Exercise 12

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b)

$$S_w = S_{0,1} = \begin{pmatrix} \frac{185}{24} & \frac{7}{2} \\ \frac{7}{2} & \frac{11}{2} \end{pmatrix}$$

$$S_B = (\vec{\mu}_0 - \vec{\mu}_1)(\vec{\mu}_0 - \vec{\mu}_1)^T$$

$$= \begin{pmatrix} \frac{23}{12} - 3 \\ 2 - \frac{3}{2} \end{pmatrix}\begin{pmatrix} \frac{23}{12} - 3 \\ 2 - \frac{3}{2} \end{pmatrix}^T$$

$$= \begin{pmatrix} \frac{23}{12} - \frac{36}{12} \\ \frac{4}{2} - \frac{3}{2} \end{pmatrix}\begin{pmatrix} \frac{23}{12} - \frac{36}{12} \\ \frac{4}{2} - \frac{3}{2} \end{pmatrix}^T$$

$$= \begin{pmatrix} \frac{-13}{12} \\ \frac{1}{2} \end{pmatrix}\begin{pmatrix} -\frac{13}{12} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} (-\frac{13}{12})^2 & -\frac{13}{24} \\ -\frac{13}{24} & \frac{1}{4} \end{pmatrix}$$

S_w^{-1}: $S_w = 1_2$

$$\left(\begin{array}{cc|cc} \frac{185}{24} & \frac{7}{2} & 1 & 0 \\ \frac{7}{2} & \frac{11}{2} & 0 & 1 \end{array}\right)$$

$$\Rightarrow S_w^{-1} = \begin{pmatrix} \frac{264}{1447} & -\frac{168}{1447} \\ -\frac{168}{1447} & \frac{370}{1447} \end{pmatrix}$$

$$S_w^{-1} S_B = \begin{pmatrix} \frac{264}{1447} & -\frac{168}{1447} \\ -\frac{168}{1447} & \frac{370}{1447} \end{pmatrix}\begin{pmatrix} \frac{169}{144} & -\frac{13}{24} \\ -\frac{13}{24} & \frac{1}{4} \end{pmatrix} = \begin{pmatrix} \frac{2405}{8682} & -\frac{185}{1447} \\ -\frac{4771}{17364} & \frac{367}{2894} \end{pmatrix}$$

$$S_w^{-1} S_B \vec{\lambda} = D \vec{\lambda}$$

calculating Eigenvalues D:

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$$\det\left[\begin{pmatrix} \frac{2405}{8682} - D & -\frac{185}{1447} \\ -\frac{4771}{17364} & \frac{367}{2894} - D \end{pmatrix}\right] \stackrel{!}{=} 0$$

$$\Leftrightarrow \left(\frac{2405}{8682} - D\right)\left(\frac{367}{2894} - D\right) - \left(-\frac{185}{1447}\right)\left(-\frac{4771}{17364}\right) \stackrel{!}{=} 0$$

$$\Leftrightarrow \left(\frac{2405}{8682}\right)\left(\frac{367}{2894}\right) - D\left(\frac{367}{2894} + \frac{2405}{8682}\right) + D^2 - \frac{882635}{25125708} = 0$$

$$\Leftrightarrow \frac{882635}{25125708} - D\left(\frac{1753}{4341}\right) + D^2 - \frac{882635}{25125708} = 0$$

$$\Leftrightarrow D\left(-\frac{1753}{4341} + D\right) = 0 \quad , \quad D_1 = 0$$

$$D_2 = \frac{1753}{4341}$$

calculating Eigenvectors $\vec{\lambda}$:

For $D_1 = 0$:

$$\begin{pmatrix} \frac{2405}{8682} & -\frac{185}{1447} \\ -\frac{4771}{17364} & \frac{367}{2894} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0}$$

$$\Leftrightarrow \begin{pmatrix} \frac{2405}{8682}x - \frac{185}{1447}y \\ -\frac{4771}{17364}x + \frac{367}{2894}y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\text{I.}: \frac{2405 \cdot 1447}{8682 \cdot 185}x = y \quad \Leftrightarrow \quad y = \frac{13}{6}x$$

$$\rightarrow \text{II.}: \frac{17364 \cdot 367}{2894 \cdot 4771}y = x$$

$$\Leftrightarrow \frac{17364 \cdot 367}{2894 \cdot 4771} \cdot \frac{13}{6}x = x$$

$$\Leftrightarrow x = x \quad , \quad x \in \mathbb{R}$$

$$\vec{\lambda}_0 = \begin{pmatrix} 1 \\ \frac{13}{6} \end{pmatrix} x$$

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For $D_1 = \frac{1753}{4341}$

$$\begin{pmatrix} \frac{2405}{8682} - \frac{1753}{4341} & -\frac{185}{1447} \\ -\frac{4771}{17364} & \frac{367}{2894} - \frac{1753}{4341} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\Leftrightarrow \quad y = -\frac{367}{370} x$

$\Leftrightarrow \quad x = x$

$$\vec{\lambda}_1 = \begin{pmatrix} 1 \\ -\frac{367}{370} \end{pmatrix} x$$