Notes Section R.2 – Integer Exponents

Lesson Objectives

1. Zero and negative exponents
2. Product, Quotient, and Power Properties for Exponents

A. ZERO and NEGATIVE exponents

Zero and negative exponents may be better understood by reviewing place value with base 10:

It turns out that **any** base (except zero) that has a zero power is equal to 1.

Property: $a^0 = \mathbf{1} \qquad (a \neq 0)$

Negative exponents do **NOT** make negative numbers! They cause a **reciprocal**.

Property: $a^{-n} = \frac{1}{a^n} \qquad (a \neq 0)$

and $\frac{1}{a^{-n}} = a^n$

"Take the Stairs" for negative exponents.

Cross the line, **change the sign**!

Powers of 10			
Power of 10	Standard Form	Fractional Form	Place Value
10^4	10,000	$\frac{10{,}000}{1}$	ten thousands
10^3	1,000	$\frac{1{,}000}{1}$	thousands
10^2	100	$\frac{100}{1}$	hundreds
10^1	10	$\frac{10}{1}$	tens
10^0	1	$\frac{1}{1}$	ones
10^{-1}	0.1	$\frac{1}{10}$	tenths
10^{-2}	0.01	$\frac{1}{100}$	hundredths
10^{-3}	0.001	$\frac{1}{1{,}000}$	thousandths
10^{-4}	0.0001	$\frac{1}{10{,}000}$	ten thousandths

- Negative Exponents and Fractions – "take the stairs" (reciprocal)

Property: $$\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$$

Property: $$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

Simplifying Exponents Tip #1: Final answer should have **no negative exponents.**

B. Product, Quotient, and Power Rules for Exponents

- **Product Rule:** $a^m \cdot a^n = a^{m+n}$

  When multiplying powers w/same base, **ADD** exponents.

- **Quotient Rule:** $\frac{a^m}{a^n} = a^{m-n}$

  When dividing powers w/same base, **SUBTRACT** exponents.

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Simplifying Exponents Tip #2:	Final answer should have **no duplicate variables** – you should see each variable only once.

- **Power Rule:** $(a^m)^n = \boldsymbol{a^{m \cdot n}}$

  When raising a power to a power, **MULTIPLY** exponents.

- **Product to Power Rule:** $(ab)^n = \boldsymbol{a^n b^n}$

  The exponent applies to each factor in the parentheses.

- **Quotient to Power Rule:** $\left(\frac{a}{b}\right)^n = \boldsymbol{\frac{a^n}{b^n}}$

  The exponent applies to each factor in numerator and denominator.

Simplifying Exponents Tip #3:	Final answer should have **no parentheses.**

Simplifying Exponents Tip #4:	A factor with no visible exponent has an **understood** (or implied) **exponent of 1**. To reduce errors, consider writing in an exponent of 1 in these situations.

Simplifying Exponents Tip #5:	Simplify exponents that have a constant (number) base. **"Do it!"** For example, don't leave 2^5 – change it to 32.

Other Tips for Success:	Remember to always reduce (simplify) fractions. Use a calculator for any numerical (non-variable) part. Watch out with the negatives! To get better with exponents, you must **PRACTICE!**

- **EXAMPLE:** Evaluate. $(-29)^0$ [R.2.21]

Any base (except zero) raised to the power of 0 equals 1. $(-29)^0 = \mathbf{1}$ **Answer**

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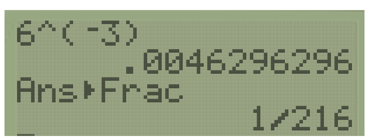
- **EXAMPLE:** Simplify the following expression. 6^{-3} [R.2.27]

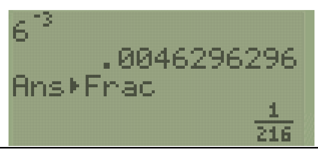
There are two common errors with this:		
1. Multiplying error	$6^{-3} = -18$	(INCORRECT!)
2. Sign error	$6^{-3} = -6^3 = -216$	(INCORRECT!)

NOTE: A negative exponent does NOT make a negative number!
A negative exponent means **reciprocal** (flip it!).

6^{-3}	$= \dfrac{6^{-3}}{1}$	$= \dfrac{1}{6^3}$	$= \mathbf{\dfrac{1}{216}}$
Write it as a fraction.	Negative exponent means **"take the stairs"** (reciprocal). BASE (6) is the SAME; EXPONENT (– 3) changes sign.	Simplify – "do it." $6^3 = 216$	**Answer**

Note also that any time you are dealing with constants (no variables), you can verify the result using your calculator:

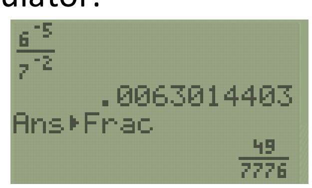


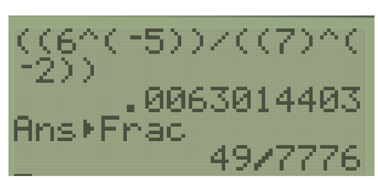


- **EXAMPLE:** Use the quotient rule to simplify the expression. $\dfrac{6^{-5}}{7^{-2}}$ [R.2.33]

$\dfrac{6^{-5}}{7^{-2}}$	$= \dfrac{7^2}{6^5}$	$= \mathbf{\dfrac{49}{7776}}$
Negative exponent means **"take the stairs"** (reciprocal). Both bases (6) and (7) will take the stairs – they will switch places.	Simplify – "do it." $7^2 = 49$ $6^5 = 7776$	(reduce fraction, if necessary.) **Answer**

Note also that any time you are dealing with constants (no variables), you can verify the result using your calculator:





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- **EXAMPLE:** Use the product rule to simplify. $8^0 \cdot 8^7 \cdot 8^9$ [R.2.39]
  (Type exponential notation with positive exponents.)

There are two common errors with this:
1. Multiplying bases error $8^0 \cdot 8^7 \cdot 8^9 = (8 \cdot 8 \cdot 8)^{0+7+9} = 512^{16}$ (INCORRECT!)
2. Multiplying exponents error $8^0 \cdot 8^7 \cdot 8^9 = 8^{(0 \cdot 7 \cdot 9)} = 8^0 = 1$ (INCORRECT!)

- **Product Rule:** $\boldsymbol{a^m \cdot a^n = a^{m+n}}$
  When multiplying powers w/same base, **ADD** exponents.

$$8^0 \cdot 8^7 \cdot 8^9 = 8^{0+7+9} = \mathbf{8^{16}}$$ **Answer**

NOTE: because the solution must be in *exponential notation*, using the calculator isn't helpful.

- **EXAMPLE:** Multiply and simplify. $3^5 \cdot 3^{-18}$ [R.2.35]
  (Simplify your answer. Type exponential notation with positive exponents.)

$3^5 \cdot 3^{-18}$	$= 3^{-13} = \dfrac{3^{-13}}{1}$	$= \dfrac{\mathbf{1}}{\mathbf{3^{13}}}$
• **Product Rule:** $\boldsymbol{a^m \cdot a^n = a^{m+n}}$ When multiplying powers w/same base, **ADD** exponents.	Can't have negative exponents! Write as fraction. "Take the stairs" (reciprocal).	**Answer** solution must be in *exponential notation* Leave answer with exponent – don't "do it."

- **EXAMPLE:** Use the product rule to simplify. $5x^{-4} \cdot 3x^8 \cdot x^5$ [R.2.37]
  (Type exponential notation with positive exponents.)

$5x^{-4} \cdot 3x^8 \cdot x^5$	$= (5 \cdot 3) \cdot (x^{-4} \cdot x^8 \cdot x^5)$	$= \mathbf{15x^9}$
Rearrange factors to multiply constants separately from variables.	Simplify. • **Product Rule:** $\boldsymbol{a^m \cdot a^n = a^{m+n}}$ When multiplying powers w/same base, **ADD** exponents.	**Answer** (No negative exponents.)

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- **EXAMPLE:** Use the quotient rule to simplify the expression. [R.2-23]

  Use positive exponents to write the answer. $\dfrac{4^{-4}}{4^{8}}$

$\dfrac{4^{-4}}{4^{8}}$	$= 4^{-4-(8)}$	$= 4^{-12} = \dfrac{4^{-12}}{1}$	$= \dfrac{\mathbf{1}}{\mathbf{4^{12}}}$
Same base (4). • **Quotient Rule:** $\dfrac{a^m}{a^n} = a^{m-n}$ When dividing powers w/same base, **SUBTRACT** exponents.	Simplify. $-4-(8) = -12$	Can't have negative exponents! Write as fraction. "Take the stairs" (reciprocal).	**Solution** Leave answer with exponent – don't "do it."

EASIER WAY? – RESET!

- **EXAMPLE:** Use the ~~quotient rule~~ to simplify the expression. [R.2-23]

  Use positive exponents to write the answer. $\dfrac{4^{-4}}{4^{8}}$

Rather than use the quotient rule, focus on the **negative exponent** in the numerator:

$\dfrac{4^{-4}}{4^{8}}$	$= \dfrac{1}{4^{8} \cdot 4^{4}}$	$= \dfrac{\mathbf{1}}{\mathbf{4^{12}}}$
Because of the negative exponent, "take the stairs" (reciprocal). Connect existing denominator 4^8 with new piece 4^4 using multiplication.	• **Product Rule:** $a^m \cdot a^n = a^{m+n}$ When multiplying powers w/same base, **ADD** exponents.	**Answer** Leave answer with exponent – don't "do it."

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- Another way to do Division: "Face-Off!"

- **EXAMPLE:** Simplify the expression. Write the answer with only positive exponents.

  All variables are nonzero. [R.2.69]

$$-\frac{24a^3b^{-2}}{18ab^{-5}}$$

To simplify this expression, work in "zones" – constants (coefficients), variable *a*, variable *b*. Then, merge (multiply) all the results together.

Constants (coefficients).	**Variable *a*.**	**Variable *b*.**	**Merge.**
$-\frac{24}{18} = -\frac{4}{3}$	$\frac{a^3}{a^1}$	1. $\frac{b^{-2}}{b^{-5}}$ 2. $= \frac{b^5}{b^2}$	Merge together the results from constants (coefficients) and variables.
Simplify the fraction of coefficients, if possible.	**"Face-off!"** 1. Do both have positive exponents? YES 2. Who has more, top or bottom? TOP 3. By how much? 2 4. Simplifies to a^2 on TOP. $\frac{a^3}{a^1} = \frac{a^2}{1} = a^2$	**"Face-off!"** 1. Do both have positive exponents? NO "take the stairs" (reciprocal) 2. Who has more, top or bottom? TOP 3. By how much? 3 4. Simplifies to b^3 on TOP. $\frac{b^5}{b^2} = \frac{b^3}{1} = b^3$	constants (coefficients): $= -\frac{4}{3}$ Variable *a*: $= a^2$ Variable *b*: $= b^3$ **Merged** $= -\frac{4}{3}a^2b^3$ or $= -\frac{4a^2b^3}{3}$ **Answer**

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- **EXAMPLE:** Use the rules of exponents to simplify the expression. [R.2.77]

$$\left(\frac{3x^6}{5y^{-3}}\right)^2$$

(Type exponential notation with positive exponents.)

$\left(\frac{3x^6}{5y^{-3}}\right)^2$	$= \left(\frac{3x^6}{5y^{-3}}\right)^2$	$= \left(\frac{3^1x^6y^3}{5^1}\right)^2$	$= \frac{(3^1)^2(x^6)^2(y^3)^2}{(5^1)^2}$	$= \frac{\mathbf{9x^{12}y^6}}{\mathbf{25}}$
Try to simplify INSIDE parentheses first. • Fraction $\frac{3}{5}$ is already simplified. • x^6 is fine … already has positive exponent	• y^{-3} needs fixing … because of the negative exponent, "take the stairs" (reciprocal).	• Coefficients always have exponent of understood 1. • **Quotient to Power Rule** $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ The exponent (2) applies to numerator and denominator.	The POSITIVE exponent, 2, applies to ALL factors. • **Power Rule:** $(a^m)^n = a^{m \cdot n}$ When raising a power to a power, **MULTIPLY** exponents. Now simplify: $(3^1)^2 = 3^2 = 9$ $(x^6)^2 = x^{12}$ $(y^3)^2 = y^6$ $(5^1)^2 = 5^2 = 25$	**Answer** Carefully merge together all the separate calculations. Be careful who goes in the numerator (TOP) and who goes in the denominator (BOTTOM).

- **EXAMPLE:** Simplify and write with positive exponents. [R.2.73]

$$(5x^{-3}y^3)^{-3}$$

$(5x^{-3}y^3)^{-3}$ $= (5^1x^{-3}y^3)^{-3}$	$= (5^1)^{-3}(x^{-3})^{-3}(y^3)^{-3}$ $= (5)^{-3}x^9y^{-9}$	$= \frac{(5)^{-3}x^9y^{-9}}{1}$	$= \frac{x^9}{5^3y^9}$
• Coefficients always have exponent of understood 1. • **Product to Power Rule:** $(ab)^n = a^nb^n$ The exponent (– 3) applies to each factor in the parentheses.	• **Power Rule:** $(a^m)^n = a^{m \cdot n}$ When raising a power to a power, **MULTIPLY** exponents. Now, write as fraction.	$(5)^{-3}$ and y^{-9} need fixing … because of the negative exponent, "take the stairs" (reciprocal).	Finally, just need to simplify $5^3 = 125$ in denominator. $= \frac{\mathbf{x^9}}{\mathbf{125y^9}}$ **Answer**

Sources Used:

1. MyLab Math for *College Algebra with Modeling and Visualization*, 6th Edition, Rockswold, Pearson Education Inc.
2. Powers of 10 chart, https://www.eduplace.com/math/mw/background/6/01/te_6_01_overview.html