Notes Section 2.1 – Equations of Lines

Lesson Objectives

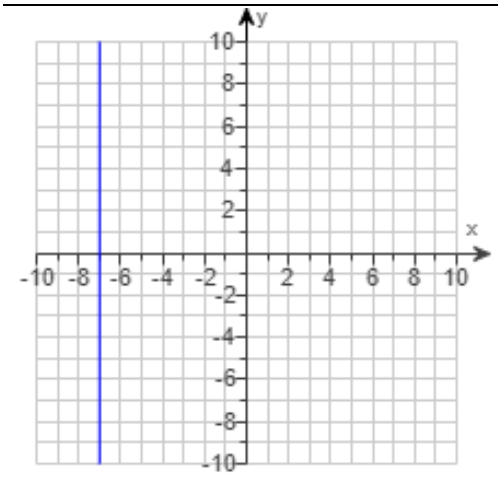
1. Vertical and Horizontal Lines
2. Find equation of a line given its slope and a point (not *y*-intercept)
3. Find equation of a line given two points
4. Find equation of a line through a given point that is parallel or perpendicular to a given line

A. Vertical and Horizontal Lines

1. Vertical Lines

- Have **UNDEFINED** slope
- Pass through the ***x*-AXIS**
- Are written in the form: $x = a$ where *a* is some constant (number)
- *a* is the *x*-intercept, located at $(a, 0)$.

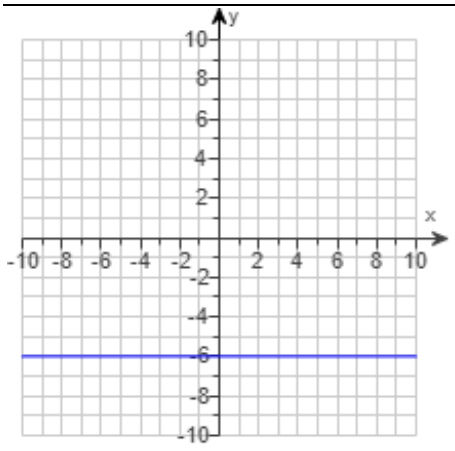
- **EXAMPLE:** Find the slope of the line in the figure. If the slope is undefined, so state. Then write an equation of the given line. [*Woodbury 3.3.37]
  This is a VERTICAL line, so the slope of the line is **UNDEFINED**.
  Since it passes through the ***x*-AXIS** at **– 7**, then the equation is $x = -7$



2. Horizontal Lines

- Have **ZERO** slope
- Pass through the ***y*-AXIS**
- Are written in the form: $y = b$, where *b* is some constant (number)
- *b* is the *y*-intercept, located at $(0, b)$

- **EXAMPLE:** Determine the equation of the given line, as well as the slope of the line. If the slope is undefined, state this. [*Woodbury 3.3.39]
  This line passes through the ***y*-AXIS** at **– 6**, so the equation of the line is $y = -6$.
  Since this is a **HORIZONTAL** line, the slope is **ZERO**.



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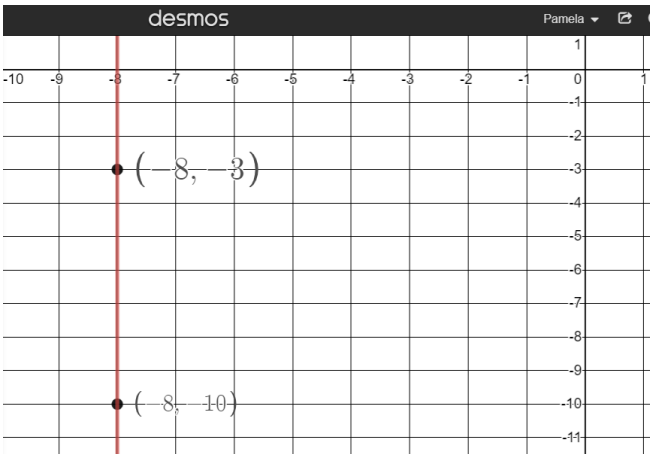
3. Other examples with horizontal and vertical lines

- **EXAMPLE:** Write an equation for the line passing through the given pair of points.
  $(-8, -3)$ and $(-8, -10)$. [2.1-14]

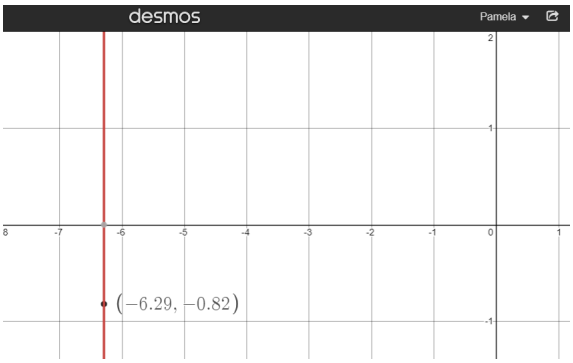
  Always examine your given points before you do any math with them.
  What do you notice? The ***x*-coordinates** are **the same**.
  The equation of the line, therefore, is simply $x = -8$.
  Recommend you make a quick SKETCH of this situation to help you understand it better.



Notice that the **red** line through those points
passes through the *x*-axis at $x = -8$.

- **EXAMPLE:** Find the equation of the line satisfying the following conditions.
  If possible, write the equation in slope-intercept form.
  Vertical, passing through $(-6.29, -0.82)$ [2.1-24]

  Vertical lines are of the form $x = a$
  Simply use the *x*-coordinate of the through-point, so the equation is $x = -6.29$.
  Recommend you make a quick SKETCH of this situation to help you understand it better.



Notice that the **red** vertical line passes through
the *x*-axis at the SAME *x*-coordinate: $x = -6.29$.

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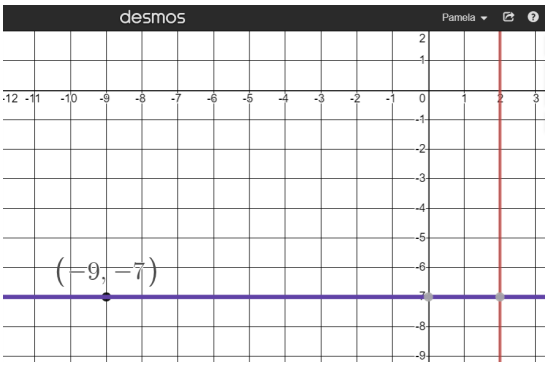
- **EXAMPLE:** Determine the equation of the line described. Put the answer in slope-intercept form, if possible. Through $(-9, -7)$ perpendicular to $x = 2$. [2.1-35]

A line perpendicular to $x = a$ (vertical line) is a line of the form $y = b$ (horizontal line)
So, use the ***y*-coordinate** of the through-point. The equation is: $y = -7$
Recommend you make a quick SKETCH of this situation to help you understand it better.



Notice that the line $x = 2$ is shown in **red**.
The line perpendicular to it that passes through $(-9, -7)$
is the **purple** line passing through the *y*-axis at $y = -7$.

B. Find equation of a line given its slope and a point (not y-intercept)

- **EXAMPLE:** Find an equation of the line that has the given slope and contains the given point. If possible, write the equation in slope-intercept form. Check that the ordered pair that represents the given point satisfies the equation. $m = \frac{7}{4}, (3, -4)$ [*Lehmann 5.4.7]

[SOLUTION] Use the **SLOPE-INTERCEPT** formula: $y = mx + b$
In words, this also means: ***y*-coordinate = slope (*x*-coordinate) + *y*-intercept**
Substitute all the given values: $y = mx + b$ $\quad -4 = \frac{7}{4}(3) + b$

Simplify: $-4 = \frac{21}{4} + b$

Solve for *b*: $-\frac{21}{4} \quad -\frac{21}{4}$

Update: $-\frac{37}{4} = b$

So, the equation is: $y = \frac{7}{4}x - \frac{37}{4}$

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C. Find Equation of Line Given Two Points

- **EXAMPLE:** Write an equation in slope-intercept form for the line described.
  x-intercept $(-4, 0)$, *y*-intercept $(0,5)$ [2.1.9]

  First, you need the **SLOPE**, so use the slope formula.

  $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 0}{0 - (-4)} = \mathbf{\frac{5}{4}}$

  Next, you need the ***y*-INTERCEPT**, which is given $(0,5)$, so $\boldsymbol{b = 5}$

  So the equation of the line is: $\boldsymbol{y = \frac{5}{4}x + 5}$

- **EXAMPLE:** Find an equation of the line containing the given pair of points.
  $(-1, -4)$ and $(-7, -8)$ [2.1.23]

  First, you need the **SLOPE**, so use the slope formula.

  $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-8-(-4)}{-7-(-1)} = \frac{-4}{-6} = \mathbf{\frac{2}{3}}$ Equation so far: $y = \frac{2}{3}x + b$

  Next, you need the ***y*-intercept**, which is not one of the two given points.
  Use the **SLOPE-INTERCEPT** formula: $\boldsymbol{y = mx + b}$, and choose either point.

Either way works - you'll get the same value for *b* choosing either point.	
Using $(-1, -4)$ and $m = \frac{2}{3}$	**Using $(-7, -8)$ and $m = \frac{2}{3}$**
$y = mx + b$	$y = mx + b$
$-4 = \left(\frac{2}{3}\right)(-1) + b$	$-8 = \left(\frac{2}{3}\right)(-7) + b$
$-4 = -\frac{2}{3} + b$	$-8 = -\frac{14}{3} + b$
$+\frac{2}{3} \quad +\frac{2}{3}$	$+\frac{14}{3} \quad +\frac{14}{3}$
$\mathbf{-\frac{10}{3} = b}$	$\mathbf{-\frac{10}{3} = b}$

Finally, write your equation using your *m* and *b*: $\boldsymbol{y = \frac{2}{3}x - \frac{10}{3}}$

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D. Parallel or Perpendicular Lines

1. Parallel lines have SAME slope (*m*)

- **EXAMPLE:** Find the slope-intercept form of the line parallel to $-2x + 3y = 6$ and passing through the point $(-3, -4)$. [2.1.45]

Given line: $-2x + 3y = 6$. This is in **STANDARD** form: $Ax + By = C$.

We need it in **SLOPE-INTERCEPT** form: $y = mx + b$, so we can know its SLOPE (*m*).

We need to convert the given equation. This is a VERY important skill you need to know how to do – it will occur in later sections as well!

Given line: $-2x + 3y = 6$ The goal is to get *y* by itself: *y* = (all the other stuff)

Add 2*x*: $+2x \quad +2x$

Simplify: $3y = 2x + 6$

Divide by 3: $\frac{3y}{3} = \frac{2}{3}x + \frac{6}{3}$

Simplified: $y = \frac{2}{3}x + 2$ This is *still* the *original* line – not the answer!

Slope of *GIVEN* line: $m = \frac{2}{3}$

Need PARALLEL (same slope), so slope of *NEW* line: $m = \frac{2}{3}$

Equation so far: $y = \frac{2}{3}x + b$ We still need the *y*-intercept, *b*.

Use through-point $(-3, -4)$ with slope $m = \frac{2}{3}$ and plug into $y = mx + b$:

$y = mx + b \quad (x, y)$

$-4 = \frac{2}{3}(-3) + b$

$-4 = -2 + b$

$+2 \quad +2$

$-2 = b$

Finally, write your equation using your *m* and *b*:

The equation of the new PARALLEL line is: $y = \frac{2}{3}x - 2$

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2. Perpendicular Lines have OPPOSITE RECIPROCAL slopes

"switch" sign & "flip" fraction

- **EXAMPLE:** Write the equation of the line containing the given point and perpendicular to the given line. Express your answer in the form $y = mx + b$. [2.1.43]

$(4, 6)$; $4x + y = 5$

Given line: $4x + y = 5$ is in standard form.
Convert to slope-intercept form to know its slope.

Given line: $4x + y = 5$ The goal is to get y by itself: y = (all the other stuff)
Subtract 4*x*: $-4x \quad -4x$
Simplified: $\boldsymbol{y = -4x + 5}$ This is *still* the *original* line – not the answer!

Slope of *GIVEN* line: $\boldsymbol{m = -4}$ Not done – don't use this slope in the *NEW* equation!

Need: PERPENDICULAR (OPPOSITE RECIPROCAL)
Switch sign and **Flip** fraction.

Convert given slope $= -4$:

Opposite (**Switch**): $m = 4$ now do reciprocal (**Flip**): $m = \frac{1}{4}$

Slope of the *NEW* line is $m = \frac{1}{4}$. Equation so far: $y = \frac{1}{4}x + b$

Use through-point $(4, 6)$ with slope $\boldsymbol{m = \frac{1}{4}}$ and plug into $y = mx + b$

$y = mx + b \quad (x, y)$

$6 = \frac{1}{4}(4) + b$

$6 = 1 + b$

$-1 \quad -1$

$5 = b$

Finally, write your equation using your *m* and *b*:

The equation of the PERPENDICULAR line is: $\boldsymbol{y = \frac{1}{4}x + 5}$.

Sources used:

1. Desmos website: www.desmos.com
2. Pearson MyLab Math *College Algebra with Modeling and Visualization, 6th Edition*, Rockswold
3. Pearson MyLab Math *Elementary and Intermediate Algebra: Functions and Authentic Apps, 2nd Edition*, Lehmann
4. Pearson MyLab Math *Elementary and Intermediate Algebra, 3rd Edition*, Woodbury.
5. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website https://archive.codeplex.com/?p=wabbit