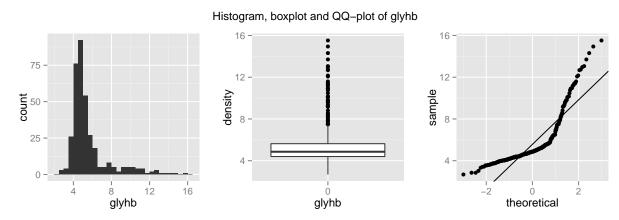
Statistics 135 – Lab Project

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1 Background

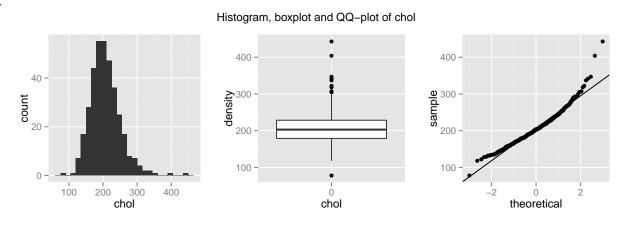
2 Accessing Data, Visualization and Summarization

1.



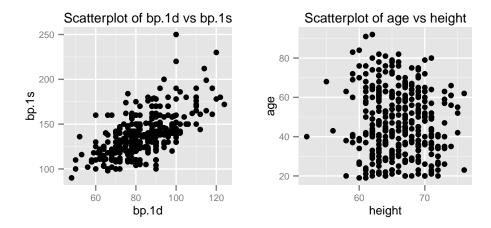
The mean, median and mode of glyhb are all approximately 5. The distribution of glyhb is left-skewed.

2.



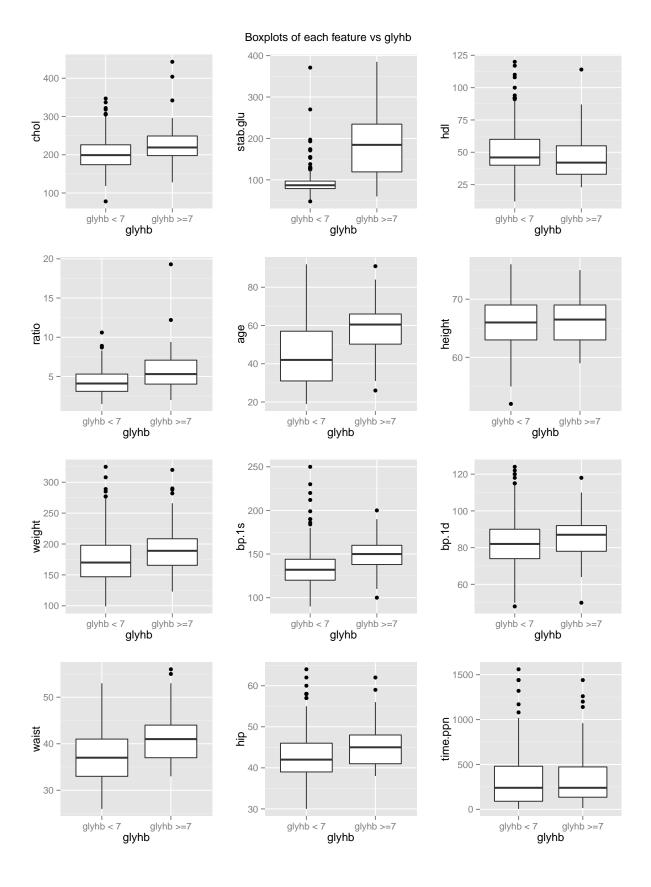
The mean, median and mode of chol are all approximately 200. The distribution of chol is better approximated with a Gaussian distribution.

3.

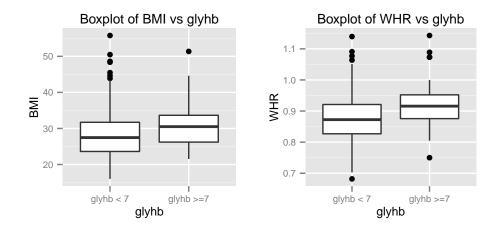


The scatterplot of bp.1s and bp.1d is near-linear, so they are approximately dependent. The scatterplot of age and weight is random, so they are approximately independent.

- 4. chol: The two distributions have small difference, so it MAY BE a relevant feature.
 - stab.glu: The two distributions have substantial difference, so it SHOULD BE a relevant feature.
 - hdl: The two distributions have small difference, so it MAY BE a relevant feature.
 - ratio: The two distributions have small difference, so it MAY BE a relevant feature.
 - age: The two distributions have substantial difference, so it SHOULD BE a relevant feature.
 - height: The two distributions have little difference, so it MAY NOT BE a relevant feature.
 - weight: The two distributions have small difference, so it MAY BE a relevant feature.
 - \bullet bp.1s: The two distributions have small difference, so it MAY BE a relevant feature.
 - bp.1d: The two distributions have small difference, so it MAY BE a relevant feature.
 - waist: The two distributions have small difference, so it MAY BE a relevant feature.
 - hip: The two distributions have small difference, so it MAY BE a relevant feature.
 - time.ppn: The two distributions have small difference, so it MAY NOT BE a relevant feature.



5.



6. In light of these first experiments, hdl, stab.glu, age, weight, bp.1s, bp.1d, waist and hip seem related to the presence of type II diabetes; chol, ratio, height and time.ppn seem unrelated to the presence of type II diabetes.

3 Parametric Inference

1.

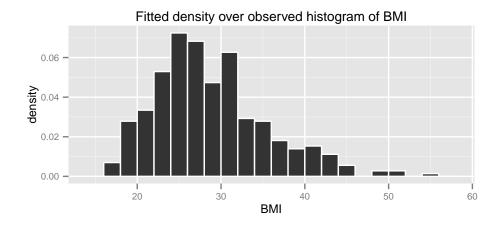
$$X \sim Gamma(\alpha, \beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha - 1} e^{-\beta x}$$

$$E(X) = \frac{\alpha}{\beta}$$

$$E(X^{2}) = Var(X) + [E(X)]^{2}$$
$$= \frac{\alpha}{\beta^{2}} + \left(\frac{\alpha}{\beta}\right)^{2}$$
$$= \frac{\alpha(\alpha + 1)}{\beta^{2}}$$

$$\begin{cases} E(X) = \frac{\alpha}{\beta} \\ E(X^2) = \frac{\alpha(\alpha+1)}{\beta^2} \end{cases} \implies \begin{cases} \alpha = \frac{[E(X)]^2}{Var(x)} \\ \beta = \frac{E(X)}{Var(x)} \end{cases} \implies \begin{cases} \hat{\alpha}_{MOM} = \frac{\overline{X}_n}{\frac{1}{n}\sum_{i=1}^n (X_i - \overline{X}_n)^2} \\ \hat{\beta}_{MOM} = \frac{\overline{X}_n^2}{\frac{1}{n}\sum_{i=1}^n (X_i - \overline{X}_n)^2} \end{cases}$$

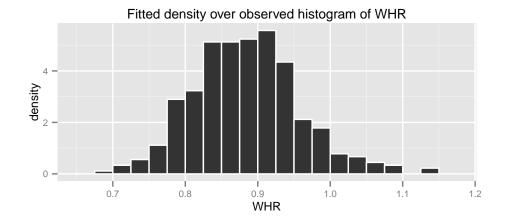
alpha beta ## 2.5% 15.87666 0.5460733 ## 97.5% 21.62832 0.7591677



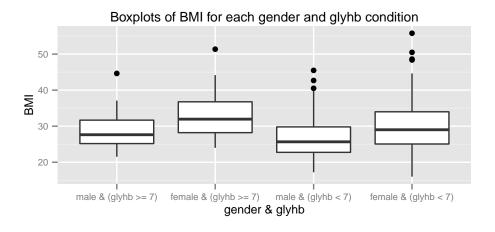
2.

$$\hat{\mu}_{MLE} = \overline{X}_n$$

$$\hat{\sigma}_{MLE}^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \overline{X}_n)^2$$

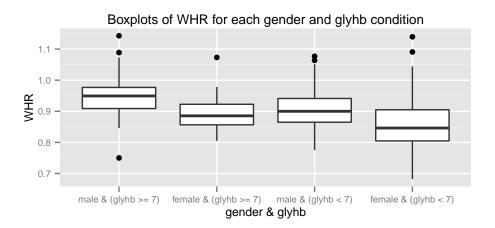


```
CIs.BMI
 ## $`male & (glyhb >= 7)`
 ##
            mu
                 sigma
 ## 2.5% 26.89463 3.533570
 ## 97.5% 30.97481 7.164682
 ##
   $`female & (glyhb >= 7)`
 ##
 ##
                 sigma
 ## 2.5% 30.90826 4.608429
 ## 97.5% 35.63231 8.207146
 ##
 ##
   $`male & (glyhb < 7)`</pre>
 ##
            mu
                 sigma
 ## 2.5% 25.57434 4.711554
 ## 97.5% 27.47041 6.358453
 ##
 ## $`female & (glyhb < 7)`
 ##
            mu
                 sigma
 ## 2.5% 28.84236 6.220170
 ## 97.5% 30.84639 7.862444
```



- On average, females have higher BMI than males.
- On average, people with type II diabetes (glyhb >= 7) have higher BMI than people without type II diabetes (glyhb < 7).
- People with type II diabetes (glyhb >= 7) have larger confidence intervals of both mean and standard deviation than people without type II diabetes (glyhb < 7), regardless of gender.

```
CIs.WHR
## $`male & (glyhb >= 7)`
##
           mu
                  sigma
## 2.5% 0.9202216 0.04735021
## 97.5% 0.9791687 0.10652492
##
##
 $`female & (glyhb >= 7)`
##
           mu
                  sigma
## 2.5% 0.8754801 0.04013498
## 97.5% 0.9139384 0.07537186
##
 $`male & (glyhb < 7)`</pre>
##
##
           mu
                  sigma
## 2.5% 0.8938359 0.05568191
## 97.5% 0.9172246 0.07075628
##
##
 $`female & (glyhb < 7)`</pre>
##
           mu
                  sigma
## 2.5% 0.8446725 0.06108226
## 97.5% 0.8644300 0.07829550
```



- On average, males have higher WHR than females.
- On average, people with type II diabetes (glyhb >= 7) have higher WHR than people without type II diabetes (glyhb < 7).
- People with type II diabetes (glyhb >= 7) have larger confidence intervals of both mean and standard deviation than people without type II diabetes (glyhb < 7), regardless of gender.

4 Testing

```
gender.glyhb.cond.table
 ##
       glyhb >= 7 glyhb < 7
            24
 ## male
                  125
 ## female
            30
                  180
 fisher.test(gender.glyhb.cond.table)
 ##
   Fisher's Exact Test for Count Data
 ##
 ##
 ## data: gender.glyhb.cond.table
 ## p-value = 0.6552
 ## alternative hypothesis: true odds ratio is not equal to 1
 ## 95 percent confidence interval:
 ## 0.6126316 2.1465820
 ## sample estimates:
 ## odds ratio
 ##
   1.151538
```

Since the p-value is 0.655, which is greater than 0.05, we fail to reject the null hypothesis that males and females are equally exposed to type II diabetes, with 5% significance level.

2. We choose to the non-parametric Kruskal-Wallis test, because it does not rely on the assumed normal distribution and less affected by outliers.

Since the p-value is 0.00475, which is smaller than 0.05, we reject the null hypothesis that hdl has equal means for those with type II diabetes and those without, with 5% significance level.

Since the p-value is 2.034e-06, which is smaller than 0.05, we reject the null hypothesis that bp.1s has equal means for those with type II diabetes and those without, with 5% significance level.

Since the p-value is 0.168, which is greater than 0.05, we fail to reject the null hypothesis that bp.1d has equal means for those with type II diabetes and those without, with 5% significance level.

Since the p-value is 0.00198, which is smaller than 0.05, we reject the null hypothesis that BMI has equal means for those with type II diabetes and those without, with 5% significance level.

Since the p-value is 9.95e-05, which is smaller than 0.05, we reject the null hypothesis that WHR has equal means for those with type II diabetes and those without, with 5% significance level.

```
pi.male.BMI
 ## [1] 0.6326667
 CI.pi.male.BMI
    2.5%
       97.5%
 ## 0.5222833 0.7420167
 pi.male.WHR
 ## [1] 0.6853333
 CI.pi.male.WHR
 ##
    2.5%
       97.5%
 ## 0.5716667 0.7950000
```

4. According to the result from part 3.2, we know that WHR has a normal distribution. We assume all patients come from the same population, so the standard deviation is constant.

$$H_0: N(\mu_0, \sigma^2)$$

 $H_1: N(\mu_1, \sigma^2)$

mean.male.glyhb.geq7.WHR

[1] 0.9489365

mean.male.glyhb.17.WHR

[1] 0.9058419

sd.male.WHR

[1] 0.06821165

$$\mu_0 = 0.906, \mu_1 = 0.949, \sigma = 0.0682$$

$$lik(x) = \frac{f_0(x)}{f_1(x)}$$

$$= \frac{\frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{(x-\mu_0)^2}{2}}}{\frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{(x-\mu_1)^2}{2}}}$$

$$= e^{-2(\mu_1 - \mu_0)x + (\mu_1^2 - \mu_0^2)}$$

Let T := X.

$$\alpha = P(T > t \mid H_0)$$

$$= P(\frac{T - \mu_0}{\sigma} > \frac{t - \mu_0}{\sigma})$$

$$= 1 - \Phi\left(\frac{t - \mu_0}{\sigma}\right)$$

$$\Longrightarrow$$

$$t = \Phi^{-1}(\alpha)\sigma + \mu_0$$

$$\beta = P(T < t \mid H_1)$$

$$= P(\frac{T - \mu_1}{\sigma} < \frac{t - \mu_1}{\sigma})$$

$$= \Phi\left(\frac{t - \mu_1}{\sigma}\right)$$

$$= \Phi\left(\frac{\Phi^{-1}(\alpha)\sigma + \mu_0 - \mu_1}{\sigma}\right)$$

$$\alpha \leqslant 5\% \implies 1 - \beta \leqslant 0.156$$

We construct a test for type II diabetes for male patient that we reject the null hypothesis if his WHR $\geqslant 1.018$. The significance of the test is $\leqslant 5\%$ and the power of the test is $\leqslant 0.156$.

```
gender.BMI.categories
 ##
          male female
 ## Underweight
           5
                3
            59
                43
 ## Healthy
 ## Overweight
            46
                66
 ## Level 1 Obese
            26
                52
 ## Level 2 Obese
            9
                22
 ## Level 3 Obese
                24
            4
 chisq.test(gender.BMI.categories)
 ##
 ##
   Pearson's Chi-squared test
 ##
 ## data: gender.BMI.categories
 ## X-squared = 25.352, df = 5, p-value = 0.0001191
```

Since the p-value is 0.000119, which is smaller than 0.05, we reject the null hypothesis that male and female population sample has homogeneous distribution of BMI categories, with 5% significance level.

```
gender.WHR.categories
##
      male female
## Low
       46
## Moderate
       66
           33
## High
       20
           76
## Very High
      17
           95
chisq.test(gender.WHR.categories)
##
##
 Pearson's Chi-squared test
##
## data: gender.WHR.categories
## X-squared = 126.99, df = 3, p-value < 2.2e-16
```

Since the p-value is 2.2e-16, which is smaller than 0.05, we reject the null hypothesis that male and female population sample has homogeneous distribution of WHR categories, with 5% significance level.

The interaction effect between BMI and WHR is not significant. BMI is more sensitive to glyhb. Overall, the result is consistent with part 3.3.

5 Regression

1. According to the result from part 2.2, we consider stab.glu, age, BMI and WHR the most relevant features for predicting type II diabetes.

The False Negatives Rate is 5.29%, which already meets the specifications. The False Positives Rate is 1.95%.

stab.glu has the largest influence.

```
summary(lm(glyhb ~ hdl + bp.1s + bp.1d, data.df))
##
## Call:
## lm(formula = glyhb ~ hdl + bp.1s + bp.1d, data = data.df)
##
## Residuals:
##
    Min
           1Q Median
                        3Q
                              Max
## -3.0818 -1.1474 -0.5394 0.2485 9.3733
##
## Coefficients:
##
            Estimate Std. Error t value Pr(>|t|)
```

Since the p-value is 9.485e-07, we reject the null hypothesis that hdl, bp.1s and bp.1d do not have any predictive value, with 5% significance level.

4. According to the result from 4.6, BMI and WHR do not interact. However, if they do interact, we would choose the feature with larger influence (BMI in this case), because the interaction effect will reduce the influence.

5.

6.

7.

A Appendix

A.1 Background

```
library("ggplot2")
library("grid")
library("gridExtra")
data.df <- na.omit(read.csv("diabetes.csv"))</pre>
```

A.2 Accessing Data, Visualization and Summarization

```
3. bp.1d.bp.1s.scatterplot <- ggplot(data.df) +
    geom_point(aes(x=bp.1d, y=bp.1s)) +
    labs(title="Scatterplot of bp.1d vs bp.1s") +
    theme(text=element_text(size=8.5))

height.age.scatterplot <- ggplot(data.df) +
    geom_point(aes(x=height, y=age)) +
    labs(title="Scatterplot of age vs height") +
    theme(text=element_text(size=8.5))

grid.arrange(bp.1d.bp.1s.scatterplot, height.age.scatterplot, ncol=2)</pre>
```

A.3 Parametric Inference

```
1. gamma.boot <- function(x) {
    alpha.mom <- mean(x)^2/var(x)
    beta.mom <- mean(x)/var(x)

bootstrap <- sapply(1:1000, function(i) {
    samples <- sample(x, length(x), replace=TRUE)
    alpha.sample <- mean(samples)^2/var(samples)
    beta.sample <- mean(samples)/var(samples)
    return(c(alpha.sample, beta.sample))
})

CIs <- sapply(1:2, function(i) {
    CI <- quantile(bootstrap[i, ], probs=c(0.025, 0.975))</pre>
```

```
return(CI)
})
colnames(CIs) <- c("alpha", "beta")

return(CIs)
}

CI.BMI <- gamma.boot(data.df$BMI)

ggplot(data.df) +
   geom_histogram(aes(x=BMI, y=..density..), binwidth=2, col="white") +
   stat_function(fun=function(x)
    dgamma(x, shape=alpha.mom, rate=beta.mom), col="white") +
   labs(title="Fitted density over observed histogram of BMI") +
   theme(text=element_text(size=8.5))</pre>
```

```
2. normal.boot <- function(x) {</pre>
    mu.mle <- mean(x)</pre>
    sigma.mle \leftarrow sd(x)
    bootstrap <- sapply(1:1000, function(i) {</pre>
       samples <- sample(x, length(x), replace=TRUE)</pre>
       mu.sample <- mean(samples)</pre>
       sigma.sample <- sd(samples)</pre>
      return(c(mu.sample, sigma.sample))
    })
    CIs <- sapply(1:2, function(i) {</pre>
      CI <- quantile(bootstrap[i, ], probs=c(0.025, 0.975))</pre>
      return(CI)
    colnames(CIs) <- c("mu", "sigma")</pre>
    return(CIs)
  CI.WHR <- normal.boot(data.df$WHR)</pre>
  ggplot(data.df) +
    geom_histogram(aes(x=WHR, y=..density..), binwidth=0.025, col="white") +
    stat_function(fun=function(x)
       dnorm(x, mean=mu.mle, sd=sigma.mle), col="white") +
    labs(title="Fitted density over observed histogram of WHR") +
    theme(text=element_text(size=8.5))
```

```
"male & (glyhb < 7)", "female & (glyhb < 7)")
gamma.boot2 <- function(x) {</pre>
  alpha.mom \leftarrow mean(x)^2/var(x)
  beta.mom \leftarrow mean(x)/var(x)
  bootstrap <- sapply(1:1000, function(i) {</pre>
    samples <- sample(x, length(x), replace=TRUE)</pre>
    mu.sample <- mean(samples)</pre>
    sigma.sample <- sd(samples)</pre>
    return(c(mu.sample, sigma.sample))
  })
  CIs <- sapply(1:2, function(i) {</pre>
    CI <- quantile(bootstrap[i, ], probs=c(0.025, 0.975))</pre>
    return(CI)
  })
  colnames(CIs) <- c("mu", "sigma")</pre>
  return(CIs)
CIs.BMI <- lapply(conditions, function(x) {</pre>
  gender.glyhb.df <- subset(data.df, gender.glyhb.cond==x)</pre>
  CIs.BMI <- gamma.boot2(gender.glyhb.df$BMI)</pre>
  return(CIs.BMI)
})
names(CIs.BMI) <- conditions</pre>
ggplot(data.df) +
  geom_boxplot(aes(x=factor(gender.glyhb.cond, levels=conditions), y=BMI)) +
  labs(title="Boxplots of BMI for each gender and glyhb condition",
       x="gender & glyhb") +
  theme(text=element_text(size=8.5))
CIs.WHR <- lapply(conditions, function(x) {</pre>
  gender.glyhb.df <- subset(data.df, gender.glyhb.cond==x)</pre>
  CIs.WHR <- normal.boot(gender.glyhb.df$WHR)</pre>
  return(CIs.WHR)
})
names(CIs.WHR) <- conditions</pre>
ggplot(data.df) +
  geom_boxplot(aes(x=factor(gender.glyhb.cond, levels=conditions), y=WHR)) +
  labs(title="Boxplots of WHR for each gender and glyhb condition",
       x="gender & glyhb") +
  theme(text=element_text(size=8.5))
```

A.4 Testing

```
colnames(gender.glyhb.cond.table) <- c("glyhb >= 7", "glyhb < 7")
rownames(gender.glyhb.cond.table) <- c("male", "female")</pre>
```

3. $pi.est \leftarrow function(x, y)$ {

```
w <- 0
    for (i in 1:length(x)) {
      for (j in 1:length(y)) {
         w \leftarrow w + (x[i] > y[j])
    return(w/(length(x)*length(y)))
  male.glyhb.geq7.BMI <-</pre>
    subset(data.df, gender.glyhb.cond=="male & (glyhb >= 7)")$BMI
  male.glyhb.17.BMI <-</pre>
    subset(data.df, gender.glyhb.cond=="male & (glyhb < 7)")$BMI</pre>
  male.glyhb.geq7.WHR <-</pre>
    subset(data.df, gender.glyhb.cond=="male & (glyhb >= 7)")$WHR
  male.glyhb.17.WHR <-
    subset(data.df, gender.glyhb.cond=="male & (glyhb < 7)")$WHR</pre>
  pi.male.BMI <- pi.est(</pre>
    male.glyhb.geq7.BMI,
    male.glyhb.17.BMI)
  pi.male.WHR <- pi.est(</pre>
    male.glyhb.geq7.WHR,
    male.glyhb.17.WHR)
  pi.male.BMI.samples <- sapply(1:1000, function(x)</pre>
    pi.est(sample(male.glyhb.geq7.BMI, length(male.glyhb.geq7.BMI), replace=TRUE),
            sample(male.glyhb.17.BMI, length(male.glyhb.17.BMI), replace=TRUE)))
  CI.pi.male.BMI <- quantile(pi.male.BMI.samples, probs=c(0.025, 0.975))
  pi.male.WHR.samples <- sapply(1:1000, function(x)</pre>
    pi.est(sample(male.glyhb.geq7.WHR, length(male.glyhb.geq7.WHR), replace=TRUE),
            sample(male.glyhb.17.WHR, length(male.glyhb.17.WHR), replace=TRUE)))
  CI.pi.male.WHR <- quantile(pi.male.WHR.samples, probs=c(0.025, 0.975))
4. mean.male.glyhb.geq7.WHR <- mean(male.glyhb.geq7.WHR)
  mean.male.glyhb.17.WHR <- mean(male.glyhb.17.WHR)</pre>
  sd.male.WHR <- sd(subset(data.df, gender=="male")$WHR)</pre>
  mu.0 <- mean.male.glyhb.17.WHR
  mu.1 <- mean.male.glyhb.geq7.WHR</pre>
  sigma <- sd.male.WHR</pre>
  alpha <- 0.05
  t <- qnorm(1-alpha)*sigma + mu.0
  beta <- pnorm((qnorm(1-alpha)*sigma + mu.0 - mu.1)/sigma)
  power <- 1 - beta
```

```
5. BMI.labels <- c("Underweight", "Healthy", "Overweight",
                          "Level 1 Obese", "Level 2 Obese", "Level 3 Obese")
  data.df$BMI.std <- cut(data.df$BMI, breaks=c(0, 18.5, 25, 30, 35, 40, Inf),
                          labels=BMI.labels, right=F)
  gender.BMI.categories <- cbind(table(subset(data.df, gender=="male")$BMI.std),</pre>
                                  table(subset(data.df, gender=="female")$BMI.std))
  colnames(gender.BMI.categories) <- c("male", "female")</pre>
  WHR.labels <- c("Low", "Moderate", "High", "Very High")
  data.df$WHR.std <- factor(NA)
  levels(data.df$WHR.std) <- WHR.labels</pre>
  gender.age.cond <-
    data.df$gender=="male" & data.df$age >= 20 & data.df$age <= 29
  data.df[gender.age.cond, ]$WHR.std <-
    cut(data.df[gender.age.cond, ]$WHR,
        breaks=c(0, 0.83, 0.88, 0.94, Inf), labels=WHR.labels)
  gender.age.cond <-
    data.df$gender=="male" & data.df$age >= 30 & data.df$age <= 39
  data.df[gender.age.cond, ]$WHR.std <-
    cut(data.df[gender.age.cond, ]$WHR,
        breaks=c(0, 0.84, 0.91, 0.96, Inf), labels=WHR.labels)
  gender.age.cond <-
    data.df$gender=="male" & data.df$age >= 40 & data.df$age <= 49
  data.df[gender.age.cond, ]$WHR.std <-
    cut(data.df[gender.age.cond, ]$WHR,
        breaks=c(0, 0.88, 0.95, 1, Inf), labels=WHR.labels)
  gender.age.cond <-</pre>
    data.df$gender=="male" & data.df$age >= 50 & data.df$age <= 59
  data.df[gender.age.cond, ]$WHR.std <-
    cut(data.df[gender.age.cond, ]$WHR,
        breaks=c(0, 0.90, 0.96, 1.02, Inf), labels=WHR.labels)
  gender.age.cond <-
    data.df$gender=="male" & data.df$age >= 60
  data.df[gender.age.cond, ]$WHR.std <-
    cut(data.df[gender.age.cond, ]$WHR,
        breaks=c(0, 0.91, 0.98, 1.03, Inf), labels=WHR.labels)
  gender.age.cond <-
    data.df$gender=="female" & data.df$age >= 20 & data.df$age <= 29
  data.df[gender.age.cond, ]$WHR.std <-
    cut(data.df[gender.age.cond, ]$WHR,
        breaks=c(0, 0.71, 0.77, 0.82, Inf), labels=WHR.labels)
  gender.age.cond <-
    data.df$gender=="female" & data.df$age >= 30 & data.df$age <= 39
  data.df[gender.age.cond, ]$WHR.std <-
    cut(data.df[gender.age.cond, ]$WHR,
```

```
breaks=c(0, 0.72, 0.78, 0.84, Inf), labels=WHR.labels)
gender.age.cond <-
  data.df$gender=="female" & data.df$age >= 40 & data.df$age <= 49
data.df[gender.age.cond, ]$WHR.std <-
  cut(data.df[gender.age.cond, ]$WHR,
      breaks=c(0, 0.73, 0.79, 0.87, Inf), labels=WHR.labels)
gender.age.cond <-</pre>
  data.df$gender=="female" & data.df$age >= 50 & data.df$age <= 59
data.df[gender.age.cond, ]$WHR.std <-
  cut(data.df[gender.age.cond, ]$WHR,
      breaks=c(0, 0.74, 0.81, 0.88, Inf), labels=WHR.labels)
gender.age.cond <-
  data.df$gender=="female" & data.df$age >= 60
data.df[gender.age.cond, ]$WHR.std <-
  cut(data.df[gender.age.cond, ]$WHR,
      breaks=c(0, 0.76, 0.83, 0.9, Inf), labels=WHR.labels)
gender.WHR.categories <- cbind(table(subset(data.df, gender=="male")$WHR.std),</pre>
                               table(subset(data.df, gender=="female")$WHR.std))
colnames(gender.WHR.categories) <- c("male", "female")</pre>
```

A.5 Regression