ex 11

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a) $f = \ln(L)$ auf dem Blatt

$$-\frac{\partial F}{\partial b} = \frac{N_{off}}{b} + \frac{N_{on}\alpha}{\alpha b} - (1+\alpha) = 0$$

$$\Rightarrow \hat{b}_0 = \frac{N_{off}+N_{on}}{1+\alpha}$$

$$\left.\frac{\partial^2 F}{\partial b^2}\right|_{b_0} = -\frac{N_{off}}{b^2} - \left.\frac{N_{on}}{b^2}\right|_{b_0} = \frac{(1+\alpha)^2}{N_{off}+N_{on}}$$

$$\sigma^2_{b_0} = \frac{N_{off}+N_{on}}{(1+\alpha)^2}$$

b) $$\lambda = \frac{L(b_0, s_0)}{L(b_1, s_1)} = \exp\left[\ln\left(\frac{L_0}{L}\right)\right]$$

$$= \exp(-F_0 - (-F))$$

$$-F(\hat{s}, \hat{b}) = N_{off}\ln(N_{off}) + N_{on}\ln(N_{on}) - (1+\alpha)N_{off}$$
$$- (N_{on} - \alpha N_{off}) - \ln(N_{off}!) - \ln(N_{on}!)$$

$$-F(\hat{b}_0) = N_{off}\ln\left(\frac{N_{on}+N_{off}}{1+\alpha}\right) + N_{on}\ln\left(\frac{\alpha(N_{on}+N_{off})}{1+\alpha}\right)$$
$$- (N_{on}+N_{off}) - \ln(N_{off}!) - \ln(N_{on}!)$$

$$-F_0 + F = N_{on}\ln\left(\frac{\alpha}{1+\alpha}\cdot\frac{N_{on}+N_{off}}{N_{on}}\right) + N_{off}\ln\left(\frac{1}{1+\alpha}\,\frac{N_{on}+N_{off}}{N_{off}}\right)$$

$$\lambda = \left[\frac{\alpha}{1+\alpha}\,\frac{N_{on}+N_{off}}{N_{on}}\right]^{N_{on}} \cdot \left[\frac{1}{1+\alpha}\,\frac{N_{on}+N_{off}}{N_{off}}\right]^{N_{off}}$$

c) $D = -2\ln\lambda \sim \chi^2(1)$

$$P(u) \sim \exp\left(-\frac{(u-\mu)^2}{2\sigma^2}\right), \quad \mu = 0$$

$$\ln(P) = -\frac{u^2}{2\sigma^2}$$

$$\boxed{\sqrt{-2\ln(P)} = \frac{u}{\sigma}}$$

$$\Rightarrow S = \sqrt{-2\ln(\lambda)} = \sqrt{-2\ln\left(\frac{L_0}{L}\right)}$$