ex 16

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a) definition of conditional probability:

$$P(S|W) = \frac{P(S \cap W)}{P(W)} \quad (1)$$

where $P(S \cap W)$ is the probability of S and W being true.
We also know that:

$$P(W|S) = \frac{P(S \cap W)}{P(S)} \quad (2)$$

$$\Leftrightarrow P(W|S) \cdot P(S) = P(S \cap W) \quad \rightarrow \text{ substituting into (1):}$$

$$P(S|W) = \frac{P(W|S) \cdot P(S)}{P(W)} \quad \square$$

b) We are looking for $P(S|W)$, which can be calculated via Bayes Theorem:

$$P(S|W) = \frac{P(W|S) \cdot P(S)}{P(W)}$$

We are looking for:

$P(S = yes \,|\, W)$; W is in table 2

① $$P(W|S = yes) = \prod_i P(x_i \,|\, S = yes) = \prod_i \frac{P(x_i \cap S = yes)}{P(S)}$$

$$= \frac{P(wind = high,\ S = yes)}{P(S = yes)} \cdot \frac{P(humidity = high \,|\, S = yes)}{P(S = yes)} \cdot \frac{P(temp = cold \,|\, S = yes)}{P(S = yes)} \cdot \frac{P(forecast = sunny \,|\, S = yes)}{P(S = yes)}$$

$$= \frac{14}{9} \cdot \frac{3}{14} \cdot \frac{14}{9} \cdot \frac{3}{14} \cdot \frac{14}{9} \cdot \frac{3}{14} \cdot \frac{14}{9} \cdot \frac{2}{14}$$

$$= \frac{3}{9} \cdot \frac{3}{9} \cdot \frac{3}{9} \cdot \frac{2}{9} = \frac{2}{243}$$

② $P(S=yes) = \frac{9}{14}$

③ $P(W) = \frac{1}{2 \cdot 2 \cdot 3 \cdot 3} = \frac{1}{36}$, since there are 36 possible weather combinations

$$\Rightarrow P(S=yes \mid W) = \frac{\frac{2}{243} \cdot \frac{9}{14}}{\frac{1}{36}} = \frac{4}{21} \approx 19{,}05\,\%$$

c) Up until now, no soccer game has been carried out during hot temperature.
This results in $P(temp=hot \mid S=yes) = 0$ and thus $P(S=yes \mid W) = 0$.

To solve this, we simply calculate $P(S=yes \mid W) = 1 - P(S=no \mid W)$:

$P(S=no \mid W)$; W is in table 2

① $P(W \mid S=no) = \prod_i P(x_i \mid S=no) = \prod_i \frac{P(x_i \cap S=no)}{P(S)}$

$$= \frac{P(wind=low, S=no)}{P(S=no)} \cdot \frac{P(humidity=high \mid S=no)}{P(S=no)} \cdot \frac{P(temp=hot \mid S=no)}{P(S=no)} \cdot \frac{P(forecast=sunny \mid S=no)}{P(S=no)}$$

$$= \frac{14}{5} \cdot \frac{2}{14} \cdot \frac{14}{5} \cdot \frac{4}{14} \cdot \frac{14}{5} \cdot \frac{1}{14} \cdot \frac{14}{5} \cdot \frac{1}{14}$$

$$= \frac{2}{5} \cdot \frac{4}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} = \frac{8}{625}$$

② $P(S=no) = \frac{5}{14}$

③ $P(W) = \frac{1}{36}$

$$\Rightarrow P(S=yes \mid W) = 1 - \frac{\frac{8}{625} \cdot \frac{5}{14}}{\frac{1}{36}} = \frac{731}{875} \approx 83{,}54\%$$