Course 02249, DTU Compute
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Computationally Hard Problems – Fall 2015
Assignment 6

Date: 03.11.2015, **Due date**: 09.11.2015, 23:59

The following exercises are **not** mandatory:

Exercise 6.1: Show that there is a **deterministic** algorithm for MaxSat with approximation ratio no worse than 2.
Hint: It is enough to consider two assignments.

End of Exercise 1

Exercise 6.2: Prove that the algorithm from Theorem 5.2 in the lecture notes has an expected approximation ratio of 4/3 (or better) if all clauses have at least 2 literals.

End of Exercise 2

Exercise 6.3: Consider the following eight clauses over the boolean variables $\{x_1, x_2, x_3\}$:

$$
\begin{aligned}
c_1 &= x_1 \vee x_2 \vee x_3 \\
c_2 &= \overline{x_1} \vee \overline{x_3} \\
c_3 &= \overline{x_2} \vee x_3 \\
c_4 &= x_2 \vee x_3 \\
c_5 &= x_1 \vee \overline{x_3} \\
c_6 &= \overline{x_1} \\
c_7 &= \overline{x_3} \\
c_8 &= x_1 \vee \overline{x_2} \vee x_3
\end{aligned}
$$

a) Determine whether there is a satisfying assignment.

b) Construct the integer program for the set of clauses.

c) Relax the integer program to a linear program as described in the notes.

d) Solve the linear program.

e) Use randomized rounding to find an assignment and determine how many clauses your rounding satisfies.

Hint: Use an off-the-shelf solver for linear programs. You can find them in Matlab and Maple and many other places.

End of Exercise 3

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Exercise 6.4: Suppose that Algorithm 5.9 from the lecture notes is run on a satisfiable 3-SAT instance with parameters $T = n/2$ and $S = \infty$. Show that it outputs a satisfying assignment in an expected number of at most $2(\sqrt{3})^n = O(1.7321^n)$ iterations of the outer loop (over s).
Hint: The random variable d^* used in the proof of Theorem 5.13 is binomially distributed with parameters n and $1/2$. In particular, this means that $d^* \leq n/2$ occurs with probability at least $1/2$.

End of Exercise 4

The following exercise is **mandatory**:

Exercise 6.5: Consider Algorithm 5.9 from the lecture notes. Suppose it is run on the following 3-SAT instance with variable set $\{x_1, \ldots, x_3\}$ and clause set

$$
\begin{gathered}
x_1 \vee x_2 \vee x_3 \\
\overline{x_1} \vee x_2 \vee x_3 \\
x_1 \vee \overline{x_2} \vee x_3 \\
x_1 \vee x_2 \vee \overline{x_3} \\
\overline{x_1} \vee \overline{x_2} \vee x_3 \\
\overline{x_1} \vee x_2 \vee \overline{x_3} \\
x_1 \vee \overline{x_2} \vee \overline{x_3}.
\end{gathered}
$$

a) Suppose the algorithm is run with $T = 1$. Find a choice of S such that the algorithm terminates with a satisfying assignment with probability at least $1/2$.

b) Suppose now $S = 1$ and that the random choice of the initial assignment in the algorithm results in $x_i = 0$ for $i \in \{1, \ldots, 3\}$. Find a choice of T such that the algorithm terminates with a satisfying assignment with probability at least $1/2$.

Justify your choice in both parts. You need not find the smallest possible S and T.

Hint: Lemma A.2 and Inequality (A.10) from the lecture notes may be useful. In part b) you may define a pessimistic sequence of events that is guaranteed to lead to a satisfying assignment.

End of Exercise 5