Course 02249, DTU Compute
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Computationally Hard Problems – Spring 2016
Assignment Exam Preparation

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This sheet contains exercises, which can be considered as possible exam problems. Some of them have already been used in the assignments in E15 or lectures. The real exam might contain different problems.

Exercise Exam Preparation.1: A polynomial with integer coefficients is a mapping $\mathbb{R} \mapsto \mathbb{R}$ defined by

$$p(x) = \sum_{i=0}^{n} a_i \cdot x^i$$

where $n \in \mathbb{N}$, $a_i \in \mathbb{Z}$ and $a_n \neq 0$. Examples are

$$p_1(x) = -7x^{29} - 3x^7 + x^2 + 3 \qquad p_2(x) = 4x^3 - 2x^2 + x$$

a) Define a language L_{poly} for polynomials with integer coefficients. Make sure that your representation is not unnecessarily long. Do not forget to specify the alphabet Σ_{poly} you use.

b) Describe how one can check for a given string $w \in \Sigma_{\text{poly}}^*$ whether $w \in L_{\text{poly}}$, and, if so, how the polynonomial can be reconstructed.

c) Show the representation of the examples p_1 and p_2 in your language.

End of Exercise 1

Exercise Exam Preparation.2: Show how one can use a decision algorithm for CLIQUE as a black box to find a clique of maximal size in a given graph. The running time of your algorithm has to be polynomial in the input size, counting each call to the black box as one computational step.

End of Exercise 2

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Exercise Exam Preparation.3: Consider the following problem:

Problem: [GRAPHCOLORING]
Input: An undirected graph $G = (V, E)$ and a number $k \in \mathbb{N}$
Output: YES if there is a k-coloring of G and NO otherwise. A *k-coloring* assigns every vertex one of k colors such that adjacent vertices have different colors. The problem can be solved in polynomial time for $k = 2$ and is $\mathcal{NP}$-complete for $k \geq 3$.

Show that GRAPHCOLORING is in the class $\mathcal{NP}$.
Note: You need not show that the problem is $\mathcal{NP}$-complete.

End of Exercise 3

Exercise Exam Preparation.4: Consider the following problem:

Problem: [SIMPLE-4-PARTITION]
Input: Given are n natural numbers $s_1, s_2, \dots, s_n$.
Output: YES if the numbers can be partitioned into four sets with equal sums, i. e., if there are disjoint sets $A_1, A_2, A_3, A_4 \subseteq \{1, \dots, n\}$ such that $A_1 \cup A_2 \cup A_3 \cup A_4 = \{1, \dots, n\}$ and

$$\sum_{i \in A_1} s_i = \sum_{j \in A_2} s_j = \sum_{k \in A_3} s_k = \sum_{\ell \in A_4} s_\ell ,$$

and NO otherwise.

Prove that SIMPLE-4-PARTITION is $\mathcal{NP}$-complete. To this end, you may assume that SIMPLE-4-PARTITION is in $\mathcal{NP}$.

End of Exercise 4

Exercise Exam Preparation.5: Consider the following program:

$i \leftarrow 3$;
while $(rand(1,5) + rand(1,2) \leq 5)$ **do**
    $i \leftarrow i + 1$;
od
$print(i)$;

a) What is the probability for the while condition to be true?

b) What are the possible values of i that could be printed?

c) What is the probability that the value 5 is printed?

d) What is the expected value of i that is printed?

End of Exercise 5