# Assignment 6
Computationally Hard Problems - Fall 2015

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# 1 a)

With $T = 1$, the algorithm simply chooses a random assignment for the variable set $\{x_1, x_2, x_3\}$, checks if all clauses evaluates to true and returns if it does. If not, it repeats, by choosing new random variables.

For the particular set of clauses, there exist exactly one truth assignment to the literals, that will cause all clauses to evaluate to true. There is $2^n$ different possible assignments, where $n$ is the number of literals. This means that in every iteration, the algorithm has a $\frac{1}{2^n}$ chance at picking the right assignment. To calculate $S$, I solve the following equation for $S$

$$\begin{aligned}
\frac{1}{2} &\leq 1-(\frac{2^n-1}{2^n})^S \Rightarrow && (1)\\
\frac{1}{2} &\leq 1-(\frac{7}{8})^S \Rightarrow && (2)\\
S &\geq \frac{ln(2)}{3ln(2)-ln(7)} \Rightarrow && (3)\\
S &\geq \sim 5.19 && (4)
\end{aligned}$$

Since $S$ is an integer, we find that $S = 6$ is the lowest $S$, where the algorithm will pick the right assignment with a probability of at least $\frac{1}{2}$.

# 2 b)

Starting from $\{x_1 = 0, x_2 = 0, x_3 = 0\}$, we have to reach the satisfying solution, which is $\{x_1 = 1, x_2 = 1, x_3 = 1\}$, which means all $x$'s has to be "flipped".

To determine a $T$, that will yield a satisfying solution with a probability of at least $\frac{1}{2}$, I look at all possible outcomes of the algorithm in each iteration. So, for $T = 1$ we start in '000' and end with three possible outcomes: '100', '010', and '001'. If we have $T = 2$, these three outcomes would each grow to three new outcomes and so on. As soon as we reach a solution, we say that it continues growing with $T$ (even though it would have stopped) to three new outcomes, that are the same solution. So '111' would grow to '111', '111', '111'.

Since we have three $x$'s, the algorithm will find the first solutions at $T = 3$. From this point on, the growth of satisfying solutions follow the following recursive formula, where we define $\Delta_T$ to be the total possible outcomes for a choice of $T$, which always grows with a factor 3.

$$s_T = \left| \begin{array}{cl} 3s_{T-1} & \text{if T is even} \\ 3s_{T-1} + \frac{2}{3}\Delta_{T-1} & \text{if T is odd} \end{array} \right.$$

With this, we find the probabilities

$$\begin{aligned}
T_1 &= \frac{0}{3} = 0\% && (5)\\
T_2 &= \frac{0}{9} = 0\% && (6)\\
T_3 &= \frac{6}{27} = 22.22...\% && (7)\\
T_4 &= \frac{18}{81} = 22.22...\% && (8)\\
T_5 &= \frac{96}{243} = 39.51\% && (9)\\
T_6 &= \frac{288}{729} = 39.51\% && (10)\\
T_7 &= \frac{1158}{2187} = 52.95\% && (11)
\end{aligned}$$

Hence, by choosing $T = 7$, more than half of all outcomes will be a satisfying solution, which means the algorithm will find a satisfying solution with a probability of at least $\frac{1}{2}$.