Mandatory Excercise 4

Computationally hard problems

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1 Excercise 4.4

1.1 Prove that Simple-3-Partition is in the class NP

We follow the approach described in lecture notes page 33.

1.1.1 Design a deterministic algorithm A which takes as input a problem instance X and a random sequence R

We define an algorithm $A_{S3P}(X, R)$, where $n = |X|$, R is a sequence of integers of length n, randomly chosen from an evenly distributed set of numbers $0, 1, 2$. Entries in R corresponds to a disjoint set in which X_i should fall, given that i is an index in R.

Having chosen R, we take the sum of our disjoint sets from X, and call these three sums A_0, A_1 and A_2.

If these sums are equal, the algorithm answers YES.

1.1.2 If the answer to X is YES, then there is a string R_0 with positive probability such that $A(X, R_0)$ = YES

Since R is chosen from an even distribution of finite size $0, 1, 2^n = 3^n$ containing all possible combinations, we know that there is a finite probability that the correct combination is chosen.

1.1.3 If the answer to X is NO, then $A(x, R)$ = NO for all R

Since A_0, A_1 and A_2 cannot be equal unless they are in fact equal, the algorithm will always answer NO if the correct answer is NO.

1.1.4 Show that A is p-bounded for some polynomial p

We can choose R in $O(n)$ time. We can choose the disjoint sets A_0, A_1 and A_2 in $O(n)$ time. We can take the sum of A_0, A_1 and A_2 respectively in $O(n)$ time. We can verify the equality of A_0, A_1 and A_2 in $O(1)$ time.

Thus we are bound by a running time of $O(n)$, ie. a 1st degree polynomial.