calculation ex.14

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a) 1. Centralazation

→ compute the mean value of the data points

2. Covariance

→ calculate the covariance of the data matrix

3. Eigenvalues and vectors

→ calculate the Eigenvalues and -vectors of the covariance matrix

4. Sorting and choosing a subset

→ choosing the k-largest Eigenvalues and Vectors

5. Transformation matrix

→ filling in the transformation matrix with the chosen Eigenvectors

6. Transformation

→ applying the transformation matrix to the dataset

b) x_1 : [1, 3, 1, 2, 3, 2]

x_2 : [1, 0, 3, 0, 1, 1]

① mean vector μ :

$$\mu = \frac{1}{6} \cdot \begin{pmatrix} 1+3+1+2+3+2 \\ 1+0+3+0+1+1 \end{pmatrix}$$

$$= \frac{1}{6} \cdot \begin{pmatrix} 12 \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

② covariance matrix:

$$cov\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right) = \begin{pmatrix} var(x_1) & cov(x_1,x_2) \\ cov(x_2,x_1) & var(x_2) \end{pmatrix}$$

$$var(x_1) = \sigma_{x_1} = \frac{1}{n} \cdot \sum_{i=1}^{n} (x_{1,i} - \bar{x}_{1,i})^2$$

$$= \frac{1}{6} \cdot [1^2 + 1^2 + 1^2 + 0^2 + 1^2 + 0^2)$$

$$= \frac{1}{6} \cdot 4 = \frac{2}{3}$$

$$var(x_2) = \sigma_{x_2} = \frac{1}{n} \cdot \sum_{i=1}^{n} (x_{2,i} - \bar{x}_{2,i})^2$$

$$= \frac{1}{6} \cdot [0^2 + 1^2 + 2^2 + 1^2 + 0^2 + 0^2)$$

$$= \frac{1}{6} \cdot 6 = 1$$

$$cov(x_1, x_2) = \frac{1}{n} \cdot \sum_{i=1}^{n} (x_{1,i} - \bar{x}_{1,i})(x_{2,i} - \bar{x}_{2,i})$$

$$= \frac{1}{6} \cdot (1 \cdot 0 + 1 \cdot 1 + 1 \cdot 2 + 0 \cdot 1 + 1 \cdot 0 + 0 \cdot 0)$$

$$= \frac{1}{6} \cdot (1 + 2) = \frac{1}{2} = cov(x_2, x_1)$$

$$\Rightarrow cov\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & \frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix}$$

③ Eigenvalues and -vectors:

$$\det\left(\operatorname{cov}\begin{pmatrix}x_1\\x_2\end{pmatrix} - \lambda E_2\right) \overset{!}{=} 0$$

$$\Leftrightarrow \det\begin{pmatrix}\frac{2}{3}-\lambda & \frac{1}{2}\\ \frac{1}{2} & 1-\lambda\end{pmatrix} = 0$$

$$\Leftrightarrow \left(\frac{2}{3}-\lambda\right)(1-\lambda) - \frac{1}{2}\cdot\frac{1}{2} = 0$$

$$\Leftrightarrow \frac{2}{3} - \frac{2}{3}\lambda - \lambda + \lambda^2 - \frac{1}{4} = 0$$

$$\Leftrightarrow \lambda^2 - \frac{5}{3}\lambda + \frac{2}{3} - \frac{1}{4} = 0$$

$$\Leftrightarrow \lambda^2 - \frac{5}{3}\lambda + \frac{5}{12} = 0$$

$$\rightarrow \lambda_{1,2} = -\frac{-\frac{5}{3}}{2} \pm \sqrt{\left(\frac{-\frac{5}{3}}{2}\right)^2 - \frac{5}{12}}$$

$$= \frac{5}{6} \pm \sqrt{\frac{25}{36} - \frac{5}{12}}$$

$$= \frac{5}{6} \pm \sqrt{\frac{10}{36}}$$

$$= \frac{5 \pm \sqrt{10}}{6} \qquad \rightarrow \lambda_1 = \frac{5-\sqrt{10}}{6} \quad ; \lambda_2 = \frac{5+\sqrt{10}}{6}$$

e.v. of λ_1: $\vec{v}_1 = \begin{pmatrix}-\frac{1+\sqrt{10}}{3}\\ 1\end{pmatrix}$

e.v. of λ_2: $\vec{v}_2 = \begin{pmatrix}\frac{-1+\sqrt{10}}{3}\\ 1\end{pmatrix}$

④ choosing a subset:

→ choosing λ_1, since $\lambda_1 < \lambda_2$

⑤ creating transformation matrix:

$W = \left(\vec{v}_1\right) = \begin{pmatrix}-\frac{1+\sqrt{10}}{3}\\ 1\end{pmatrix}$

$$W = \left(v_1 \right) = \begin{pmatrix} -\frac{1+\sqrt{10}}{3} \\ 1 \end{pmatrix}$$

⑥ transforming the dataset:

$$X' = X \cdot W = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \cdot W$$

$$\Rightarrow X' = \left[\frac{2-\sqrt{10}}{3}, -1-\sqrt{10}, \frac{8-\sqrt{10}}{3}, -\frac{2+2\sqrt{10}}{3}, -\sqrt{10}, \frac{1-2\sqrt{10}}{3} \right]$$

$$\approx \underline{\left[-0.39, -4.16, 1.61, -2.77, -3.16, -1.77 \right]}$$