Notes Section R.4 – Factoring Polynomials

The **greatest common factor (GCF)** of a group of natural numbers is the largest number that is a factor of all of the numbers in the group.

The questions in MyMathLab may ask you to use a variety of methods, but we will use 2 methods:

- Prime Factorization Method (Factor Tree)
- Calculator Method (the gcd command)

Finding the Greatest Common Factor (Prime Factors Method)

Step 1 Write the prime factorization of each number.

Step 2 Choose all primes common to *all* factorizations, with each prime raised to the *least* exponent that appears.

Step 3 Form the product of *all* the numbers in Step 2; this product is the greatest common factor.

Example: Greatest Common Factor by Prime Factors Method (Factor Tree)

Find the greatest common factor of 360 and 1350.

Solution

The prime factorizations are below.

$360 = 2^3 \cdot 3^2 \cdot 5$

$1350 = 2 \cdot 3^3 \cdot 5^2$

So, the GCF is $2 \cdot 3^2 \cdot 5 = 90$

Greatest Common Factor – Calculator Method
Using the gcd command (greatest common divisor)

A couple of **limitations** of using the calculator:

1. The gcd command can only do **2 numbers at a time**. If more than 2 numbers are involved, the gcd command must be used multiple times, or "nested" – more on that later.

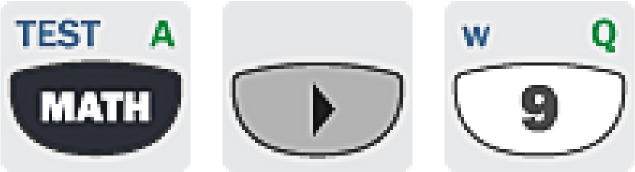
2. The gcd command can only do **POSITIVE numbers**. If negatives are involved, they must be ignored on the calculator.

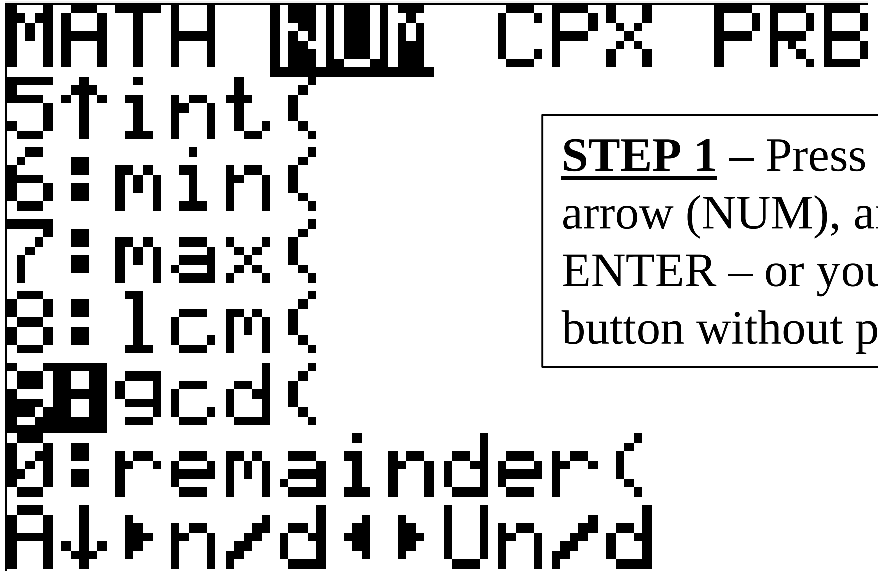
3. The gcd command **cannot do variables** – only numbers (coefficients).

Greatest Common Factor – Calculator Method
Using the gcd command (greatest common divisor)

Let’s look at the previous example:

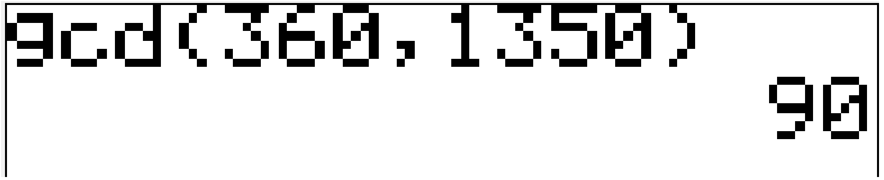
Find the greatest common factor of 360 and 1350.





<u>STEP 1</u> – Press MATH, the RIGHT arrow (NUM), and scroll to #9, ENTER – or you can just press the #9 button without pressing enter.

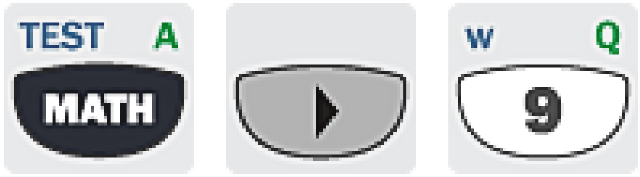
<u>STEP 2</u> – It will start out reading just **gcd(** on the screen.
Enter your first given number, then a comma (over the #7 key), then type your second given number, then type a close-parentheses (above the #9 key), and press ENTER. That’s it!

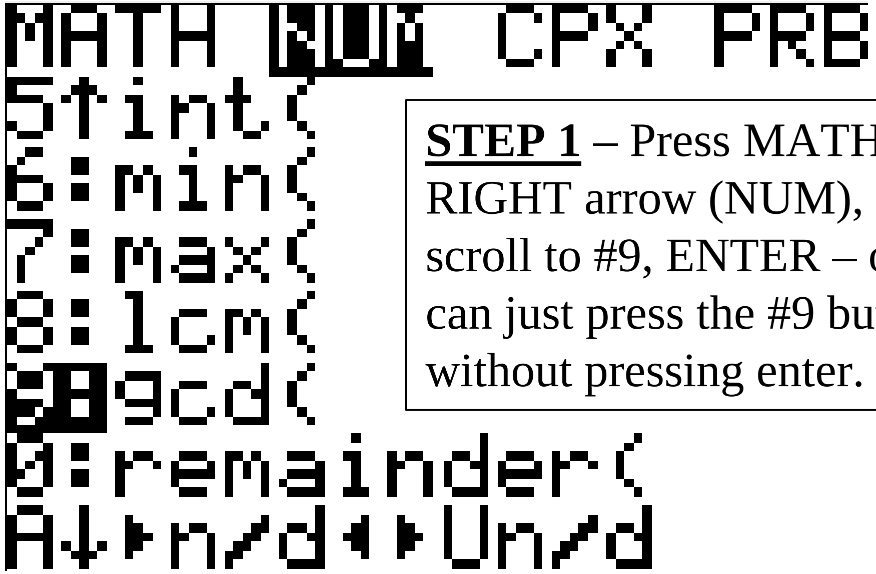


Greatest Common Factor – Calculator Method
Using the gcd command (greatest common divisor)

Now let’s look at an example involving 3 numbers.

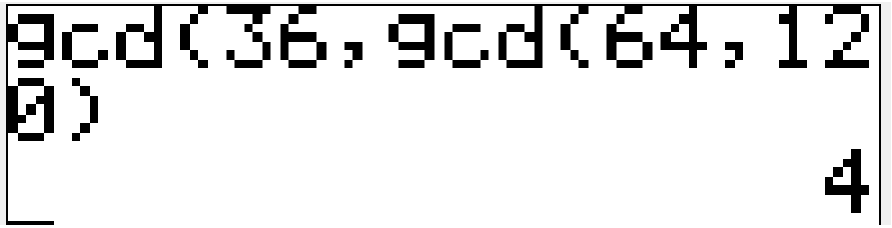
Find the greatest common factor of 36, 64, and 120.





<u>STEP 1</u> – Press MATH, the RIGHT arrow (NUM), and scroll to #9, ENTER – or you can just press the #9 button without pressing enter.

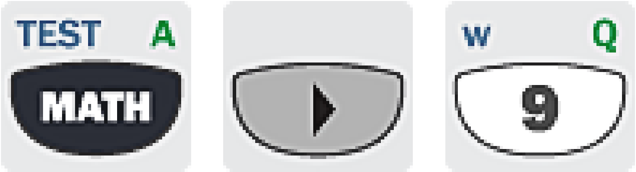
<u>STEP 2</u> – It will start out reading just **gcd(** on the screen. Type your first given number, then a comma (over the #7 key), then call up the gcd command again (see **<u>STEP 1</u>**). Type your second number, then a comma (over the #7 key), then type your third number then type a close-parentheses (above the #9 key), and press ENTER. That’s it!

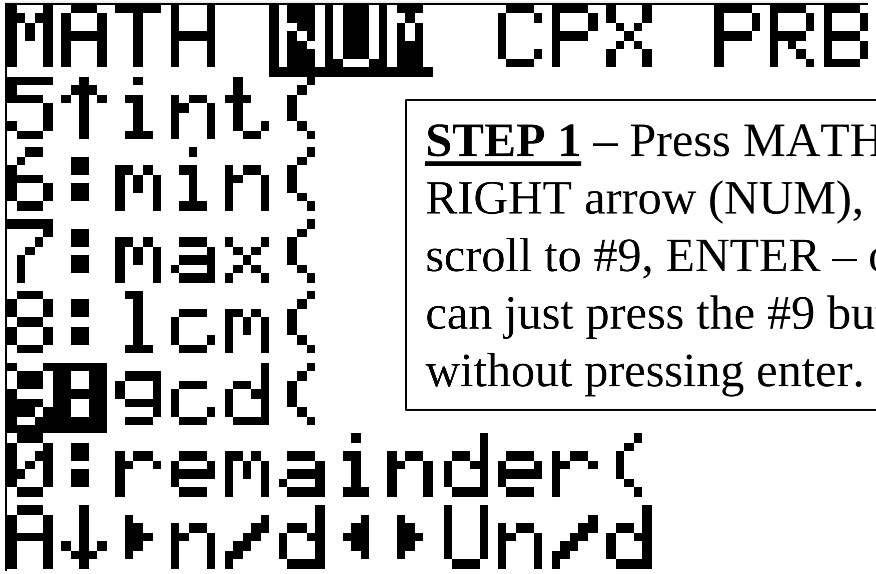


Greatest Common Factor – Calculator Method
Using the gcd command (greatest common divisor)

Now let's look at an example involving 3 numbers.

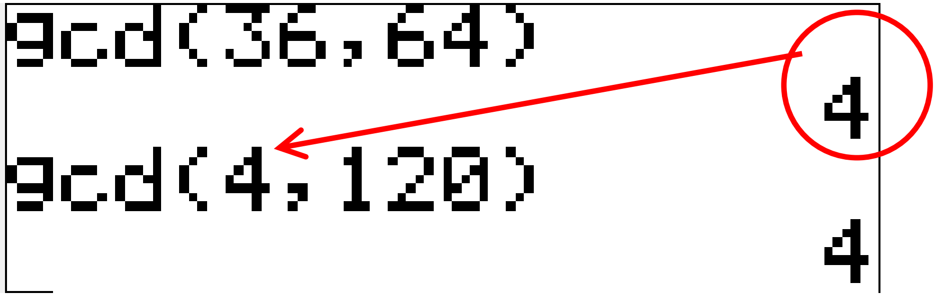
Find the greatest common factor of 36, 64, and 120. **(alternate method)**





<u>STEP 1</u> – Press MATH, the RIGHT arrow (NUM), and scroll to #9, ENTER – or you can just press the #9 button without pressing enter.

<u>STEP 2</u> – The alternate way to do 3 numbers at a time is to do the first 2 numbers, then take that answer and do the gcd command again with that answer and the third number.



Factoring Out the GCF

Definition

The **greatest common factor (GCF)** of two or more terms is the monomial with the largest coefficient and the highest degree that is a factor of all the terms.

A polynomial is said to be ***prime*** if the GCF for all its terms is 1.

Factoring Out the GCF

Greatest Common Factor

Example Factor: $18x^4 - 30x^2$.

Solution

1. **Find GCF of coefficients.** GCF of 18 and 30 is 6.

2. **Find GCF of variables.** - only if ALL terms have the same variable. Pick the variable with smallest exponent. Pick the "runt," or the "baby." Repeat the process for however many variables are present.

GCF of x^4 and x^2 is x^2 .

3. **Multiply coefficient and variable GCF's together.** The final overall GCF is $6 \cdot x^2$, or simply $6x^2$.

Factoring Out the GCF

Example Factor: $18x^4 - 30x^2$.

Solution $6x^2\ (\qquad\qquad\qquad)$

4. **Write the GCF** (from Step 3) on the next line underneath your given expression to factor, and **open up a set of parentheses the same width** as your given expression.

5. To fill **inside the parentheses**, use the GCF and ask "**times-what**, equals my starting amount?" Do coefficients (numbers) first, followed by variables. (see continued example, next slide).

Factoring Out the GCF

Greatest Common Factor

Example

Factor: $18x^4 - 30x^2$.

Solution

$6x^2\ (\qquad\qquad)$

$$18x^4 - 30x^2$$

$$6x^2\,(3x^2 - 5)$$

Factoring Out the GCF

Factoring Out a Common Factor

Example

Factor

1. $10x^2 - 15x$
2. $8x^3 + 20x^2$

Solution

1. $$\begin{aligned} 10x^2 - 15x &= 5x \cdot 2x - 5x \cdot 3 && \text{5x is a common factor.} \\ &= 5x(2x - 3) && \text{Factor out 5x.} \end{aligned}$$

2. $$\begin{aligned} 8x^3 + 20x^2 &= 4x^2 \cdot 2x + 4x^2 \cdot 5 && 4x^2 \text{ is a common factor.} \\ &= 4x^2(2x + 5) && \text{Factor out } 4x^2. \end{aligned}$$

Not completely factored: $8x^3 + 20x^2 = 2x(4x^2 + 10x)$

Multiplying Polynomials Versus Factoring Polynomials

Multiplying and factoring are reverse processes,
For example

$$(x+2)(x+3) \underset{\text{Factoring}}{\overset{\text{Multiplying}}{\rightleftarrows}} x^2+5x+6$$

Shall We Play a Game?

Find two numbers that *multiply* to make 6 but also *add* to make 5. ________________

Find two numbers whose *product* is 24 and whose *sum* is 10. ________________

Find two numbers that *multiply* to make 36 but also *add* to make –15 . ________________

Find two numbers whose *product* is 48 and whose *sum* is –19. ________________

Shall We Play a Game?

Find two numbers that *multiply* to make –6 but also *add* to make 5. ________________

Find two numbers whose *product* is –24 and whose *sum* is 10. ________________

Find two numbers that *multiply* to make –36 but also *add* to make –9. ________________

Find two numbers whose *product* is –48 and whose *sum* is –13. ________________

Factoring a Trinomial

Factoring $x^2 + b + c$

Process

To factor $x^2 + bx + c$, look for two integers p and q whose product is c and whose sum is b. That is, $pq = c$ and $p + q = b$. If such integers exist, the factored polynomial is

$$(x + p)(x + q)$$

Factoring a Trinomial – **Positive** Constant Term *c*

Example

Factor $x^2 + 11x + 24$.

- We need two integers whose product is 24 and whose sum is 11

Solution

Product = 24	**Sum = 11?**
$1(24) = 24$	$1 + 24 = 25$
$2(12) = 24$	$2 + 12 = 14$
$3(8) = 24$	$3 + 8 = 11 \leftarrow$ Success!
$4(6) = 24$	$4 + 6 = 10$

- $3(8) = 24$ and $3 + 8 = 11$: last terms are 3 and 8

$$x^2 + 11x + 24 = (x + 3)(x + 8)$$

- Check:

$$(x + 3)(x + 8) = x^2 + 8x + 3x + 24 = x^2 + 11x + 24$$

Example

Factor $x^2 - 12x + 36$.

Solution

Product = 36	Sum = −12?
$-1(-36) = 36$	$-1 + (-36) = -37$
$-2(-18) = 36$	$-2 + (-18) = -20$
$-3(-12) = 36$	$-3 + (-12) = -15$
$-4(-9) = 36$	$-4 + (-9) = -13$
$-6(-6) = 36$	$-6 + (-6) = -12 \longleftarrow$ Success!

$$x^2 - 12x + 36 = (x - 6)(x - 6) = (x - 6)^2$$

Factoring a Trinomial – **<u>Positive</u>** Constant Term *c*

Factoring a Trinomial of the Form x^2 + *b* + c

Process

To factor a trinomial of the form $x^2 + bx + c$ with a **<u>positive</u>** constant term c,

- If ***<u>b</u>*** **<u>is positive</u>** (middle term), look for two positive integers whose product is c and whose sum is b.

$$x^2 + 10x + 16 = (x + 2)(x + 8)$$

Positive b

Positive c

Both last terms are positive.

Factoring a Trinomial – **Positive** Constant Term *c*

Process

- If ***b* is negative** (middle term), look for two negative integers whose product is *c* and whose sum is *b*.

$$x^2 - 9x + 18 = (x - 3)(x - 6)$$

Negative *b*

Positive *c*

Both last terms are negative.

I feel a song coming on…

(*To the tune of "If You're Happy and You Know It"*)

If the last term is **positive**, they're the **SAME**.

You look to the middle for the charge (sign).

If it's **positive**, "**plus…plus**" (__ + __) (__ + __)

If it's **negative**, "**minus…minus**" (__ – __) (__ – __)

If the last term is **positive**, they're the **SAME**.

Factoring with *c* Negative

Process

To factor a trinomial of the form $x^2 + bx + c$ with a **negative** constant term c, look for two integers with *different* signs whose product is c and whose sum is b.

For example,

$$x^2 + 2x - 24 = (x - 4)(x + 6)$$

Negative c

The last terms have different signs.

If the last term is **negative**, they're **DIFFERENT**

"One of each" (__ – __) (__ + __) or switched.

Trinomials with Negative Constant Terms

Example

Factor $w^2 - 3w - 18$.

Solution

Product $= -18$	Sum $= -3$?
$1(-18) = -18$	$1 + (-18) = -17$
$2(-9) = -18$	$2 + (-9) = -7$
$3(-6) = -18$	$3 + (-6) = -3 \leftarrow$ Success!
$6(-3) = -18$	$6 + (-3) = 3$
$9(-2) = -18$	$9 + (-2) = 7$
$18(-1) = -18$	$18 + (-1) = 17$

$$\boxed{w^2 - 3w - 18 = (w + 3)(w - 6)}$$

Check:

$$(w + 3)(w - 6) = w^2 - 6w + 3w - 18 = w^2 - 3w - 18$$

Prime Polynomials

Definition

A polynomial that cannot be factored is called **prime.**

Example

Factor $-14 + 6x + x^2$.

Solution

Product = −14	**Sum = 6?**
$1(-14) = -14$	$1 + (-14) = -13$
$2(-7) = -14$	$2 + (-7) = -5$
$7(-2) = -14$	$7 + (-2) = 5$
$14(-1) = -14$	$14 + (-1) = 13$

None of the sums equal 6. Thus, prime.

Here’s a “Special Case…”

The Difference of Two Squares

Property

$$A^2 - B^2 = (A + B)(A - B)$$

In words: The difference of the squares of two terms is the product of the sum of the terms and the difference of the terms.

The Difference of Two Squares

Example Factor.

1. $x^2 - 49$

Since $x^2 - 49 = (x)^2 - (7)^2$, we substitute x for A and 7 for B:

$$A^2 - B^2 = (A + B)(A - B)$$

$$x^2 - 49 = x^2 - 7^2 = (x + 7)(x - 7)$$

2. $36w^2 - 25y^2$

Since $36w^2 - 25y^2 = (6w)^2 - (5y)^2$, we substitute $6w$ for A and $5y$ for B:

$$A^2 - B^2 = (A + B)(A - B)$$

$$36w^2 - 25y^2 = (6w)^2 - (5y)^2 = (6w + 5y)(6w - 5y)$$