Turing Machine

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Turing machines *(source)*

A Turing machine M consists of a finite state control (i.e., a finite program) attached to a read/write head moving on an infinite tape.

- The tape is divided into squares, each capable of storing one symbol from a finite alphabet Γ that includes the blank symbol b.
- Each machine M has a specified input alphabet Σ, which is a subset of Γ, not including the blank symbol b.
  - At each step in a computation, M is in some state q in a specified finite set Q of possible states. Initially, a finite input string over Σ is written on adjacent squares of the tape, all other squares are blank (contain b), the head scans the left-most symbol of the input string, and M is in the initial state q_0.
  - At each step M is in some state q and the head is scanning a tape square containing some tape symbol s, and the action performed depends on the pair (q, s) and is specified by the machine's transition function (or program) δ.
- The action consists of printing a symbol on the scanned square, moving the head left or right one square, and assuming a new state.

Formally, a Turing machine M is a tuple $\langle \Sigma, \Gamma, Q, \delta \rangle$, where Σ, Γ, Q are finite nonempty sets with $\Sigma \subseteq \Gamma$ and $b \in \Gamma - \Sigma$. The state set Q contains three special states q_0, q_{accept}, q_{reject}. The transition function δ satisfies

$\delta : (Q - \{q_{accept}, q_{reject}\}) \times \Gamma ß Q \times \Gamma \times \{-1, 1\}$.

If $\delta(q, s) = (q', s', h)$, the interpretation is that, if M is in state q scanning the symbol s, then q' is the new state, s' is the symbol printed, and the tape head moves left or right one square depending on whether h is -1 or 1.

- We assume that the sets Q and Γ are disjoint.
- A *configuration* of M is a string xqy with $x, y \in \Gamma^*, y$ not the empty string, and $q \in Q$.
- The interpretation of the configuration xqy is that M is in state q with xy on its tape, with its head scanning the left-most symbol of y.

- If C and C' are configurations, then $CMßC'$ if $C = xqsy$ and $\delta(q, s) = (q', s', h)$ and one of the following holds:
  - $C' = xs'q'y$ and $h = 1$ and y is nonempty.
  - $C' = xs'q'b$ and $h = 1$ and y is empty.
  - $C' = x'q'as'y$ and $h = -1$ and $x = x'a$ for some $a \in \Gamma$.
  - $C' = q'bs'y$ and $h = -1$ and x is empty.
- A configuration xqy is halting if $q \in \{q_{accept}, q_{reject}\}$. Note that for each nonhalting configuration C there is a unique configuration C' such that $C \xrightarrow{M} C'$.
- The *computation* of M on input $w \in \Sigma^*$ is the unique sequence $C_0, C_1, ...$ of configurations such that $C_0 = q_0 w$ (or $C_0 = q_0 b$ if w is empty) and $C_i \xrightarrow{M} C_{i+1}$ for each i with C_{i+1} in the computation, and either the sequence is infinite or it ends in a halting configuration.
- If the computation is finite, then the number of steps is one less than the number of configurations; otherwise the number of steps is infinite. We say that M accepts w iff the computation is finite and the final configuration contains the state q_{accept}.