Course 02249, DTU Compute
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Computationally Hard Problems – Fall 2015
Assignment Exam Preparation

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This sheet contains only non-mandatory exercises, which were exam problems before or can be considered as such. Some of them have already been used in the assignments or lectures. The real exam may contain different problems.

The following exercises are **not** mandatory:

Exercise Exam Preparation.1: A monoalphabetic cipher is a bijective (one-to-one) substitution of the 26 latin upper-case letters `A`–`Z` by letters from the same alphabet. One example is given by the Caesar cipher, which replaces each letter by its successor in the alphabet, except for `Z`, which is replaced by `A`. The aim is to describe monoalphabetic ciphers as a formal language.

a) Specify the alphabet Σ_{cipher} you use.

b) Specify how the language L_{cipher} is defined.

c) Describe how one can check whether a given word $w \in \Sigma^*_{cipher}$ is in L_{cipher}.

d) How would the Caesar cipher be encoded in your language?

End of Exercise 1

Exercise Exam Preparation.2: Show how one can use a decision algorithm for Clique as a black box to find a clique of maximal size in a given graph. The running time of your algorithm has to be polynomial in the input size, counting each call to the black box as one computational step.

End of Exercise 2

Continued on next page.

Exercise Exam Preparation.3: Consider the following problem:

Problem: [GraphColoring]
Input: An undirected graph $G = (V, E)$ and a number $k \in \mathbb{N}$
Output: YES if there is a k-coloring of G and NO otherwise. A *k-coloring* assigns every vertex one of k colors such that adjacent vertices have different colors. The problem can be solved in polynomial time for $k = 2$ and is $\mathcal{NP}$-complete for $k \geq 3$.

Show that GraphColoring is in the class $\mathcal{NP}$.
Note: You need not show that the problem is $\mathcal{NP}$-complete.

End of Exercise 3

Exercise Exam Preparation.4: Consider the following problem:

Problem: [Simple-4-Partition]
Input: Given are n natural numbers $s_1, s_2, \ldots, s_n$.
Output: YES if the numbers can be partitioned into four sets with equal sums, i. e., if there are disjoint sets $A_1, A_2, A_3, A_4 \subseteq \{1, \ldots, n\}$ such that $A_1 \cup A_2 \cup A_3 \cup A_4 = \{1, \ldots, n\}$ and

$$\sum_{i \in A_1} s_i = \sum_{j \in A_2} s_j = \sum_{k \in A_3} s_k = \sum_{\ell \in A_4} s_\ell \ ,$$

and NO otherwise.

Prove that Simple-4-Partition is $\mathcal{NP}$-complete. To this end, you may assume that Simple-4-Partition is in $\mathcal{NP}$.

End of Exercise 4

Exercise Exam Preparation.5: Consider the following program:

$i \leftarrow 3$;
while ($rand(1,5) + rand(1,2) \leq 5$) **do**
    $i \leftarrow i + 1$;
od
$print(i)$;

a) What is the probability for the while condition to be true?

b) What are the possible values of i that could be printed?

c) What is the probability that the value 5 is printed?

d) What is the expected value of i that is printed?

End of Exercise 5