Exercise 12

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a)

Die Zählraten der einzelnen Bins folgen einer Poisson-Verteilung.

Die PDF lauten also $\frac{e^{-Np_i}(Np_i)^{n_i}}{n_i!}$ & $\frac{e^{-Mp_i}(Mp_i)^{m_i}}{m_i!}$

b)

Die Likelihood ist das Produkt der beiden PDF:

$$L(p_i) = \frac{e^{-Np_i}(Np_i)^{n_i} \cdot e^{-Mp_i}(Mp_i)^{m_i}}{n_i!\, m_i!} = \frac{1}{n_i! m_i!} e^{-p_i(N+M)} \cdot N^{n_i} M^{m_i} (p_i)^{n_i+m_i}$$

Maximiere $F = \ln(L) = -p_i(N+M) + (n_i+m_i)\cdot \ln(p_i) + \text{const}$

$$\left.\frac{\partial F}{\partial p_i}\right|_{\hat{p}_i} = 0$$

$$\Rightarrow -(N+M) + \frac{n_i+m_i}{\hat{p}_i} = 0 \quad \Leftrightarrow \quad \hat{p}_i = \frac{n_i+m_i}{N+M}$$

c)

$$\chi^2 = \sum_{i=1}^{r} \frac{(n_i - n_i^{Model})^2}{n_i^{Model}} \qquad n_i^{Model} = N\hat{p}_i \quad (\text{analog für } M)$$

$$\Rightarrow \chi^2 = \sum_{i=1}^{r}\left[\frac{\left(n_i - N\cdot\frac{n_i+m_i}{N+M}\right)^2}{\frac{n_i+m_i}{N+M}\cdot N} + \frac{\left(m_i - M\frac{n_i+m_i}{N+M}\right)^2}{\frac{n_i+m_i}{N+M}\cdot M}\right]$$