Notes Section 3.5 – Transformations of Graphs

Lesson Objectives

1. Parent Functions
2. Vertical and Horizontal Translations (shifts)

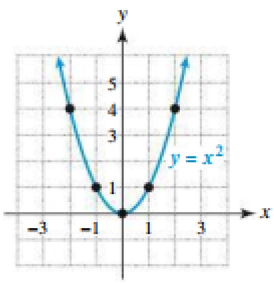
A. Parent Functions

1. **Quadratic (or _____________) Function:**

$f(x) = x^2$ or $y = x^2$

Square Function: $f(x) = x^2$

x	-2	-1	0	1	2
$y = x^2$	4	1	0	1	4



$D = (-\infty, \infty)$

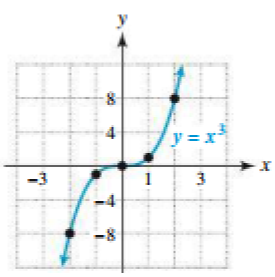
$R = [0, \infty)$

2. **___________ Function:**

$f(x) = x^3$ or $y = x^3$

Cube Function: $f(x) = x^3$

x	-2	-1	0	1	2
$y = x^3$	-8	-1	0	1	8



$D = (-\infty, \infty)$

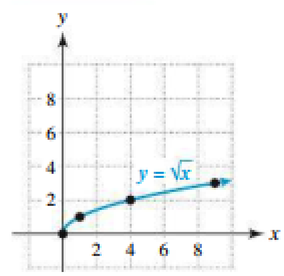
$R = (-\infty, \infty)$

3. **____________ __________ Function:**

$f(x) = \sqrt{x}$ or $y = \sqrt{x}$

Square Root Function: $f(x) = \sqrt{x}$

x	0	1	4	9
$y = \sqrt{x}$	0	1	2	3



$D = [0, \infty)$

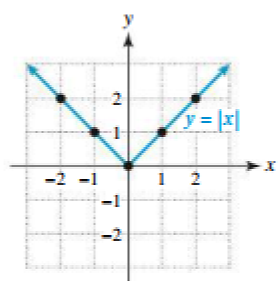
$R = [0, \infty)$

4. **______________ __________ Function:**

$f(x) = |x|$ or $y = |x|$

Absolute Value Function: $f(x) = |x|$

x	-2	-1	0	1	2
$y = \|x\|$	2	1	0	1	2



$D = (-\infty, \infty)$

$R = [0, \infty)$

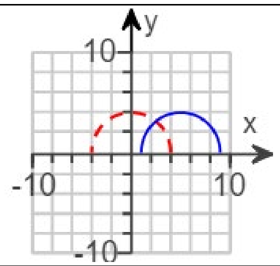
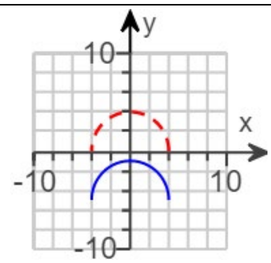
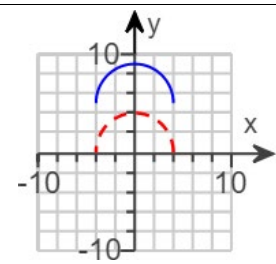
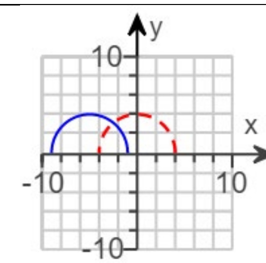
B. Vertical and Horizontal Shifts

<table>
<tr><td colspan="3">Let f be a function, and let c be a positive number.</td></tr>
<tr><th>To Graph</th><th>Shift the Graph of $y = f(x)$ by c Units</th><th>(NOTES below)</th></tr>
<tr><td>$y = f(x) + c$</td><td></td><td rowspan="2">Adding or subtracting a number OUTSIDE parentheses with y causes a ______________________________ SHIFT in the SAME direction as that number.
OUTSIDE – y is “do what you see.” (↓ , ↑)</td></tr>
<tr><td>$y = f(x) - c$</td><td></td></tr>
<tr><td>$y = f(x - c)$</td><td></td><td rowspan="2">Adding or subtracting a number INSIDE parentheses with x causes a ______________________________ SHIFT in the OPPOSITE direction of that number.
INSIDE – x goes _______________! (← , →)</td></tr>
<tr><td>$y = f(x + c)$</td><td></td></tr>
</table>

Notes Section 3.5 – Transformations of Graphs

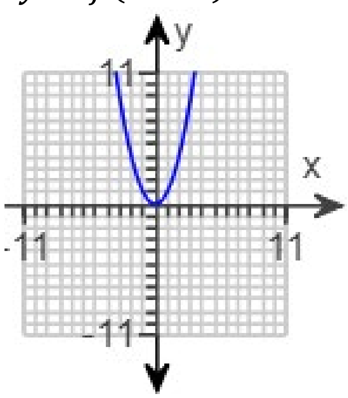
- **EXAMPLE:** The graph of $y = f(x)$ is shown with dashed (red) lines. Graph $y = f(x) + 5$. Choose the correct graph in solid (blue). [3.5-31]

$y = f(x) + 5$ _____________ **– *y* is "do what you see."** + 5 means __________Graph ___.

A.	**B.**	**C.**	**D.**
			

- **EXAMPLE:** Determine which graph indicates the shift in the indicated equation. [3.5-6]

$y = f(x - 3) - 5$

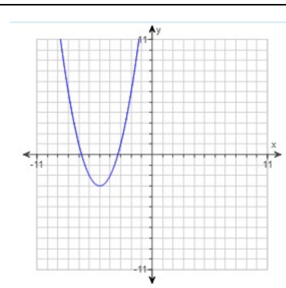
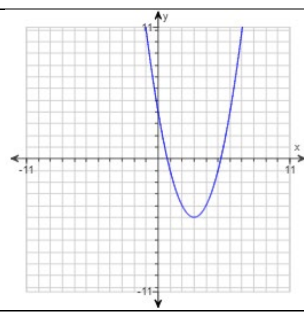
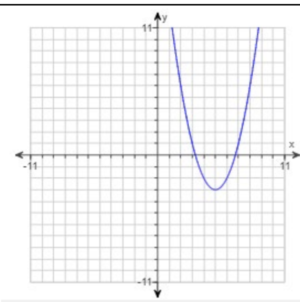
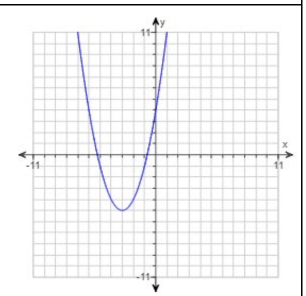


The graph to the left is the given graph of *y* = *f*(*x*).
To graph *y* = *f*(*x* – 3) – 5

INSIDE parentheses:
***x* goes OPPOSITE!**
I see – 3 with *x*, so the shift is to the ______, not the left.

OUTSIDE parentheses:
***y* is "do what you see"**
I see – 5 outside parentheses, so the shift is also ________ 5.

So, together, the shift for $y = f(x - 3) - 5$ is __________________________________. Correct answer is ____.

A.	**B.**	**C.**	**D.**
			

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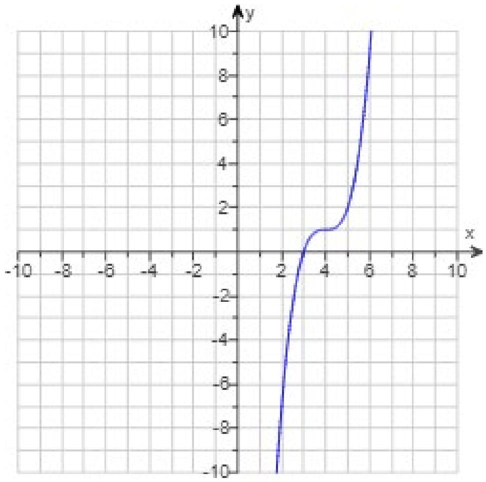
Notes Section 3.5 – Transformations of Graphs

- **EXAMPLE:** The graph is a translation of one of the basic functions $y = x^2, y = x^3, y = \sqrt{x}, y = |x|$. Find the equation that defines the function. [3.5.1] (Type an expression using *x* as the variable. Do not simplify).



The graph to the left is a translation of $y =$ ______.
The INFLECTION point for $y = x^3$ is normally at the origin.
In this graph, though, it has moved: ________________

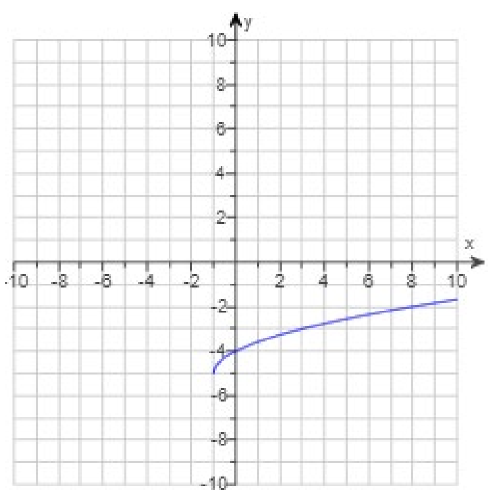
RIGHT 4 is a change in ____, so be sure to _____________ the value. (remember: **INSIDE – *x* goes OPPOSITE**!)
RIGHT 4 is written as __________________

UP 1 is a change in _____, so that is written as _____.
(remember: **OUTSIDE – *y* is "do what you see."**)

When $y = x^3$ that goes RIGHT 4, UP 1,

the equation is *y* = ___________________________

- **EXAMPLE:** The graph is a translation of one of the basic functions $y = x^2, y = x^3, y = \sqrt{x}, y = |x|$. Find the equation that defines the function. [3.5.5] (Type an expression using *x* as the variable. Do not simplify).



The graph to the left is a translation of $y =$ _____.
The starting point for $y = \sqrt{x}$ is normally at the origin.
In this graph, though, it has moved: _______________

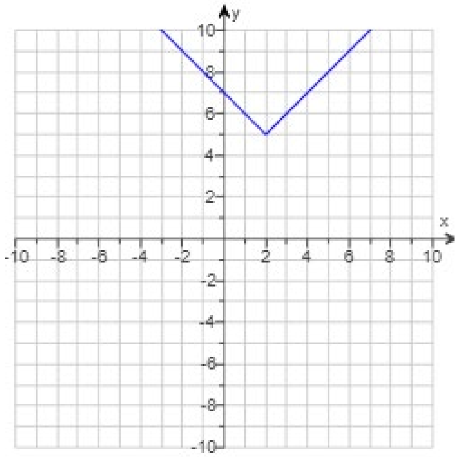
LEFT 1 is a change in ____ so be sure to SWITCH the value.
(remember: **INSIDE – *x* goes OPPOSITE**!)
So, LEFT 1 is written as _____________

DOWN 5 is a change in *y*, so that is written as _____.
(remember: **OUTSIDE – *y* is "do what you see."**)

When $y = \sqrt{x}$ goes LEFT 1, DOWN 5,
The equation is *y* = ___________________________

Notes Section 3.5 – Transformations of Graphs

- **EXAMPLE:** The graph is a translation of one of the basic functions $y = x^2, y = x^3, y = \sqrt{x}, y = |x|$. Find the equation that defines the function. [3.5.7] (Type an expression using *x* as the variable. Do not simplify).



The graph to the left is a translation of $y =$ _____.
The vertex for $y = |x|$ is normally at the origin.
In this graph, though, it has moved: _______________.

RIGHT 2 is a change in ____, so be sure to SWITCH the value. (remember: **INSIDE – *x* goes OPPOSITE**!)
So, RIGHT 2 is written as ________________.

UP 5 is a change in ___, so that would be written as ____
(remember: **OUTSIDE – *y* is "do what you see."**)

When $y = |x|$ that goes RIGHT 2, UP 5,

the equation is *y* = ___________________________

- **EXAMPLE:** Find the equation that shifts the graph of *f* by the indicated amounts.
  $f(x) = x^4$ right 8 units, up 7 units [3.5-1]

Right 8 units is a change in ______, so be sure to SWITCH the value.

So, RIGHT 8 is written as ______________. (remember: **INSIDE – *x* goes OPPOSITE**!)

Up 7 units is a change in ______, so UP 7 is written with _____ at the end.
(remember: **OUTSIDE – *y* is "do what you see."**)

A.	$y = -(x-8)^4 + 7$	
B.	$y = -(x-8)^4 + 56$	
C.	$y = (x-8)^4 + 7$	
D.	$y = (x+8)^4 - 7$	

Notes Section 3.5 – Transformations of Graphs

- **EXAMPLE:** Use transformations to explain how the graph of f can be found using the graph of $y = x^2$.

$$f(x) = (x-3)^2 + 2 \qquad [3.5.53]$$

I see ______ with x, so the shift is to the _____________, not the ____________.
(remember: **INSIDE – *x* goes OPPOSITE**!)

So $(x - 3)$ means it moves _______________ **3**.

I see _______ outside parentheses, so the shift is also _______________.
(remember: **OUTSIDE – *y* is "do what you see."**)

Together, the shift for $f(x) = (x-3)^2 + 2$ from $y = x^2$ is _______________________.

- **EXAMPLE:** Use transformations of the graphs of $y = x^2$ or $y = |x|$ to sketch a graph of f by hand.

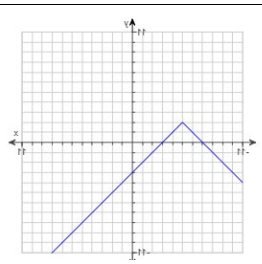
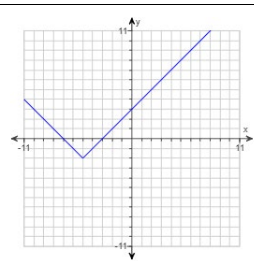
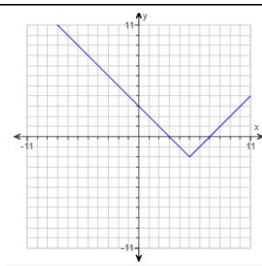
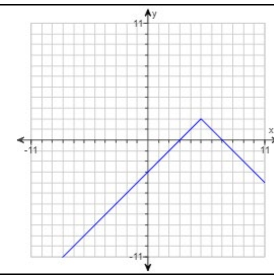
$$f(x) = |x - 5| - 2 \qquad [3.5\text{-}11]$$

I see _____ with ____, so the shift is to the ____________, not the ___________.
(remember: **INSIDE – *x* goes OPPOSITE**!)

So $|x - 5|$ means it moves _____________ **5**.

I see _____ outside parentheses, so the shift is also ________________.
(remember: **OUTSIDE – *y* is "do what you see."**)

Together, the shift for $f(x) = |x-5| - 2$ from $y = |x|$ is _______________________.

A.	B.	C.	D.
			

Notes Section 3.5 – Transformations of Graphs

- **EXAMPLE:** Find the equation that shifts the graph of *f* by the desired amounts. Graph *f* and the shifted graph in the same *xy*-plane. [3.5.15]

  $f(x) = x^2 - 2x + 2$ right 5 units, upward 3 units

Right 5 units is a change in _____, so be sure to SWITCH the value.

RIGHT 5 is written as: ________________ (remember: **INSIDE – *x* goes OPPOSITE**!)

Use __________________everywhere you see an _____ in the function.

$x^2 \quad - \; 2 \quad x \quad + 2$ changes to

$(\qquad)^2 - 2(\qquad) \quad + 2$

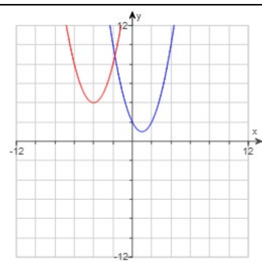
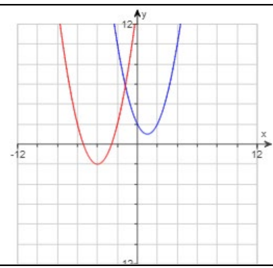
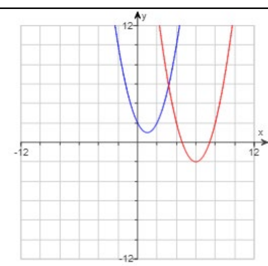
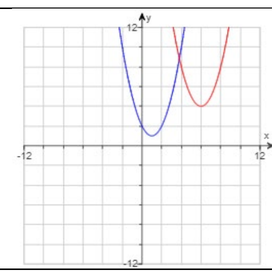
Upward 3 units is a change in ______, so jut include a ________ at the end.
(remember: **OUTSIDE – *y* is "do what you see."**)

$(x-5)^2 - 2(x-5) \; + 2$ ____ Combine like terms on the end

Updated: $y =$ ____
(Make sure BOTH sets of parentheses have the SAME value!)

Notice that in all four graphs, one of the graphs is always in the same place, with vertex at about (1,1). Remember that the overall shift is:

right 5 units and upward 3 units, or more simply: **______________ and _______.**

A.	**B.**	**C.**	**D.**
			

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Notes Section 3.5 – Transformations of Graphs

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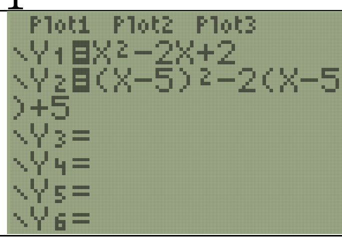
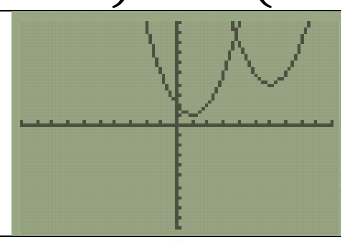
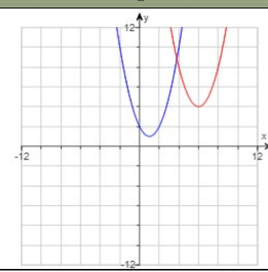
- **EXAMPLE:** Find the equation that shifts the graph of *f* by the desired amounts. Graph *f* and the shifted graph in the same *xy*-plane. [3.5.15]

  $f(x) = x^2 - 2x + 2$ right 5 units, upward 3 units

NOTE: You can also verify the graph on your graphing calculator:

Original function	Modified function: right 5, upward 3
$f(x) = x^2 - 2x + 2$	$y = (x-5)^2 - 2(x-5) + 5$
$Y_1 = x^2 - 2x + 2$	$Y_2 = (x-5)^2 - 2(x-5) + 5$
	
The graph on the calculator matches the answer we got on the previous page:	

Sources Used:

1. MyLab Math for *College Algebra with Modeling and Visualization*, 6th Edition, Rockswold, Pearson Education Inc.
2. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website https://archive.codeplex.com/?p=wabbit