calculation

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a) $$PDF(\Delta\psi) = \begin{cases} N \cdot \exp(-|\Delta\psi| \cdot k) & ; \Delta\psi \in [-\pi, \pi) \\ 0 ; \text{ sonst} \end{cases}$$

Determining N:

$$\int_{-\pi}^{\pi} N \cdot e^{-|\Delta\psi| \cdot k} \, d\Delta\psi \overset{!}{=} 1$$

$$\Leftrightarrow \int_{-\pi}^{\pi} e^{-|\Delta\psi| \cdot k} \, d\Delta\psi = \frac{1}{N}$$

$$\Leftrightarrow \int_{-\pi}^{0} e^{\Delta\psi \cdot k} d\Delta\psi + \int_{0}^{\pi} e^{-\Delta\psi \cdot k} d\Delta\psi = \frac{1}{N}$$

$$\Leftrightarrow \frac{1}{k} \cdot \left[e^{\Delta\psi \cdot k} \right]_{-\pi}^{0} - \frac{1}{k} \cdot \left[e^{-\Delta\psi \cdot k} \right]_{0}^{\pi} = \frac{1}{N}$$

$$\Leftrightarrow (1 - e^{-\pi k}) - (e^{-\pi k} - 1) = \frac{k}{N}$$

$$\Leftrightarrow 2 - 2e^{-\pi k} = \frac{k}{N}$$

$$\Leftrightarrow \underline{N = \frac{k}{2(1 - e^{-\pi k})}}$$

b) $$CDF(\Delta\psi) = \int_{-\pi}^{\Delta\psi} PDF(\Delta\psi') \, d\Delta\psi' \quad ; \quad \Delta\psi \leq \pi$$

$$= N \cdot \int_{-\pi}^{\Delta\psi} e^{-|\Delta\psi| \cdot k} \, d\Delta\psi$$

$\rightarrow$ 2 possible cases

① $\underline{\Delta\psi < 0}$: $CDF(\Delta\psi) = N \cdot \int_{-\pi}^{\Delta\psi} e^{\Delta\psi \cdot k} \, d\Delta\psi$

① $\underline{\Delta\psi < 0:}$ $CDF(\Delta\psi) = N \cdot \int_{-\pi} e^{\Delta\psi \cdot k} \, d\Delta\psi$

$$= \frac{N}{k}\left(e^{\Delta\psi \cdot k} - e^{-\pi k}\right)$$

$$= \frac{e^{\Delta\psi \cdot k} - e^{-\pi k}}{2(1-e^{-\pi k})}$$

② $\underline{\Delta\psi > 0:}$ $CDF(\Delta\psi) = \underbrace{N \cdot \int_{-\pi}^{0} e^{-\Delta\psi \cdot k} d\Delta\psi}_{=0{,}5} + N \cdot \int_{0}^{\Delta\psi} e^{-\Delta\psi \cdot k} \, d\Delta\psi)$

$$= -\frac{N}{k}\left(e^{-\Delta\psi \cdot k} - 1\right) + 0{,}5$$

$$= -\frac{e^{-\Delta\psi \cdot k} - 1}{2(1-e^{-\pi k})} + 0{,}5$$

c) PPF ist defined as $(CDF)^{-1}$, it is it's inversion:

$\underline{\text{case } 1:}$ $q = 0 \rightarrow \Delta\psi = -\pi$

$\underline{\text{case } 2:}$ $0 < q < 0{,}5$:

$$q = \frac{e^{\Delta\psi \cdot k} - e^{-\pi k}}{2(1-e^{-\pi k})} \Leftrightarrow 2q(1-e^{-\pi k}) = e^{\Delta\psi \cdot k} - e^{-\pi k}$$

$$\Leftrightarrow e^{\Delta\psi k} = 2q(1-e^{-\pi k}) + e^{-\pi k}$$

$$\Leftrightarrow \Delta\psi = \frac{1}{k} \cdot \ln\left[2q(1-e^{-\pi k}) + e^{-\pi k}\right]$$

$\underline{\text{case } 3:}$ $0{,}5 \leq q < 1$:

$$q = -\frac{e^{-\Delta\psi \cdot k} - 1}{2(1-e^{-\pi k})} + 0{,}5 \Leftrightarrow 2q(1-e^{-\pi k}) = -e^{-\Delta\psi \cdot k} + 1 + (1-e^{\pi k})$$

$$\Leftrightarrow e^{-\Delta\psi k} = -2q(1-e^{-\pi k}) + 1 + (1-e^{\pi k})$$

$$\Leftrightarrow e^{-\Delta\varphi k} = -2q(1-e^{-\pi k}) + 1 + (1-e^{-\pi k})$$

$$\Leftrightarrow \Delta\varphi = -\frac{1}{k} \cdot \ln\left[-2q(1-e^{-\pi k}) + 2 - e^{-\pi k})\right]$$

<u>case 4</u>: $q = 1 \rightarrow \Delta\varphi = \pi$