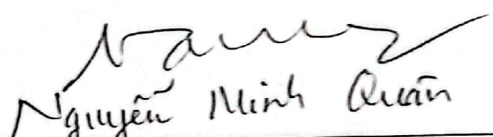



THE INTERNATIONAL UNIVERSITY (IU)  
VIETNAM NATIONAL UNIVERSITY - HCMC

**FINAL EXAMINATION**  
**PROBABILITY, STATISTICS AND RANDOM PROCESS**  
Semester 2, 2022-23 • June 2023 • Total duration: 90 minutes

Chair of Mathematics Department	Lecturer
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**INSTRUCTIONS:** Each student is allowed calculators, statistical tables and one double-sided sheet of reference material (size A4 or similar) marked with their name and ID. All other documents and electronic devices are forbidden.

1. (30 points) (Gambling problem) Consider an amount \$3 is shared between player A and player B. At each round, player A may earn \$1 with probability 0.6 and in this case player B loses \$1. Conversely, player A may loses \$1 with probability 0.6, in which case player B wins \$1. Suppose that the successive rounds are independent. The game terminates whenever one of the players loses all his/she money. The wealth of player A is modelled by a Markov chain  $(X_n)_{n \geq 0}$  with state space  $S = \{0, 1, 2, 3\}$  and transition matrix

$$P = \begin{array}{c} \text{To} \\ \begin{array}{c} 0 \quad 1 \quad 2 \quad 3 \\ \text{From } \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \end{pmatrix} \end{array} \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.4 & 0 & 0.6 & 0 \\ 0 & 0.4 & 0 & 0.6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- (a) Compute  $P(X_2 = 0 | X_0 = 1)$ .  
 (b) Evaluate  $P(X_2 = 0, X_1 = 0 | X_0 = 1)$ .  
 (c) Given  $X_0 = 1$ , determine probability that the game terminates after 2 rounds.  
 (d) Which state(s) are recurrent? Which states are transient? Find all recurrent class(s) of this Markov chain.
2. (10 points) Consider a Markov chain  $(X_n)_{n \geq 0}$  with state space  $S = \{1, 2, 3\}$  and transition matrix

$$P = \begin{array}{c} \text{To} \\ \begin{array}{c} 1 \quad 2 \quad 3 \\ \text{From } \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \end{array} \end{array} \begin{bmatrix} 0.25 & 0.5 & 0.25 \\ 0.1 & 0.3 & 0.6 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}$$

Find the stationary distribution of the Markov chain.

PLEASE TURN OVER



3. (10 points) Data for weight of salmon (in kilograms) in a sample is given by

1 2 3 4 5 6 7 8 9 10  
1.72 1.79 1.80 1.74 1.82 1.79 1.82 1.78 1.60 1.75

Compute sample mean, sample median and the sample standard deviation of the above data.

4. (10 points) Assume that the gain in a circuit on a semiconductor device. Assume that gain is normally distributed with standard deviation  $\sigma = 15$ . In a sample of 15 circuit, the average value of gain is 1000. Construct a 99% confidence interval for the true mean.
5. (10 points) The diameter of ring piston produced in a manufacture is normally distributed with standard deviation  $\sigma = 0.002$  millimeters. It is desired to estimate the mean of diameter with an error is within 0.0005 millimeters at 95% confidence. What sample size is required?
6. (10 points) Sugar content of syrup in canned peaches is normally distributed. A random sample of 20 cans yields a standard deviation  $s = 5.2$  milligrams. Construct 95% confidence interval for variance  $\sigma^2$ .
7. (10 points) The sodium content of twenty 300-gram boxes of organic cornflakes was determined. The data (in milligrams) are as follows: 131.15, 130.69, 130.91, 129.54, 129.64, 128.77, 130.72, 128.33, 128.24, 129.65, 130.14, 129.29, 128.71, 129.00, 129.39, 130.42, 129.53, 130.12, 129.78, 130.92.  
At level of significant  $\alpha = 1\%$ , do the data support the claim that that mean sodium content of this brand of cornflakes differs from 132 milligrams?
8. (10 points) Running time in a sample of 17 players yields a standard deviation  $s = 0.09$  seconds.  
At level of confidence  $\alpha = 0.05$ , is there enough evidence to conclude that the standard deviation of running time exceeds the historical value of 0.8 seconds?

THE END