## Exercise 11 $\gamma$ -Astronomy 2

This task is a continuation of the task  $\gamma$ -Astronomy from the last sheet. Now it is to be determined whether there really is a  $\gamma$  source at the position at which the telescope was pointed. As a reminder the likelihood function was

$$\ln L = -F = N_{\text{off}} \ln(b) + N_{\text{on}} \ln(s + \alpha b) - (1 + \alpha)b - s - \ln(N_{\text{off}}!) - \ln(N_{\text{on}}!)$$
(1)

and the following values for s and b maximized this likelihood:

$$\hat{s} = N_{\rm on} - \alpha N_{\rm off} \tag{2}$$

$$\hat{b} = N_{\text{off}} \tag{3}$$

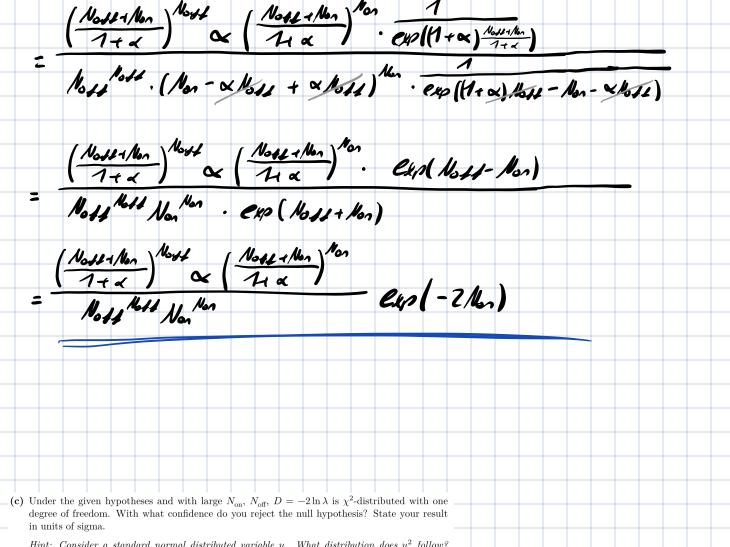
(a) The null hypothesis states that there is no  $\gamma$  source:  $s_0 = 0$ . Under this assumption, what value and what error result for  $b_0$  according to the maximum likelihood method?

## la L = Kalla (b) + Non h (ab) - (1+a)b - la (Noss!) - la (Noss!)

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(b) What is the ratio  $\lambda$  of the two likelihoods?

$$-N_{\mathrm{off}} \ln(b) + N_{\mathrm{on}} \ln(s + \alpha b) - (1 + \alpha)b - s - \ln(N_{\mathrm{off}}!) - \ln(N_{\mathrm{on}}!)$$



Hint: Consider a standard normal distributed variable u. What distribution does  $u^2$  follow? Compare with D.

$$D = -2 \ln \lambda$$

$$= -2 \ln \lambda$$

$$= -2 \ln \left( \frac{5^{NoH} \cdot \alpha + \frac{1}{NoH^{1/N_{1}}} - exp((1+\alpha)5)}{(5^{NoH} \cdot (5+\alpha5)^{Non} \frac{1}{NoH^{1/N_{1}}}) - exp((1+\alpha)5-5)} \right)$$

$$= -2 \ln \left( \frac{5^{NoH} \cdot (5+\alpha5)^{Non} \frac{1}{NoH^{1/N_{1}}} - exp((1+\alpha)5)}{(5^{NoH} \cdot (5+\alpha5)^{Non} \frac{1}{NoH^{1/N_{1}}}) - exp((1+\alpha)5)} \right)$$

$$= -2 \ln \left( \frac{5^{NoH} \cdot (5+\alpha5)^{Non} \frac{1}{NoH^{1/N_{1}}}}{\sqrt{5^{NoH} \cdot (5+\alpha5)^{Non} \frac{1}{NoH^{1/N_{1}}}}} - exp((1+\alpha)5-5) \right)$$

$$\alpha = \sqrt{2\pi\sigma} \quad e \quad | \quad Sids \ dalib \ q., \ also$$