

Notes Section 2.3 – Linear Inequalities

Lesson Objectives

1. Overview of an Inequality Relationship
 2. **Remember** the **Reverse Rule** for Inequalities!
 3. Basic Linear Inequalities
 4. Compound (Three-Part) Linear Inequalities
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A. Overview of an Inequality Relationship

Let's examine a simple inequality relationship.

$$-2 < 3$$

That's certainly **TRUE**: negative 2 is less than 3.

- If you multiply both sides by **+5**, for example:

$$-2(\mathbf{5}) < 3(\mathbf{5}) \quad \text{updates to} \quad \underline{\hspace{2cm}}$$

So that new inequality relationship $-10 < 15$ Is that still **TRUE**?

Now let's reset back to the original inequality:

$$-2 < 3$$

- Let's examine what happens when we multiply by **-5**:

$$-2(\mathbf{-5}) < 3(\mathbf{-5}) \quad \text{updates to} \quad \underline{\hspace{2cm}}$$

Is this inequality relationship $10 < -15$ **TRUE** now?

In order to keep the inequality **TRUE**, we need to the direction of the inequality symbol. This preserves the smaller-bigger or bigger-smaller relationship:

$$10 > -15$$

Is this inequality relationship $10 > -15$ **TRUE** now?

That is a *really, really* **BIG IDEA** when solving inequalities!

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B. Remember the **Reverse Rule** for Inequalities!

Whenever you _____ or _____ an inequality by a _____ number, you must **remember** to _____ the direction of the **inequality symbol**!

This is true for *any* type of inequality, no matter how simple or complex.

You do **NOT** reverse the inequality when:

- Multiplying or dividing by a _____ number
- Using the _____ *Property* with a negative number (on one side of equation)
- Adding or subtracting

C. Basic Linear Inequalities

Solving a linear inequality involves the same steps as solving a linear equation (Section 2.2), but you now must **Remember** the **Reverse Rule** and its possibility of being used.

Here is a revised checklist to solve basic linear inequalities:

1. ****Combine Like Terms**, if you can.
2. **Undo Parentheses**, using the Distributive Property, then ****** (see #1).
3. (if necessary) **Clear out fractions** – multiply all terms by the common denominator (also known as the Least Common Multiple, or LCM), then ****** (see #1).

Remember the **Reverse Rule** – if you **MULTIPLY** by a **NEGATIVE** number, **REVERSE** it!

4. **Letters go LEFT** – use ADD or SUBTRACT to move variable terms to the LEFT side of the equation, then ****** (see #1).
5. **Numbers go RIGHT** – use ADD or SUBTRACT to move constant terms to the RIGHT side of the equation, then ****** (see #1).
6. **Divide** – last step is to DIVIDE by the coefficient of your variable and simplify.

Remember the **Reverse Rule** – if you **DIVIDE** by a **NEGATIVE** number, **REVERSE** it!

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- **EXAMPLE:** Solve the inequality. Write the solution set in interval notation. [2.3.15]

$$x - 2 \leq 4x + 7$$

We can't combine like terms yet. There are no parentheses nor fractions to deal with.

$$x - 2 \leq 4x + 7$$

Letters go LEFT:

Combine like terms:

Numbers go RIGHT:

Combine like terms:

Divide by coefficient:

Remember the Reverse Rule!

Simplify:

Is variable on the LEFT? _____

Graph:



Direction of shade? _____

Is endpoint included? _____

Interval Notation:

(go on to the next page)

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- **EXAMPLE:** Solve the inequality symbolically. Express the solution in interval notation.

$$6x - 4 < \frac{-36 - x}{-2} \quad [2.3.21]$$

We can't combine like terms at this point. There are no parentheses to deal with yet. There is one fraction (denominator), which is -2 .

To clear out fractions, we **MULTIPLY** both sides by the -2 . **Remember the Reverse Rule!**

$$6x - 4 < \frac{-36 - x}{-2}$$

MULTIPLY both sides by -2

Remember the Reverse Rule!

Simplify – **Divide out common factors.**

Use the **Distributive Property.**

Letters go LEFT.

Simplify – **Combine like terms.**

Numbers go RIGHT.

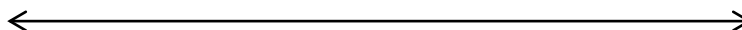
Simplify – **Combine like terms.**

Divide – Remember the Reverse Rule!

Simplify.

Is variable on the left? _____

Graph.



Direction to shade? _____

Is endpoint included? _____

Interval Notation:

(go on to the next page)

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D. Compound (_____-Part) Linear Inequalities

Recall the basic structure of an “in-between” inequality (or the Three-Part Inequality):

$$\begin{array}{ccccc} & < & & < & \\ \text{smaller number} & \text{or } (variable) & \text{or} & \text{larger number} & \\ & \leq & & \leq & \end{array}$$

Here’s a summary of how it looks in interval notation. Assume x as our variable.

Let a represent the smaller number and let b represent the larger number.

Inequality	Interval Notation
$a < x < b$	
$a < x \leq b$	
$a \leq x < b$	
$a \leq x \leq b$	

When you solve a Three-Part Inequality, focus on the _____, where the variable is. Keep your variable in the middle, and UNDO anything attached to it, in this order

1. If a common denominator, clear out fractions first – **Remember the Reverse Rule!**
2. Undo addition or subtraction next
3. Undo multiplication (coefficient) last – **Remember the Reverse Rule!**

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- **EXAMPLE:** Solve the inequality symbolically. Express the solution set in interval notation.

$$-3 \leq -8 - 4x \leq 15 \qquad [2.3-13]$$

We cannot combine like terms yet. There are no parentheses nor fractions to deal with.

Focus on the **middle**, where the variable is.

$$-3 \leq -8 - 4x \leq 15$$

First, undo the subtract 8 with add 8 on all 3 sides:

Simplify – **Combine Like Terms.**

Divide by the coefficient.

Remember the Reverse Rule!

Simplify.

Inspect for proper format:

Is smaller number on left, larger on right? _____

Are inequality symbols pointing LEFT? _____

We need to “pivot” or reverse the entire inequality:

Interval Notation:

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- **EXAMPLE:** Solve the inequality symbolically. Express the solution in interval notation.

$$-6 \leq \frac{1-2x}{6} < 12 \quad [2.3.33]$$

(Simplify your answer. Use integers or decimals for any numbers in the expression.)

We cannot combine like terms yet. There are no parentheses to deal with, but there is a denominator of 6 that is controlling the middle. First, we will clear out fractions.

Multiply all three regions by ____:

Do we REVERSE the inequalities? _____

Simplify – **Divide out common factors.**

Undo positive 1 with subtract 1 all 3 sides.

Simplify – **Combine Like Terms.**

Divide by the coefficient.

Remember the Reverse Rule!

Simplify. (We're using decimals)

Inspect for proper format:

Is smaller number on left, larger on right? _____

Are inequality symbols pointing LEFT? _____

We need to “pivot” or reverse the entire inequality:

Interval Notation:

Source Used:

1. Pearson MyLab Math *College Algebra with Modeling and Visualization*, 6th Edition, Rockswold