# Notes Section 1.3 A – Interval Notation

## Lesson Objectives

1. Writing and Graphing Inequalities
2. Interval Notation – Overview
3. Convert among inequality, graph, and interval notation (any variation)
   - **Non-ending** interval (involves positive or negative infinity)
4. Compound Inequalities
   - **Open or Closed** interval (in BETWEEN 2 endpoints – infinity is NOT involved)
   - **Two non-ending intervals** together (2 endpoints going AWAY from each other)

## A. Writing & Graphing Inequalities

- Warm-Up

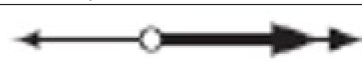
**Directions:** Compare. Write <, >, or = in the blank.

1) $-3$ _____ $2$
2) $6.5$ _____ $6.3$
3) $\frac{1}{2}$ _____ $\frac{3}{4}$
4) $0.25$ _____ $\frac{1}{4}$

- Graphing Inequalities

To write and graph inequalities, you must connect words with symbols.

Look at the information in the table below.

| Symbol | Words | Graph |
|---|---|---|
| $<$ | "is less than" |  Shades to the LEFT<br>Endpoint EXCLUDED |
| $\leq$ | "is less than or equal to"<br>"at most"<br>"no more than" |  Shades to the LEFT<br>Endpoint INCLUDED |
| $>$ | "is greater than" |  Shades to the RIGHT<br>Endpoint EXCLUDED |
| $\geq$ | "is greater than or equal to"<br>"at least"<br>"no less than" |  Shades to the RIGHT<br>Endpoint INCLUDED |

# Notes Section 1.3 A – Interval Notation

## B. Interval Notation - Overview

- Domain and range do not always involve a **discrete** (countable) number of elements.
- **Interval notation** is used for **infinite** elements (not countable).
- Set-builder notation (inequality) can convert to interval notation, and vice-versa.

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**Interval Notation** is describing how the number line is shaded (or “painted”), written as:

- the smaller value first, and
- the larger value second.

It’s identifying the starting and ending points of the interval.
All points in between are shaded (included).

When using interval notation, the starting and ending points also contain a symbol:

**(** or **)** means “not included” or “open”

**[** or **]** means “included” or “closed”

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- General format for Interval Notation

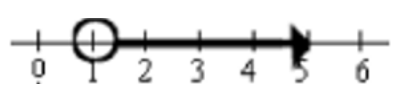
The general format for Interval Notation looks like this:

**( or [ smaller number, larger number ] or )**

Note that the smaller number could be $-\infty$, or the larger number could be $+\infty$.
Remember to ALWAYS use PARENTHESES with either positive or negative INFINITY.

**Non-ending interval:** $(a, \infty)$ is interpreted as $x > a$, where *a* is NOT included and infinity is always expressed as being “open” (not included).

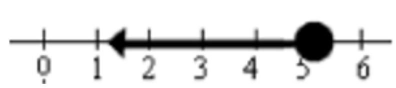
Example: $x > 1$ graph:




Interval Notation: $(1, \infty)$

**Non-ending interval:** $(-\infty, b]$ is interpreted as $x < b$, where *b* IS included and again, infinity is always expressed as being “open” (not included).

Example: $x \leq 5$ graph:




Interval Notation: $(-\infty, 5]$

# Notes Section 1.3 A – Interval Notation

## C. Convert among Inequality, Graph, and Interval Notation (any variation)

- Non-ending interval (involves positive or negative infinity)
- **EXAMPLE:** Write the set $\{x|x < 5\}$ in interval notation. [2.3.1]
  [SOLUTION] The expression there at the end is read as:

  "The set of all values *x*, such that *x* is less than 5."

That expression $\{x|x < 5\}$ is written in **set-builder notation**.

For ease of use, you can IGNORE the brackets and the initial "$x|$" part.
Focus just on the INEQUALITY part: $x < 5$

- Ensure the variable is on the **LEFT** of the symbol (it IS) $x < 5$
  - If it isn't, you need to **REVERSE** the inequality (not needed here): $x < 5$
  - By far the most common error students make – forgetting to reverse it.
- $x < 5$ is read as "*x* is less than 5"
- First, look at the **graph** of this inequality:


- What is the SMALLER number? $-\infty$
  - Is it INCLUDED or EXCLUDED? EXCLUDED
  - How do you know? Infinity goes on forever
  - Use BRACKET or PARENTHESIS? PARENTHESIS

- What is the LARGER number? 5
  - Is it INCLUDED or EXCLUDED? EXCLUDED
  - How do you know? Symbol is < (missing equals sign)
  - Use BRACKET or PARENTHESIS? PARENTHESIS

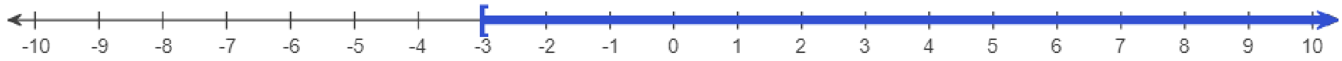
- Interval notation is always done as: **[** or **(** **smaller number , larger number )** or **]**

ANSWER: Given the set-builder notation, $\{x|x < 5\}$

In interval notation: $(-\infty, \mathbf{5})$

# Notes Section 1.3 A – Interval Notation

- **EXAMPLE:** Write the interval notation for the graph. [*Beecher JIT.6.8]




[SOLUTION]

- What is the SMALLER number? $-3$
  - Is it INCLUDED or EXCLUDED? INCLUDED
  - How do you know? It shows a BRACKET

- What is the LARGER number? $+\infty$
  - Is it INCLUDED or EXCLUDED? EXCLUDED
  - How do you know? Infinity goes on forever
  - Use BRACKET or PARENTHESIS? PARENTHESIS

- Interval notation is always done as: **[** or **(** **smaller number , larger number** **)** or **]**

In interval notation: $[-3, \infty)$

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- **EXAMPLE:** Write the interval notation for the graph. [*Beecher JIT.6.10]


[SOLUTION]

- What is the SMALLER number? $-\infty$
  - Is it INCLUDED or EXCLUDED? EXCLUDED
  - How do you know? Infinity goes on forever

- What is the LARGER number? $4$
  - Is it INCLUDED or EXCLUDED? EXCLUDED
  - How do you know? It shows a PARETHESIS

- Interval notation is always done as: **[** or **(** **smaller number , larger number** **)** or **]**

In interval notation: $(-\infty, 4)$

(go on to the next page)

# Notes Section 1.3 A – Interval Notation

- **EXAMPLE:** Write the inequality in interval notation. $\{x| - 3 \leq x\}$ [1.3.5]

The expression there at the end is read as:

"The set of all values *x*, such that negative 3 is less than or equal to *x*."

That expression $\{x| - 3 \leq x\}$ is written in **set-builder notation**.

For ease of use, you can IGNORE the brackets and the initial "$x|$" part.

Focus just on the INEQUALITY part: $-3 \leq x$

- Ensure the variable is on the **LEFT** of the symbol (it's NOT). $-3 \leq x$
  - If it isn't, you need to **REVERSE** the inequality: $x \geq -3$
  - Keep the "nose" of the inequality pointed to the same object.
  - By far the **most common error students make – forgetting to reverse it**.
- $x \geq -3$ is read as "*x* is greater than or equal to negative 3"
- First, look at the **graph** of this inequality:


- What is the SMALLER number? – 3
  - Is it INCLUDED or EXCLUDED? INCLUDED
  - How do you know? Symbol is ≥ which has equals
  - Use BRACKET or PARENTHESIS? BRACKET

- What is the LARGER number? $+\infty$
  - Is it INCLUDED or EXCLUDED? EXCLUDED
  - How do you know? Infinity goes on forever
  - Use BRACKET or PARENTHESIS? PARENTHESIS

▪ Interval notation is always done as: **[** or **(** **smaller number , larger number )** or **]**

ANSWER: Given the set-builder notation, $\{x| - 3 \leq x\}$

In interval notation: $[\mathbf{-3}, \infty)$

# Notes Section 1.3 A – Interval Notation

## D. Compound Inequalities

- Open or Closed interval (in BETWEEN two endpoints – infinity is NOT involved)

- **EXAMPLE:** Write the interval notation for the set $\{x| -10 < x < 10$ [*Beecher JIT.6.6]

  [SOLUTION]

  That expression $\{x| -10 < x < 10\}$ is written in **set-builder notation**.

  For ease of use, you can IGNORE the brackets and the initial "$x|$" part.
  Focus just on the INEQUALITY part: $-10 < x < 10$

  This is one type of **COMPOUND INEQUALITY**, because it involves more than one endpoint. Notice that the variable is **IN BETWEEN** the two endpoints.

  There is a common structure with this "in-between" inequality:

  - The SMALLER number is always on the LEFT
  - The LARGER number is always on the RIGHT
  - (This mimics how they truly are on the number line as well.)
  - BOTH symbols are pointing LEFT (less-than type)
    - The symbols can use any combination of $<$ or $\leq$.

Returning to the inequality: $-10 < x < 10$

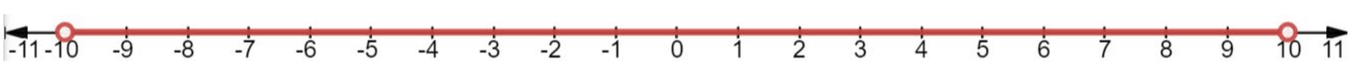
Pull these apart into 2 separate inequalities: $-10 < x$ and $x < 10$

Let's graph these TOGETHER on the same number line.

$-10 < x$ REVERSE ***before*** you graph it: $-10 < x$ reverses to $x > -10$

You need to graph TOGETHER these 2 inequalities: $x > -10$ and $x < 10$

– 10 (excluded) shading RIGHT and +10 (excluded) shading LEFT




ANSWER: The set $\{x| -10 < x < 10$ in interval notation is:
$\mathbf{(-10, 10)}$

# Notes Section 1.3 A – Interval Notation

You can graph compound inequalities MUCH FASTER than the previous example, by streamlining the process. Let's look at another example:

- **EXAMPLE:** Write the inequality in interval notation. $-3 < x \leq 2$ [1.3.3]

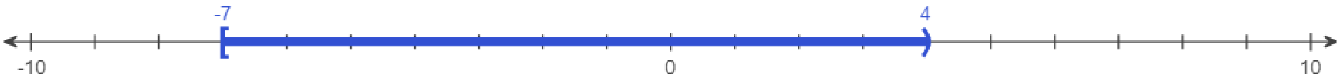
[SOLUTION]

The smaller number – 3 has the symbol < (– 3 excluded = PARENTHESIS)

The larger number 2 has the symbol ≤ (2 included = BRACKET)

**ANSWER**: The inequality $-3 < x \leq 2$ in interval notation is $\mathbf{(-3, 2]}$

- **EXAMPLE:** Write the inequality in interval notation. [1.3.9]




[SOLUTION]

This time the inequality is not given – only the graph is given. This is even easier!

- Smaller number = – 7 BRACKET (included)
- Larger number = 4 PARENTHESIS (excluded)

**ANSWER**: That inequality as graphed in interval notation is $\mathbf{[-7, 4)}$

- Two non-ending intervals together (2 endpoints going AWAY from each other)

The other type of COMPOUND INEQUALITY is where the shading starts at 2 endpoints and they go AWAY from each other. A big giveaway is that you'll see the word **"OR"** or the "UNION" symbol, ∪.

- **EXAMPLE:** Express the set in interval notation. [1.3.7]

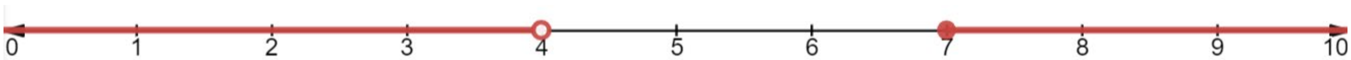
$\{x|x < 4 \text{ or } x \geq 7\}$

[SOLUTION]

There are 2 separate inequalities separated by **"or"**: $x < 4$ $x \geq 7$

Make sure the SMALLER endpoint is written on the LEFT, the LARGER on the RIGHT.

Graph these 2 inequalities TOGETHER on the SAME number line:




| **Left piece** | **Right piece** |
|---|---|
| Smaller number: $-\infty$ (parenthesis) | Smaller number: 7 (bracket) |
| Larger number: 4 (parenthesis) | Larger number: $+\infty$ (parenthesis) |
| Interval notation: $(-\infty, \mathbf{4})$ | Interval notation: $[\mathbf{7}, \infty)$ |

MERGE these two pieces TOGETHER, remove the word "or" and use "UNION" symbol **(**∪**)**:

ANSWER: The interval notation for $\{x|x < 4 \text{ or } x \geq 7\}$ is $(-\infty, 4) \cup [7, \infty)$

# Notes Section 1.3 A – Interval Notation


Sources used:

1. Desmos online graphing calculator, located at www.desmos.com/calculator
2. Pearson *MyLab Math*: *College Algebra with Integrated Review*, *5th Edition*, Beecher
3. Pearson *MyLab Math*: *College Algebra with Modeling and Visualization, 6th Edition*, Rockswold