Exercise 11

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(a) The null hypothesis states that there is no γ source: $s_0 = 0$. Under this assumption, what value and what error result for b_0 according to the maximum likelihood method?

$$\ln L = N_{off} \ln(b) + N_{on} \ln(\alpha b) - (1+\alpha) b - \ln(N_{off}!) - \ln(N_{on}!)$$

$$\frac{d(\ln L)}{db} = \frac{N_{off}}{b} + \frac{N_{on}}{b} - (1+\alpha) \overset{!}{=} 0$$

$$\Leftrightarrow \frac{N_{off}+N_{on}}{b} = 1+\alpha$$

$$\Rightarrow \hat{b} = \frac{N_{off}+N_{on}}{1+\alpha}$$

$$\frac{d^2(\ln L)}{db^2} = \frac{N_{off}+N_{on}}{b^2} = \frac{1+\alpha}{N_{off}+N_{on}}$$

$$\Rightarrow b_0 = \frac{N_{off}+N_{on}}{1+\alpha} \pm \frac{1+\alpha}{N_{off}+N_{on}}$$

b)

$$\lambda = \frac{L(b_0, s_0)}{L(b_1, s_1)} \qquad b_0 = \frac{N_{off}+N_{on}}{1+\alpha}, \quad s_0 = 0$$

$$b_1 = N_{off}, \quad s_1 = N_{on} - \alpha N_{off}$$

$$L(b,s) = \frac{b^{N_{off}} (\alpha b + s)^{N_{on}}}{N_{off}! \, N_{on}!} e^{-(1+\alpha)b - s}$$

$$\Rightarrow \lambda = \frac{\left(\alpha \frac{N_{off}+N_{on}}{1+\alpha}\right)^{N_{on}} e^{-\cancel{(1+\alpha)} \frac{N_{off}+N_{on}}{\cancel{1+\alpha}}}}{(N_{on} - \alpha N_{off})^{N_{off}} (\cancel{\alpha N_{off}} + N_{on} - \cancel{\alpha N_{off}})^{N_{on}} e^{-\cancel{(1+\alpha) N_{off} - N_{on} + \alpha N_{off}}}}$$

$$= \left(\frac{\alpha}{N_{on}} \frac{N_{off}+N_{on}}{1+\alpha}\right)^{N_{on}} \cdot \left(\frac{1}{N_{on} - \alpha N_{off}}\right)^{N_{off}}$$

c)

$$D = -2 \ln(\lambda) = -2 N_{on} \ln\left(\frac{\alpha}{N_{on}} \frac{N_{off}+N_{on}}{1+\alpha}\right) + 2 N_{off} \ln(N_{on} - \alpha N_{off})$$

Vergleiche: $\mathcal{N}(\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{u^2}{2\sigma^2}}$

u^2 ist χ^2 verteilt, $-2 \ln(\lambda)$ auch