Course 02249, DTU Compute
Carsten Witt, Paul Fischer

Computationally Hard Problems – Spring 2016
Assignment Exam Preparation, Part 2

Date: Spring 2016

This sheet contains exercises, which can be considered as possible exam problems. Some of them have already been used in the assignments in E15 or lectures. The real exam might contain different problems.

Exercise Exam Preparation.6: Consider the following problem.

Problem: [LazyTSP]
Input:

- a number $n \in \mathbb{N}$,
- a matrix $d\colon (i,j) \to \mathbb{N}$, $1 \leq i,j \leq n$;
- a number $B \in \mathbb{N}$,

Output: YES if there is a sequence $i_1 = 1, i_2, \ldots, i_{n-1}$ of pairwise distinct positive integers from $\{1, \ldots, n\}$ such that

$$\left(\sum_{j=1}^{n-2} d(i_j, i_{j+1})\right) + d(i_{n-1}, 1) \;\leq\; B \qquad (*)$$

Otherwise output NO.

What you have to do:

a) Show that LazyTSP is in the class $\mathcal{NP}$.

b) Show that LazyTSP is $\mathcal{NP}$-complete.

End of Exercise 6

Exercise Exam Preparation.7: Suppose you run the algorithm from Section 5.2 in the lecture notes on a complete undirected graph on n vertices using $p_i = 1/(d_{\text{avg}})$ for $i \in \{1, \ldots, n\}$. Let $|I^*|$ be the random size of the independent set computed by the algorithm.

1. What values can $|I^*|$ take with positive probability and what are the exact probabilities for these values?

2. What is the expected size of $|I^*|$?

End of Exercise 7

Exercise Exam Preparation.8: Show that the problem KNAPSACK is $\mathcal{NP}$-complete. In this exercise you may assume that KNAPSACK is in the class $\mathcal{NP}$.

End of Exercise 8

Exercise Exam Preparation.9: Let the following instance to the KNAPSACK problem be given. There are $n = 3$ items. The weights are $w_1 = 10$, $w_2 = 20$, and $w_3 = 10$. The values are $s_1 = 3$, $s_2 = 3$, and $s_3 = 2$. Finally, the capacity of the knapsack is $B = 30$.

a) Show the table computed by the algorithm from Section 4.5.2 of the lecture notes. Which objects will the optimal solution contain?

b) Now divide all object values by 2 and round them down to the nearest integer afterwards. Keep the weights and the capacity. Run the algorithm again on the instance obtained in this way and show the table and solution it computes. What is the approximation ratio of this solution (with respect to the optimal solution with the original values?)

End of Exercise 9

Exercise Exam Preparation.10: Show how to convert a decision algorithm for VERTEXCOVER into an optimization algorithm, i. e., one that finds a vertex cover of minimal size. The resulting optimization algorithm should have polynomial running time if a call of the decision algorithm is counted as one basic computational step.
Hint: You might want to introduce extra vertices and edges.

End of Exercise 10