Mandatory Excercise 6

Computationally hard problems

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1 Excercise 6.5

1.0.1 a

The general form of the probability for $T = 1$ can be described as follows.

$$p = 1 - \left(\frac{2^n - 1}{2^n}\right)^S \tag{1}$$

In order to find the S where $p \geq \frac{1}{2}$ we simply solve for

$$\begin{aligned}
\frac{1}{2} &\leq 1 - \left(\frac{2^3 - 1}{2^3}\right)^S && (2)\\
\frac{1}{2} &\leq 1 - \left(\frac{7}{8}\right)^S && (3)\\
S &\geq \frac{ln(2)}{3ln(2) - log(7)} && (4)\\
S &\geq \sim 5.19 && (5)
\end{aligned}$$

Which tells us that we need at least 6 iterations of the S-loop for a probability of at least $\frac{1}{2}$ of all clauses being satisfied.

1.0.2 b

Starting from $\{x_1 = 0, x_2 = 0, x_3 = 0\}$, we have to reach the satisfying solution $\{x_1 = 1, x_2 = 1, x_3 = 1\}$ having all of the x's inversed.

To determine a transformation T for reaching the solution with a probability of at least 12, we look at all possible outcomes of the algorithm in each iteration. So, for $T = 1$ we start in '000' and end with three possible outcomes: '100', '010', and '001'.

If we have $T = 2$, these three outcomes would each grow to three new outcomes and so on. As soon as we reach a solution, we say that it continues growing with T to three new outcomes (by a factor of 3), that are the same solution.

Since we have three x's the algorithm will find the first solutions at $T = 3$. By choosing $T = 7$, more than half of all outcomes will be a satisfying solution, which means the algorithm will find a satisfying solution with a probability of at least $\frac{1}{2}$