Notes Section R.4 – Factoring Polynomials

Lesson Objectives

1. Greatest Common Factor
2. Factor Out a Greatest Common Factor
3. Factor a Quadratic Trinomial $x^2 + bx + c$
4. Factor a Difference of Squares $x^2 - m^2$

A. Greatest Common Factor

Greatest: the biggest **Common**: shared

Factor: numbers that "_______________________-into" (also called **divisors**)

Easiest way to see if a number is a factor is to **divide** it in your head or on a calculator.

For example, with $8 \div 2 = 4$, since there is no decimal part or remainder, then both 2 and 4 are ________________________ of 8.

- **EXAMPLE:** Find the greatest common factor for the list of terms: $30x^5, 110x^7, 60x^9$ [*Akst 16.1.9]

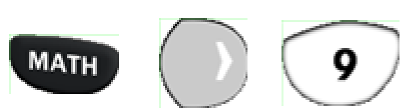
I recommend if necessary, use the calculator for the coefficients. Here's how you do that:

Greatest Common Factor (GCF) on calculator (TI-84 Plus or TI-83 Plus)
is actually called the **Greatest Common** _____________________ (abbreviated **gcd**)

The **gcd(** command on the calculator (TI-84 Plus or TI-83 Plus) has limitations:

1. only _________________________, not variables
2. only ____ numbers at a time
3. only ______________________ numbers

To access Greatest Common Divisor (gcd) on calculator:



press **MATH**, →(NUM), **9**:gcd(

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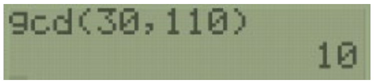
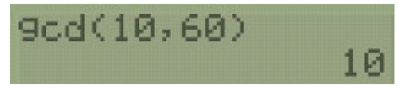
Here's the problem again for reference:

- **EXAMPLE:** Find the greatest common factor for the list of terms: $30x^5, 110x^7, 60x^9$

To find the GCF of the coefficients 30, 110, 60, we will use calculator (TI-83/84 Plus):

gcd(30,110) = 10 then take that answer and do it again gcd(10,60) = 10





The GCF of the coefficients 30, 110, and 60 is ________.

Here's the problem again for reference:

- **EXAMPLE:** Find the greatest common factor for the list of terms: $30x^5, 110x^7, 60x^9$

For the variable part: x^5, x^7, x^9

You can only include variables in GCF if ______ the terms include that __________ variable.

Since all 3 terms have ___, use the _______________ listed (only what's shared), which is ____.

The overall GCF for $30x^5, 110x^7, 60x^9$ is __________.

B. Factor Out the Greatest Common Factor

Factoring out the GCF should always be tried _____________, before trying other methods.

Factoring out the GCF is sort of like doing the ______________________ **property** in reverse.

- **EXAMPLE:** Factor out the greatest common factor.

$$12x^3 + 8x^2 - 16x \qquad \text{[R.4.9]}$$

- **STEP 1. Find GCF of coefficients.** GCF coeff. = ___
  - $\gcd(12,8) =$ ______
  - $\gcd(4,16) =$ ______
- **STEP 2. Find GCF of variables.**
  - Do all terms have same variable? _______. All have an ________
  - If YES, what is the **SMALLEST** of the ones listed? smallest of x^3, x^2, and x is ___.

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(continued from the previous page … here is the problem again for reference)

- **EXAMPLE:** Factor out the greatest common factor.

  $12x^3 + 8x^2 - 16x$ [R.4.9]

- **STEP 3. Multiply the coefficient and variable GCF's together.**
  - Coefficient GCF = ____, variable GCF = ___ Product = ________
  - The overall GCF is ________.

- **STEP 4. Skip a line** and **write the GCF** with a **"reverse-indent**."

  Open a set of **parentheses** the **SAME WIDTH** as the expression.

  $12x^3 + 8x^2 - 16x$

  ______ (   )

- **STEP 5.** To determine what goes INSIDE the parentheses, simply ________________ each term of the expression **by the GCF** and simplify. Write the simplified result in parentheses.

  $$\frac{12x^3}{} + \frac{8x^2}{} - \frac{16x}{}$$

  $\mathbf{4x}$ (+   −   ) **(ANSWER)**

  The entire expression is in "factored form."

$12x^3 + 8x^2 - 16x$	$4x\,(\,3x^2 + 2x \;-\; 4\;)$
Original expression	Factored expression
_____ terms	_____ terms
Addition & Subtraction	Multiplication

Factoring is a process that converts addition & subtraction into ____________________.

This allows the opportunity to ________________ – most common are fractions and roots.

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C. Factor a trinomial of the form $x^2 + bx + c$

- Review of Multiplying Binomials $(x+p)(x+q)$ – Use the FOIL method

- **EXAMPLE:** Multiply. $(x+5)(x-3)$ [R.3.55]

$$(x+5)(x-3)$$

F: ________________ ___ · ___ = ______ Write all the terms connected together:
O: ________________ ___ · ___ = ______ ________________________________
I: _________________ ___ · ___ = ______ Simplify – combine like terms:
L: _________________ ___ · ___ = ______ **ANSWER:** ____________________

- Factor a trinomial of the form $x^2 + bx + c$

Factoring a trinomial in this form is sort of like doing FOIL ____ ________________.

- **EXAMPLE:** Find the binomial factors for the trinomial. [*Akst *16.2.7]

$x^2 + 17x + 16$

F O + I L

$x^2 + 17x + 16$ = (___)(___)

F (Firsts): Open up 2 sets of parentheses, with your variable in the ____________position.

Next, we need two integers whose ______ is 17 and whose _________________ is 16.

O + I **(sum of Outers and Inners)**

L **(Lasts)**

To finish factoring, we need **2 numbers:**		
Product = ______	**Sum = _____**	**Winner?**

ANSWER: $x^2 + 17x + 16$ = $(\boldsymbol{x}\quad)(\boldsymbol{x}\quad)$ or $(\boldsymbol{x}\quad)(\boldsymbol{x}\quad)$

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- **EXAMPLE:** Factor the expression. $r^2 - 18r + 81$ [R.4.81]

Open 2 sets of parentheses with variable in the **first** position:

$$r^2 - 18r + 81 \quad = \quad (\_\_\_ \quad)(\_\_\_ \quad)$$

Next, we need 2 integers whose **SUM** is _____ and whose **PRODUCT** is _____

To finish factoring, we need **2 numbers:**		
Product = 81	**Sum = -18**	**Winner?**

ANSWER: $r^2 - 18r + 81 \quad = \quad (\boldsymbol{r} \quad)(\boldsymbol{r} \quad)$ or ________________________

- **EXAMPLE:** Factor the expression completely. [R.4.37]

$v^2 + v - 72$

Open 2 sets of parentheses with variable in the **first** position:

$$v^2 + 1v - 72 \quad = \quad (\_\_\_\_ \quad)(\_\_\_\_ \quad)$$

Next, we need 2 integers whose **SUM** is ____ and whose **PRODUCT** is ______

To finish factoring, we need **2 numbers:**		
Product = -72 (______________ signs)	**Sum = +1** (opposite signs means _________________)	**Winner?**
$\pm \quad \cdot \mp \quad = -72$	$\pm \quad + (\mp \quad) = \mp$	
$\pm \quad \cdot \mp \quad = -72$	$\pm \quad + (\mp \quad) = \mp$	
$\pm \quad \cdot \mp \quad = -72$	$\pm \quad + (\mp \quad) = \mp$	
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$\pm \quad \cdot \mp \quad = -72$	$\pm \quad + (\mp \quad) = \mp$	

ANSWER: $v^2 + v - 72 \quad = \quad (\boldsymbol{v} \quad)(\boldsymbol{v} \quad)$ **or** $(\boldsymbol{v} \quad)(\boldsymbol{v} \quad)$

- **EXAMPLE:** Factor completely, if possible. [*Akst 16.2-15)

$$x^2 - x - 48$$

Open 2 sets of parentheses with variable in the **first** position:

$$x^2 - 1x - 48 \qquad = \qquad (\_\_\_\_ \qquad)(\_\_\_\_ \qquad)$$

Next, we need 2 integers whose **SUM** is ____ and whose **PRODUCT** is _____

To finish factoring, we need **2 numbers:**		
Product = -48 (opposite signs)	**Sum = -1 (opposite signs means SUBTRACT)**	**Winner?**
$\pm \quad \cdot \mp \quad = -48$	$\pm \quad + (\mp \quad) = \mp$	
$\pm \quad \cdot \mp \quad = -48$	$\pm \quad + (\mp \quad) = \mp$	
$\pm \quad \cdot \mp \quad = -48$	$\pm \quad + (\mp \quad) = \mp$	
$\pm \quad \cdot \mp \quad = -48$	$\pm \quad + (\mp \quad) = \mp$	
$\pm \quad \cdot \mp \quad = -48$	$\pm \quad + (\mp \quad) = \mp$	

D. Factor a Difference of Squares $x^2 - m^2$

- **EXAMPLE:** Factor. $s^2 - 81$ [R.4.59]

It's missing the middle term. Rewrite it with zero: $s^2 \_\_\_\_\_\_\_ - 81$

Open 2 sets of parentheses with variable in the **first** position:

$$s^2 + 0s - 81 \qquad = \qquad (\_\_\_\_ \qquad)(\_\_\_\_ \qquad)$$

To finish factoring, we need **2 numbers:**		
Product = -81 (opposite signs)	**Sum = 0 (opposite signs means SUBTRACT)**	**Winner?**
$\pm \quad \cdot \mp \quad = -81$	$\pm \quad + (\mp \quad) = \mp$	
$\pm \quad \cdot \mp \quad = -81$	$\pm \quad + (\mp \quad) = \mp$	
$\pm \quad \cdot \mp \quad = -81$	$\pm \quad + (\mp \quad) = \mp$	

ANSWER: $s^2 - 81 \qquad = \qquad (s \qquad)(s \qquad)$ or $(s \qquad)(s \qquad)$

Square Root is

Square Root is

Opposite Signs

Opposite Signs

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- FORMULA for the Difference of Squares: $\boldsymbol{x^2 - m^2 = (x - m)(x + m)}$

Works as long as the two terms are ____________________ ___________________ and they are ____________________________.

- **EXAMPLE:** Factor the binomial completely. [R.4.61]

$$64r^2 - 49$$

Perfect Square .

Perfect Square .

SUBTRACT

Perfect Square .

ANSWER: $64r^2 - 49$ = ($-$)($+$) or($+$)($-$)

- **EXAMPLE:** Factor the expression completely, if possible. [R.4-27]

$$100s^2 + 81t^4$$

Perfect Square .

Perfect Square .

ADD

(To factor, it MUST be _________________)

Perfect Square

$9t^2 \cdot 9t^2$

The **SUM** (addition) of perfect squares is always _____________ – it **DOES** ______ **FACTOR !!**

Sources Used:

1. MyLab Math for *Developmental Mathematics through Applications*, 1st Edition, Akst, Pearson Education Inc.
2. MyLab Math for *College Algebra with Modeling and Visualization*, 6th Edition, Rockswold, Pearson Education Inc.
3. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website https://archive.codeplex.com/?p=wabbit