Exercise 12

Sonntag, 29. Mai 2022 12:14

b)

$$S_w = S_{0,1} = \begin{pmatrix} \frac{185}{24} & \frac{7}{2} \\ \frac{7}{2} & \frac{11}{2} \end{pmatrix}$$

$$S_B = (\vec{\mu}_0 - \vec{\mu}_1)(\vec{\mu}_0 - \vec{\mu}_1)^T$$

$$= \begin{pmatrix} \frac{23}{12} - 3 \\ 2 - \frac{3}{2} \end{pmatrix} \begin{pmatrix} \frac{23}{12} - 3 \\ 2 - \frac{3}{2} \end{pmatrix}^T$$

$$= \begin{pmatrix} \frac{23}{12} - \frac{36}{12} \\ \frac{4}{2} - \frac{3}{2} \end{pmatrix} \begin{pmatrix} \frac{23}{12} - \frac{36}{12} \\ \frac{4}{2} - \frac{3}{2} \end{pmatrix}^T$$

$$= \begin{pmatrix} -\frac{13}{12} \\ \frac{1}{2} \end{pmatrix} \begin{pmatrix} -\frac{13}{12} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \left(-\frac{13}{12}\right)^2 & -\frac{13}{24} \\ -\frac{13}{24} & \frac{1}{4} \end{pmatrix}$$

S_w^{-1} : $\quad S_w = 1_2$

$$\left(\begin{array}{cc|cc} \frac{185}{24} & \frac{7}{2} & 1 & 0 \\ \frac{7}{2} & \frac{11}{2} & 0 & 1 \end{array}\right)$$

$$\Rightarrow S_w^{-1} = \begin{pmatrix} \frac{264}{1447} & -\frac{168}{1447} \\ -\frac{168}{1447} & \frac{370}{1447} \end{pmatrix}$$

$$\vec{\lambda}^* = \arg\max \left[\frac{\vec{\lambda}^T S_B \vec{\lambda}}{\vec{\lambda}^T S_w \vec{\lambda}} \right] = S_w^{-1} (\mu_0 - \mu_1)$$

$$= \begin{pmatrix} \frac{264}{1447} & -\frac{168}{1447} \\ -\frac{168}{1447} & \frac{370}{1447} \end{pmatrix} \begin{pmatrix} \frac{23}{12} - 3 \\ 2 - \frac{3}{2} \end{pmatrix} = \frac{1}{1447} \begin{pmatrix} -370 \\ 367 \end{pmatrix}$$