INTEREST

If we borrow an amount of money today, we will repay a larger amount later. The increase in value is known as interest. The money *gains value over time.*

The amount of a loan or a deposit is called the principal (the amount of money you begin with). The interest is usually computed as a *percent of the principal.* This percent is called the rate of interest or interest rate and is assumed to be an annual rate unless otherwise stated.

I. Simple Interest: Interest calculated only on the **principal** amount.

The formula for **simple interest** is:

$$I = Prt$$

I = simple interest **P** = principal **r** = annual interest rate (decimal) **t** = time (in years)

NOTE: Be careful if time is given in months. Formula is based on **years**, so convert: $\frac{\text{months}}{12}$

EXAMPLE: Find the simple interest paid in order for you to borrow $4800 for 6 months at 7%.

P = $4800 r = .07 t = $\frac{6}{12} = \frac{1}{2}$

Interest = $168

Using formula for **simple interest**: $I = Prt = 4800(.07)(6/12)$

(FYI) You would have to repay (round to the nearest cent) $4800 + 168 = \$4968$ to repay

EXAMPLE: Find the simple interest (round to the nearest cent) for the following:
Principal: $8000 Rate: 6% Time in Months: 3

P = 8000 r = .06 t = $\frac{3}{12} = \frac{1}{4}$

Using formula for **simple interest**: $I = Prt = 8000(.06)(1/4) =$ $120 Interest

(FYI) You would have to repay (round to the nearest cent) $8000 + 120 = \$8120$ to repay

II. Future and Present Value for Simple Interest:

The total amount repaid, ***A***, when you borrow money is called the **maturity value** or the **value** of the loan. We will refer to it as the future value or future amount

The original principal (amount of money originally borrowed), ***P***, can also be thought of as present value.

P = principal, the present value of your money. Amount of money you (deposit) borrow or begin with.

A = the future value of your money. The total amount of money you will pay back.

The formula for the future value for **simple interest** is:

$$A = P(1 + rt)$$

A = future value of money
r = annual interest rate (decimal)
P = principal or present value of money borrowed
t = time (in years)

EXAMPLE: Find the future value of $460 in 8 months, if the annual interest rate is 12%.

P = 460 r = .12 t = 8/12 = 2/3

Using formula for future value for **simple interest**: $A = P(1+rt) = 460(1+.12(2/3))$

You would have to repay (round to the nearest cent) $496.80

EXAMPLE: Find the future value of the deposit if the account pays simple interest. Round to the nearest cent. $1920 at 2.3% for 4 years

P = 1920 r = .023 t = 4

Using formula for **future value for simple interest**: $A = P(1+rt) = 1920(1+.023*4)$

You would have to repay (round to the nearest cent) $2096.64

III. Future and Present Value for Compound Interest:

Interest paid on principal plus any previously earned interest is called Compound interest. After a certain period, the interest earned so far is *credited* (added) to the account, and the sum (principal plus interest) then earns interest during the next period. Interest can be credited to an account at time intervals other than 1 year. For example, it can be done semiannually, quarterly, monthly, or daily. This time interval is called the Compounding period.

The formula for future value for **compounded interest** is: $A = P\left(1 + \frac{r}{n}\right)^{nt}$

A = future value of money **P** = principal or present value of money deposited
r = annual interest rate **n** = number of periods (per year) **t** = time (in years)

NOTE: Be careful if time is given in months. Formula is based on **years**, so convert: $\frac{\text{months}}{12}$

compounding period	annually	semi-annually	quarterly	monthly	daily
number of periods (*n*)	1	2	4	12	365 or 360

EXAMPLE: Find the future value of $8560 at 4% compounded quarterly for 8 years.

P = 8560 r = .04 n = 4 t = 8

Using formula for **future** value for **compound interest**: $A = 8560\left(1 + \frac{.04}{4}\right)^{4*8}$

The future value will be: $11769.49 (round to the nearest cent)

EXAMPLE: What amount must be deposited today, at 5% compounded monthly, so that it will be $18,000 in 20 years?

"deposited today" means **present value** (P) from compound interest formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$

To get P by itself, simply divide both sides by $\left(1 + \frac{r}{n}\right)^{nt}$ to get: $P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}}$

This is the formula to find the **present** value for **compound interest**.

A = 18,000 r = .05 n = 12 t = 20

The deposit today should be $6635.60 (round to the nearest cent)

$\frac{18000}{\left(1 + \frac{.05}{12}\right)^{12*20}}$

IV. Effective annual yield

Banks, credit unions, and other financial institutions often advertise two rates: first, the actual annualized interest rate, or **nominal rate** (the "named" or "stated" rate), and second, the rate that would produce the same final amount, or future value, at the end of 1 year if the interest being paid were simple rather than compound. This is called the "effective rate", or more commonly the **effective annual yield**. (It may be denoted **APY** for **"annual percentage yield."**)

A nominal interest rate of r (as a decimal), compounded n times per year, is equivalent to the following **effective annual yield.**

$$Y = \left(1 + \frac{r}{n}\right)^n - 1$$

NOTE: Be sure to multiply by 100 to convert this to a percent.

compounding period	**annually**	**semi-annually**	**quarterly**	**monthly**	**daily**
number of periods (n)	**1**	**2**	**4**	**12**	**365** or 360

EXAMPLE: Suppose a savings and loan pays a nominal rate of 3.5% on savings deposits. Find the effective annual yield if interest is compounded daily. Assume that the year is not a leap year. (Round to the nearest thousandth of a percent as needed.)

$r = .035$ $n = 365$ effective annual yield formula: $Y = \left(1 + \frac{.035}{365}\right)^{365} - 1$

$.0356179711 \times 100$

$= 3.562\%$

EXAMPLE: Find the effective annual interest rate for the given nominal interest rate. Round your answers to the nearest hundredth of a percent. 4.1% compounded monthly

$r = .041$ $n = 12$ effective annual yield formula: $Y = \left(1 + \frac{.041}{12}\right)^{12} - 1$

$.041779308 \times 100$

$= 4.178\%$