

1. In the movie, *Field of Dreams*, a voice from the sky says, "If you build it, he will come." The movie is referring to the main character and if he builds a baseball field, then the ghost of a baseball player will come and play on it.

The quote from the movie is a conditional statement. Conditional statements use the connective, **if...then**.

Using **p** and **q** as our statements, the compound statement "**if p then q**", means "**p implies q**"

We write it in symbol form as  $p \rightarrow q$

before "If" is the Antecedent or condition and "then" is the consequence

\* The words after "**IF**" give a condition the makes the statement after "**THEN**" to be true. This does not mean it is a cause and effect situation. For example,

If I pass this math class, then the sun will rise the next day.

In general, "**IF**" a condition is met, "**THEN**" this statement becomes is true.

In everyday language, we speak in conditional statements all the time, however, sometimes we use *if...then* and sometimes we do not. Sometimes the connective is "hidden" in everyday expressions or statements.

**EXAMPLE:** Rewrite the following statements as a conditional statement using **if...then**. Rearrange the words and add words if needed but keep the same meaning of the original statement.

1. All Marines love boot camp. If you are a Marine, then you love boot camp.
2. Big girls don't cry. If you are a big girl, then you don't cry.
3. Doing my homework will help me pass math class. If I do my homework, then it will help me pass math class.
4. Texting during class might cause me to fail math class. If I text during class then it might cause me to fail math class.
5. It's hard to study when I'm distracted. If I'm distracted, then it's hard to study.

## 2. If...then

Find the truth value,  $T$  or  $F$ , of compound statements using the connective, **if...then**. Let's use ordinary language and statements to create the rules for finding truth values. Then we will write the rule in symbol form.

**If...then** ( $\rightarrow$ ) is a connective that "**implies**", "**IF**" a condition is met, "**THEN**" this statement is true.

**For example**, "If it rains then the ballgame will be cancelled." The weather is the condition for the ball game happening or not.

**EXAMPLE:** "If I get paid, then I will give you twenty dollars." This statement implies that you get twenty dollars depending on the condition of me getting paid.

$p$  = If I get paid (the condition)

$q$  = then I will give you twenty dollars (the consequence or maybe a promise)

$p$	$q$	$p \rightarrow q$
paid?	give \$	If I get paid, I give money
yes- $T$	yes- $T$	$T$
yes- $T$	no- $F$	$F$
no- $F$	yes- $T$	$T$
no- $F$	no- $F$	$T$

Truth Table RULE for **IF...THEN**

$p$	$q$	$p \rightarrow q$
$T$	$T$	$T$
$T$	$F$	$F$
$F$	$T$	$T$
$F$	$F$	$T$

\* If...then has "order", left to right because of the arrow pointing left to right.

**EXAMPLE:** If  $p$  and  $r$  are a false statement and  $q$  is a true statement, find the truth value for the given compound statements.

$$1. \sim r \rightarrow q = T \rightarrow T = T$$

$$2. q \rightarrow p = T \rightarrow F = F$$

$$3. \sim p \rightarrow (q \wedge r) = T \rightarrow F = F$$

$F$

### TRUTH TABLES:

1. Draw a basic table
2.  $p$  and  $q$  are the statements and go on the left side of the table.

\* You need to know how many basic combinations of TRUE and FALSE will be in your truth table (left side/column). In other words, how many rows are in the truth table? Use the same formula from Chapter 2, when we looked for the number of subsets created from one given set:  $2^n$  where  $n$  was the number of elements in a set. Now  $n$  will be the number of statements.

The formula:  $2^n$       How many rows?  $2^2 = 4$        $2^3 = 8$

$pq$        $pqr$

3. In many mathematical problems, sometimes you have several steps to get to the final answer. Create a column for each step, the last column being your final answer.
4. We solve math problems using Order of Operations, so you must fill in truth tables in a particular order, too:
  - ① Parentheses
  - ② Not
  - ③ And, Or, If...Then
5. Refer back the basic truth table rules to follow the pattern and find your answers.

**EXAMPLE:** Using the four basic truth table rules, (rules for and, or, not, if...then) construct and complete a truth table for the following compound statements:

$$1. \sim q \rightarrow p$$

$$2. (p \vee q) \rightarrow (q \vee p)$$

Tautology is when the last column (final answer) in a truth table has all TRUE values

$$3. (\sim p \rightarrow q) \rightarrow p$$



### 3.3 Continued

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1)  $\sim q \rightarrow p$

p	q	$\sim q$	$\sim q \rightarrow p$
T	T	F	T
T	F	T	T
F	T	F	T
F	F	T	F

order  
does matter

$F \rightarrow T = T$

$T \rightarrow T = T$

$F \rightarrow F = T$

$T \rightarrow F = F$

2)  $(p \vee q) \rightarrow (q \vee p)$

p	q	$(p \vee q)$	$(q \vee p)$	$(p \vee q) \rightarrow (q \vee p)$
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T



Final answer  
is a tautology  
(All true)

### 3.3 continued

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3)  $(\sim p \rightarrow q) \rightarrow p$

$p$	$q$	$\sim p$	$(\sim p \rightarrow q)$	$(\sim p \rightarrow q) \rightarrow p$
T	T	F	$F \rightarrow T = T$	$T \rightarrow T = T$
T	F	F	$F \rightarrow F = T$	$T \rightarrow T = T$
F	T	T	$T \rightarrow T = T$	$T \rightarrow F = F$
F	F	T	$T \rightarrow F = F$	$F \rightarrow F = T$

↑ final answer

4)  $r \rightarrow (p \wedge \sim q)$

$p$	$q$	$r$	$\sim q$	$(p \wedge \sim q)$	$r \rightarrow (p \wedge \sim q)$
T	T	T	F	F	$T \rightarrow F = F$
T	T	F	F	F	$F \rightarrow F = T$
T	F	T	T	T	$T \rightarrow T = T$
T	F	F	T	T	$F \rightarrow T = T$
F	T	T	F	F	$T \rightarrow F = F$
F	T	F	F	F	$F \rightarrow F = T$
F	F	T	T	F	$T \rightarrow F = F$
F	F	F	T	F	$F \rightarrow F = T$

↑ final answer