Notes Section 1.3 B – Function Basics

Lesson Objectives

1. The basics of function
2. Function notation
3. Understand the relationship between function notation and its graph
4. Determine whether a relation is a function
5. Evaluate a function from a formula or a graph
6. Determine the domain and range of a function

A. Functions – The Basics

Ways to Represent a function

1. **Symbolic** – a formula (equation)
2. **Numerical** – a table of values
3. **Graphical** – a visual display of points
4. **Verbal** – a description in words

The variables of a function

- The set of all valid **inputs** for a function is called the **domain**, and the input variable is called the **independent** variable **– *x***.
- The set of all valid **outputs** for a function is called the **range**, and the output variable is called the **dependent** variable – ***y***.

B. Function Notation – the symbolic representation

Function notation looks like this: $\boldsymbol{y = f(x)}$ and is read as "***y* equals *f* of *x***."
The reverse is also true: $f(x) = y$
It does **NOT** mean multiplication! (It is NOT "*f* times *x* equals *y*.")
Another way: ***f*(input) = output**
The **name** of the function is ***f***, but a function can be called *g*(*x*) or *h*(*x*), etc.

C. Function Notation and its Graph: *f*(Input) = Output

When evaluating a graphically, first locate the **input** value on the ***x*-axis**, then determine the corresponding **output** value on the ***y*-axis**.

- **EXAMPLE:** If *f*(-5) = 3, identify a point on the graph of *f*. (Type an ordered pair.)[1.3.23, Q4]

  - The number in **parentheses** is the ***x***, and the number **by itself** is the ***y***.
  - Together, they make the ordered pair, or the point, **(*x*, *y*)**.
  - So, if *f*(-5) = 3, that means a point on the graph of *f* is **(-5,3)**.

Notes Section 1.3 B – Function Basics

- **EXAMPLE:** If (5,27) lies on the graph of f, then $f($__**5**__$)$ = _____**27**_____. [1.3.25]

  - This is the reverse idea of the previous example.
  - Any point in the graph of a function can be written in function notation.

- **EXAMPLE:** A function g is defined as follows: $g(-4) = -6$, $g(0) = -9$, $g(4) = -4$, $g(8) = -6$
  (a) Write g as a set of ordered pairs.
  (b) Give the domain and range of g.

  [1.3.47]

  [solution] **(a)** g = { (-4,-6), (0,-9), (4,-4), (8,-6) }
  (b) D = {-4, 0, 4, 8} and R = {-6, -9, -4}

D. Function – special kind of relation (set of ordered pairs)

Each element in the domain corresponds to exactly one element in the range.
A function can only have one output for each input.
The ***x*-coordinates cannot repeat** in a function.

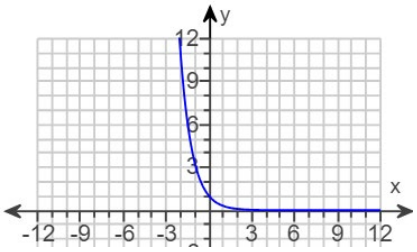
- **EXAMPLE:** Determine whether the relation S is a function. [1.3.107]
  S = { (a,5), (b,5), (c,9), (d,5), (e,5) }.

  - Each x-coordinate is unique; none of the x-coordinates repeat.
  - **YES** – the relation **IS** a function (Doesn't matter if y-coordinates repeat)

Vertical Line Test – used to tell if a graph is a function

- Scan with a vertical line from left to right along the graph
- Must maintain EXACTLY **one** point of contact throughout the scan.
  - If it maintains **exactly one** point of contact, then YES, it's a function
  - If it makes **two or more** points of contact anywhere, then NO, it's not a function.

- **EXAMPLE:** Is the relation a function? [1.3-45]



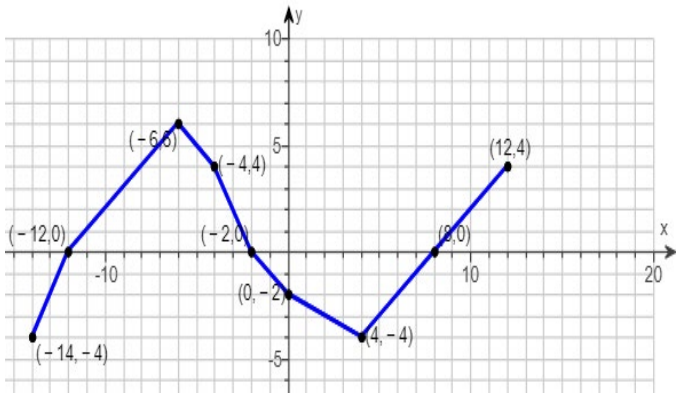
Although the left side looks like it's going vertical, in reality, it's not.

Notes Section 1.3 B – Function Basics

E. Evaluate a Function from its Graph

- **EXAMPLE:** [*Consortium 3.1.12]

Use the graph of the function *f* shown to the right to answer parts **(a)** through **(d)**.



(a) Find $f(-14)$.
The number in parentheses is always the ***x*-coordinate**. So, $f(-14)$ means find the ***y*-coordinate** that goes with $x = -14$.
The graph contains the point $(-14,-4)$, so
Find y in $(-14, y)$ $f(-14) =$ **-4**.

(b) Find $f(-6)$.
The graph contains the point $(-6,6)$, so
Find y in $(-6, y)$ $f(-6) =$ **6**.

(c) Find $f(12)$.

The graph contains the point (12,4), so
Find y in $(12, y)$ $f(12) =$ **4**.

(d) For what number(s) of x is $f(x) = -4$?

This problem is different – the number in parentheses, x, is not given.
f*(*x*)** is another name for ***y, so $f(x) = -4$ really means that ***y*-coordinate** is **-4**.
The graph contains the point $(-14, -4)$, so
Find x in $(x, -4)$ So, $x =$ **-14**

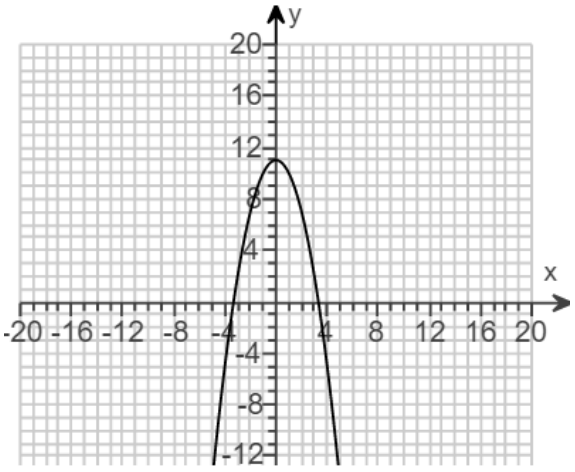
F. Determine Domain and Range of a Function in a Graph

- **EXAMPLE:** Use the graph of the function *f* to estimate its domain and range.
  Evaluate $f(0)$. [1.3.73]

Assume graph goes on forever, unless it has a big fat dot or open dot (endpoint).
Domain: all the ***x*-coordinates** shown in graph.
Domain is *x*, and it moves left – to – right.
(Answer) The domain is: $(-\infty, \infty)$

Range: all the ***y*-coordinates** shown in graph.
Range is *y*, and it moves low – to – high.
(Answer) The range is: $(-\infty, \mathbf{11}]$

Use **bracket** for range because 11 is **included**.



Evaluate $f(0)$.
That's the point (0,11)

Find y in $(0, y)$
(Answer)
$f(0) =$ **11**

Notes Section 1.3 B – Function Basics

G. Evaluate a Function from its Formula

- **EXAMPLE:** Given that $f(x) = |2x - 8| + 7$, find $f(3)$. [*Akst Appendix.G-28]

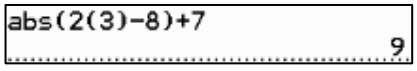
$f(3)$ means find y when $x = 3$. Plug $x = 3$ into the function formula.
Remember to always use **parentheses** to avoid messing up.

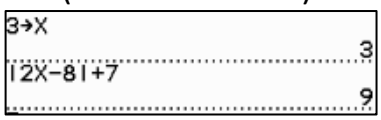
$$f(x) = |2x - 8| + 7$$
$$f(3) = |2(3) - 8| + 7$$
$$f(3) = |6 - 8| + 7$$
$$f(3) = |-2| + 7$$
$$f(3) = 2 + 7 = \mathbf{9}$$

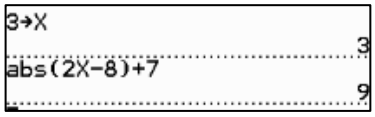
By the way – together, that makes the point on the graph: $(3,9)$

There are two main ways you can do this on calculator:

<table>
<tr>
<td>1. Direct Substitution
To get absolute value bars, press MATH, →, ENTER
 
</td>
<td colspan="2">2. “Go to the STO”
You are going to store the number 3 for the variable x.
Press: 3 ; STO> ; X,T,θ,n ; ENTER
 
</td>
</tr>
<tr>
<td>(from TI-84 Plus)
 

(from TI-83 Plus)
 
</td>
<td>Type in your function formula
$|2x - 8| + 7$

as is, press ENTER.
Use variables – don’t plug in anything.</td>
<td>(from TI-84 Plus)
 

(from TI-83 Plus)
 
</td>
</tr>
</table>

(go on to the next page)

Notes Section 1.3 B – Function Basics

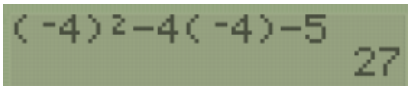
- **EXAMPLE:** Find $f(-4)$ when $f(x) = x^2 - 4x - 5$. [1.3-12]

  $f(-4)$ means find y when $x = -4$. Plug $x = -4$ into the function formula. Remember to always use **parentheses** to avoid messing up.

$$f(x) = x^2 - 4x - 5$$
$$f(-4) = (-4)^2 - 4(-4) - 5$$
$$f(-4) = 16 + 16 - 5$$
$$f(-4) = 32 - 5 = \mathbf{27}$$

By the way – together, that makes the point on the graph: $(-4,27)$

There are two main ways you can do this on calculator:

1. Direct Substitution	**2. "Go to the STO"**	
	Press **(–)**; **4** ; **STO>** ; **X,T,θ,n** ; **ENTER**  Type in your function formula $x^2 - 4x - 5$ as is, press **ENTER**. Use variables – don't plug in anything.	

Sources used:

1. Pearson MyLab Math *College Algebra with Modeling and Visualization, 6th Edition*, Rockswold
2. Pearson MyLab Math *Consortium: MIA: Intro to Algebraic, Graphical, & Numerical Problem Solving, 6th Edition*.
3. Pearson MyLab Math *Developmental Mathematics through Applications, 1st Edition*, Akst.
4. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website https://archive.codeplex.com/?p=wabbit