(0) = \mathbf{2}$

Source Used: MyLab Math for *College Algebra with Modeling and Visualization*, 6th Edition, Rockswold, Pearson Education Inc.