Notes Section 5.3 – Exponential Functions and Their Representation

Lesson Objectives

1. The Basics of an Exponential Function
2. Graph an Exponential Function (use calculator!)
3. Evaluating a function using the Natural base, *e*
4. Applications Involving Compound Interest
    a. Regular compounding
    b. Continuous compounding

A. The **Basics** of an **Exponential Function**

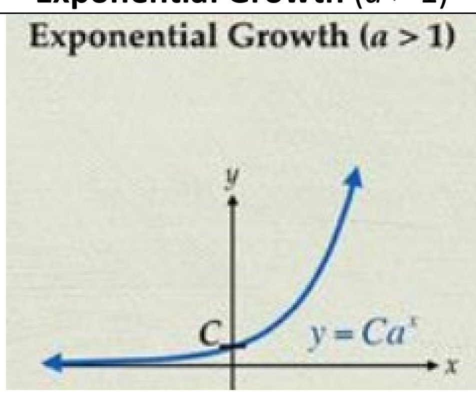
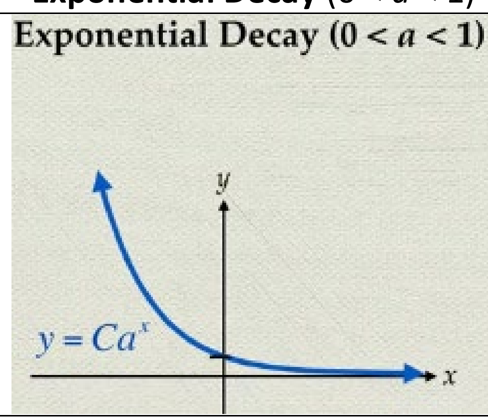
An **exponential function** is of the form: $f(x) = \boldsymbol{Ca^x}$,

Where *C* is called the ***initial amount*** (starting amount) and is the *y*-intercept, and

a is called the **growth factor** (if $a > 1$) or the **decay factor** (if $0 < a < 1$)

B. The **Graph** of an **Exponential Function**

The graph on an exponential function has two general types, depending on whether it is growth or decay.

Exponential Growth ($a > 1$)	**Exponential Decay** ($0 < a < 1$)
	
INCREASING	**DECREASING**

(go on to the next page)

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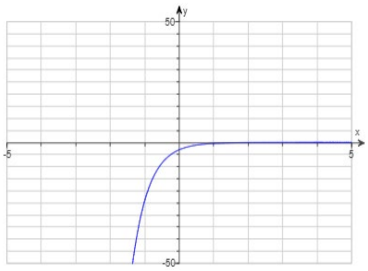
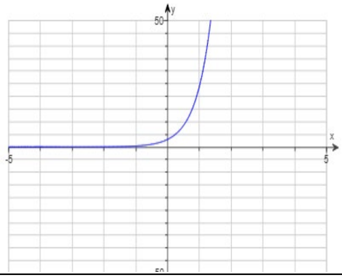
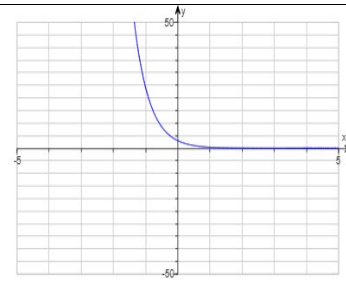
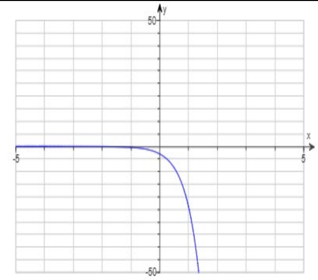
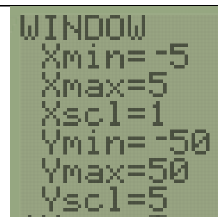
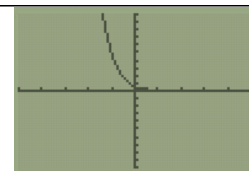
- **EXAMPLE:** Sketch a graph of $y = f(x)$. $f(x) = 3\left(\frac{1}{8}\right)^x$ [5.3.61]

A.	**B.**	**C.**	**D.**
			

$f(x) = 3\left(\frac{1}{8}\right)^x$. The base (1/8) is between 0 and 1, so this is a **DECAY** function (decreasing).

You can easily verify the correct graph using your graphing calculator – PLEASE do this!!

 1. Press **Y=** button and enter your function.	 2. Press **WINDOW** button to adjust graph settings. Then press **GRAPH** button.	 3. This is an exponential decay function. Correct answer is: **C**

C. Evaluating a Function using the **Natural** base, *e*

The **natural** base, ***e***, is an irrational number (similar to pi, or π).

The value of it is $e \approx 2.718281828\ldots$

To do graphs and/or calculations with the **natural** base ***e***, you can use your calculator.

The button for *e* can be found in two places:

 above the LN key (used for e^x) or  above the divide key

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- **EXAMPLE:** Approximate *f*(*x*) to four decimal places. [5.3.49]

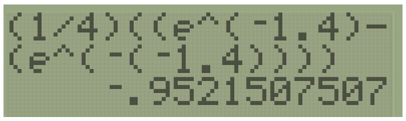
$$f(x) = \frac{1}{4}(e^x - e^{-x}) \qquad x = -1.4$$

Use your **calculator** for this one!

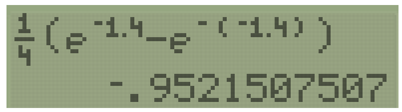
There are 2 ways to do this:

direct substitution (w/parentheses) or the "go to the STO→" method

Direct substitution: $f(-1.4) = \left(\frac{1}{4}\right)\left(\left(e^{(-1.4)}\right) - \left(e^{(-(-1.4))}\right)\right)$

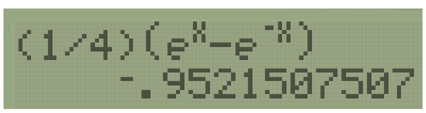


or

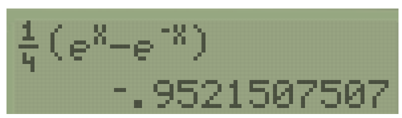


rounds to **– 0.9522**

"**Go to the STO→**" method (plug in – 1.4 for *x* in calculator)



or



same answer

- **EXAMPLE:** A sample of 250 grams of a radioactive substance decays according to the function $A(t) = 250e^{-0.046t}$, where *t* is the time in years. How much of the substance will be left in the sample after 30 years? Round your answer to the nearest whole gram.

[*Lial 10.6-30, Q10]

Define your variables.

A(*t*): **amount of substance** and *t*: **time in years**

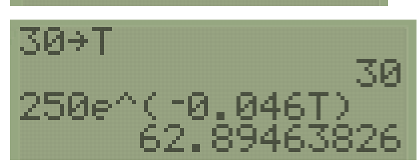
To find out how much substance is left after 30 years (*t* = 30), calculate ***A*(30)**, which simply means plug in $t = 30$ into the given function formula $A(t) = 250e^{-0.046t}$ (Use calculator).

Direct Substitution:

or

"**Go to the STO→**"

or



Answer: after 30 years, the amount of substance is approximately **63** grams.

D. Applications Involving Compound Interest

1. Regular Compoundings Formula: $$A = P\left(1+\frac{r}{n}\right)^{nt}$$

Where: ***A*** is the future value, or final value (**amount**)

P is the **principal** (initial amount, starting amount, deposit, etc.)

r is the interest **rate**, converted from percent to decimal (just divide by 100)

n is the number of interest-compoundings per year:

n **= 1**	***n*** **= 2**	***n*** **= 4**	***n*** **= 12**	***n*** **= 365**
annually or yearly	semi-annually	quarterly	monthly	daily

t is the time in years

- **EXAMPLE:** Use the compound interest formula to determine the final value of the given amount. $1,000 at 15% compounded semiannually for 8 years [5.3-21]

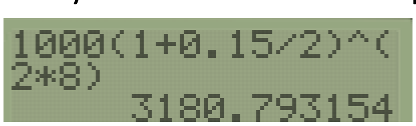
P = 1000 **r = 0.15** **n = 2** **t = 8**

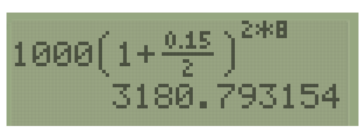
$1000 Principal **rate is 15%** **semiannually** **8 years**

Using the Compound Interest formula: $$A = P\left(1+\frac{r}{n}\right)^{n\,t}$$

Substitute your given information: $$A = 1000\left(1+\frac{0.15}{2}\right)^{2\cdot 8}$$

Use your calculator to compute the final amount:





Because it's money, it rounds to 2 decimal places: $3180.79 **Answer**

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2. Continuous Compounding Formula: $$A = Pe^{rt}$$

Where: ***A*** is the future value, or final value (**amount**)

P is the **principal** (initial amount, starting amount, deposit, etc.)

r is the interest **rate**, converted from percent to decimal (just divide by 100)

t is the time in years

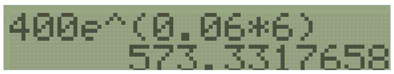
- **EXAMPLE:** Use the compound interest formula to determine the final value of the given amount. $400 at 6% compounded continuously for 6 years [5.3.103]

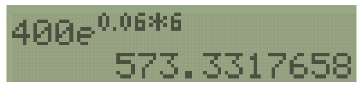
  P **= 400** ***r*** **= 0.06 continuous compounding** ***t*** **= 6**

  Using the Continuous Compound Interest formula: $\boldsymbol{A = Pe^{rt}}$

  Substitute your given information: $A = 400e^{0.06*6}$

  Use your calculator to compute the final amount:

  Because it's money, it rounds to 2 decimal places: **$573.33** **Answer**

Sources Used:

1. MyLab Math for *Algebra for College Students*, 8th Edition, Lial, Pearson Education Inc.
2. MyLab Math for *College Algebra with Modeling and Visualization*, 6th Edition, Rockswold, Pearson Education Inc.
3. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website https://archive.codeplex.com/?p=wabbit