# Notes Section R.4 – Factoring Polynomials

## Lesson Objectives

1. Greatest Common Factor
2. Factor Out a Greatest Common Factor
3. Factor a Quadratic Trinomial $x^2 + bx + c$
4. Factor a Difference of Squares $x^2 - m^2$

## A. Greatest Common Factor

**Greatest:** the biggest  **Common**: shared

**Factor:** numbers that "divide-into" (also called **divisors**)

Easiest way to see if a number is a factor is to **divide** it in your head or on a calculator.

For example, with $8 \div 2 = 4$, since there is no decimal part or remainder, then both 2 and 4 are **factors** of 8.

- **EXAMPLE:** Find the greatest common factor for the list of terms: $30x^5, 110x^7, 60x^9$ [*Akst 16.1.9]

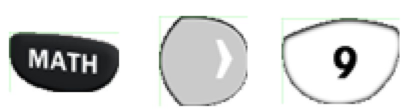
I recommend if necessary, use the calculator for the coefficients. Here's how you do that:

Greatest Common Factor (GCF) on calculator (TI-84 Plus or TI-83 Plus)
is actually called the **Greatest Common Divisor** (abbreviated **gcd**)

The **gcd(** command on the calculator (TI-84 Plus or TI-83 Plus) has limitations:

1. only numbers, not variables
2. only 2 numbers at a time
3. only POSITIVE numbers

To access Greatest Common Divisor (gcd) on calculator:



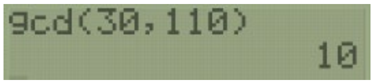

press **MATH**, →(NUM), **9**:gcd(

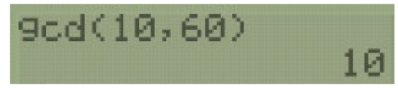
Here's the problem again for reference:

- **EXAMPLE:** Find the greatest common factor for the list of terms: $30x^5, 110x^7, 60x^9$

To find the GCF of the coefficients 30, 110, 60, we will use calculator (TI-83/84 Plus):

gcd(30,110) = 10 then take that answer and do it again gcd(10,60) = 10




The GCF of the coefficients 30, 110, and 60 is **10**.

Here's the problem again for reference:

- **EXAMPLE:** Find the greatest common factor for the list of terms: $30x^5, 110x^7, 60x^9$

For the variable part: $x^5$, $x^7$, $x^9$

You can only include variables in GCF if ALL the terms include that same variable.

Since all 3 terms have $x$, use the **SMALLEST** listed (only what's shared), which is $\boldsymbol{x^5}$.

The overall GCF for $30x^5, 110x^7, 60x^9$ is $\mathbf{10}\boldsymbol{x^5}$.

## B. Factor Out the Greatest Common Factor

Factoring out the GCF should always be tried FIRST, before trying other methods.

Factoring out the GCF is sort of like doing the **distributive property** in reverse.

- **EXAMPLE:** Factor out the greatest common factor.

  $12x^3 + 8x^2 - 16x$ [R.4.9]

  - **STEP 1. Find GCF of coefficients.** GCF coeff. = **4**
    - $\text{gcd}(12,8) = 4$
    - $\text{gcd}(4,16) = 4$
  - **STEP 2. Find GCF of variables.**
    - Do all terms have same variable? **YES**. All have an $x$
    - If YES, what is the **SMALLEST** of the ones listed? smallest of $x^3, x^2$, and $x$ is $\boldsymbol{x}$.

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(continued from the previous page … here is the problem again for reference)

- **EXAMPLE:** Factor out the greatest common factor.

  $12x^3 + 8x^2 - 16x$ [R.4.9]

- **STEP 3. Multiply the coefficient and variable GCF's together.**
  - Coefficient GCF = 4, variable GCF = *x* Product = $4 \cdot x = \mathbf{4x}$
  - The overall GCF is **4x**.

- **STEP 4. Skip a line** and **write the GCF** with a **"reverse-indent**."

  Open a set of **parentheses** the **SAME WIDTH** as the expression.

  $12x^3 + 8x^2 - 16x$

  $\mathbf{4x}\ ( \qquad ? \qquad )$

- **STEP 5.** To determine what goes INSIDE the parentheses, simply **divide** each term of the expression **by the GCF** and simplify. Write the simplified result in parentheses.

  $$\frac{12x^3}{\mathbf{4x}} + \frac{8x^2}{\mathbf{4x}} - \frac{16x}{\mathbf{4x}}$$

  $$\mathbf{4x\,(3x^2 + 2x - 4)}$$ **(ANSWER)**

  The entire expression is in "factored form."

| | |
|---|---|
| $12x^3 + 8x^2 - 16x$ | $4x\,(3x^2 + 2x - 4)$ |
| Original expression | Factored expression |
| 3 terms | 1 term |
| Addition & Subtraction | Multiplication |

Factoring is a process that converts addition & subtraction into multiplication.

This allows the opportunity to SIMPLIFY – most common are fractions and roots.

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## C. Factor a trinomial of the form $x^2 + bx + c$

- Review of Multiplying Binomials $(x+p)(x+q)$ – Use the FOIL method

- **EXAMPLE:** Multiply. $(x+5)(x-3)$ [R.3.55]

$$(x+5)(x-3)$$

F: Firsts $x \cdot x = x^2$
O: Outers $x \cdot (-3) = -3x$
I: Inners $5 \cdot x = 5x$
L: Lasts $5 \cdot (-3) = -15$

Write all the terms connected together:

$$x^2 - 3x + 5x - 15$$

Simplify – combine like terms:

**ANSWER:** $\mathbf{x^2 + 2x - 15}$

- Factor a trinomial of the form $x^2 + bx + c$

Factoring a trinomial in this form is sort of like doing FOIL in reverse.

- **EXAMPLE:** Find the binomial factors for the trinomial. [*Akst *16.2.7]

$$x^2 + 17x + 16$$

F O + I L

$$x^2 + 17x + 16 \quad = \quad (x \qquad)(x \qquad)$$

F (Firsts): Open up 2 sets of parentheses, with your variable in the **first** position.

Next, we need two integers whose **SUM** is 17 and whose **PRODUCT** is 16.

**O + I**
**(sum of Outers and Inners)**

**L**
**(Lasts)**

| To finish factoring, we need **2 numbers:** | | |
|---|---|---|
| **Product = 16** | **Sum = 17** | **Winner?** |
| 1(16) = 16 | 1 + 16 = 17 | **YES – use + 1 and + 16** |
| 2(8) = 16 | 2 + 8 = 10 | NO |
| 4(4) = 16 | 4 + 4 = 8 | NO |

**ANSWER:** $x^2 + 17x + 16 \quad = \quad \mathbf{(x+1)(x+16)}$ or $\mathbf{(x+16)(x+1)}$

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- **EXAMPLE:** Factor the expression. $r^2 - 18r + 81$ [R.4.81]

Open 2 sets of parentheses with variable in the **first** position:

$$r^2 - 18r + 81 \quad = \quad (r \qquad)(r \qquad)$$

Next, we need 2 integers whose **SUM** is – 18 and whose **PRODUCT** is 81

| To finish factoring, we need **2 numbers:** | | |
|---|---|---|
| **Product = 81** | **Sum = -18** | **Winner?** |
| -1(-81) = 81 | -1 + (-81) = -81 | NO |
| -3(-27) = 81 | -3 + (-27) = -30 | NO |
| -9(-9) = 81 | -9 + (-9) = -18 | **YES –<br>Use -9 and -9** |

**ANSWER:** $r^2 - 18r + 81 \quad = \quad (\boldsymbol{r-9})(\boldsymbol{r-9})$ or $(\boldsymbol{r-9})^{\mathbf{2}}$

- **EXAMPLE:** Factor the expression completely. [R.4.37]

$$v^2 + v - 72$$

Open 2 sets of parentheses with variable in the **first** position:

$$v^2 + v - 72 \quad = \quad (v \qquad)(v \qquad)$$

Next, we need 2 integers whose **SUM** is +1 and whose **PRODUCT** is -72

| To finish factoring, we need **2 numbers:** | | |
|---|---|---|
| **Product = -72<br>(opposite signs)** | **Sum = +1<br>(opposite signs means SUBTRACT)** | **Winner?** |
| $\pm 1 \cdot \mp 72 = -72$ | $\pm 1 + (\mp 72) = \mp 71$ | NO |
| $\pm 2 \cdot \mp 36 = -72$ | $\pm 2 + (\mp 36) = \mp 34$ | NO |
| $\pm 3 \cdot \mp 24 = -72$ | $\pm 3 + (\mp 24) = \mp 21$ | NO |
| $\pm 4 \cdot \mp 18 = -72$ | $\pm 4 + (\mp 18) = \mp 14$ | NO |
| $\pm 6 \cdot \mp 12 = -72$ | $\pm 6 + (\mp 12) = \mp 6$ | NO |
| $\pm 8 \cdot \mp 9 = -72$ | $\pm 8 + (\mp 9) = \mp 1$<br>$8 + (-9) = -1$ or $-8 + 9 = +1$ | **YES –<br>Use -8 and +9** |

**ANSWER:** $v^2 + v - 72 \quad = \quad (\boldsymbol{v-8})(\boldsymbol{v+9})$ **or** $(\boldsymbol{v+9})(\boldsymbol{v-8})$

# Notes Section R.4 – Factoring Polynomials

- **EXAMPLE:** Factor completely, if possible. [*Akst 16.2-15)

$$x^2 - x - 48$$

Open 2 sets of parentheses with variable in the **first** position:

$$x^2 - x - 48 \quad = \quad (x \qquad)(x \qquad)$$

Next, we need 2 integers whose **SUM** is -1 and whose **PRODUCT** is -48

| To finish factoring, we need **2 numbers:** | | |
|---|---|---|
| **Product = -48**<br>**(opposite signs)** | **Sum = -1**<br>**(opposite signs means SUBTRACT)** | **Winner?** |
| $\pm 1 \cdot \mp 48 = -48$ | $\pm 1 + (\mp 48) = \mp 47$ | NO |
| $\pm 2 \cdot \mp 24 = -48$ | $\pm 2 + (\mp 24) = \mp 22$ | NO |
| $\pm 3 \cdot \mp 16 = -48$ | $\pm 3 + (\mp 16) = \mp 13$ | NO |
| $\pm 4 \cdot \mp 12 = -48$ | $\pm 4 + (\mp 12) = \mp 8$ | NO |
| $\pm 6 \cdot \mp 8 = -48$ | $\pm 6 + (\mp 8) = \mp 2$ | NO |
| NONE of the pairs work – therefore, $x^2 - x - 48$ is **NOT FACTORABLE** or **PRIME**. | | |

## D. Factor a Difference of Squares $x^2 - m^2$

- **EXAMPLE:** Factor. $s^2 - 81$ [R.4.59]

It's missing the middle term. Rewrite it with zero: $s^2 + 0s - 81$

Open 2 sets of parentheses with variable in the **first** position:

$$s^2 + 0s - 81 \quad = \quad (s \qquad)(s \qquad)$$

| To finish factoring, we need **2 numbers:** | | |
|---|---|---|
| **Product = -81**<br>**(opposite signs)** | **Sum = 0**<br>**(opposite signs means SUBTRACT)** | **Winner?** |
| $\pm 1 \cdot \mp 81 = -81$ | $\pm 1 + (\mp 81) = \mp 80$ | NO |
| $\pm 3 \cdot \mp 27 = -81$ | $\pm 3 + (\mp 27) = \mp 24$ | NO |
| $\pm 9 \cdot \mp 9 = -81$ | $\pm 9 + (\mp 9) = 0$<br>$+9 - 9 = 0$ or $-9 + 9 = 0$ (same thing) | **YES –**<br>**Use – 9 and +9** |

**ANSWER:** $s^2 - 81 \quad = \quad (\boldsymbol{s - 9})(\boldsymbol{s + 9})$ or $(\boldsymbol{s + 9})(\boldsymbol{s - 9})$

**Square Root is *s***

**Square Root is 9**

**Opposite Signs**

**Opposite Signs**

# Notes Section R.4 – Factoring Polynomials

- FORMULA for the Difference of Squares: $\boldsymbol{x^2 - m^2 = (x - m)(x + m)}$

Works as long as the two terms are **PERFECT SQUARES** and they are **SUBTRACTED**.

- **EXAMPLE:** Factor the binomial completely. [R.4.61]

$$64r^2 - 49$$

Perfect Square $\boldsymbol{8 \cdot 8}$

Perfect Square $\boldsymbol{r \cdot r}$

SUBTRACT

Perfect Square $\boldsymbol{7 \cdot 7}$

**ANSWER:** $64r^2 - 49$ = $(\boldsymbol{8r - 7})(\boldsymbol{8r + 7})$ or $(\boldsymbol{8r + 7})(\boldsymbol{8r - 7})$

- **EXAMPLE:** Factor the expression completely, if possible. [R.4-27]

$$100s^2 + 81t^4$$

Perfect Square $\boldsymbol{10 \cdot 10}$

Perfect Square $\boldsymbol{s \cdot s}$

ADD

(To factor, it MUST be SUBTRACT !!)

Perfect Square $\boldsymbol{9t^2 \cdot 9t^2}$

The **SUM** (addition) of perfect squares is always **PRIME** – it **DOES NOT FACTOR !!**

Sources Used:


1. MyLab Math for *Developmental Mathematic through Applications*, 1st Edition, Akst, Pearson Education Inc.
2. MyLab Math for *College Algebra with Modeling and Visualization*, 6th Edition, Rockswold, Pearson Education Inc.
3. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website https://archive.codeplex.com/?p=wabbit