

calculation ex.14

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a) 1. Centralization

→ compute the mean value of the data points

2. Covariance

→ calculate the covariance of the data matrix

3. Eigenvalues and vectors

→ calculate the Eigenvalues and -vectors of the covariance matrix

4. Sorting and choosing a subset

→ choosing the k-largest Eigenvalues and Vectors

5. Transformation matrix

→ filling in the transformation matrix with the chosen Eigenvectors

6. Transformation

→ applying the transformation matrix to the dataset

6) $x_1 : [1, 3, 1, 2, 3, 2]$

$x_2 : [1, 0, 3, 0, 1, 1]$

① mean vector μ :

$$\mu = \frac{1}{6} \cdot \begin{pmatrix} 1+3+1+2+3+2 \\ 1+0+3+0+1+1 \end{pmatrix}$$

$$= \frac{1}{6} \cdot \begin{pmatrix} 12 \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

② covariance matrix:

$$\text{cov}\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right) = \begin{pmatrix} \text{var}(x_1) & \text{cov}(x_1, x_2) \\ \text{cov}(x_2, x_1) & \text{var}(x_2) \end{pmatrix}$$

$$\begin{aligned} \text{var}(x_1) - \sigma_{x_1}^2 &= \frac{1}{n} \cdot \sum_{i=1}^n (x_{1,i} - \bar{x}_{1,i})^2 \\ &= \frac{1}{6} \cdot [1^2 + 1^2 + 1^2 + 0^2 + 1^2 + 0^2] \\ &= \frac{1}{6} \cdot 6 = \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \text{var}(x_2) - \sigma_{x_2}^2 &= \frac{1}{n} \cdot \sum_{i=1}^n (x_{2,i} - \bar{x}_{2,i})^2 \\ &= \frac{1}{6} \cdot [0^2 + 1^2 + 2^2 + 1^2 + 0^2 + 0^2] \\ &= \frac{1}{6} \cdot 6 = 1 \end{aligned}$$

$$\begin{aligned} \text{cov}(x_1, x_2) &= \frac{1}{n} \cdot \sum_{i=1}^n (x_{1,i} - \bar{x}_{1,i})(x_{2,i} - \bar{x}_{2,i}) \\ &= \frac{1}{6} \cdot (1 \cdot 0 + 1 \cdot 1 + 1 \cdot 2 + 0 \cdot 1 + 1 \cdot 0 + 0 \cdot 0) \\ &= \frac{1}{6} \cdot (1 + 2) = \frac{1}{2} = \text{cov}(x_2, x_1) \end{aligned}$$

$$\Rightarrow \text{cov}\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right) = \begin{pmatrix} \frac{2}{3} & \frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix}$$

③ Eigenvalues and -vectors:

$$\det(\text{cov}(\vec{x}_i) - \lambda E_i) = 0$$

$$\Leftrightarrow \det \begin{pmatrix} \frac{2}{3} - \lambda & \frac{1}{2} \\ \frac{1}{2} & 1 - \lambda \end{pmatrix} = 0$$

$$\Leftrightarrow \left(\frac{2}{3} - \lambda \right) (1 - \lambda) - \frac{1}{2} \cdot \frac{1}{2} = 0$$

$$\Leftrightarrow \frac{2}{3} - \frac{2}{3}\lambda - \lambda + \lambda^2 - \frac{1}{4} = 0$$

$$\Leftrightarrow \lambda^2 - \frac{5}{3}\lambda + \frac{2}{3} - \frac{1}{4} = 0$$

$$\Leftrightarrow \lambda^2 - \frac{5}{3}\lambda + \frac{5}{12} = 0$$

$$\Rightarrow \lambda_{1,2} = -\frac{-\frac{5}{3}}{2} \pm \sqrt{\left(\frac{-5}{3}\right)^2 - \frac{5}{12}}$$

$$= \frac{5}{6} \pm \sqrt{\frac{25}{36} - \frac{5}{12}}$$

$$= \frac{5}{6} \pm \sqrt{\frac{10}{36}}$$

$$= \frac{5 \pm \sqrt{10}}{6} \quad \Rightarrow \lambda_1 = \frac{5 - \sqrt{10}}{6}, \lambda_2 = \frac{5 + \sqrt{10}}{6}$$

$$\text{e.v. of } \lambda_1: \vec{v}_1 = \begin{pmatrix} -\frac{1+\sqrt{10}}{3} \\ 1 \end{pmatrix}$$

$$\text{e.v. of } \lambda_2: \vec{v}_2 = \begin{pmatrix} -\frac{1+\sqrt{10}}{3} \\ 1 \end{pmatrix}$$

④ choosing a subset:

→ choosing λ_2 , since $\lambda_1 < \lambda_2$

⑤ creating transformation matrix:

$$W = (\vec{v}_i) = \begin{pmatrix} \frac{\sqrt{10}-1}{3} \\ 1 \end{pmatrix}$$

$$W = \begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{10}-1}{3} \\ \frac{3}{3} \\ 1 \end{pmatrix}$$

⑥ transforming the dataset:

$$X' = X \cdot W = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \cdot W$$

$$\rightarrow X' = \left[\frac{2+\sqrt{10}}{3}, -1+\sqrt{10}, \frac{8+\sqrt{10}}{3}, \frac{-2+2\sqrt{10}}{3}, \sqrt{10}, \frac{1+2\sqrt{10}}{3} \right]$$

$$\approx \underline{\left[1.72, 2.16, 3.72, 1.44, 3.16, 2.44 \right]}$$