Notes Section 3.2 – Quadratic Equations

Lesson Objectives

1. Zero Product Property
2. Solve by Factoring GCF
3. Solve by Factoring $x^2 + bx + c = 0$ into binomial factors
4. Solve by Square Root Method
5. Solve by the Quadratic Formula
6. Using the Discriminant
7. Solve by Graphing (find *x*-intercepts) on Calculator

A. Zero Product Property

If *a* · *b* = 0, then either *a* = 0 or *b* = 0.

- **EXAMPLE:** Solve. $(13s + 8)(5s - 15) = 0$ [*Beecher 3.2.1]

By the Zero Product Property,
Set each factor (parentheses) **equal to zero**: $13s + 8 = 0$ or $5s - 15 = 0$
Solve each equation.

$$13s + 8 = 0 \qquad \text{or} \qquad 5s - 15 = 0$$
$$-8 \;\; -8 \qquad\qquad +15 \;\; +15$$
$$13s = -8 \qquad\qquad 5s = 15$$
$$\frac{13s}{13} = \frac{-8}{13} \qquad\qquad \frac{5s}{5} = \frac{15}{5}$$

(both are solutions) $\boldsymbol{s = -\frac{8}{13}}$ or $\boldsymbol{s = 3}$

B. Solve by **Factoring GCF**

- **EXAMPLE:** Solve the quadratic equation. $9x^2 = 54x$ [3.2-1]

NEVER divide by a variable….ever! Don't do this: $\cancel{\frac{9x^2}{9x} = \frac{54x}{9x}}$ no, No, NO!!! Bad! Stop it! Very illegal!

Set your equation **EQUAL** to **ZERO**! $9x^2 = 54x$ (subtract 54*x*)

Then, you can **FACTOR out the GCF:** $9x^2 - 54x = 0$ (GCF is 9*x*)

$9x(x - 6) = 0$ (GCF outside on the LEFT)

Now, use the **Zero Product Property**: $9x = 0$ or $x - 6 = 0$
Solve each equation: $\frac{9x}{9} = \frac{0}{9}$ $\qquad +6 \;\; +6$

$\boldsymbol{x = 0}$ **or** $\boldsymbol{x = 6}$ (both are solutions)

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C. Solve by Factoring $x^2 + bx + c = 0$ into Binomial Factors

- **EXAMPLE:** Solve the equation by factoring. $x^2 = 3x + 40$ [*Blitzer 1.5.3-Setup & Solve]

Set your equation **EQUAL** to **ZERO**! (subtract 3x and subtract 40)

$x^2 = 3x + 40 \quad \rightarrow \quad x^2 - 3x - 40 = 0$

Try to factor: $x^2 - 3x - 40$

Open 2 sets of parentheses with variable in the first position:

$x^2 - 3x - 40 \quad = \quad (x \quad\quad)(x \quad\quad)$

Next, we need 2 integers whose SUM is -3 and whose PRODUCT is -40

To finish factoring, we need 2 numbers:		
Product = -40 (opposite signs)	**Sum = -3** (opposite signs means SUBTRACT)	**Winner?**
$\pm 1 \cdot \mp 40 = -40$	$\pm 1 + (\mp 40) = \mp 39$	NO
$\pm 2 \cdot \mp 20 = -40$	$\pm 2 + (\mp 20) = \mp 18$	NO
$\pm 4 \cdot \mp 10 = -40$	$\pm 4 + (\mp 10) = \mp 6$	NO
$\pm 5 \cdot \mp 8 = -40$	$\pm 5 + (\mp 8) = \mp 3$	YES. Use + 5 – 8

$x^2 - 3x - 40$ factors into: $(x + 5)(x - 8)$

Rewrite the equation in factored form

$$x^2 - 3x - 40 = 0$$
$$(x + 5)(x - 8) = 0$$

By the Zero Product Property, set each factor (parentheses) equal to zero:

$x + 5 = 0$ or $x - 8 = 0$

So, $x = -5$ or $x = 8$ (both are solutions)

The solution set is $\{\mathbf{-5}, \mathbf{8}\}$.

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D. Solve by the **Square Root** Method

The **Square Root Method** is used when only the **SQUARED** term and the **CONSTANT** term are present. That is, the **Square Root Method** is used when your equation is of the form:

$$\boldsymbol{ax^2 - c = 0}.$$

There is no *x* term – only an x^2 term and a constant term.

- **EXAMPLE:** Solve the quadratic equation. Check the answer. $4x^2 - 256 = 0$ [3.2.5]

Because no "*x*" term, **ISOLATE** the **SQUARED** part: $4x^2 = 256$ (add 256)

Continue to **ISOLATE** the **SQUARED** part: $\dfrac{4x^2}{4} = \dfrac{256}{4}$ (divide by 4)

$x^2 = 64$ (take square root)

(What number could you square to get 64?) $\sqrt{x^2} = \sqrt{64}$

REALLY IMPORTANT! Don't forget the $\pm$ symbol! $\boldsymbol{x = \pm 8}$ or $\{\boldsymbol{-8, 8}\}$

- **EXAMPLE:** Solve the following equation. $(x + 21)^2 = 11$ [3.2.29]

First, **ISOLATE** the **SQUARED** part. (DONE!) $(x + 21)^2 = 11$

Take the **SQUARE ROOT** both sides: $\sqrt{(x + 21)^2} = \sqrt{11}$

Simplify square root, if needed. Don't forget the "plus or minus" $x + 21 = \pm\sqrt{11}$

Solve for *x* by subtracting 21: $-21 \quad -21$

$$\boldsymbol{x = -21 \pm \sqrt{11}}$$

(proper format is rational part first, followed by the irrational part)

Can also be written as: $\left\{-21 - \sqrt{11}, -21 + \sqrt{11}\right\}$

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E. Solve by the Quadratic Formula

The **Quadratic Formula:** Given $ax^2 + bx + c = 0$ (with $a \neq 0$)

the solutions are: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ or $x = \dfrac{-(b) \pm \sqrt{(b)^2 - 4 \cdot a \cdot c}}{(2 \cdot a)}$

Make sure you do the following:

1. Set your equation **EQUAL** to **ZERO**, if needed.
2. Correctly identify the values for *a*, *b*, and *c*.
3. Watch out for negatives! (Use parentheses)

- **EXAMPLE:** Solve the quadratic equation. $x^2 + 6x + 9 = 14$ [3.2-4]

Set your equation **EQUAL** to **ZERO**! $x^2 + 6x - 5 = 0$ (subtracted 14)

You can try to factor first. If it doesn't factor, use the **Quadratic Formula**.

NOTE: You can ALWAYS use Q.F. for ANY quadratic equation, even if other methods do (or don't) work.

Use **Quadratic Formula**: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ with $a = 1, b = 6, c = -5$

Plug in your values: $x = \dfrac{-(6) \pm \sqrt{(6)^2 - 4 \cdot 1 \cdot -5)}}{(2 * 1)}$

Simplify inside the square root (no decimals!) $x = \dfrac{-6 \pm \sqrt{36 + 20}}{2} = \dfrac{-6 \pm \sqrt{56}}{2}$

Simplify the square root itself: $\sqrt{56} = \sqrt{2 \cdot 28} = \sqrt{2 \cdot 4 \cdot 7} = \sqrt{2 \cdot 2 \cdot 2 \cdot 7} = 2\sqrt{2 \cdot 7} = 2\sqrt{14}$

(pairs and spares, Section R.7)

Now update the solution above: $x = \dfrac{-6 \pm \sqrt{56}}{2} = \dfrac{-6 \pm 2\sqrt{14}}{2}$

The common denominator is 2. **PULL them APART!** $x = \dfrac{-6 \pm 2\sqrt{14}}{2} = \dfrac{-6}{2} \pm \dfrac{2\sqrt{14}}{2}$

Reduce each fraction (ignore square root part) $x = \dfrac{-6}{2} \pm \dfrac{2\sqrt{14}}{2} = -3 \pm \sqrt{14}$

The solutions are: $\boldsymbol{x = -3 \pm \sqrt{14}}$ or $\boldsymbol{\{-3 - \sqrt{14}, -3 + \sqrt{14}\}}$

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Mrs. E! Is there…maybe…an EASIER way to do that last example? Let's revisit it:

- **EXAMPLE:** Solve the quadratic equation. $x^2 + 6x + 9 = 14$ [3.2-4]

  There is an interesting opportunity here! Look at just the LEFT side of the equation – do NOT set it equal to zero.

$$x^2 + 6x + 9$$

Let's **factor** that. $x^2 + 6x + 9 = (x + 3)(x + 3) = (x + 3)^2$

Revisit the equation: $x^2 + 6x + 9 = 14$

Put factored form on the LEFT. $(x + 3)^2 = 14$ Use **square root** property.

$$\sqrt{(x + 3)^2} = \sqrt{14}$$

Simplify. Don't forget the $\pm$ symbol. $x + 3 = \pm\sqrt{14}$ Subtract 3.

The solutions are: $\boldsymbol{x = -3 \pm \sqrt{14}}$ or $\boldsymbol{\{-3 - \sqrt{14}, -3 + \sqrt{14}\}}$

F. Using the Discriminant

Recall the **Quadratic Formula:** Given $ax^2 + bx + c = 0$ (with $a \neq 0$)

the solutions are: $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The expression inside the square root (the *radicand*), the $\boldsymbol{b^2 - 4ac}$, is called the **discriminant**, which can determine the **number of solutions** to the quadratic equation.

QUADRATIC EQUATIONS AND THE DISCRIMINANT

To determine the number of real solutions to $ax^2 + bx + c = 0$ with $a \neq 0$, evaluate the discriminant $b^2 - 4ac$.

1. If $b^2 - 4ac > 0$, there are two real solutions.
2. If $b^2 - 4ac = 0$, there is one real solution.
3. If $b^2 - 4ac < 0$, there are no real solutions.

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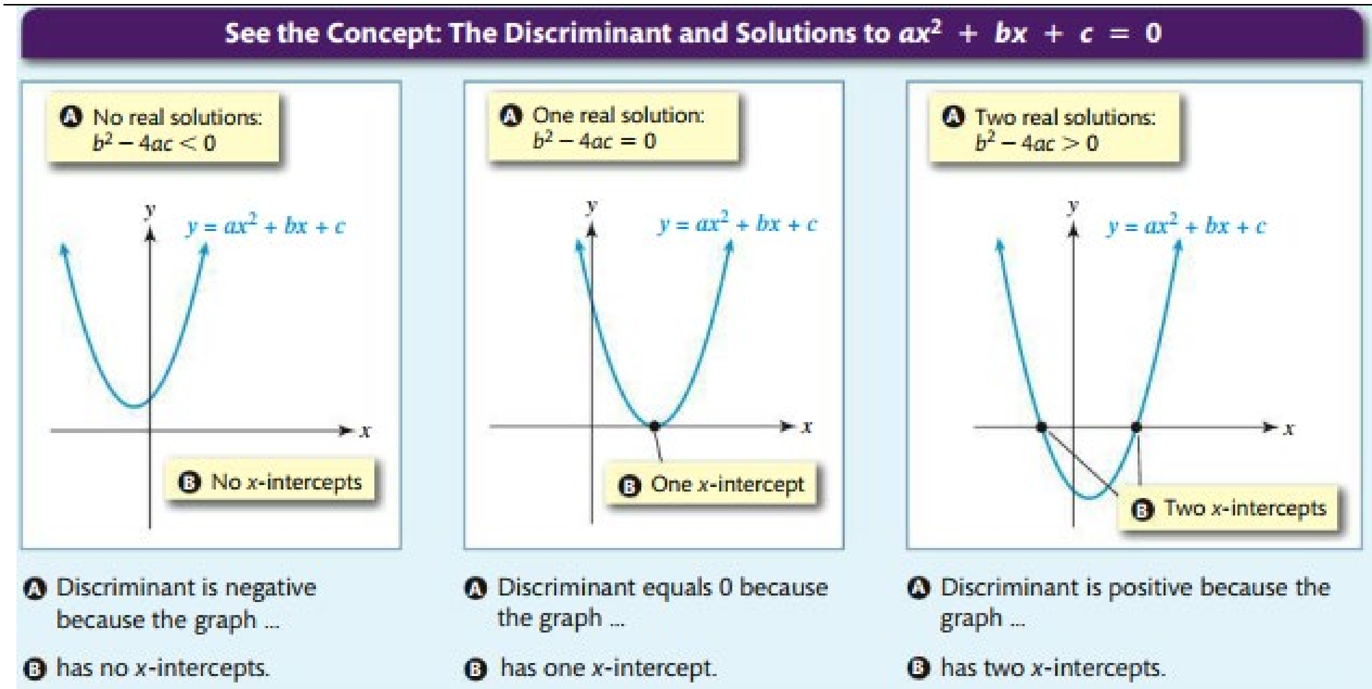


- **EXAMPLE:** Use the discriminant to determine the number of real solutions.

$$w^2 - 2w + 3 = 0 \qquad [3.2\text{-}29]$$

$$a = 1, b = -2, c = 3$$

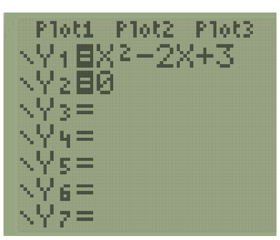
$$b^2 - 4ac = (-2)^2 - 4 \cdot 1 \cdot 3 = 4 - 12 = -8$$

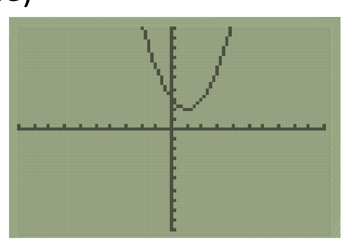
Since the discriminant $b^2 - 4ac < 0$ (negative), the equation will have:

NO real solutions.

Another (easier?) way: **GRAPH** the equation $w^2 - 2w + 3 = 0$ on calculator

(use *x* as your variable)





(Put left side equation in Y1, right side in Y2) (standard window Zoom 6)

Because the parabola does **NOT** have any *x*-intercepts, then that also means it only has **NO real solutions**.

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G. Solve by **Graphing** (**finding *x*-intercepts**) on **Calculator**

To solve a quadratic equation by graphing:

1. Set your equation **EQUAL** to **ZERO**, if needed. Go to **Y=** on calculator. 

2. Put **left** side of equation into **Y1** and
   right side (zero) into **Y2** on calculator (use *x* as your variable).
3. Graph starting with standard window, ZOOM 6.
   You may need to Zoom In or Out (ENTER), if needed.
4. Does your graph (parabola) cross or touch *x*-axis?
   If YES, go to STEP 5 to find *x*-intercepts.
   If NO, then stop – your equation has **no real solutions**.
5. Press **2ND**, **TRACE**, **5** (Intersect). 

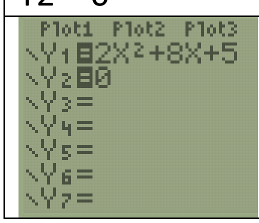
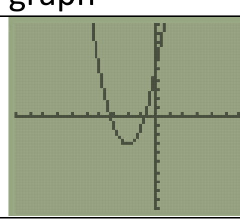
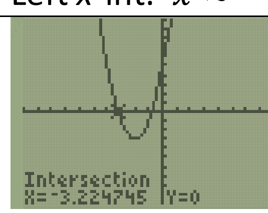
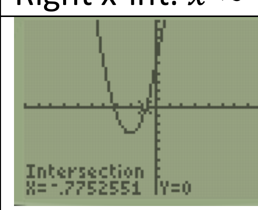
6. Press DOWN Arrow to switch to Y2=0
   and move cursor to where the parabola is touching *x*-axis.
7. Press **ENTER** three times.
8. You should see the word INTERSECTION with
   x = some number and *y* = 0. This is an ***x*-intercept**.
9. The **solution** is the ***x*-coordinate** of that *x*-intercept (round the amount accordingly).
10. Repeat STEPS 5 through 9 if there is a second *x*-intercept. It will be the second **solution**.

- **EXAMPLE:** Use a calculator to find the graphical solution to the equation.
  Round to the nearest thousandth.

  $2n^2 = -8n - 5$ [3.2-16]

  Set your equation equal to zero (add 8*n* and add 5)

  $2n^2 + 8n + 5 = 0$ Go to Y= on calculator. Use *x* as your variable.

Y1 = $2x^2 + 8x + 5$ Y2 = 0	ZOOM 6 to graph	2nd TRACE 5 Left *x*-int. $x \approx -3.225$	2nd TRACE 5 again Right x-int. $x \approx -0.775$
			

The solutions to the equation $2n^2 = -8n - 5$ are $\boldsymbol{n \approx -3.225}$ **or** $\boldsymbol{-0.775}$ (rounded)

Sources Used:

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2. MyLab Math for *College Algebra*, 7th Edition, Blitzer, Pearson Education Inc.
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4. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website https://archive.codeplex.com/?p=wabbit