Notes Section 2.4 – More Modeling with Functions

Lesson Objectives

1. Modeling with Linear Functions
2. Applications with Linear Models
3. Piecewise-Defined (Linear) Functions

A. Modeling with Linear Functions

To model a quantity that is changing at a constant rate with $f(x) = mx + b$, the following formula may be used:

$\boldsymbol{f(x)}$ **= (constant rate of change, changing amount, or rate)·*x* + (initial amount)**

- The constant **rate** of change (changing amount) corresponds to the **slope** of the graph of *f*.
- The **initial amount** (starting or fixed amount) corresponds to the ***y*-intercept**.

- **EXAMPLE:** Brand C soup contains 889 milligrams of sodium. Find a linear function *f* that computes the number of milligrams of sodium in *x* cans of Brand C soup. [2.2-28]

A. **Define your variables**. What information are we tracking?
Let *x* = **number of cans of soup**
Let *f*(*x*) = **milligrams of sodium in *x* total cans of soup**

B. **Identify the initial amount (when *x* is zero)**. Initial amount = **0**
Notice the initial (or starting) amount is not explicitly stated in this problem.
Because we are counting *x* cans of soup, there isn't fewer than **zero** cans of soup.
Also, with zero cans of soup, there's also **zero** sodium.
So, it's reasonable to assume that the initial amount must be zero.

C. **Identify the changing amount (rate)**. Changing amount = **889** (increasing)
Since each can of soup has **889** mg of sodium, then the total sodium *f*(*x*) INCREASES by that amount, 889 mg, for each can of soup *x*.
The changing amount, then is +889 mg sodium.

D. **Write the formula for the linear function.**
$f(x) =$ **(changing amount)·*x* + initial amount**
$\boldsymbol{f(x) = 889x + 0}$ or more simply: $\boldsymbol{f(x) = 889x}$

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B. Applications with Linear Models

- **EXAMPLE:** A 900-gallon tank is initially full of water and is being drained at a rate of 30 gallons per minute. Complete parts (a) through (d) below. [2.4.19]

(a) Write a linear function *W* that gives the gallons of water in the tank after *t* minutes.

A. **Define your variables:**

Let t = **time** in minutes

Let $W(t)$ = **gallons in tank** after t minutes

B. **Identify initial amount:**

When t = **0**, draining hasn't started yet, so tank is full at 900 gallons.

Initial amount = **900** (keywords: 900-gallon tank is initially **full**)

C. **Identify changing amount (rate):**

The tank is **draining** at a **rate** of **30** gallons per minute.

Changing amount = **– 30** (keyword: draining, which is **decreasing**)

D. **Write the formula:** $f(x)$ = (changing amount)·x + initial amount

So, the formula is: $\boldsymbol{W(t) = -30t + 900}$

(b) How much water is in the tank after 4 minutes?

Use the formula you just found to compute this.

After 4 minutes means t = **4**:

Use $\boldsymbol{W(t) = -30t + 900}$, with $t = 4$.

$W(4) = -30(4) + 900 = -120 + 900 = 780$

So, after 4 minutes, there are **780** gallons of water in the tank.

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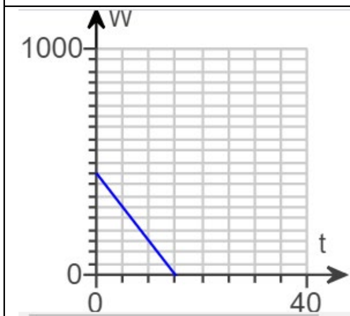
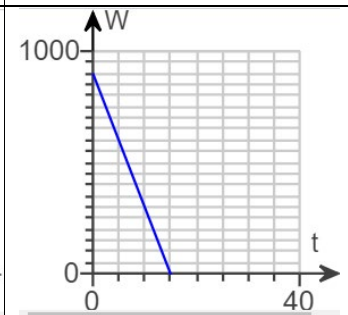
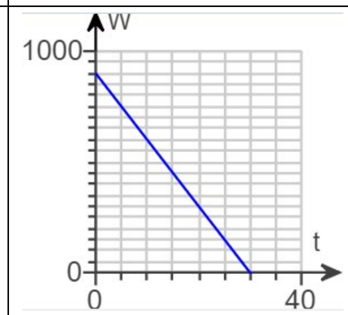
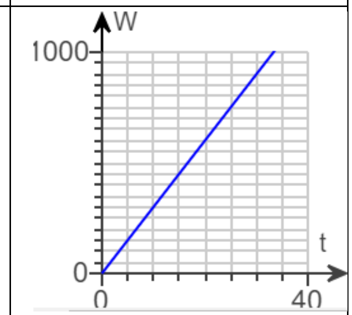
(c) **Graph function *W* and identify and interpret the intercepts. Choose the correct graph below.**

Recall from previous page that the function is: ***W*(*t*) = – 30*t* + 900**

A.	B.	C.	D.
			
Incorrect – initial amount is too low (should be 900)	Incorrect slope. $\frac{\mathbf{DOWN\ 900}}{\mathbf{RIGHT\ 15}} = -60$ Should be – 30	CORRECT	Incorrect –graph is increasing, which is not “draining”

The ***t*-intercept** is where *W*(*t*) (same thing as *y*) must be zero (that is *y* = 0).

Refer to the formula found earlier: $W(t) = -30t + 900$

Set *W*(*t*) equal to zero to find the *t*-intercept. $\mathbf{0} = -30t + 900$

Now, solve the equation for *t*.

$$\mathbf{-900 \qquad\quad -900}$$

$$\mathbf{-900 = -30t}$$

$$\frac{-900}{\mathbf{-30}} = \frac{-30}{\mathbf{-30}}t$$

So, the *t*-intercept is at $(\mathbf{30}, \mathbf{0})$. $\mathbf{30 = t}$

The ***W*-intercept** is where *t* = 0. Time = zero sec. (initial amount of water in tank).

Refer to the formula found earlier: $W(t) = -30t + 900$

The *W*-intercept (or the *y*-intercept, *b*) is: $(\mathbf{0}, \mathbf{900})$.

(d) **Find the domain of *W*.** (Use set-builder notation)

Remember that domain is *x*, but with this function, the domain involves *t* (time).

The tank starts draining at *t* = 0 minutes and it stops draining at *t* = 30 minutes.

So, the domain is: {t| **0 ≤ *t* ≤ 30** }. (the time is **between** zero and 30 minutes)

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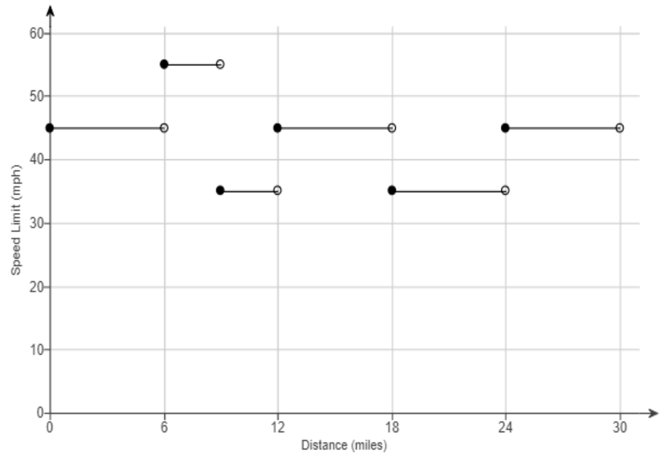
C. Piecewise-Defined (Linear) Functions

- **EXAMPLE:** The graph of $y = f(x)$ gives the speed limit y along a rural highway x miles from its starting point. [2.4.27]

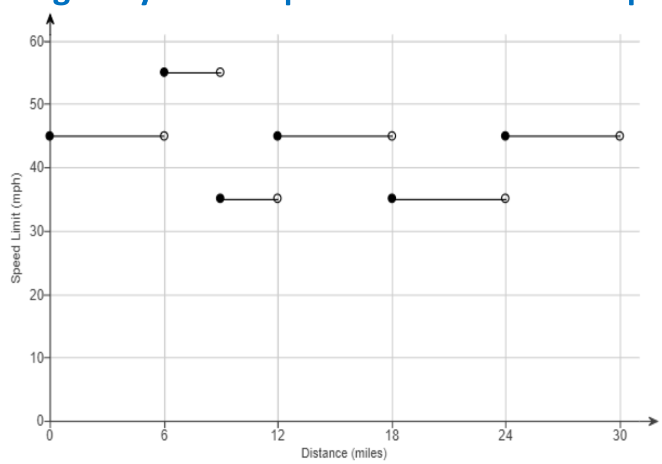
  (a) What are the maximum and minimum speed limits along this stretch of highway?
  (b) Estimate the miles of highway with a speed limit of 45 miles per hour.
  (c) Evaluate $f(9)$, $f(24)$, and $f(3)$.
  (d) At what x-values is the graph discontinuous? Interpret each discontinuity.

(a) What are the maximum and minimum speed limits along this stretch of highway?



Maximum speed limit = **55** mph Minimum speed limit = **35** mph

(b) Estimate the miles of highway with a speed limit of 45 miles per hour.



Estimated miles of highway with speed of 45 mph = **3** pieces × **6** miles = **18** miles

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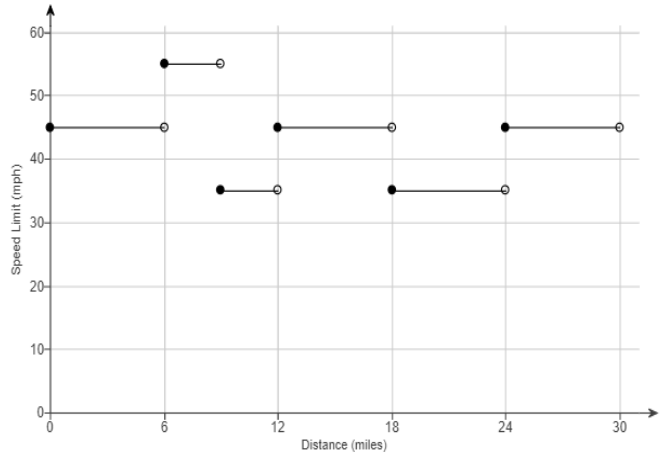
(c) Evaluate $f(9)$, $f(24)$, and $f(3)$.

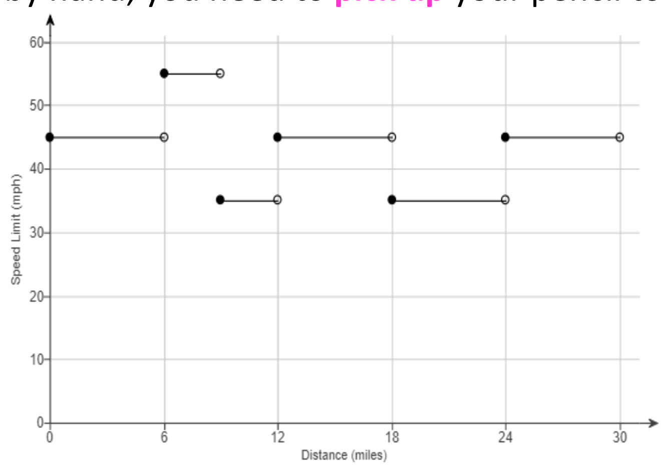


Use the **CLOSED** dot, not the open dot!

$f(9) = \mathbf{35}$	The ***y***-coordinate when *x* = 9
$f(24) = \mathbf{45}$	The ***y***-coordinate when *x* = 24
$f(3) = \mathbf{45}$	The ***y***-coordinate when *x* = 3

(d) At what x-values is the graph discontinuous? Interpret each discontinuity.

discontinuous: where there is a "**break**" in the graph, with another value **following** it. If you were to graph by hand, you need to **pick up** your pencil to continue the graph.



The graph is discontinuous at $x = \mathbf{6, 9, 12, 18, 24}$

(NOTE: it is NOT discontinuous at $x = \mathbf{30}$ because there is nothing following it)

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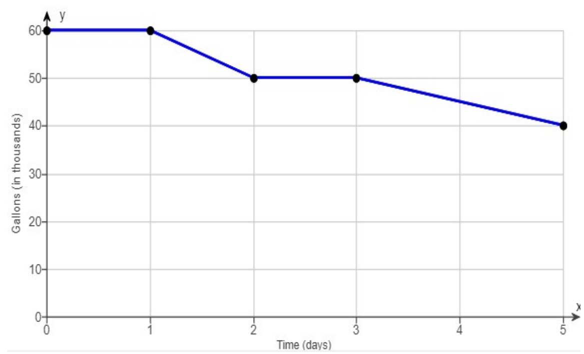
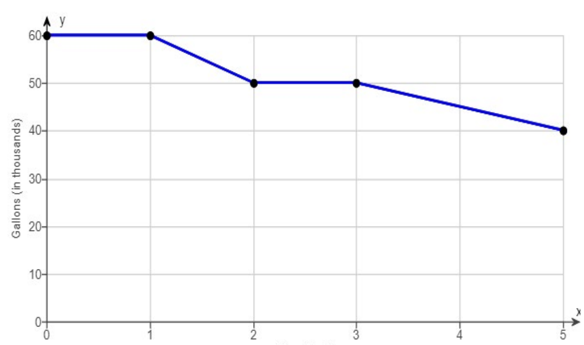
- **EXAMPLE:** The graph of $y = f(x)$ shows the amount of water y in thousands of gallons remaining in a swimming pool after x days. [2.4.32]

(a) Estimate the initial and final amounts of water in the pool. (Type a whole number.)
(b) When did the amount of water in the pool remain constant?
(c) Approximate $f(2)$ and $f(4)$.
(d) At what rate was water being drained from the pool when $3 \le x \le 5$?



(a) Estimate the initial and final amounts of water in the pool. (Type a whole number.)
Recall that the initial amount is the *y*-intercept, *b*, so find that location in the graph:
The initial amount of water in the pool was____60,000_______ gallons.
The final amount is the number of gallons seen at the end (far right) of the graph:
The final amount of water in the pool was ___40,000_____ gallons.

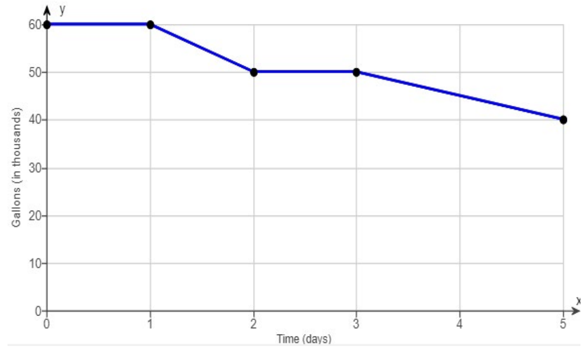
(b) When did the amount of water in the pool remain constant?
In the graph, a **constant** amount is horizontal because it isn't changing.



Choose the correct answer below

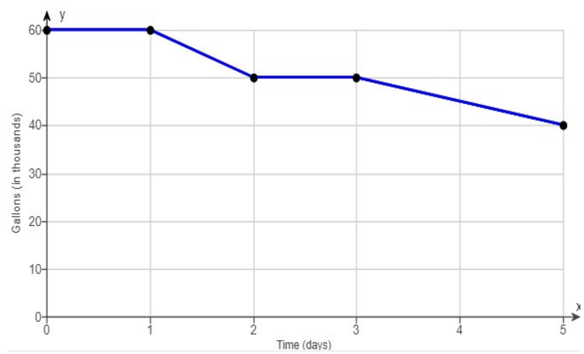
A. The amount of water in the pool was constant when $0 \le x \le 2$ and $2 \le x \le 4$.
B. The amount of water in the pool was constant when $0 \le x \le 1$ and $2 \le x \le 4$.
C. **The amount of water in the pool was constant when $0 \le x \le 1$ and $2 \le x \le 3$.**
D. The amount of water in the pool was constant when $0 \le x \le 2$ and $2 \le x \le 3$.

(c) Approximate $f(2)$ and $f(4)$.



$f(2)$ means find y when $x = 2$. (Use the graph) $f(2) =$ __50__
This means that after 2 days, there are about 50,000 gallons in the pool.
$f(4)$ means find y when $x = 4$. (Use the graph) $f(4) =$ __45__
This means that after 4 days, there are about 45,000 gallons in the pool.

(d) At what rate was water being drained from the pool when $3 \le x \le 5$?



When $3 \le x \le 5$, that's between days 3 and 5. The **rate** is its slope.

$$\text{rate} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\textbf{down 10,000 gallons}}{\textbf{right 2 days}} = \frac{\mathbf{-10,000}}{\mathbf{2}} = \mathbf{-5000}\,\frac{\text{gallons}}{\text{day}}$$

The water drained at a rate of __5000__ gallons per day when $3 \le x \le 5$.

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- **EXAMPLE:** For the following function find the values of

  (a) $G(-18)$ **(b)** $G(3)$ **(c)** $G(-1)$ [*Bittinger 2.2.97]

$$G(x) = \begin{cases} x-4, & \text{if } x \le -1 \\ x, & \text{if } x > -1 \end{cases}$$

These are **DOMAIN** restrictions

ALWAYS start here **first** with **inequality** part!

- Only **ONE** row "works" – the row you use **depends** on the **value of x** involved.
- Work backwards – test your value for x on the **RIGHT** side of each row

(a) $G(-18)$ means $x = -18$. **Test it** in the **domain** (inequality) part first.

$$G(x) = \begin{cases} x-4, & \text{if } x \le -1 \\ x, & \text{if } x > -1 \end{cases}$$

$-18 \overset{?}{\le} -1$ **TRUE** – use the **FIRST** row to plug in $x = -18$

$-18 \overset{?}{>} -1$ **FALSE** – do **NOT** use the second row for $x = -18$

Using the FIRST row of the function, $G(x) = x - 4$

$G(-18) = -18 - 4 = -22$

(b) $G(3)$ means $x = 3$. **Test it** in the **domain** (inequality) part first.

$$G(x) = \begin{cases} x-4, & \text{if } x \le -1 \\ x, & \text{if } x > -1 \end{cases}$$

$3 \overset{?}{\le} -1$ **FALSE** – do **NOT** use the first row for $x = -18$

$3 \overset{?}{>} -1$ **TRUE** – use the **SECOND** row to plug in $x = -18$

Using the SECOND row of the function, $G(x) = x$

$G(3) = 3$

(c) $G(-1)$ means $x = -1$. **Test it** in the **domain** (inequality) part first.

$$G(x) = \begin{cases} x-4, & \text{if } x \le -1 \\ x, & \text{if } x > -1 \end{cases}$$

$-1 \overset{?}{\le} -1$ **TRUE** – use the **FIRST** row to plug in $x = -1$

$-1 \overset{?}{>} -1$ FALSE – do NOT use the second row for $x = -1$

Using the FIRST row of the function, $G(x) = x - 4$

$G(-1) = -1 - 4 = -5$

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- **EXAMPLE:** The charges for renting a moving van are \$75 for the first 30 miles and \$5 for each additional mile. Assume that a fraction of a mile is rounded up.
  - **(i)** Determine the cost of driving the van 84 miles.
  - **(ii)** Find a symbolic representation for a function *f* that computes the cost of driving the van *x* miles, where $0 < x \leq 100$.

    (Hint: Express *f* as a piecewise-defined function) [*Lial 2.6-30]

[SOLUTION]

(Total = **84** miles)

<u>Price</u>

First **30** miles → → → → → \$**75**

Miles remaining:

84 – **30** = **54** \$**5** × **54** =

After first 30 miles, price is \$**270**

\$**5** each additional mile

Total price: \$**75** + \$**270**

= \$**345**

ANSWER:

C

$$f(x) = \begin{cases} 75 & \text{if } 0 < x \leq 30 \\ 75 + 5(x - 30) & \text{if } 30 < x \leq 100 \end{cases}$$

Use **2nd** row: $f(x) = 75 + 5(x - 30)$

Test it!

$30 < 84 \leq 100$

is **TRUE**

need to use **2nd** row because $x = \mathbf{84}$ miles

$$f(84) = 75 + 5(84 - 30)$$
$$= 75 + 5(54)$$
$$= 75 + 270 = \$345$$

A. \$6570; $f(x) = \begin{cases} 75x & \text{if } 0 < x \leq 30 \\ 75 + 5(x - 30) & \text{if } 30 < x \leq 100 \end{cases}$

B. \$645; $f(x) = \begin{cases} 75 & \text{if } 0 < x \leq 30 \\ 75 + 5(x + 30) & \text{if } 30 < x \leq 100 \end{cases}$

C. \$345; $f(x) = \begin{cases} 75 & \text{if } 0 < x \leq 30 \\ 75 + 5(x - 30) & \text{if } 30 < x \leq 100 \end{cases}$

D. \$645; $f(x) = \begin{cases} 75 & \text{if } 0 < x \leq 30 \\ 75 + 5(x - 30) & \text{if } 30 < x \leq 100 \end{cases}$

Sources Used:

1. Pearson MyLab Math *College Algebra with Integrated Review, 12th Edition*, Lial
2. Pearson MyLab Math *College Algebra with Modeling and Visualization, 6th Edition*, Rockswold
3. Pearson MyLab Math *Intermediate Algebra: Concepts and Applications, 10th Edition*, Bittinger