Notes Section 2.5 – Absolute Value Equations and Inequalities

Lesson Objectives

1. Basic Terms Involving Absolute Value
2. Solve an Absolute Value Equation
3. Solve an Absolute Value Inequality

A. Basic Terms Involving Absolute Value

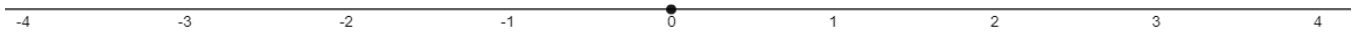
The **ABSOLUTE VALUE** of a number **IS** its _____ from _____ on a number line.

ABSOLUTE VALUE IS DISTANCE

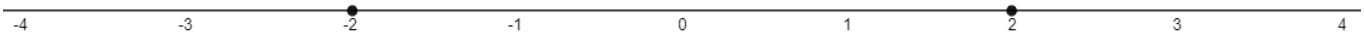
What is always true about a DISTANCE? It is always _____! It can NEVER be negative.

The same is true about the absolute value of a number. It is always positive.

Suppose you are standing at ZERO on a basic horizontal number line:



Where do you go so that you are a DISTANCE of 2 units away from zero? You go to _____.



That is to say, you can go either 2 units to the _____ (– 2) or 2 units to the _____ (+2).

This situation can be modeled using an absolute value equation: $|x| = 2$

Where do you go… …the distance is 2 from zero

So with the equation: $|x| = 2$, you are to find numbers that are ____ units from _____.

The solutions can be modeled using two equations: $x = -2$ or $x = 2$

Go 2 units to the LEFT of zero or go 2 units to the RIGHT

This type of solution, $x = -2$ or $x = 2$, can also be written in a simplified version: $x = \pm 2$.

That format, $x = \pm 2$, is read as "*x* equals plus or minus 2."

So, remember the following: _____ _____ ___ _____

- **EXAMPLE:** Solve the absolute value equation: $|-4x| = -3$ [2.5.27]

Remember that absolute value is a distance. In this equation, the distance number is _____.
But distance can't be _____, so this equation has _____ **SOLUTION.**

In general:	$\|\text{"stuff"}\| = \text{negative}$	means	**"NO SOLUTION"**

Notes Section 2.5 – Absolute Value Equations and Inequalities

B. Solve an Absolute Value Equation (by hand – symbolically)

- **Step 1 – _____** the absolute value part, if needed.

- **Step 2 – _____** the distance number (opposite the A.V. stuff).
  - If distance number is **NEGATIVE** – STOP! Equation has **NO SOLUTION**.
  - If distance number is _____ – ignore A.V. bars; make ____ equation (**ONE** solution).
  - If distance number is **POSITIVE** – keep going; there will be _____ equations (**TWO** solutions).

- **Step 3 – "_____ _____"** and make 2 separate equations
  .
- **Step 4 – _____** each equation.

- **Step 5 – _____** your solution(s).

- **EXAMPLE:** Solve the equation for *x*. $|3 + 6x| = 0$ [2.5.39]

  Step 1 – ISOLATE. There is _____ attached to the outside of the A.V. part (nothing multiplied; nothing added or subtracted)

  Step 2 – INSPECT. Distance number, ___, means ignore A.V. bars & make ___ equation

  Step 3 – BRANCH. **(**Not needed, since there is only one equation.)

  Step 4 – SOLVE. $|3 + 6x| = 0$ converts to

  Step 5 – WRITE. The solution is: $x =$ _____ .

- **EXAMPLE:** Solve for *b*. $|b + 9| + 8 = 10$ [2.5-8]

  Step 1 – ISOLATE. Given: $|b + 9| + 8 = 10$ (Subtract 8)

  (do NOT subtract 9…yet!)

  Step 2 – INSPECT. The distance number, ___, is POSITIVE – make 2 equations

Step 3 – BRANCH. Make 2 separate equations: $|b + 9| = 2$

Start BOTH equations with _____ :

Step 4 – SOLVE. (Subtract 9)

Step 5 – WRITE. $\boldsymbol{b} =$ _____ **or** $\boldsymbol{b} =$ _____

- **EXAMPLE:** Find the solution set for the equation. $4|3x| + 5 = 37$ [*Blitzer 4.3.19]

Step 1 – ISOLATE. Given: $4|3x| + 5 = 37$ (Subtract 5)

(Divide by 4)

(do NOT divide by 3….yet!)

Step 2 – INSPECT. The distance number, ___, is POSITIVE – make 2 equations

Step 3 – BRANCH. Make 2 separate equations: $|3x| = 8$

Start BOTH equations with _____

Step 4 – SOLVE. Solve each equation. (divide by 3)

Step 5 – WRITE. Write your solutions. $\boldsymbol{x} =$ ____ **or** $\boldsymbol{x} =$ ___ also $\boldsymbol{x} =$ ____

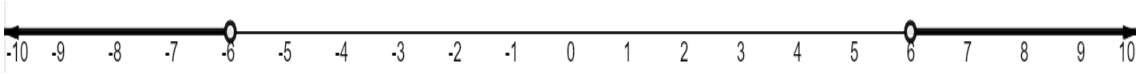
Notes Section 2.5 – Absolute Value Equations and Inequalities

C. Solve an Absolute Value Inequality (by hand – symbolically)

1. Solving a Greater-Than type Absolute Value Inequality (_____ is "____")

- **EXAMPLE:** Solve the inequality. $|1-2x|>6$ [2.5.83]

Focus for now on the information outside the inequality: _____.
Where would you need to be so that you are **greater than 6** units away from zero?



Left piece is _____ or Right piece is _____

In EACH piece graphed above, the distance is **greater than 6** units away from zero on the number line.
This is what we'll use to set up the solution process for this inequality. **(More is "or")**

$$|1-2x|>6$$

(BOTH inequalities will start with _____)

(left piece) _____ or _____ (right piece)

(Solve each inequality)

(subtract 1)

or

(Divide by – 2)

(_____ **!!** – remember, you don't always reverse)

or

(**VERY IMPORTANT!** Swap places to mimic number line; smaller on left, larger on right)

or

WRITE the solution. Set Builder Notation: $\{x|$ _____$\}$

Interval Notation: _____

So, in general:	If given $\|A\| > B$	setup is:	$A < -B$	or $A > B$
Solution is: $(-\infty, \text{smaller}) \cup (\text{larger}, \infty)$	parentheses are used on 2 solutions			
If given $\|A\| \geq B$	setup is:	$A \leq -B$	or	$A \leq B$
Solution is: $(-\infty, \text{smaller}] \cup [\text{larger}, \infty)$	brackets are used on 2 solutions			

Notes Section 2.5 – Absolute Value Equations and Inequalities

2. Solving a Less-Than type Absolute Value Inequality (_____ is "_____")

- **EXAMPLE:** Solve the inequality. $|3 - 5x| \leq 8$ [2.5.75]

Focus for now on the information outside the inequality: _____.
Where would you need to be so that you are **less than or equal to 8** units away from zero?

This segment above shows staying a distance of **less than or equal to 8** units from zero.

So, notice that this graph shows all points with a distance _____ **– 8** and **+8**.

That is written as a compound inequality: $-8 \leq \text{distance} \leq 8$

This is what we'll use to set up the solution process for this inequality.

Given: $|3 - 5x| \leq 8$ (**Less is "Nest"**)

Setup to Solve: "Nest" the 3 – 5x in the middle

Subtract 3:

Divide by – 5 and **REVERSE!**:

(Remember, you won't always reverse)

You need to "pivot" (or "dab") this inequality, to match the number line:
smaller ≤ *x* ≤ larger

WRITE the solution. Set Builder Notation: $\{x|$_____$\}$

Interval Notation: _____

So, in general:	If given $\|A\| < B$	setup is:	$-B < A < +B$
Solution is: $(\text{ smaller}, \text{larger }) $		parentheses are used	
If given $\|A\| \leq B$		setup is:	$-B \leq A \leq +B$
Solution is: $[\text{ smaller}, \text{larger }]$		brackets are used	

Notes Section 2.5 – Absolute Value Equations and Inequalities

3. Solve Applications with Absolute Value Inequalities

- **EXAMPLE:** The inequality describes the range of monthly average temperatures *T* in degrees Fahrenheit at a certain location. Find an equivalent expression and monthly average temperatures.

$$|T - 49| \leq 29 \qquad [2.5.117] \qquad \text{(Less is "nest")}$$

Setup to solve:
Add 49:

The inequality is equivalent to:_____ interval notation: _____

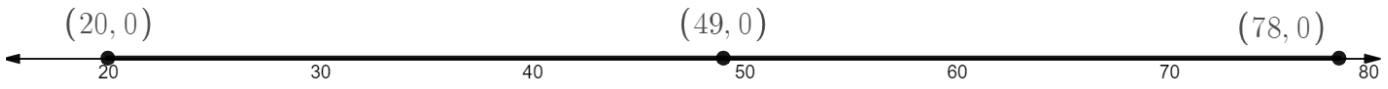
Interpret this solution: If the high and low monthly average temperatures satisfy the inequality, then the monthly averages are always within _____ degrees of ____° F.

How do we get this? The "within" 29 degrees part represents a _____, so use the **DISTANCE** number (away from the inequality) for **"_____"**.

The "of" 49° F part represents a _____ (center point), so use the value _____ the absolute value.

$$|T - 49| \leq 29$$

"____" (center point, location) "_____" (distance)



Sources Used:

1. Solving Absolute Value Inequalities, https://www.youtube.com/watch?v=BhFj7Rkyc5E&t=180s, Gdawg Enterprises, © 2009, using phrases "More is 'OR', Less is 'Nest'".
2. Number Line Inequalities (modified) from Desmos, https://www.desmos.com/calculator/evxn1e1njv, © 2019, Desmos, Inc.
3. Wrong Way sign, Driver's Prep website, https://driversprep.com/wrong-way/
4. Dab icon, ICONFINDER website, https://www.iconfinder.com/icons/2657453/celebrate_celebration_dab_dabbing_pose_posture_icon
5. MyLab Math for *Algebra for College Students*, 8th Edition, Blitzer, Pearson Education Inc.
6. MyLab Math for *College Algebra with Modeling and Visualization*, 6th Edition, Rockswold, Pearson Education Inc.