

Mandatory Exercise 1

Computationally hard problems

September 3, 2015

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1 Exercise 1.4

1.1 A

The alphabet used to describe the undirected hypergraph language L_{hg} can be denoted as:

$$\Sigma_{hg} = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, -, :, \# \quad (1)$$

1.2 B

We can encode an undirected hypergraph with the following notation

$$< \text{number of vertices} > : \text{vertex}_x - \text{vertex}_y - \text{vertex}_z \# \text{vertex}_a - \text{vertex}_b \# \dots \quad (2)$$

Here we implicitly assume that vertices are labelled from 1 to n . This assumption enables us to simply specify the number of vertices.

The syntax can be defined as a the following regular expression:

$$[0 - 9]^+ : ([0 - 9]^+)(-[0 - 9]^+)^* (\#[0 - 9]^+ (-[0 - 9]^+)^*)^* \quad (3)$$

And example of this notation would be:

$$4 : 1 - 2 - 3 \# 1 - 4 \quad (4)$$

1.3 C

We can check if a given word $w \in \Sigma_{hg}^*$ is in the language L_{hg} , if it starts with a number, and only contains numbers lower than this first number. Furthermore it must match the regular expression described above. If these conditions are not met, we discard the hypergraph.

We can reconstruct the hypergraph by first creating the given number of vertices. Then we split the word at each $\#$ to get the vertex sets. Each vertex set is then split at the

—, breaking the set down into individual vertices. We can now create an edge between each of the vertices in the given set. We repeat this step for all vertex sets and upon completion we have our hypergraph constructed.

1.4 D

We can encode the given hypergraph $V = \{1 \dots 10\}$ and $E = \{\{1, 2, 5\}, \{4\}, \{3, 5, 6, 9\}, \emptyset\}$ in the following manner:

$$10 : 1 - 2 - 5 \# 4 \# 3 - 5 - 6 - 9 \tag{5}$$