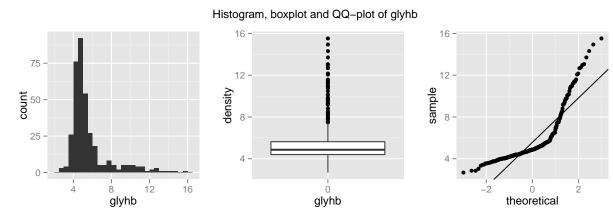
Statistics 135 – Lab Project

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1 Background

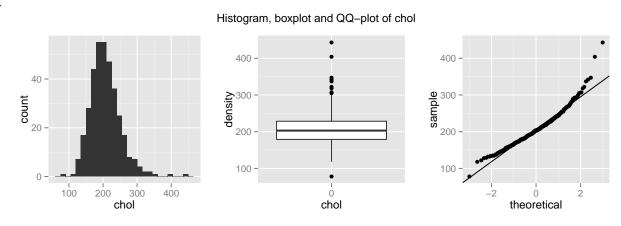
2 Accessing Data, Visualization and Summarization

1.



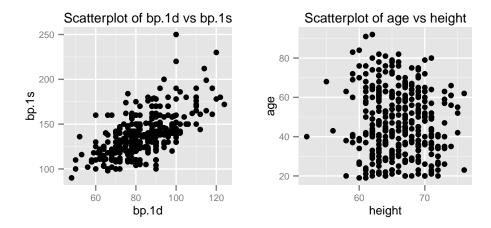
The mean, median and mode of glyhb are all approximately 5. The distribution of glyhb is left-skewed.

2.



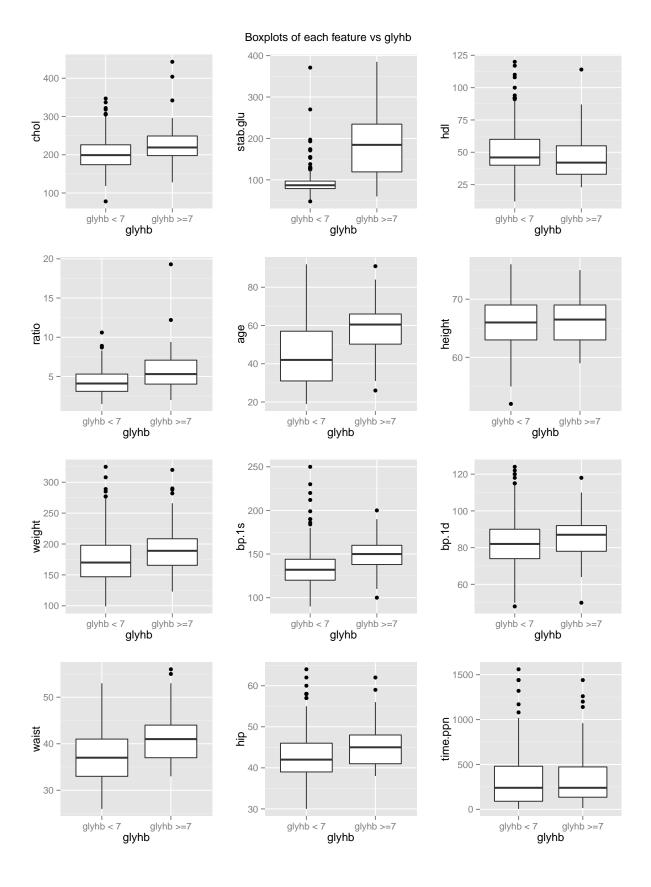
The mean, median and mode of chol are all approximately 200. The distribution of chol is better approximated with a Gaussian distribution.

3.

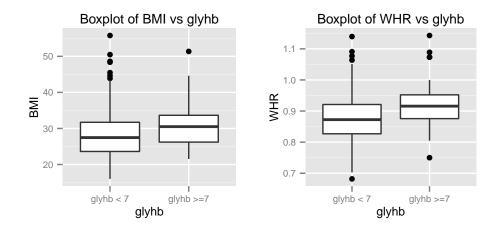


The scatterplot of bp.1s and bp.1d is near-linear, so they are approximately dependent. The scatterplot of age and weight is random, so they are approximately independent.

- 4. chol: The two distributions have small difference, so it MAY BE a relevant feature.
 - stab.glu: The two distributions have substantial difference, so it SHOULD BE a relevant feature.
 - hdl: The two distributions have small difference, so it MAY BE a relevant feature.
 - ratio: The two distributions have small difference, so it MAY BE a relevant feature.
 - age: The two distributions have substantial difference, so it SHOULD BE a relevant feature.
 - height: The two distributions have little difference, so it MAY NOT BE a relevant feature.
 - weight: The two distributions have small difference, so it MAY BE a relevant feature.
 - bp.1s: The two distributions have small difference, so it MAY BE a relevant feature.
 - bp.1d: The two distributions have small difference, so it MAY BE a relevant feature.
 - waist: The two distributions have small difference, so it MAY BE a relevant feature.
 - hip: The two distributions have small difference, so it MAY BE a relevant feature.
 - time.ppn: The two distributions have small difference, so it MAY NOT BE a relevant feature.



5.



6. In light of these first experiments, hdl, stab.glu, age, weight, bp.1s, bp.1d, waist and hip seem related to the presence of type II diabetes; chol, ratio, height and time.ppn seem unrelated to the presence of type II diabetes.

3 Parametric Inference

1.

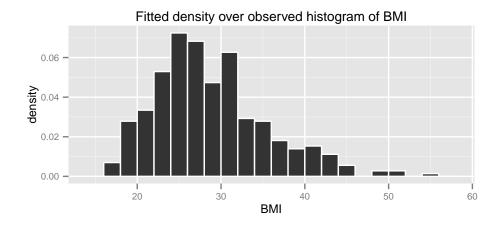
$$X \sim Gamma(\alpha, \beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha - 1} e^{-\beta x}$$

$$E(X) = \frac{\alpha}{\beta}$$

$$E(X^{2}) = Var(X) + [E(X)]^{2}$$
$$= \frac{\alpha}{\beta^{2}} + \left(\frac{\alpha}{\beta}\right)^{2}$$
$$= \frac{\alpha(\alpha + 1)}{\beta^{2}}$$

$$\begin{cases} E(X) = \frac{\alpha}{\beta} \\ E(X^2) = \frac{\alpha(\alpha+1)}{\beta^2} \end{cases} \implies \begin{cases} \alpha = \frac{[E(X)]^2}{Var(x)} \\ \beta = \frac{E(X)}{Var(x)} \end{cases} \implies \begin{cases} \hat{\alpha}_{MOM} = \frac{\overline{X}_n}{\frac{1}{n}\sum_{i=1}^n (X_i - \overline{X}_n)^2} \\ \hat{\beta}_{MOM} = \frac{\overline{X}_n^2}{\frac{1}{n}\sum_{i=1}^n (X_i - \overline{X}_n)^2} \end{cases}$$

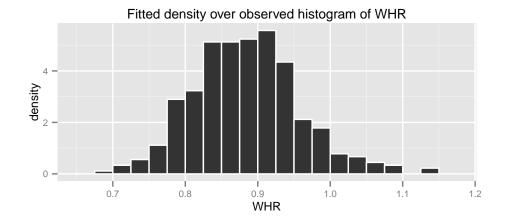
alpha beta ## 2.5% 15.87666 0.5460733 ## 97.5% 21.62832 0.7591677



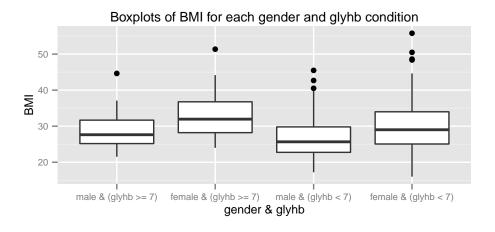
2.

$$\hat{\mu}_{MLE} = \overline{X}_n$$

$$\hat{\sigma}_{MLE}^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \overline{X}_n)^2$$

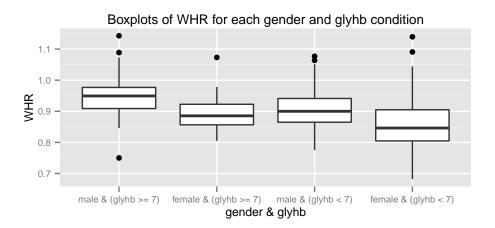


```
CIs.BMI
 ## $`male & (glyhb >= 7)`
 ##
            mu
                 sigma
 ## 2.5% 26.89463 3.533570
 ## 97.5% 30.97481 7.164682
 ##
   $`female & (glyhb >= 7)`
 ##
 ##
                 sigma
 ## 2.5% 30.90826 4.608429
 ## 97.5% 35.63231 8.207146
 ##
 ##
   $`male & (glyhb < 7)`</pre>
 ##
            mu
                 sigma
 ## 2.5% 25.57434 4.711554
 ## 97.5% 27.47041 6.358453
 ##
 ## $`female & (glyhb < 7)`
 ##
            mu
                 sigma
 ## 2.5% 28.84236 6.220170
 ## 97.5% 30.84639 7.862444
```



- On average, females have higher BMI than males.
- On average, people with type II diabetes (glyhb >= 7) have higher BMI than people without type II diabetes (glyhb < 7).
- People with type II diabetes (glyhb >= 7) have larger confidence intervals of both mean and standard deviation than people without type II diabetes (glyhb < 7), regardless of gender.

```
CIs.WHR
## $`male & (glyhb >= 7)`
##
           mu
                  sigma
## 2.5% 0.9202216 0.04735021
## 97.5% 0.9791687 0.10652492
##
##
 $`female & (glyhb >= 7)`
##
           mu
                  sigma
## 2.5% 0.8754801 0.04013498
## 97.5% 0.9139384 0.07537186
##
 $`male & (glyhb < 7)`</pre>
##
##
           mu
                  sigma
## 2.5% 0.8938359 0.05568191
## 97.5% 0.9172246 0.07075628
##
##
 $`female & (glyhb < 7)`</pre>
##
           mu
                  sigma
## 2.5% 0.8446725 0.06108226
## 97.5% 0.8644300 0.07829550
```



- On average, males have higher WHR than females.
- On average, people with type II diabetes (glyhb >= 7) have higher WHR than people without type II diabetes (glyhb < 7).
- People with type II diabetes (glyhb >= 7) have larger confidence intervals of both mean and standard deviation than people without type II diabetes (glyhb < 7), regardless of gender.

4 Testing

```
gender.glyhb.cond.table
 ##
       glyhb >= 7 glyhb < 7
            24
 ## male
                  125
 ## female
            30
                  180
 fisher.test(gender.glyhb.cond.table)
 ##
   Fisher's Exact Test for Count Data
 ##
 ##
 ## data: gender.glyhb.cond.table
 ## p-value = 0.6552
 ## alternative hypothesis: true odds ratio is not equal to 1
 ## 95 percent confidence interval:
 ## 0.6126316 2.1465820
 ## sample estimates:
 ## odds ratio
 ##
   1.151538
```

Since the p-value is 0.655, which is greater than 0.05, we fail to reject the null hypothesis that males and females are equally exposed to type II diabetes, with 5% significance level.

2. We choose to the non-parametric Kruskal-Wallis test, because it does not rely on the assumed normal distribution and less affected by outliers.

Since the p-value is 0.00475, which is smaller than 0.05, we reject the null hypothesis that hdl has equal means for those with type II diabetes and those without, with 5% significance level.

Since the p-value is 2.034e-06, which is smaller than 0.05, we reject the null hypothesis that bp.1s has equal means for those with type II diabetes and those without, with 5% significance level.

Since the p-value is 0.168, which is greater than 0.05, we fail to reject the null hypothesis that bp.1d has equal means for those with type II diabetes and those without, with 5% significance level.

Since the p-value is 0.00198, which is smaller than 0.05, we reject the null hypothesis that BMI has equal means for those with type II diabetes and those without, with 5% significance level.

Since the p-value is 9.95e-05, which is smaller than 0.05, we reject the null hypothesis that WHR has equal means for those with type II diabetes and those without, with 5% significance level.

```
pi.male.BMI
 ## [1] 0.6326667
 CI.pi.male.BMI
    2.5%
       97.5%
 ## 0.5222833 0.7420167
 pi.male.WHR
 ## [1] 0.6853333
 CI.pi.male.WHR
 ##
    2.5%
       97.5%
 ## 0.5716667 0.7950000
```

4. According to the result from part 3.2, we know that WHR has a normal distribution. We assume all patients come from the same population, so the standard deviation is constant.

$$H_0: N(\mu_0, \sigma^2)$$

 $H_1: N(\mu_1, \sigma^2)$

mean.male.glyhb.geq7.WHR

[1] 0.9489365

mean.male.glyhb.17.WHR

[1] 0.9058419

sd.male.WHR

[1] 0.06821165

$$\mu_0 = 0.906, \mu_1 = 0.949, \sigma = 0.0682$$

$$lik(x) = \frac{f_0(x)}{f_1(x)}$$

$$= \frac{\frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{(x-\mu_0)^2}{2}}}{\frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{(x-\mu_1)^2}{2}}}$$

$$= e^{-2(\mu_1 - \mu_0)x + (\mu_1^2 - \mu_0^2)}$$

Let T := X.

$$\alpha = P(T > t \mid H_0)$$

$$= P(\frac{T - \mu_0}{\sigma} > \frac{t - \mu_0}{\sigma})$$

$$= 1 - \Phi\left(\frac{t - \mu_0}{\sigma}\right)$$

$$\Longrightarrow$$

$$t = \Phi^{-1}(\alpha)\sigma + \mu_0$$

$$\beta = P(T < t \mid H_1)$$

$$= P(\frac{T - \mu_1}{\sigma} < \frac{t - \mu_1}{\sigma})$$

$$= \Phi\left(\frac{t - \mu_1}{\sigma}\right)$$

$$= \Phi\left(\frac{\Phi^{-1}(\alpha)\sigma + \mu_0 - \mu_1}{\sigma}\right)$$

$$\alpha \leqslant 5\% \implies 1 - \beta \leqslant 0.156$$

We construct a test for type II diabetes for male patient that we reject the null hypothesis if his WHR $\geqslant 1.018$. The significance of the test is $\leqslant 5\%$ and the power of the test is $\leqslant 0.156$.

```
gender.BMI.categories
 ##
          male female
 ## Underweight
           5
                3
            59
                43
 ## Healthy
 ## Overweight
            46
                66
 ## Level 1 Obese
            26
                52
 ## Level 2 Obese
            9
                22
 ## Level 3 Obese
                24
            4
 chisq.test(gender.BMI.categories)
 ##
 ##
   Pearson's Chi-squared test
 ##
 ## data: gender.BMI.categories
 ## X-squared = 25.352, df = 5, p-value = 0.0001191
```

Since the p-value is 0.000119, which is smaller than 0.05, we reject the null hypothesis that male and female population sample has homogeneous distribution of BMI categories, with 5% significance level.

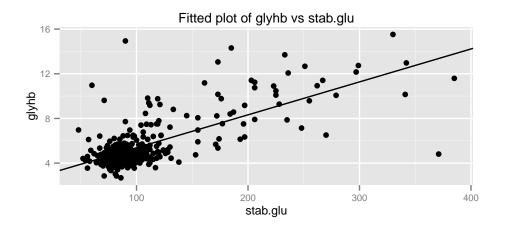
```
gender.WHR.categories
##
      male female
## Low
       46
## Moderate
       66
           33
## High
       20
           76
## Very High
      17
           95
chisq.test(gender.WHR.categories)
##
##
 Pearson's Chi-squared test
##
## data: gender.WHR.categories
## X-squared = 126.99, df = 3, p-value < 2.2e-16
```

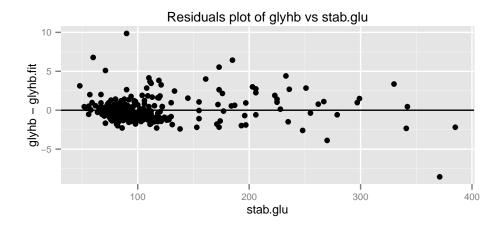
Since the p-value is 2.2e-16, which is smaller than 0.05, we reject the null hypothesis that male and female population sample has homogeneous distribution of WHR categories, with 5% significance level.

The interaction effect between BMI and WHR is not significant. BMI is more sensitive to glyhb. Overall, the result is consistent with part 3.3.

5 Regression

1. According to the result from part 2.2, we consider stab.glu the most relevant feature for predicting type II diabetes.





The False Negatives Rate is 5.57%, which already meets the specifications. The False Positives Rate is 1.95%.

- 3.
- 4.
- 5.
- 6.
- 7.

A Appendix

A.1 Background

```
library("ggplot2")
library("grid")
library("gridExtra")
data.df <- na.omit(read.csv("diabetes.csv"))</pre>
```

A.2 Accessing Data, Visualization and Summarization

```
3. bp.1d.bp.1s.scatterplot <- ggplot(data.df) +
    geom_point(aes(x=bp.1d, y=bp.1s)) +
    labs(title="Scatterplot of bp.1d vs bp.1s") +
    theme(text=element_text(size=8.5))

height.age.scatterplot <- ggplot(data.df) +
    geom_point(aes(x=height, y=age)) +
    labs(title="Scatterplot of age vs height") +
    theme(text=element_text(size=8.5))

grid.arrange(bp.1d.bp.1s.scatterplot, height.age.scatterplot, ncol=2)</pre>
```

A.3 Parametric Inference

```
1. gamma.boot <- function(x) {
    alpha.mom <- mean(x)^2/var(x)
    beta.mom <- mean(x)/var(x)

bootstrap <- sapply(1:1000, function(i) {
    samples <- sample(x, length(x), replace=TRUE)
    alpha.sample <- mean(samples)^2/var(samples)
    beta.sample <- mean(samples)/var(samples)
    return(c(alpha.sample, beta.sample))
})

CIs <- sapply(1:2, function(i) {
    CI <- quantile(bootstrap[i, ], probs=c(0.025, 0.975))</pre>
```

```
return(CI)
})
colnames(CIs) <- c("alpha", "beta")

return(CIs)
}

CI.BMI <- gamma.boot(data.df$BMI)

ggplot(data.df) +
   geom_histogram(aes(x=BMI, y=..density..), binwidth=2, col="white") +
   stat_function(fun=function(x)
    dgamma(x, shape=alpha.mom, rate=beta.mom), col="white") +
   labs(title="Fitted density over observed histogram of BMI") +
   theme(text=element_text(size=8.5))</pre>
```

```
2. normal.boot <- function(x) {</pre>
    mu.mle <- mean(x)</pre>
    sigma.mle \leftarrow sd(x)
    bootstrap <- sapply(1:1000, function(i) {</pre>
       samples <- sample(x, length(x), replace=TRUE)</pre>
       mu.sample <- mean(samples)</pre>
       sigma.sample <- sd(samples)</pre>
      return(c(mu.sample, sigma.sample))
    })
    CIs <- sapply(1:2, function(i) {</pre>
      CI <- quantile(bootstrap[i, ], probs=c(0.025, 0.975))</pre>
      return(CI)
    colnames(CIs) <- c("mu", "sigma")</pre>
    return(CIs)
  CI.WHR <- normal.boot(data.df$WHR)</pre>
  ggplot(data.df) +
    geom_histogram(aes(x=WHR, y=..density..), binwidth=0.025, col="white") +
    stat_function(fun=function(x)
       dnorm(x, mean=mu.mle, sd=sigma.mle), col="white") +
    labs(title="Fitted density over observed histogram of WHR") +
    theme(text=element_text(size=8.5))
```

```
"male & (glyhb < 7)", "female & (glyhb < 7)")
gamma.boot2 <- function(x) {</pre>
  alpha.mom \leftarrow mean(x)^2/var(x)
  beta.mom \leftarrow mean(x)/var(x)
  bootstrap <- sapply(1:1000, function(i) {</pre>
    samples <- sample(x, length(x), replace=TRUE)</pre>
    mu.sample <- mean(samples)</pre>
    sigma.sample <- sd(samples)</pre>
    return(c(mu.sample, sigma.sample))
  })
  CIs <- sapply(1:2, function(i) {</pre>
    CI <- quantile(bootstrap[i, ], probs=c(0.025, 0.975))</pre>
    return(CI)
  })
  colnames(CIs) <- c("mu", "sigma")</pre>
  return(CIs)
CIs.BMI <- lapply(conditions, function(x) {</pre>
  gender.glyhb.df <- subset(data.df, gender.glyhb.cond==x)</pre>
  CIs.BMI <- gamma.boot2(gender.glyhb.df$BMI)</pre>
  return(CIs.BMI)
})
names(CIs.BMI) <- conditions</pre>
ggplot(data.df) +
  geom_boxplot(aes(x=factor(gender.glyhb.cond, levels=conditions), y=BMI)) +
  labs(title="Boxplots of BMI for each gender and glyhb condition",
       x="gender & glyhb") +
  theme(text=element_text(size=8.5))
CIs.WHR <- lapply(conditions, function(x) {</pre>
  gender.glyhb.df <- subset(data.df, gender.glyhb.cond==x)</pre>
  CIs.WHR <- normal.boot(gender.glyhb.df$WHR)</pre>
  return(CIs.WHR)
})
names(CIs.WHR) <- conditions</pre>
ggplot(data.df) +
  geom_boxplot(aes(x=factor(gender.glyhb.cond, levels=conditions), y=WHR)) +
  labs(title="Boxplots of WHR for each gender and glyhb condition",
       x="gender & glyhb") +
  theme(text=element_text(size=8.5))
```

A.4 Testing

```
colnames(gender.glyhb.cond.table) <- c("glyhb >= 7", "glyhb < 7")
rownames(gender.glyhb.cond.table) <- c("male", "female")</pre>
```

3. $pi.est \leftarrow function(x, y)$ {

```
w <- 0
    for (i in 1:length(x)) {
      for (j in 1:length(y)) {
         w \leftarrow w + (x[i] > y[j])
    return(w/(length(x)*length(y)))
  male.glyhb.geq7.BMI <-</pre>
    subset(data.df, gender.glyhb.cond=="male & (glyhb >= 7)")$BMI
  male.glyhb.17.BMI <-</pre>
    subset(data.df, gender.glyhb.cond=="male & (glyhb < 7)")$BMI</pre>
  male.glyhb.geq7.WHR <-</pre>
    subset(data.df, gender.glyhb.cond=="male & (glyhb >= 7)")$WHR
  male.glyhb.17.WHR <-
    subset(data.df, gender.glyhb.cond=="male & (glyhb < 7)")$WHR</pre>
  pi.male.BMI <- pi.est(</pre>
    male.glyhb.geq7.BMI,
    male.glyhb.17.BMI)
  pi.male.WHR <- pi.est(</pre>
    male.glyhb.geq7.WHR,
    male.glyhb.17.WHR)
  pi.male.BMI.samples <- sapply(1:1000, function(x)</pre>
    pi.est(sample(male.glyhb.geq7.BMI, length(male.glyhb.geq7.BMI), replace=TRUE),
            sample(male.glyhb.17.BMI, length(male.glyhb.17.BMI), replace=TRUE)))
  CI.pi.male.BMI <- quantile(pi.male.BMI.samples, probs=c(0.025, 0.975))
  pi.male.WHR.samples <- sapply(1:1000, function(x)</pre>
    pi.est(sample(male.glyhb.geq7.WHR, length(male.glyhb.geq7.WHR), replace=TRUE),
            sample(male.glyhb.17.WHR, length(male.glyhb.17.WHR), replace=TRUE)))
  CI.pi.male.WHR <- quantile(pi.male.WHR.samples, probs=c(0.025, 0.975))
4. mean.male.glyhb.geq7.WHR <- mean(male.glyhb.geq7.WHR)
  mean.male.glyhb.17.WHR <- mean(male.glyhb.17.WHR)</pre>
  sd.male.WHR <- sd(subset(data.df, gender=="male")$WHR)</pre>
  mu.0 <- mean.male.glyhb.17.WHR
  mu.1 <- mean.male.glyhb.geq7.WHR</pre>
  sigma <- sd.male.WHR</pre>
  alpha <- 0.05
  t <- qnorm(1-alpha)*sigma + mu.0
  beta <- pnorm((qnorm(1-alpha)*sigma + mu.0 - mu.1)/sigma)
  power <- 1 - beta
```

```
5. BMI.labels <- c("Underweight", "Healthy", "Overweight",
                          "Level 1 Obese", "Level 2 Obese", "Level 3 Obese")
  data.df$BMI.std <- cut(data.df$BMI, breaks=c(0, 18.5, 25, 30, 35, 40, Inf),
                          labels=BMI.labels, right=F)
  gender.BMI.categories <- cbind(table(subset(data.df, gender=="male")$BMI.std),</pre>
                                  table(subset(data.df, gender=="female")$BMI.std))
  colnames(gender.BMI.categories) <- c("male", "female")</pre>
  WHR.labels <- c("Low", "Moderate", "High", "Very High")
  data.df$WHR.std <- factor(NA)</pre>
  levels(data.df$WHR.std) <- WHR.labels</pre>
  gender.age.cond <-
    data.df$gender=="male" & data.df$age >= 20 & data.df$age <= 29
  data.df[gender.age.cond, ]$WHR.std <-
    cut(data.df[gender.age.cond, ]$WHR,
        breaks=c(0, 0.83, 0.88, 0.94, Inf), labels=WHR.labels)
  gender.age.cond <-
    data.df$gender=="male" & data.df$age >= 30 & data.df$age <= 39
  data.df[gender.age.cond, ]$WHR.std <-
    cut(data.df[gender.age.cond, ]$WHR,
        breaks=c(0, 0.84, 0.91, 0.96, Inf), labels=WHR.labels)
  gender.age.cond <-
    data.df$gender=="male" & data.df$age >= 40 & data.df$age <= 49
  data.df[gender.age.cond, ]$WHR.std <-
    cut(data.df[gender.age.cond, ]$WHR,
        breaks=c(0, 0.88, 0.95, 1, Inf), labels=WHR.labels)
  gender.age.cond <-</pre>
    data.df$gender=="male" & data.df$age >= 50 & data.df$age <= 59
  data.df[gender.age.cond, ]$WHR.std <-
    cut(data.df[gender.age.cond, ]$WHR,
        breaks=c(0, 0.90, 0.96, 1.02, Inf), labels=WHR.labels)
  gender.age.cond <-
    data.df$gender=="male" & data.df$age >= 60
  data.df[gender.age.cond, ]$WHR.std <-
    cut(data.df[gender.age.cond, ]$WHR,
        breaks=c(0, 0.91, 0.98, 1.03, Inf), labels=WHR.labels)
  gender.age.cond <-
    data.df$gender=="female" & data.df$age >= 20 & data.df$age <= 29
  data.df[gender.age.cond, ]$WHR.std <-
    cut(data.df[gender.age.cond, ]$WHR,
        breaks=c(0, 0.71, 0.77, 0.82, Inf), labels=WHR.labels)
  gender.age.cond <-
    data.df$gender=="female" & data.df$age >= 30 & data.df$age <= 39
  data.df[gender.age.cond, ]$WHR.std <-
    cut(data.df[gender.age.cond, ]$WHR,
```

```
breaks=c(0, 0.72, 0.78, 0.84, Inf), labels=WHR.labels)
gender.age.cond <-</pre>
  data.df$gender=="female" & data.df$age >= 40 & data.df$age <= 49
data.df[gender.age.cond, ]$WHR.std <-
  cut(data.df[gender.age.cond, ]$WHR,
      breaks=c(0, 0.73, 0.79, 0.87, Inf), labels=WHR.labels)
gender.age.cond <-
  data.df$gender=="female" & data.df$age >= 50 & data.df$age <= 59
data.df[gender.age.cond, ]$WHR.std <-
  cut(data.df[gender.age.cond, ]$WHR,
      breaks=c(0, 0.74, 0.81, 0.88, Inf), labels=WHR.labels)
gender.age.cond <-
  data.df$gender=="female" & data.df$age >= 60
data.df[gender.age.cond, ]$WHR.std <-
  cut(data.df[gender.age.cond, ]$WHR,
      breaks=c(0, 0.76, 0.83, 0.9, Inf), labels=WHR.labels)
gender.WHR.categories <- cbind(table(subset(data.df, gender=="male")$WHR.std),</pre>
                               table(subset(data.df, gender=="female")$WHR.std))
colnames(gender.WHR.categories) <- c("male", "female")</pre>
```

A.5 Regression

```
1. glyhb.stab.glu.lm <- lm(glyhb ~ stab.glu, data.df)
  coeff <- unname(coefficients(glyhb.stab.glu.lm))</pre>
  glyhb.fit <- predict(glyhb.stab.glu.lm, data.df)</pre>
  ggplot(data.df) +
    geom_point(aes(x=stab.glu, y=glyhb)) +
    geom_abline(aes(intercept=coeff[1], slope=coeff[2])) +
    labs(title="Fitted plot of glyhb vs stab.glu") +
    theme(text=element_text(size=8.5))
  ggplot(data.df) +
    geom_point(aes(x=stab.glu, y=glyhb-glyhb.fit)) +
    geom_abline(aes(intercept=0, slope=0)) +
    labs(title="Residuals plot of glyhb vs stab.glu") +
    theme(text=element_text(size=8.5))
  threshold <- function(y, lambda) {</pre>
    return(ifelse(y > lambda, 1, 0))
  err.rate <- mean(sapply(data.df$glyhb, function(x) threshold(x, 7)) ==
                     sapply(glyhb.fit, function(x) threshold(x, 7)))
```