Notes Section 1.1 – Numbers, Data, and Problem Solving

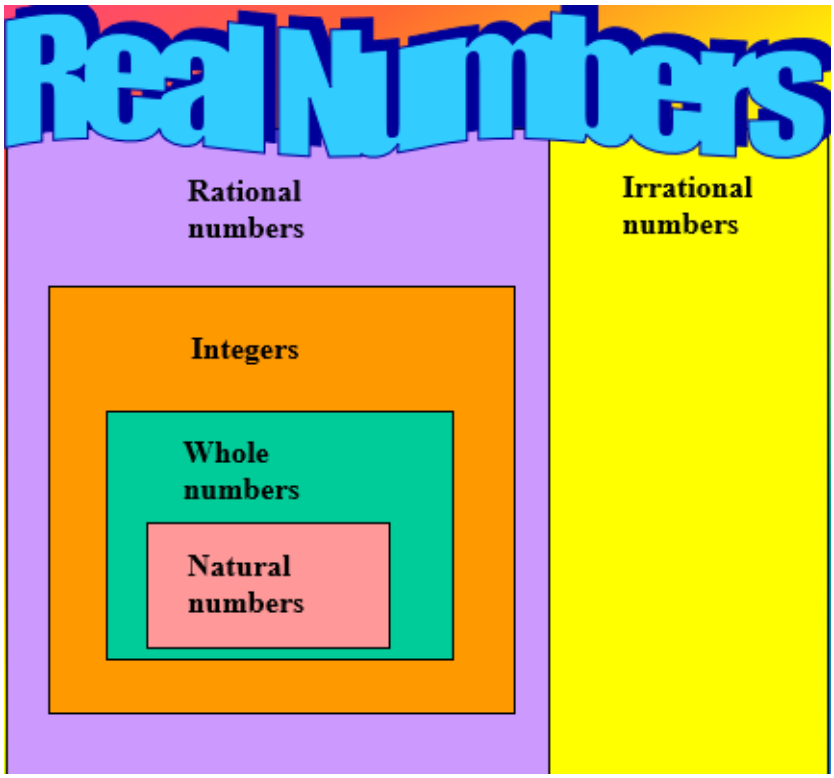
Lesson Objectives

1. Classify real numbers
2. Convert a number from standard notation to scientific notation
3. Convert a number from scientific notation to standard notation
4. Use a calculator for computations involving scientific notation
5. Calculate Percent Change

A. Classifying Real Numbers

1. Natural numbers – the counting numbers {1, 2, 3,…, ∞}
   - Includes any number that can **simplify** to a natural number
     - Examples: $\frac{4}{2}, \sqrt{9}, \frac{25}{5}, \sqrt{49}$, etc.
2. Whole numbers – include zero and the natural numbers {0, 1, 2, 3, …, ∞}
   - Think "wh0le" – write the "o" in the word "whole" with a zero instead
   - Includes any number that can **simplify** to a whole number
3. Integers – whole numbers and opposites (negatives) {–∞, …, –3, –2, –1, 0, 1, 2, 3, …, ∞}
   - Sort of like "mile markers" on a number line; often used for reference in graphing
   - Includes any number that can **simplify** to an integer
     - Examples: $-\sqrt{4}, -\frac{33}{11}, -\sqrt{121}, -\frac{48}{6}$, etc., or examples from **natural** numbers
   - CAUTION! Just because a number is *negative* doesn't necessarily mean it's an integer!
   - They're "pretty" numbers. With integers, remember "ahh!," not "eww!"
4. Rational numbers – can be written as a ratio (fraction, denominator not zero)
   - Includes all **integers**
   - Includes some (not all!) of the numbers *in between* the integers
   - Includes **all** forms of **fractions** (that don't contain irrational numbers), positive or negative
     - Proper fraction (smaller/larger) – Examples: $-\frac{4}{7}, \frac{2}{5}$, etc.
     - Improper fraction (larger/smaller) – Examples: $-\frac{123}{6}, \frac{1245}{1108}$, etc.
     - Mixed number – Examples: $-7\frac{3}{11}, 29\frac{17}{67}$, etc.
   - Includes some (not all) types of decimal numbers, positive or negative
     - All **terminating decimals** – they stop eventually
       - Examples: -12.93, -6.1, -0.4556, 0.23, 4.51, 67.88421, etc.
     - All **repeating decimals** – go on forever, but in a repeatable pattern
       - Examples: $-0.\overline{4}, -0.\overline{26}, -0.14\overline{67}, 0.\overline{43}, 2.\overline{7}$, etc.

5. Irrational numbers – number that is not rational (can't be written as a fraction)
   - Includes any **root that doesn't "work," or simplify** to a rational number, positive or negative
     - Examples: $-\sqrt{88}, -\sqrt{50}, -6\sqrt{20}, -\sqrt[3]{12}, \sqrt[4]{2}, \sqrt{15}, 3\sqrt{62}$, etc.
   - Includes other numbers, like π, e the golden ratio (called "phi" or φ), and anything that includes them, positive or negative.
     - Examples: $-\frac{\pi}{2}, \frac{3\pi}{4}, e^2, 100e^{-0.02293}$, etc.
   - Includes decimals that are both non-terminating AND non-repeating, positive or negative
     - Examples: 0.1010010001…, or 0.101101110…, etc.



A Real number has more than one "nested" category:
Natural-Whole-Integer-Rational-Real
Whole-Integer-Rational-Real
Integer-Rational-Real
Rational-Real
Irrational-Real

- **EXAMPLE:** Classify each real number as one or more of the following: natural number, whole number, integer, rational number, or irrational number. [1.1.7]

  $\frac{6}{1}, -\frac{6}{7}, \sqrt{11}, 0.\overline{25}, \pi$

  Let's go through this list of numbers one at a time, considering all the categories.

Number	**Does it Simplify?**	**Categories of Numbers it's Classified**
$\frac{6}{1}$	simplifies to 6	**Natural, whole, integer, rational, real**
$-\frac{6}{7}$	doesn't simplify	Fraction = **rational, real**
$\sqrt{11}$	doesn't simplify	Root that doesn't simplify = **irrational, real**
$0.\overline{25}$	doesn't simplify	Repeating decimal = **rational, real**
π	doesn't simplify	Has π = **irrational, real**

- **EXAMPLE:** Choose to which group of sets the following number belongs. Be sure to account for ALL sets. $-\frac{1}{3}$ Choose the correct answer below. [1.1.1]

  A. real numbers, rational numbers
  B. real numbers, irrational numbers
  C. real numbers, rational numbers, natural numbers
  D. rational numbers, natural numbers, integers
  E. irrational numbers, natural numbers

  Common error: "Since $-\frac{1}{3}$ is negative, then it is an integer." (INCORRECT)
  Remember that an integer can be either positive or negative, but they are "pretty" numbers, not fractions like this one. So, Answer D is incorrect.

  Notice that Answers C and E include natural numbers, but those are just the basic counting numbers 1, 2, 3, …. But $-\frac{1}{3}$ is definitely not a counting number!
  Since $-\frac{1}{3}$ is a simplified fraction, it MUST be a *rational* number.

B. Scientific Notation

1. Format for scientific notation: $c \times 10^n$ ("stem" times "power of 10")

- For the "stem," c is somewhere between 1 (included) and 10 (excluded).
  - Written more simply: $1 \le c < 10$
- For the "power of 10", n is an integer.

2. Convert standard notation ("regular" number) to scientific notation

- **EXAMPLE:** Write the following number in scientific notation 276,000 [1.1.29]
  - Step 1. Start with all the leading nonzero digits; ignore trailing zeros: 276
  - Step 2. Insert a decimal point to create a stem between 1 and 10: 2.76
  - Step 3. Move from the *stem* decimal location to the *actual* decimal location.
    - Count the **number** of positions moved: 5 places
    - Note the **direction** moved: to the RIGHT (positive)
  - Step 4. Value of n (exponent on base 10) comes from Step 3: n = +5 or just 5
  - Step 5. Write the number in scientific notation: 2.76×10^5

- **EXAMPLE:** Convert to scientific notation 0.000000051 [1.1.31]
  - Follow the same steps in a similar fashion as previous example.
  - Stem: 5.1; move 8 places LEFT to get to actual decimal point. (ignore zero to left of decimal)
  - In scientific notation: 5.1×10^{-8}

3. Convert scientific notation to standard notation ("regular" number)

- **EXAMPLE:** The distance from the planet Drendal to its moon Kronotor is about 9×10^5 meters. Express this distance in standard form. [1.1.43]
  - Step 1. Write down just the "stem" number. 9
  - Step 2. Use *n* (the "power of 10") to move (or "hop") the corresponding direction (positive RIGHT, negative LEFT) and number of decimal places. "hop" 5 places to the RIGHT
  - Step 3. Drop a decimal point after completing Step 2.
  - Step 4. Fill in any vacant place values with a zero.
  - Step 5. Remove ("kill") the decimal point from the original stem.
  - In standard notation: 900,000.

- **EXAMPLE:** Write the number in standard form. 7.902×10^{-6} [1.1-25]
  - Follow the same steps in a similar fashion as previous example.
  - From decimal point between 7 and 9, move 6 places to the LEFT, then fill with zeros.
  - In standard notation: 0.000007902
    (note that the zero to the left of decimal point is a matter of visual clarity only)

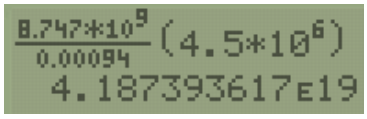
4. Calculator computations involving scientific notation

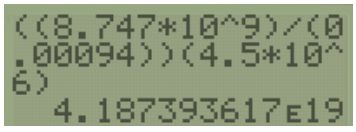
- **EXAMPLE:** Use a calculator to approximate the expression. Write the result in scientific notation.

$$\frac{8.747 \times 10^9}{0.00094}(4.5 \times 10^6)$$

(Round to two decimal places as needed. Use scientific notation. Use the multiplication symbol in the math palette as needed.) [1.1.61]

  - TI-84 has fraction feature: press ALPHA, Y=, ENTER 



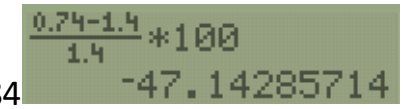
  - TI-83 should use extra parentheses with fraction. Best to use separate ones for numerator, for denominator, and for entire fraction itself, so BE CAREFUL!


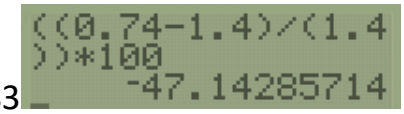
  - The short capital letter E with 18 at end is the power of 10, so "E19" means "$\times 10^{19}$ "
  - Rounded to two decimal places: 4.19×10^{19}

C. Percent Change – formula: Percent Change = $\left(\frac{new-old}{old}\right) \times 100\%$

- **Example:** Find the percent change if a quantity changes from P_1 to P_2.
  $P_1 = 1.4$ and $P_2 = 0.74$ (round to the nearest tenth as needed) [App.D.3]

  - Since from P_1 to P_2 is **decreasing**, we expect the percent change to be NEGATIVE.
  - Use formula:
    Percent Change = $\left(\frac{P_2-P_1}{P_1}\right) \times 100 = \left(\frac{0.74-1.4}{1.4}\right) \times 100$
  - Use calculator:

TI-84 

TI-83 

  - Rounded to the nearest tenth (one decimal place): -47.1 %

Sources used:

1. Pearson MyMathLab *College Algebra with Modeling and Visualization, 6th Edition*, Rockswold
2. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website https://archive.codeplex.com/?p=wabbit