1. Fundamental Counting Principle

Counting is much more than counting from 1-100. It includes counting how many ways things occur. You can make a list, a chart, or even a tree diagram so you can count things in a systematic order but these things can take time and are not always practical. So we can use the Fundamental Counting Principle to count things quicker. It involves multiplying which is a much quicker way of adding or counting things.

Fundamental Counting Principle: (Gina's interpretation)
you have n_1 number of ways to do one thing and
you have n_2 number of ways to do a second thing and
you have n_3 number of ways to do a third thing,
then the total number of ways to do all things can be found by multiplying:

$$n_1 \cdot n_2 \cdot n_3 \cdot \ldots\ldots\ldots \cdot n_k = \text{total}$$

EXAMPLES: Use the Fundamental Counting Principle to count the items:

(1) If three ordinary dice are rolled, one red and one white and one blue, then how many result are possible?

Red Dice – 6
White Dice – 6
Blue Dice – 6

$6 \cdot 6 \cdot 6 = 216$

(2) Sometimes license plates have 3 letters (A – Z) followed by 3 numbers (0 – 9) for example, ABC 123. Would this provide enough different license plates for a state with 8 million vehicles?

yes

$\boxed{26}\ \boxed{26}\ \boxed{26}\quad \boxed{10}\ \boxed{10}\ \boxed{10} = 17,576,000$

$26^3 \cdot 10^3 =$

(3) How many 3-digit numbers can be created using only the numbers 4, 5, 6 if repeated numbers are allowed?

$3 \cdot 3 \cdot 3 = 27$

Sometimes restrictions may apply:

(4) How many 3-digit numbers can be created using only the numbers 4, 5, 6 if the **numbers cannot be repeated**?

$3 \cdot 2 \cdot 1 = 6$

Sometimes restrictions may apply:

(5) How many 7-digit telephone numbers are possible if the **first digit *cannot* be a 0**?

$\boxed{9}\ \boxed{10}\ \boxed{10}\quad \boxed{10}\ \boxed{10}\ \boxed{10}\ \boxed{10}$ = 9,000,000

$9 \cdot 10^6 =$

(6) If an ice-cream shop has 6 of my favorite flavors of ice-cream and 3 waffle cones, plus 5 delicious toppings, how many different ice-cream cone sundaes can I create?

Ice cream – 6
Cones – 3
Toppings – 5

$6 \cdot 3 \cdot 5 = 90$

(7) Five students win an award: 3 guys and 2 girls. How many ways can all five winners line up for a photograph for the newspaper?

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

(8) How many ways can we select 1 guy and 1 girl from the award winners above to plan a reception for family and friends?

$3 \cdot 2 = 6$

2. Factorial

Did you notice in example #7 above the pattern of the numbers? Five students win an award. How many ways can all five winners line up for a photograph for the newspaper?

First Person	Second Person	Third Person	Fourth Person	Fifth Person	
5	4	3	2	1	= 120

Numbers that are multiplied like this are common in counting, probability, and statistics. This product has a special name called a ***factorial*** and the symbol used is an exclamation point **!** Factorial is an ***arrangement*** of one distinct group of items.

Factorial Formula: For any counting number, ***n***, the quantity/answer for n factorial is:

$$n! = n(n-1)(n-2)\ldots(3)(2)(1) \qquad \text{and} \qquad 0! = 1$$

meaning that ***n*** is the given number, them multiply by one less, until you reach the number 1

For example: What is 6! (6 factorial)

$$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$$

EXAMPLE: Try these factorial problems. Use the factorial button on your calculator. Hit your **MATH** button, then arrow over to **PRB** (probability), then your factorial button should be **#4: !**

(1) $8! = 40{,}320$

```
8!
                40320
13!
           6227020800
```

(2) $13! = 6{,}227{,}020{,}800$

(3) $\frac{9!}{7!} = 72 \leftarrow \frac{9\,8\,\cancel{7\,6\,5\,4\,3\,2\,1}}{\cancel{7\,6\,5\,4\,3\,2\,1}}$

(4) $\frac{5!}{(5-2)!} = \frac{5!}{3!} = 20$ $\qquad \frac{5\,4\,\cancel{3\,2\,1}}{\cancel{3\,2\,1}}$

(5) $\frac{5!}{(10!\,10!)} = 9.11 \times 10^{-12}$

(6) Determine the number of distinguishable arrangements of the letters in each word: (a) ATTRACT (b) NIGGLING

Each letter can only be used one time, so this is a factorial problem. However, some letter are already repeated so we divide by the repeated letters so there are no repeated arrangements of letters.

(a) 7 letters 3 T's 2 A's $\frac{7!}{(3!\,2!)} = 420$

(b) 8 letters 2 N's 2 I's 3 G's

$$\frac{8!}{(2!\,2!\,3!)} =$$