THE INTERNATIONAL UNIVERSITY(IU)
VIETNAM NATIONAL UNIVERSITY - HCMC

FINAL EXAMINATION
PROBABILITY, STATISTICS AND RANDOM PROCESS

Semester 2, 2022-23 • June 2023 • Total duration: 90 minutes

Chair of Mathematics Department	Lecturer
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INSTRUCTIONS: Each student is allowed calculators, statistical tables and one double-sided sheet of reference material (size A4 or similar) marked with their name and ID. All other documents and electronic devices are forbidden.

1. *(30 points)* **(Gambling problem)** Consider an amount \$3 is shared between player A and player B. At each round, player A may earn \$1 with probability 0.6 and in this case player B loses \$1. Conversely, player A may loses \$1 with probability 0.6, in which case player B wins \$1. Suppose that the succesive rounds are independent. The game terminates whenever one of the players loses all his/she money.
The wealth of player A is modelled by a Markov chain $(X_n)_{n\geq 0}$ with state space $\mathbb{S} = \{0,1,2,3\}$ and transition matrix

$$P = \begin{array}{cc} & \text{To} \\ & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \text{From} \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.4 & 0 & 0.6 & 0 \\ 0 & 0.4 & 0 & 0.6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

(a) Compute $P(X_2 = 0|X_0 = 1)$.

(b) Evaluate $P(X_2 = 0, X_1 = 0|X_0 = 1)$

(c) Given $X_0 = 1$, determine probability that the game terminates after 2 rounds.

(d) Which state(s) are recurrent? Which states are transient? Find all recurrent class(s) of this Markov chain.

2. *(10 points)* Consider a Markov chain $(X_n)_{n\geq 0}$ with state space $\mathbb{S} = \{1,2,3\}$ and transition matrix

$$P = \begin{array}{cc} & \text{To} \\ & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \text{From} \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0.25 & 0.5 & 0.25 \\ 0.1 & 0.3 & 0.6 \\ 0.2 & 0.3 & 0.5 \end{bmatrix} \end{array}$$

Find the stationary distribution of the Markov chain.

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3. *(10 points)* Data for weight of salmon (in kilograms) in a sample is given by

1 2 3 4 5 6 7 8 9 10

1.72 1.79 1.80 1.74 1.82 1.79 1.82 1.78 1.60 1.75

Compute sample mean, sample median and the sample standard deviation of the above data.

4. *(10 points)* Assume that the gain in a circuit on a semiconductor device. Assume that gain is normally distributed with standard deviation $\sigma = 15$. In a sample of 15 circuit, the average value of gain is 1000. Contruct a 99% confidence interval for the true mean.

5. *(10 points)* The diameter of ring piston produced in a manufacture is normally distributed with standard deviation $\sigma = 0.002$ milimeters. It is desired to estimate the mean of diameter with an error is within 0.0005 milimeters at 95% confidence. What sample size is required?

6. *(10 points)* Sugar content of syrup in canned peaches is normally distributed. A random sample of 20 cans yields a standard deviation $s = 5.2$ miligrams. Construct 95% confidence interval for variance σ^2.

7. *(10 points)* The sodium content of twenty 300-gram boxes of organic cornflakes was determined. The data (in milligrams) are as follows: 131.15, 130.69, 130.91, 129.54, 129.64, 128.77, 130.72, 128.33, 128.24, 129.65, 130.14, 129.29, 128.71, 129.00, 129.39, 130.42, 129.53, 130.12, 129.78, 130.92.

   At level of significant $\alpha = 1\%$, do the data support the claim that that mean sodium content of this brand of cornflakes differs from 132 milligrams?

8. *(10 points)* Running time in a sample of 17 players yields a standard deviation $s = 0.09$ seconds.

   At level of confidence $\alpha = 0.05$, is there enough evidence to conclude that the standard deviation of running time exceeds the historical value of 0.8 seconds?

THE END