Notes Section 2.5 – Absolute Value Equations and Inequalities

Lesson Objectives

1. Basic Terms Involving Absolute Value
2. Solve an Absolute Value Equation
3. Solve an Absolute Value Inequality

A. **Basic Terms** Involving Absolute Value

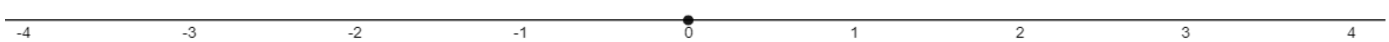
The **ABSOLUTE VALUE** of a number **IS** its **DISTANCE** from zero on a number line.

ABSOLUTE VALUE IS DISTANCE

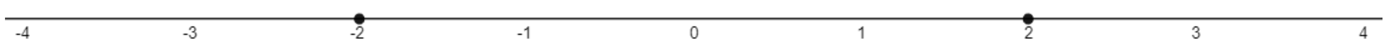
What is always true about a DISTANCE? It is always POSITVE! It can NEVER be negative.

The same is true about the absolute value of a number. It is always positive.

Suppose you are standing at ZERO on a basic horizontal number line:



Where do you go so that you are a DISTANCE of 2 units away from zero? You go to either – 2 or 2.



That is to say, you can go either 2 units to the LEFT (– 2) or 2 units to the RIGHT (+2).

This situation can be modeled using an absolute value equation: $|x| = 2$

Where do you go... ...the distance is 2 from zero

So with the equation: $|x| = 2$, you are to find numbers that are 2 units from zero.

The solutions can be modeled using two equations: $x = -2$ or $x = 2$

Go 2 units to the LEFT of zero or go 2 units to the RIGHT

This type of solution, $x = -2$ or $x = 2$, can also be written in a simplified version: $x = \pm 2$.

That format, $x = \pm 2$, is read as "*x* equals plus or minus 2."

So, remember the following: **ABSOLUTE VALUE IS DISTANCE**

- **EXAMPLE:** Solve the absolute value equation: $|-4x| = -3$ [2.5.27]

  Remember that absolute value is a distance. In this equation, the distance number is – 3. But distance can't be negative, so this equation has **NO SOLUTION.**

In general:	$\mathbf{|"stuff"| = negative}$	means	**"NO SOLUTION"**

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B. Solve an Absolute Value Equation (by hand – symbolically)

- **Step 1 – ISOLATE** the absolute value part, if needed.

- **Step 2 – INSPECT** the distance number (opposite the A.V. stuff).
    - If distance number is **NEGATIVE** – STOP! Equation has **NO SOLUTION**.
    - If distance number is **ZERO** – ignore A.V. bars; make **ONE** equation (**ONE** solution).
    - If distance number is **POSITIVE** – keep going; there will be **TWO** equations (**TWO** solutions).

- **Step 3 – "BRANCH OFF"** and make 2 separate equations
.
- **Step 4 – SOLVE** each equation.

- **Step 5 – WRITE** your solution(s).

- **EXAMPLE:** Solve the equation for *x*. $|3+6x|=0$ [2.5.39]

Step 1 – ISOLATE. There is nothing attached to the outside of the A.V. part (nothing multiplied; nothing added or subtracted)

Step 2 – INSPECT. Distance number, 0, means ignore A.V. bars & make 1 equation

Step 3 – BRANCH. **(**Not needed, since there is only one equation.)

Step 4 – SOLVE. $|3+6x|=0$ converts to $3+6x=0$

$6x=-3$ so $x=-\frac{3}{6}=-\boldsymbol{\frac{1}{2}}$

Step 5 – WRITE. The solution is: $\boldsymbol{x=-\frac{1}{2}}$.

- **EXAMPLE:** Solve for *b*. $|b+9|+8=10$ [2.5-8]

Step 1 – ISOLATE. Given: $|b+9|+8=10$ (Subtract 8)

$|b+9|=2$ (do NOT subtract 9…yet!)

Step 2 – INSPECT. The distance number, 2, is POSITIVE – make 2 equations

Step 3 – BRANCH. Make 2 separate equations: $|b+9|=2$

Start BOTH equations with $+9$: $b+9=-2$ $b+9=2$

Step 4 – SOLVE. (Subtract 9)

Step 5 – WRITE. $\boldsymbol{b=-11}$ **or** $\boldsymbol{b=-7}$

- **EXAMPLE:** Find the solution set for the equation. $4|3x|+5=37$ [*Blitzer 4.3.19]

Step 1 – ISOLATE. Given: $4|3x|+5=37$ (Subtract 5)

$4|3x|=32$ (Divide by 4)

$|3x|=8$ (do NOT divide by 3….yet!)

Step 2 – INSPECT. The distance number, 8, is POSITIVE – make 2 equations

Step 3 – BRANCH. Make 2 separate equations: $|3x|=8$

Start BOTH equations with $3x$ $3x=-8$ or $3x=8$

Step 4 – SOLVE. Solve each equation. (divide by 3)

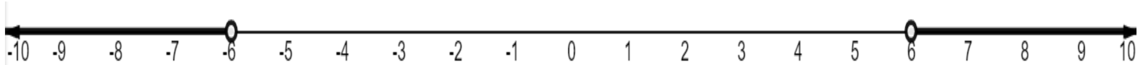
Step 5 – WRITE. Write your solutions. $\boldsymbol{x=-\frac{8}{3}}$ **or** $\boldsymbol{x=\frac{8}{3}}$ also $\boldsymbol{x=\pm\frac{8}{3}}$

C. Solve an Absolute Value Inequality (by hand – symbolically)

1. Solving a Greater-Than type Absolute Value Inequality (More is "or")

- **EXAMPLE:** Solve the inequality. $|1-2x|>6$ [2.5.83]

Focus for now on the information outside the inequality: **greater than 6**.
Where would you need to be so that you are **greater than 6** units away from zero?



Left piece is **less than – 6** or Right piece is **greater than 6**

In EACH piece graphed above, the distance is **greater than 6** units away from zero on the number line.

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This is what we'll use to set up the solution process for this inequality. **(More is "or")**

$$|1 - 2x| > 6$$

(BOTH inequalities will start with $1 - 2x$)

(left piece) $1 - 2x < -6$ or $1 - 2x > 6$ (right piece)

(Solve each inequality)

(subtract 1)

$-2x < -7$ or $-2x > 5$

(Divide by – 2)

(**REVERSE !!** – remember, you don't always reverse)

$x > \frac{7}{2}$ or $x < -\frac{5}{2}$

(**VERY IMPORTANT!** Swap places to mimic number line; smaller on left, larger on right)

$x < -\frac{5}{2}$ or $x > \frac{7}{2}$

WRITE the solution. Set Builder Notation: $\left\{x \middle| \, x < -\frac{5}{2} \text{ or } x > \frac{7}{2}\right\}$

Interval Notation: $\left(-\infty, -\frac{5}{2}\right) \cup \left(\frac{7}{2}, \infty\right)$

So, in general: If given $|A| > B$ setup is: $A < -B$ or $A > B$

Solution is: $(-\infty, \text{smaller}\,) \cup (\,\text{larger}, \infty\,)$ parentheses are used on 2 solutions

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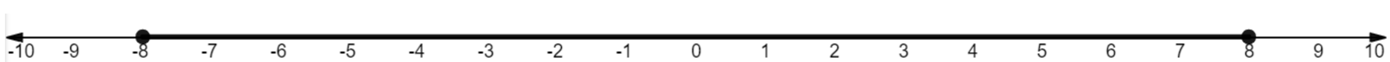
If given $|A| \geq B$ setup is: $A \leq -B$ or $A \leq B$

Solution is: $(-\infty, \text{smaller}\,] \cup [\,\text{larger}, \infty\,)$ brackets are used on 2 solutions

2. Solving a **Less-Than** type **Absolute Value Inequality** (Less is "Nest")

- **EXAMPLE:** Solve the inequality. $|3 - 5x| \leq 8$ [2.5.75]

Focus for now on the information outside the inequality: **less than or equal to 8**.
Where would you need to be so that you are **less than or equal to 8** units away from zero?



This segment above shows staying a distance of **less than or equal to 8** units from zero.

So, notice that this graph shows all points with a distance **in between – 8** and **+8**.

That is written as a compound inequality: $-8 \leq \text{distance} \leq 8$

This is what we'll use to set up the solution process for this inequality.

Given: $|3 - 5x| \le 8$ (**Less is "Nest"**)

Setup to Solve: $-8 \le 3 - 5x \le +8$ "Nest" the 3 – 5x in the middle

Subtract 3:

Divide by – 5 and **REVERSE!**: $-11 \le -5x \le 5$

(Remember, you won't always reverse)

$$\frac{11}{5} \ge x \ge -1$$

You need to "pivot" (or "dab") this inequality, to match the number line:

smaller ≤ *x* ≤ larger

$$-1 \le x \le \frac{11}{5}$$

WRITE the solution. Set Builder Notation: $\left\{x \middle| -1 \le x \le \frac{11}{5}\right\}$

Interval Notation: $\left[-1, \frac{11}{5}\right]$

So, in general:	If given $|A| < B$	setup is:	$-B < A < +B$
Solution is:	$(\text{ smaller}, \text{larger })$	parentheses are used	
If given $|A| \le B$	setup is:	$-B \le A \le +B$	
Solution is:	$[\text{ smaller}, \text{larger }]$	brackets are used	

3. Solve Applications with Absolute Value Inequalities

- **EXAMPLE:** The inequality describes the range of monthly average temperatures *T* in degrees Fahrenheit at a certain location. Find an equivalent expression and monthly average temperatures.

$|T - 49| \le 29$ [2.5.117] (Less is "nest")

Setup to solve: $-29 \le T - 49 \le +29$

Add 49:

The inequality is equivalent to: $\boldsymbol{20 \le T \le 78}$ interval notation: $[20{,}78]$

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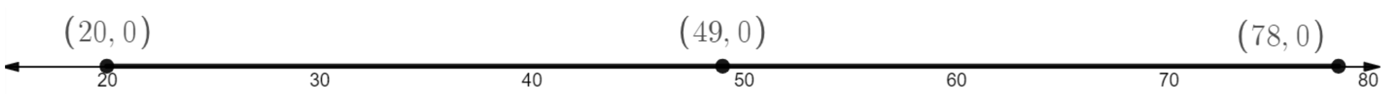
Interpret this solution: If the high and low monthly average temperatures satisfy the inequality, then the monthly averages are always within **29** degrees of **49**° F.

How do we get this? The “within” 29 degrees part represents a **DISTANCE**, so use the **DISTANCE** number (away from the inequality) for **“within”**.

The “of” 49° F part represents a **LOCATION** (center point), so use the value **inside** the absolute value.

$$|T - 49| \leq 29$$

“of” (center point, location) “within” (distance)



Sources Used:

1. Solving Absolute Value Inequalities, https://www.youtube.com/watch?v=BhFj7Rkyc5E&t=180s, Gdawg Enterprises, © 2009, using phrases “More is ‘OR’, Less is ‘Nest’”.
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