Ch. 4 Exercises: Probability

Thea Sukianto

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Exercise 4.4

Consider a spinner with a $[0, 1]$ scale on its circumference. Suppose that the spinner is slanted or magnetized or bent in some way such that it is biased, and its probability density function is $p(x) = 6x(1 - x)$ over the interval $x \in [0, 1]$.

(A) Adapt the program `IntegralOfDensity.R` to plot this density function and approximate its integral. Comment your code. Be careful to consider values of x only in the interval $[0, 1]$. Hint: You can omit the first couple of lines regarding `meanval` and `sdval`, because those parameter values pertain only to the normal distribution. Then set `xlow = 0` and `xhigh = 1` ,and set `dx` to some small value.

```r
# source("../DBDA2E-utilities.R")
# Graph of normal probability density function, with comb of intervals.
#meanval = 0.0               # Specify mean of distribution.
#sdval = 0.2                 # Specify standard deviation of distribution.
xlow  = 0 # Specify low end of x-axis.
xhigh = 1 # Specify high end of x-axis.
dx = 0.01                 # Specify interval width on x-axis
# Specify comb of points along the x axis:
x = seq( from = xlow , to = xhigh , by = dx )
# Compute y values, i.e., probability density at each value of x:
y = 6*x*(1-x)
# Plot the function. "plot" draws the intervals. "lines" draws the bell curve.
#openGraph(width=7,height=5)
#plot( x , y , type="h" , lwd=1 , cex.axis=1.5
    #, xlab="x" , ylab="p(x)" , cex.lab=1.5 ,
    #, main="Probability Density of X" , cex.main=1.5 )
#lines( x , y , lwd=3 ,  col="skyblue" )
# Approximate the integral as the sum of width * height for each interval.
area = sum( dx * y )
# Display info in the graph.
#text( meanval-sdval , .9*max(y) , bquote( paste(mu ," = " ,.(meanval)) )
      #, adj=c(1,.5) , cex=1.5 )
#text( meanval-sdval , .75*max(y) , bquote( paste(sigma ," = " ,.(sdval)) )
      #, adj=c(1,.5) , cex=1.5 )
#text( .75 , .9*max(y) , bquote( paste(Delta , "x = " ,.(dx)) )
      #, adj=c(0,.5) , cex=1.5 )
#text( .75 , .75*max(y) ,
      #bquote(
        #paste( sum(,x,) , " " , Delta , "x p(x) = " , .(signif(area,3)) )
      #) , adj=c(0,.5) , cex=1.5 )
# Save the plot to an PNG  file.
#saveGraph( file = "IntegralOfDensity" , type="png" )
```

(B) Derive the exact integral using calculus. Hint: See the example, Equation 4.7.

$$\int_0^1 6x(1-x)dx = 6\int_0^1 x - x^2 dx = 6[\tfrac{x^2}{2} - \tfrac{x^3}{3}|_0^1] = 6 \times \tfrac{1}{6} = 1$$

(C) Does this function satisfy Equation 4.3?

Yes, the above integral is equal to 1, so the function is a legitimate PDF.

(D) From inspecting the graph, what is the maximal value of $p(x)$?

1.5 which occurs at x = 0.5.

Exercise 4.6

School children were surveyed regarding their favorite foods. Of the total sample, 20% were 1st graders, 20% were 6th graders, and 60% were 11th graders. For each grade, the following table shows the proportion of respondents that chose each of three foods as their favorite:

	Ice cream	Fruit	French fries
1st graders	0.3	0.6	0.1
6th graders	0.6	0.3	0.1
11th graders	0.3	0.1	0.6

From that information, construct a table of joint probabilities of grade and favorite food. Also, say whether grade and favorite food are independent or not, and how you ascertained the answer. Hint: You are given p(grade) and p(food|grade). You need to determine p(grade, food).

Discussion

By the definition of conditional probability, p(grade, food) = p(grade)p(food|grade), so we have:

	Ice cream	Fruit	French fries	Total
1st graders	0.06	0.12	0.02	0.2
6th graders	0.12	0.06	0.02	0.2
11th graders	0.18	0.06	0.36	0.6
Total	0.36	0.24	0.40	1

We know that Grade and Food are independent iff p(grade, food) = p(grade)p(food).

But p(1st grade, Ice cream) = 0.06, which is not equal to p(1st grade)p(Ice cream) = 0.2 x 0.36 = 0.072.

Therefore, Grade and Food are not independent.